

MATHCOUNTS®

2020

■ Chapter Competition ■ Sprint Round Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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02-C20SPR

1. _____ minutes How many minutes are in 4.5 hours?

2. _____ apples Herbert's gift basket contains three oranges for every five apples.
If the basket has nine oranges, how many apples does it have?



3. _____ If $x = \frac{1}{2}$ and $y = 6$, what is the value of $12xy$?

4. _____ mi/h The table shows the minimum and maximum wind speeds for four categories of hurricanes. What is the absolute difference between the minimum wind speed of a category four hurricane and the maximum wind speed of a category one hurricane?

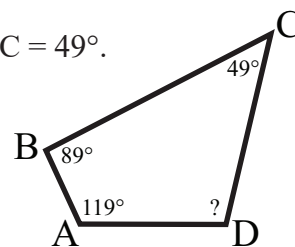
HURRICANE WIND SPEEDS (mi/h)

Category	Min Speed	Max Speed
Four	130	156
Three	111	129
Two	96	110
One	74	95

5. _____ cm What is the perimeter of a square whose area is 144 cm^2 ?

6. _____ furlongs If 3 miles equal 1 league and 1 league equals 24 furlongs, how many furlongs are equal to 1 mile?

7. _____ degrees In quadrilateral ABCD, $m\angle A = 119^\circ$, $m\angle B = 89^\circ$ and $m\angle C = 49^\circ$. What is the degree measure of $\angle D$?



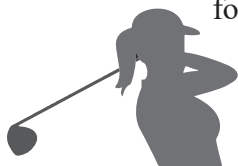
8. _____ If the first four terms of a geometric sequence are 2, 4, 8, 16 what is the fifth term of this sequence?

9. _____ sides Gladys draws two polygons. Her second polygon has two fewer than twice as many sides as her first polygon. If Gladys' first polygon is a triangle, how many sides does her second polygon have?

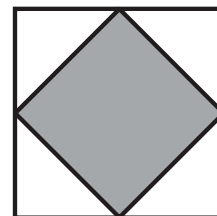
10. \$ _____ Lolli's Candy Store sells fruit-flavored candy rope, priced by length, at \$4.00 per meter. At this rate, how much will it cost to buy 50 cm of candy rope?



11. _____ Misko had an average score of 70 for her first three rounds of golf. If her scores for the first two rounds were 68 and 72, what was her score for the third round?



12. _____ cm^2 In the figure shown, the shaded inner square has area 36 cm^2 , and each of its vertices intersects the midpoint of a side of the outer square. What is the area of the outer square?



13. _____ meters Rafa and Sascha played a long 320-point tennis match. If Rafa ran an average of 12.7 meters per point and Sascha ran an average of 11.8 meters per point, how many more meters did Rafa run over the course of the match?



14. _____ units^2 The length and width of a rectangle add up to 16 units, and the length is three times the width. What is the area of the rectangle?

15. _____ What is the value of $\sqrt{5 \cdot 6 \cdot 10 \cdot 12}$?

16. _____ in²



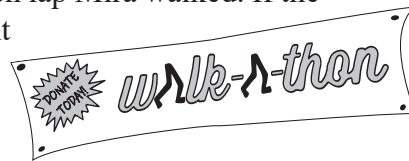
The figure shown is a square with sides of length 5 inches. The shaded stripes run parallel to the sides of the square, and they divide the bottom and left-hand sides of the square into segments of length 1 inch. What is the total area of the shaded stripes?

17. _____ numbers

How many two-digit prime numbers have 1 as their units digit?

18. _____ laps

For a charity walk-a-thon, Jen donated \$10 and pledged to donate 10¢ for each lap Mira walked. Joy pledged to donate 35¢ for each lap Mira walked. If the total amount Jen donated equals the total amount Joy donated, how many laps did Mira walk?



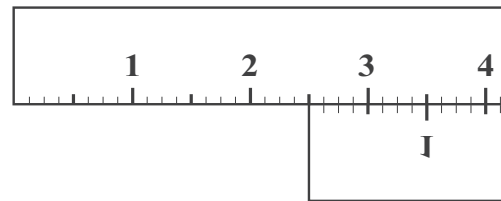
19. _____ values

$$\begin{array}{r} 4A \\ + 53 \\ \hline 1 \end{array}$$

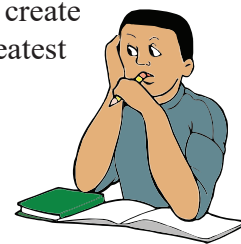
Some of the digits in the following correctly-worked arithmetic problem are missing. How many possible values are there for the digit represented by A?

