Readington Township Public Schools

Algebra 1

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I. OVERVIEW

This full-year Algebra 1 course is designed to provide Advanced 8th grade and Honors 7th grade students with the opportunity to be introduced to algebra skills in the areas of linear, quadratic, and exponential functions, while extending their content base and knowledge to include higher-degree polynomial functions, rational exponents and radical functions, solving exponential and logarithmic equations, rational functions, arithmetic and geometric sequences and series, algebraic ratios and proportions, functions, probability, data analysis and statistics.

In addition to these topics, and in keeping with the New Jersey Student Learning Standards, students will experience the course content as an integrated, useful, and coherent whole, continually refining their abilities to model with mathematics, reason abstractly and quantitatively while attending to precision both in calculations and vocabulary, and to make sense of problem situations as an essential part of the solution process.

II. STUDENT OUTCOMES (Linked to New Jersey Student Learning Standards for Mathematics 2016)

Interpret the structure of expressions.

NJSLS.MATH.CONTENT.HSA.SSE.A.1
Interpret expressions that represent a quantity in terms of its context.

NJSLS.MATH.CONTENT.HSA.SSE.A.1.A
Interpret parts of an expression, such as terms, factors, and coefficients.

NJSLS.MATH.CONTENT.HSA.SSE.A.1.B
Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P*.

NJSLS.MATH.CONTENT.HSA.SSE.A.2
Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

NJSLS.MATH.CONTENT.HSA.SSE.B.3
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

NJSLS.MATH.CONTENT.HSA.SSE.B.3.A
Factor a quadratic expression to reveal the zeros of the function it defines.

NJSLS.MATH.CONTENT.HSA.SSE.B.3.B
Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

NJSLS.MATH.CONTENT.HSA.SSE.B.3.C
Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%*.

NJSLS.MATH.CONTENT.HSA.SSE.B.4
Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*[*]

Perform arithmetic operations on polynomials.

NJSLS.MATH.CONTENT.HSA.APR.A.1

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

NJSLS.MATH.CONTENT.HSA.APR.B.2

Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

NJSLS.MATH.CONTENT.HSA.APR.B.3

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

NJSLS.MATH.CONTENT.HSA.APR.C.4

Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

NJSLS.MATH.CONTENT.HSA.APR.C.5

(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

Rewrite rational expressions.

NJSLS.MATH.CONTENT.HSA.APR.D.6

Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + ^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

NJSLS.MATH.CONTENT.HSA.APR.D.7

(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Create equations that describe numbers or relationships.

NJSLS.MATH.CONTENT.HSA.CED.A.1

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.

NJSLS.MATH.CONTENT.HSA.CED.A.2

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

NJSLS.MATH.CONTENT.HSA.CED.A.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.

NJSLS.MATH.CONTENT.HSA.CED.A.4
Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R*.

Understand solving equations as a process of reasoning and explain the reasoning.
NJSLS.MATH.CONTENT.HSA.REI.A.1
Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

NJSLS.MATH.CONTENT.HSA.REI.A.2
Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.
NJSLS.MATH.CONTENT.HSA.REI.B.3
Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
NJSLS.MATH.CONTENT.HSA.REI.B.4
Solve quadratic equations in one variable.

NJSLS.MATH.CONTENT.HSA.REI.B.4.A
Use the method of completing the square to transform any quadratic equation in $x$into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

NJSLS.MATH.CONTENT.HSA.REI.B.4.B
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations.
NJSLS.MATH.CONTENT.HSA.REI.C.5
Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

NJSLS.MATH.CONTENT.HSA.REI.C.6
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

NJSLS.MATH.CONTENT.HSA.REI.C.7
Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

NJSLS.MATH.CONTENT.HSA.REI.C.8
(+) Represent a system of linear equations as a single matrix equation in a vector variable.

NJSLS.MATH.CONTENT.HSA.REI.C.9
(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Represent and solve equations and inequalities graphically.

NJSLS.MATH.CONTENT.HSA.REI.D.10

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

NJSLS.MATH.CONTENT.HSA.REI.D.11

Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

NJSLS.MATH.CONTENT.HSA.REI.D.12

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

III. ESSENTIAL QUESTIONS AND CONTENT

Seeing Structure in Expressions and Writing Equivalent Forms
Why do we need to use exponential notation to model situations?
Why should we factor?
How does the graph of a quadratic function relate to its algebraic equation?

