Summit Public Schools

Summit, New Jersey

Grade Level 10-11 / Content Area: Mathematics

Length of Course: Full Year

Algebra 2/Trigonometry Honors Curriculum

(Adam Leaman 2019)

Course Description:

The algebraic properties of the real number system, equations and inequalities of the first and second degree, and elementary functions are reviewed. The real number system is extended to the complex number system. Coordinate geometry, systems of equations, and exponential, logarithmic, polynomial, trigonometric, and circular functions are studied in detail. Probability is covered as time permits. A graphing calculator is required for use throughout the course.

Course Outline

1. Prerequisites/Functions
2. Linear Functions/Systems/Matrices
3. Quadratic Functions
4. Radical/Exponential Functions
5. Logarithmic/Exponential Functions
6. Polynomial Functions
7. Rational Functions
8. Trigonometric Functions

Prerequisites/Functions

In this unit, students will explore number sets, relations, functions, domain and range, inverses, and graph based on tables, mapping diagrams, and equations.

Anchor Standard: Please see "Progress Indicators" for unit-specific standards	
Big Ideas:*Course Objectives/Content Statement(s)* ● Classify numbers into sets ● Explore the connection between relations and functions ● Domain and range are integral to the study of functions	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● How have number classifications changed over time in an attempt to make them make sense in the real world (negative numbers could signal debt), and to solve mathematical problems? ● How and why do we represent relations in multiple ways? ● What symbols do we use to differentiate between equations and inequalities? ● How can we classify functions, and what are the key characteristics of functions? ● What does it mean to be an inverse, and what symbols are involved? Does a function always have an inverse function?	● Number classifications have changed to adapt to needs. For example, creating a set of numbers to allow for the square root of negative numbers allow the Fundamental Theorem of Algebra to hold true ● Relations can be represented in multiple ways, including mapping diagrams, tables, ordered pairs, graphs, and equations. Functions are specific types of relations where no one input has more than one output ● Functions can be classified as even, odd, or neither, depending on symmetry. You can check algebraically by plugging in -x for x and comparing the result to the original function ● Domain is the set of all input values and Range is the set of all output values specific to the function being analyzed ● An inverse is found by switching the input with the output (typically switching "x" and "y"). The resulting relation is reflected over the line y=x. A function is invertible if it is one-to-one, that is, each output only has no more than one input.

Areas of Focus: Proficiencies (Progress Indicators)	Examples, Outcomes, Assessments
Students will: F-IF A 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F-IF B 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F-BF B 3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	Instructional Focus: SWBAT: ● Create a graph (labeling the dependent and independent axis) that represents a written scenario ● Classify numbers (integer, whole, natural..) ● Simplify algebraic expressions and solve algebraic equations ● Work with algebraic axioms and properties ● Solve inequalities and absolute value inequalities, and represent the solution set on a number line ● Represent relations in multiple ways (table, graph, mapping diagram) ● Work with functions ● Define and use domain and range ● Identify parent graphs of various functions ● Tell if a function is even, odd, or neither algebraically and graphically ● Find the inverse of a function and use inverse notation ● Define a one-to-one function and be able to determine if a function is one-to-one based on a graph

4. Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write

an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠1.*

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

N-RN

B. Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

N-Q

A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations

Sample Assessments:

- Classwork and homework
- Exit tickets/warm-ups/short quizzes
- Long quizzes
- Unit exam

Sample items:

- ***Simplify***: $6-2[x-3-(x+4)+3(x-2)]$
- $f(x)=4x+5$
  $Find\,(f\circ f)(7)$
- ***Provide a rough sketch for the following scenario:***

The temperature of your coffee is related to how long it has been cooling.

- ***Restrict the domain of the function so that it is one-to-one and has an inverse function***
  $f(x)=|x-2|+1$

Projects/Post Assessment

- Students work in groups to create scenarios that can be represented by graphs (for example, see above "The temperature of your coffee is related to how long it has been cooling.") These can be traded between groups to solve, and then shared with the class

Instructional Strategies

- Interdisciplinary Connections

Students will be creating sketches of functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics

on measurement when reporting quantities.