20. _____

This figure shows two transparent foot-long rulers; the numerical markings on each ruler are in inches. If the marking on the top ruler for 6 inches will line up with the marking on the bottom ruler for q inches, what is the value of q ? Express your answer as a mixed number.

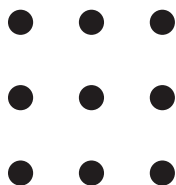


21. _____ Noah wants to fill in the two blanks in the numeral 5_1_2 to create a five-digit positive integer that is divisible by 6. What is the greatest five-digit multiple of 6 that he can create?



22. _____ What is the integer nearest to x if $3^x = 1500$?

23. _____ lines

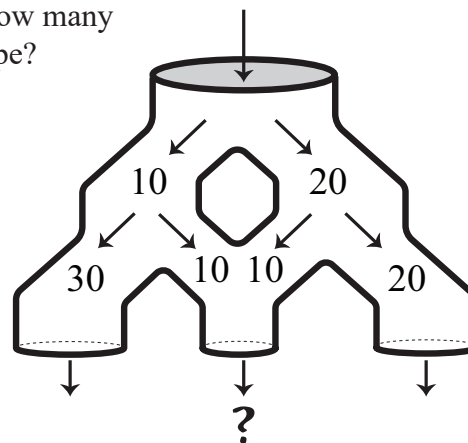


How many different lines pass through at least two of the nine points in the grid below?

24. _____ If $(2x - 3)^3 = a_3x^3 + a_2x^2 + a_1x + a_0$, what is the value of $a_3 + a_2 + a_1 + a_0$?

25. _____ marbles

Ten thousand marbles are released into the top pipe as shown and roll down the pipe system. Anytime the pipe forks, the marbles split in proportion to the cross-sectional areas of the pipes. All pipes have circular cross-sections with diameters as indicated in the figure. How many marbles exit through the bottom, middle pipe?



26. _____ prisms

A rectangular prism of size $3 \times 4 \times 5$ is made from 60 unit cubes. Including the full $3 \times 4 \times 5$ prism and the 60 unit prisms, how many total rectangular prisms can be found in the large rectangular prism by taking a subset of the 60 cubes?

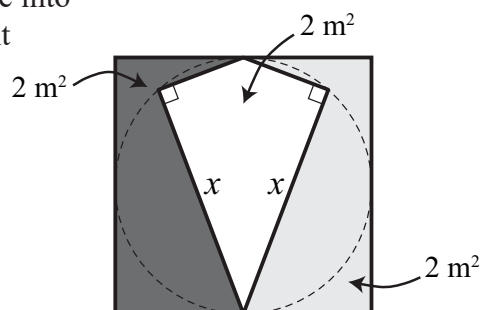
27. _____



Ms. Pauling's chemistry class has 5 lab benches, each of which seats 2 students. If 6 students file into her otherwise empty classroom, and each student picks a random available open seat, what is the probability that at least one of the lab benches is completely empty? Express your answer as a common fraction.

28. _____

In the figure, a kite with two right angles is circumscribed by a circle. The circle is then circumscribed by a square such that the diagonals of the kite are parallel to the sides of the square as shown. The longer sides of the kite each have length x meters. The sides of the kite divide the square into three regions, shaded dark gray, white and light gray as shown, with each region of area 2 m^2 . If $x^2 = a + \sqrt{b}$, what is the value of $a + b$?



29. (_____ , _____)

When $9!$ is expressed as an integer in base 9, the result ends in m zeros, and the last nonzero digit immediately preceding the m zeros is n . What is the value of the ordered pair (m, n) ?

30. _____

Triangle ABC has vertices $A(0, 5)$, $B(12, 0)$ and $C(0, 0)$ in the coordinate plane. The image when triangle ABC is rotated clockwise about the origin is triangle $A'B'C'$ with vertex $A'(3, 4)$. What fraction of the area of triangle $A'B'C'$ is below the x -axis? Express your answer as a common fraction.