- Interpret and understand the parts of an expression, such as the terms, factors and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity. (i.e. P(1+r) as the product of P and a factor not depending on P).
- Using the structure of an expression and properties of operations to rewrite the expression in a different form.
- Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- Factor a quadratic expression to reveal the zeros of the function it defines.
- Complete the square in a quadratic expression to reveal the max or min value of the function it defines.
- Use the properties of exponents to transform expressions for exponential functions.
- Derive the formula for the sum of a finite geometric

Arithmetic with Polynomial, Rational, and Radical Expressions
How are rational and irrational numbers the same and different?
Why should we solve rational equations?

- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.
- Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x-a) is a factor of p(x).
- Identify zeros of polynomials when suitable factorizations are available, and use the zeroes to construct a rough graph of the function defined by the polynomial.
- Prove polynomial identities and use them to describe numerical relationships.
- Know and apply the Binomial Theorem for the expansion of (x+y) ^n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
- Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system.

- Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

Creating Equations

How do we create equations to represent what we see in the real world?

- Create equations and inequalities in one variable and use them to solve problems.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in modeling context.
- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning With Equations and Inequalities

What can we do with a system of equations/inequalities that we cannot do with a single equation/inequality?

- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- Solve quadratic equations in one variable.
- Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- Solve systems of linear equations exactly and approximately, focusing on pairs of linear equations in two variables.
- Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- Represent a system of linear equations as a single matrix equation in a vector variable.
- Find the inverse of a matrix if it exists and use it to solve systems of linear equations.
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve.
- Explain why the x-coordinates of the points where the graphs of two equations intersect are the solutions of the equation when both are set equal.
- Graph the solutions to a linear inequality in two variables as a half-plane and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

IV. STRATEGIES

Intellectual engagement and active involvement of students in daily lessons will be achieved by a variety of teaching strategies, including but not limited to:

- Comprehensive direct instruction utilizing Interactive SMARTBoard technology
- Guided practice using worked-out study examples, followed immediately by
- Independent Practice using 'Now You Try' examples
- Student presentation of work/solutions, with explanation and justification of solutions
- High-level questioning and encouragement of student participation
- Student group discussion and mutual help as part of point 3 above

- Regular written 'check for understanding' assessments.
- End of unit formative assessments to check for PARCC readiness and to guide instructional choices.

V. EVALUATION

Chapter Tests and Quizzes
Mid-Term Exam
Final Exam
Class Participation
Homework Assignments
Individual-lesson and unit-based formative assessments

VI. REQUIRED RESOURCES

Textbook for course:
McDougal Littell Algebra 1. Larson, R., Boswell, L., Kanold, T.D., & Stiff, L. Copyright 2004 by McDougal Littell, a Houghton Mifflin Company, Evanston, IL.

Supplemental resources for course:
Chapter Resource Books. Larson, R., Boswell, L., Kanold, T.D., & Stiff, L. Copyright 2001 by McDougal Littell, a Houghton Mifflin Company, Evanston, IL.

Standardized Test Practice Workbook. Larson, R., Boswell, L., Kanold, T.D., & Stiff, L. Copyright 2001 by McDougal Littell, a Houghton Mifflin Company, Evanston, IL.

Big Ideas MATH Algebra 1, Larson, R., Boswell, L. Copyright 2015 by Big Ideas Learning, LLC.

Suggested supplemental companion Web sites:
McDougal Littell eEdition Plus Online (online textbook: www.classzone.com)

Online Courses including Algebra 1: www.shmoop.com/common-core-standards/

Math problems explained in detail with corresponding NJSLSM references : www.illustrativemathematics.org

Learnzillion video tutorials: https://learnzillion.com/

Khan Academy video tutorials: www.khanacademy.org

Gizmos

VII. SCOPE AND SEQUENCE

A. Expressions, Equations, and Functions (14 days)
1. Evaluate Expressions
2. Apply Order of Operations
3. Write Expressions
4. Write Equations and Inequalities
5. Use a Problem Solving Plan
6. Represent Functions as Rules and Tables
7. Represent Functions as Graphs