A-CED

A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

A-REI

B. Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters

- Technology Integration

TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the exploration of parent functions analysis, and complex calculations

- Media Literacy Integration

Students will use technology to investigate interdisciplinary connections (above) and global perspectives (below). Students will have the opportunity to express results on Google Classroom, Ed Puzzle, Google Docs, interactive presentations, and self-made videos.

- Global Perspectives

Real data that describes different nations' economies and populations, as well as natural phenomenon such as earthquakes, will be modeled by linear, radical, exponential, logarithmic, polynomial, rational, and periodic functions.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triands or small groups
Pictures	Tables	In a whole group
Illustrations, diagrams & drawings	Graphs	Using cooperative group
Magazines & Newspapers	Timelines	Structures
Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations

Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

Career-Ready Practices:
CRP1: Act as a responsible and contributing citizen and employee.
CRP2: Apply appropriate academic and technical skills.
CRP3: Attend to personal health and financial well-being.
CRP4: Communicate clearly and effectively and with reason.
CRP5: Consider the environmental, social and economic impacts of decisions.
CRP6: Demonstrate creativity and innovation.
CRP7: Employ valid and reliable research strategies.
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9: Model integrity, ethical leadership and effective management.
CRP10: Plan education and career paths aligned to personal goals.
CRP11:. Use technology to enhance productivity.
CRP12: Work productively in teams while using cultural global competence.

Linear Functions/Systems/Matrices

Students will extend their knowledge of systems learned in Algebra 1 by using matrices and technology to solve, and solve problems using matrices and determinants. The "z" axis is introduced, and serves as a platform for solving three variable systems.

Anchor Standard: Please see "Progress Indicators" for unit-specific standards	
Big Ideas:*Course Objectives/Content Statement(s)* ● Systems of equations can be used to model and solve real-world problems ● Matrices can be used as an alternative to solving a system using traditional methods	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● What is the significance of the point of intersection of two or more functions? What methods can be used to find these intersections? ● How are matrices related to systems? ● What does the "z" axis represent, and what is its significance when solving a system of three equations in three variables? ● How can technology be used to simplify processes of solving systems learned in previous math courses?	● Graphically, the point of intersection of functions is where those two functions are equal to each other. In context, this could mean a break-even point in financial analysis, or solve problems where two quantities are unknown ● Matrices are an array of numbers that can represent systems. ● The "z" axis creates the 3-dimensional plane. The solution of a system of three equations in three variables is where, if at all, planes intersect. ● Graphing programs like desmos, or TI-83/84 commands like RREF (reduced row-echelon form) can help us solve systems quickly.

Areas of Focus: Proficiencies (Progress Indicators)	Examples, Outcomes, Assessments
Students will: N-Q **A. Reason quantitatively and use units to solve problems.** 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N-VM **C. Perform operations on matrices and use matrices in applications.** 6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. 7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. 8. (+) Add, subtract, and multiply matrices of appropriate dimensions. 9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. 10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The	Instructional Focus: SWBAT: ● Graph lines in slope-intercept, standard and point slope form. ● Solve systems of linear equations in two variables, three variables, and nonlinear systems algebraically and on the calculator ● Add, subtract, multiply, scale, and find the determinant of a matrix, and use it to solve problems ● Use matrices to solve systems in two or more variables ● Use the concept of linear programming to optimize in real-world scenarios Sample Assessments: ● Classwork and homework ● Exit tickets/warm-ups/short quizzes ● Long quizzes ● Unit exam Sample items: ● Suppose you own a car that is presently 40 months old. The "Blue Book" value of the car claims that the present trade-in value is $3300. From and old Blue Book, you find that the trade in value 10 months ago was $4700. Assume that its trade-in value decreases linearly with time. a) Write the particular equation expressing trade-in value of your car as a function of its age in months. b) You plan to get rid of the car when its trade-in value drops to $1000. How much longer can you keep the car?

determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

A-REI

C. Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y=–3x*andthecircle *x*2+*y*2 *=3.*

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. 9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

D. Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

12. Graph the solutions to a linear inequality in two

- Using determinants, determine if the points (1,2), (3,4) and (5,6) are collinear or not.