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2020 ■ Chapter Competition ■ Target Round Problems 1 & 2

Name _____

School _____

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This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

Problem 1	Problem 2	Scorer's Initials

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02-C20TAR1

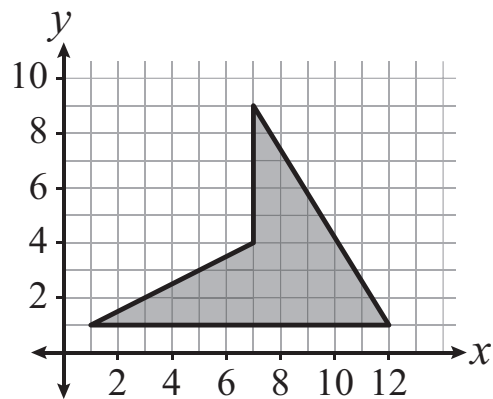
1. _____ $\frac{\text{candy}}{\text{bars}}$

Natasha received 14 candy bars at Halloween, while her younger brother Soren received 8. How many candy bars must Natasha give Soren to ensure that they both have the same number of candy bars?



2. _____ units^2

In the coordinate grid shown, what is the area of the shaded quadrilateral, all of whose vertices are on lattice points of the grid?



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2020
■ Chapter Competition ■
Target Round
Problems 3 & 4

Name _____

School _____

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Problem 3	Problem 4	Scorer's Initials

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3. _____ If the polynomial $5x - 3$ is a factor of the polynomial $5x^2 + 7x + k$, what is the value of the constant k ?

4. \$ _____ The movie sequel *Cat Lawyer II: The Purrfect Crime* grossed \$35.3 million during its opening weekend by selling 4.29 million tickets. Based on this, what was the average price per ticket during opening weekend for this movie?



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2020
■ Chapter Competition ■
Target Round
Problems 5 & 6

Name _____

School _____

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Problem 5	Problem 6	Scorer's Initials

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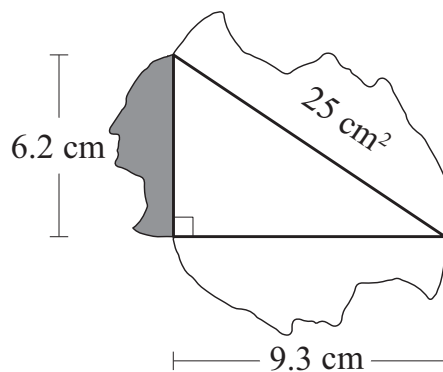
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5. _____ Addi, Subbi, Multi and Divi, are given two distinct nonzero numbers. Addi adds the two numbers. Subbi subtracts the lesser number from the greater. Multi multiplies the two numbers. Divi divides the greater number by the lesser. If the results obtained by Addi, Multi and Divi are the same, what is Subbi's result? Express your answer as a common fraction.

6. _____ cm^2 Three geometrically similar George Washington profiles are attached to the sides of a right triangle with leg lengths of 6.2 cm and 9.3 cm. The height of each profile is equal to the length of the side to which it is attached. If the largest profile has an area of 25 cm^2 , what is the area of the smallest profile, shown here shaded? Express your answer as a common fraction.



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2020
■ Chapter Competition ■
Target Round
Problems 7 & 8

Name _____

School _____

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Problem 7	Problem 8	Scorer's Initials

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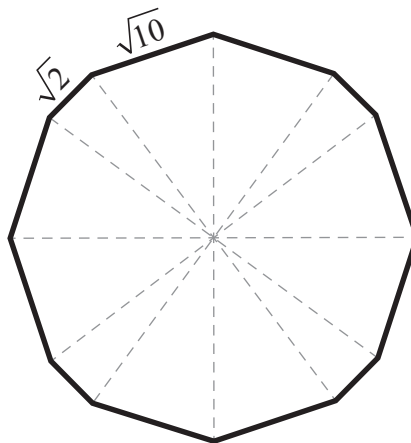
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7. _____ m²

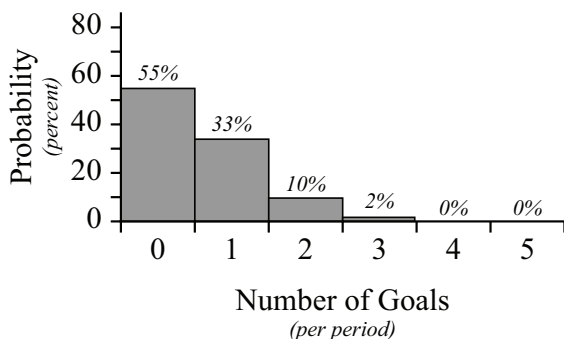
The polygon shown is a dodecagon with six diagonals, each of length 10 meters and intersecting at its center. This polygon has 8 sides of length $\sqrt{10}$ meters each and 4 sides of length $\sqrt{2}$ meters each. What is the area of this dodecagon?