B. Properties of Real Numbers (12 days)

1. Use Integers and Rational Numbers
2. Add Real Numbers
3. Subtract Real Numbers
4. Multiply Real Numbers
5. Apply the Distributive Property
6. Divide Real Numbers
7. Find Square Roots and Compare Real Numbers

C. Solving Linear Equations (15 days)
1. Solve One-Step Equations
2. Solve Two-Step Equations
3. Solve Multi-Step Equations
4. Solve Equations with Variables on Both Sides
5. Write Ratios and Proportions
6. Solve Proportions Using Cross Products
7. Solve Percent Problems
8. Rewrite Equations and Formulas

D. Graphing Linear Equations and Functions (15 days)
1. Plot Points in a Coordinate Plane
2. Graph Linear Equations
3. Graph Using Intercepts
4. Find Slope and Rate of Change
5. Graph Using Slope-Intercept Form
6. Model Direct Variation
7. Graph Linear Functions

E. Writing Linear Equations (12 days)
1. Write Linear Equations in Slope-Intercept Form
2. Use Linear Equations in Slope-Intercept Form
3. Write Linear Equations in Point-Slope Form
4. Write Linear Equations in Standard Form
5. Write Equations of Parallel and Perpendicular Lines
6. Fit a Line to Data
7. Predict with Linear Models

F. Solving and Graphing Linear Inequalities (13 days)
1. Solve Inequalities Using Addition and Subtraction
2. Solve Inequalities Using Multiplication and Division
3. Solve Multi-Step Inequalities
4. Solve Compound Inequalities
5. Solve Absolute Value Equations
6. Solve Absolute Value Inequalities
7. Graph Linear Inequalities in Two Variables

G. Systems of Equations and Inequalities (12 days)
1. Solve Linear Systems by Graphing
2. Solve Linear Systems by Substitution
3. Solve Linear Systems by Adding or Subtracting
4. Solve Linear Systems by Multiplying First
5. Solve Special Types of Linear Systems

6. Solve Systems of Linear Inequalities

H. Exponents and Exponential Functions (15 days)
   1. Apply Exponent Properties Involving Products
   2. Apply Exponent Properties Involving Quotients
   3. Define and Use Zero and Negative Exponents
   4. Use Scientific Notation
   5. Write and Graph Exponential Growth Functions
   6. Write and Graph Exponential Decay Functions

I. Polynomials and Factoring (15 days)
   1. Add and Subtract Polynomials
   2. Multiply Polynomials
   3. Find Special Products of Polynomials
   4. Solve Polynomial Equations in Factored Form
   5. Factor x^2 + bx + c
   6. Factor ax^2 + bx + c
   7. Find Special Products
   8. Factor Polynomials Completely

J. Quadratic Equations and Functions (16 days)
   1. Graph y = ax^2 + c
   2. Graph y = ax^2 +bx + c
   3. Solve Quadratic Equations by Graphing
   4. Use Square Roots to Solve Quadratic Equations
   5. Solve Quadratic Equations by Completing the Square
   6. Solve Quadratic Equations by the Quadratic Formula
   7. Interpret the Discriminant
   8. Compare Linear, Exponential, and Quadratic Models

K. Radicals and Geometry Connections (10 days)
   1. Graph Square Root Functions
   2. Simplify Radical Expressions
   3. Solve Radical Equations
   4. Apply the Pythagorean Theorem in Its Converse
   5. Apply the Distance and Midpoint Formulas

L. Rational Equations and Functions (14 days)
   1. Model Inverse Variation
   2. Graph Rational Functions
   3. Divide Polynomials
   4. Simplify Rational Expressions
   5. Multiply and Divide Rational Expressions
   6. Add and Subtract Rational Expressions
   7. Solve Rational Equations

M. Probability and Data Analysis (13 days)
   1. Find Probabilities and Odds
   2. Find Probabilities Using Permutations
   3. Find Probabilities Using Combinations
   4. Find Probabilities of Compound Events

5. Analyze Surveys and Samples
6. Use Measures of Central Tendency and Dispersion
7. Interpret Stem-and-Leaf Plots and Histograms
8. Interpret Box-and-Whisker Plots