- The sum of three numbers is 12. The first is five times the second, and the sum of the first and the third is 9. Find the numbers

Projects/Post Assessment

- Students work in groups of three to create their own systems of equation problems and answer key. The problems are shared with other groups which have to solve the problem using substitution, elimination, graphing, matrices and technology. At least one problem should be a three variable system.

Instructional Strategies

- Interdisciplinary Connections

Students will be solving equations that are mathematical models for scenarios in the social sciences, physics, biology, and economics.

- Technology Integration

TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in solving systems using matrices and RREF, finding intersections of linear and nonlinear systems, and graphing lines

variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. F-IF **C. Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	● Media Literacy Integration Students will be asked to use resources online and in other textbooks and publications to find alternative ways to solve equations. Students will be asked to analyze these resources in terms of their usefulness and ease. These resources can be compiled into an online catalogue for future classes to use and add to. ● Global Perspectives Real data that describes different nations' economies and populations, as well as natural phenomenon such as earthquakes, will be modeled by linear, radical, exponential, logarithmic, polynomial, rational, and periodic functions.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
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Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
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Models & Figures		

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations

Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

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CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
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CRP11: Use technology to enhance productivity.
CRP12: Work productively in teams while using cultural global competence.

Quadratics

Students will explore a new number system, the imaginary numbers, thus extending their knowledge of the roots of quadratics. The Fundamental Theorem of Algebra will be introduced.

Anchor Standard: Please see "Progress Indicators" for unit-specific standards	
Big Ideas:*Course Objectives/Content Statement(s)* ● Quadratic functions can represent real-world scenarios ranging from sales/profit to velocity ● Examine the attributes and characteristics of the graphs of quadratic functions ● The Fundamental Theorem of Algebra ensures that each quadratic has two solutions (including repeated and imaginary) and two linear factors ● Work with the set of imaginary numbers	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● How does The Fundamental Theorem of Algebra extend what we have learned about the nature of the roots of quadratics in the past? ● How can the process completing the square be used to reveal not only the zeros of a quadratic function, but also the minimum or maximum value of that quadratic? ● Can we extend the number system to find the square root of a negative number? ● How can technology help us solve real-world scenarios involving quadratics?	● The Fundamental Theorem of Algebra ensures that a quadratic equation will always have two solutions (and corresponding linear factors), if you include repeated and imaginary solutions. ● Completing the square can be used as a method to solve a quadratic equation. Additionally, the process of completing the square can transform a standard form quadratic into a vertex form quadratic, revealing the minimum or maximum value. ● To satisfy the Fundamental Theorem of Algebra, we use the imaginary unit "i" to represent the square root of negative numbers. ● Using graphing technology such as desmos, or the TI-83/84 calculator, attributes of the graph such as: zeros, minima, maxima, and intersections can be found. These all have meaning in real-world contexts.

Areas of Focus: Proficiencies (Progress Indicators)	Examples, Outcomes, Assessments
Students will: N-Q **A. Reason quantitatively and use units to solve problems.** 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N-CN **A. Perform arithmetic operations with complex numbers.** 1. Know there is a complex number *i* such that *i*2 = –1, and every complex number has the form*a* + *bi* with *a* and *b* real. 2. Use the relation *i*2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. 3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **C. Use complex numbers in polynomial identities and equations.** 7. Solve quadratic equations with real coefficients that have complex solutions. 8. (+) Extend polynomial identities to the complex	Instructional Focus: SWBAT: ● Graph quadratics in standard and vertex form ● Convert quadratics from standard form to vertex form by completing the square ● Find the minimum or maximum value by hand and using technology ● Find zeros of a function by hand and by using technology ● Work with the imaginary unit "i" and perform operations on complex numbers involving "i" ● Solve word problems, including vertical motion problems by hand and using technology Sample Assessments: ● Classwork and homework ● Exit tickets/warm-ups/short quizzes ● Long quizzes ● Unit exam Sample items: A rectangular corral is to be built by stringing an electric fence as shown with y feet for the side parallel to the river and x feet for each of the two sides perpendicular to the river. The total length of the fence is 900 feet. a) Write an equation expressing y in terms of x. What kind of function is this? b) Let A(x) be the number of square feet area taken up by the corral. Write the particular equation for A(x). What kind of function is this?

numbers. *For example, rewrite* $x2 + 4$ *as* $(x + 2i)(x – 2i)$.