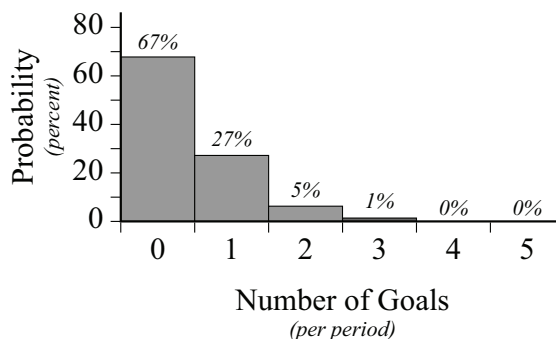
8. _____ percent

The bar graphs shown give the probability for Team North and Team South to each score a specified integer number of goals per period. Based on this data and assuming the numbers of goals scored by each team are independent of each other, what is the probability that Team South will be in the lead at the end of the first period? Express your answer as a percent to the nearest whole number.

TEAM NORTH



TEAM SOUTH



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2020

■ Chapter Competition ■
Team Round
Problems 1–10

School _____

Team
Members _____, Captain

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

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02-C20TEA

0
1
2
3
4
5
6
7
8
9

1. \$ _____

Jack goes on a road trip across the Southwest, during which he buys fuel in several cities. The table shows the amounts and prices of the fuel Jack bought in each city. What is the greatest amount Jack spent on fuel in any of these cities?

Road Trip Fuel Purchases

CITY	GALLONS	PRICE (per gallon)
Las Cruces, NM	10.44	2.47
Tucson, AZ	10.01	2.41
Flagstaff, AZ	10.39	2.65
Las Vegas, NV	9.16	2.62

2. _____

23				
			26	

For the 5×5 array shown, Chad fills in each empty cell with a positive integer so that any two cells that are vertically or horizontally adjacent contain numbers that differ by exactly 1. What is the greatest number that can appear in any of these cells?

3. _____ times

Marko's heart beats 72 times per minute during normal activity. It beats 65 times per minute during sleep, and 118 times per minute when Marko is exercising. During a 24-hour period, Marko slept from 10:00 p.m. until 5:00 a.m. and then exercised from 5:30 a.m. to 7:30 a.m. For the remaining time he had normal activity. During that 24-hour period, how many times did Marko's heart beat?

4. _____ years

Kepler's third law of planetary motion states that the square of the amount of time it takes for a planet to orbit the Sun is proportional to the cube of the planet's greatest distance from the Sun. Based on this and the data for Mars, Jupiter, Uranus and Neptune included in the table shown, how many Earth years does it take for Neptune to orbit the Sun? Express your answer to the nearest whole number.

Sun Distances and Orbit Times

PLANET	GREATEST DISTANCE (kilometers to Sun)	ORBIT TIME (Earth years)
Mars	2.28×10^8	1.882
Jupiter	7.78×10^8	11.86
Uranus	2.87×10^9	84.01
Neptune	4.50×10^9	?

5. _____

An ordered triple (a, b, c) is randomly chosen from the set of all ordered triples for which a, b and c are nonnegative integers that satisfy $a + b + c = 22$. What is the probability that $a < b < c$? Express your answer as a common fraction.

6. sequences Lincoln stands at vertex A of hexagon ABCDEF and rolls a die three times. For each roll, if the number rolled is even, he moves clockwise that number of vertices. If the number rolled is odd, he moves counter-clockwise that number of vertices. How many different sequences of three rolls will result in Lincoln ending at vertex A?
7. _____ An abundant number is a positive integer n for which the sum of the positive integer factors of n is greater than $2n$. What is the sum of all the abundant numbers that are less than 60?
8. lengths Suppose 7 points lie in a plane, and a line segment is drawn connecting each pair of points, forming a total of 21 line segments. What is the minimum number of distinct lengths among the 21 line segments?
9. _____ A standard 52-card deck of playing cards has thirteen ranks in each of four suits. If five cards are randomly selected from the deck, without replacement, what is the probability that they are all the same suit? Express your answer as a decimal to the nearest thousandth.
10. units² Let R be the set of all points (x, y) satisfying $x^2 + y^2 \leq 100$. What is the least possible area of a right triangle with three integer side lengths that cannot fit inside R ?