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A-SSE

A. Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

For example, interpret $P(1+r)n$ *as the product of P and a factor not depending on P*

2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x4 – y4$ *as* $(x2)2 – (y2)2$, *thus recognizing it as a difference of squares that can be factored as* $(x2 –y2)(x2 + y2)$.

B. Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

A-REI

B. Solve equations and inequalities in one variable

4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform

c) Find A(100) and A(300)

d) What value of x makes A(x) = 0?

e) Sketch the graph of function A in a reasonable domain.

Projects/Post Assessment

- Students will be asked to research, discuss, and present the history of number systems. The development of the number 0 and the applications of *i* will be emphasized.

Instructional Strategies

- Interdisciplinary Connections

Students will learn that complex numbers are widely used in the fields of physics and engineering, as well as in advanced mathematics. Students will learn the history of complex numbers, as well as the connections between complex numbers and fractal geometry, which appears in both art and nature.

- Technology Integration

TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in finding roots, relative and absolute extrema, and intersections

- Media Literacy Integration

Students will use technology to investigate interdisciplinary connections (above) and global perspectives (below). Students will have the opportunity to express results on Google Classroom, Ed Puzzle, Google Docs, interactive presentations, and self-made videos.

any quadratic equation in x into an equation of the form $(x - p)2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. F-IF **C. Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum	● Global Perspectives Real data that describes different nations' economies and populations, as well as natural phenomenon such as earthquakes, will be modeled by linear, radical, exponential, logarithmic, polynomial, rational, and periodic functions.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triands or small groups
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Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations

Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

Career-Ready Practices:
CRP1: Act as a responsible and contributing citizen and employee.
CRP2: Apply appropriate academic and technical skills.
CRP3: Attend to personal health and financial well-being.
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CRP5: Consider the environmental, social and economic impacts of decisions.
CRP6: Demonstrate creativity and innovation.
CRP7: Employ valid and reliable research strategies.
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9: Model integrity, ethical leadership and effective management.
CRP10: Plan education and career paths aligned to personal goals.
CRP11: Use technology to enhance productivity.
CRP12: Work productively in teams while using cultural global competence.

Radical/Exponential Functions

Students will begin to see radicals as exponents (their alternate form). Students will use the properties of exponents previously learned to solve more complex problems involving radicals.

Anchor Standard: Please see "Progress Indicators" for unit-specific standards	
Big Ideas:*Course Objectives/Content Statement(s)* ● Explore the connection between radicals and exponents ● Properties of exponents are critical to topics in advanced mathematics	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● How are the properties of exponents, that we learned in the past, related to radical expressions? ● Why are there restrictions on the domain of radical functions, and how do we find them? ● Why is it important to check answers when solving a radical equation?	● Using rational exponents, a radical can be represented in a different form. For example, the square root of 5 can be represented as 5 to the one-half. ● In the real-number system, you cannot take the square root (or any even index) of a negative number. Therefore, we must restrict the domain, as we will not get an output if there is a negative input under the radical (in other words, the radicand cannot be negative) ● Solving a radical equation may require the process of squaring both sides to remove the radical. This, in effect, changes the domain of the problem, and could yield solutions that do not work in the original equation. Keeping in mind that radicals have domain restrictions, we must check all solutions.

Areas of Focus: Proficiencies (Progress Indicators)	Examples, Outcomes, Assessments
Students will: N-RN **A. Extend the properties of exponents to rational exponents.** 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.* 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. N-Q **A. Reason quantitatively and use units to solve problems.** 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N-CN **A. Perform arithmetic operations with complex numbers.** 1. Know there is a complex number *i* such that *i*2 = –1, and every complex number has the form*a* + *bi* with *a*	Instructional Focus: SWBAT: ● Simplify expressions with rational, integer, and radical exponents, using properties ● Solve exponential equations ● Evaluate expressions using rational exponents (for example, 9 to the ½ = 3) ● Perform operations on radical expressions ● Graph square root, cube root, and cubic functions ● Solve radical equations Sample Assessments: ● Classwork and homework ● Exit tickets/warm-ups/short quizzes ● Long quizzes ● Unit exam Sample items: ● Rationalize the denominator $\frac{\sqrt{5x^4}+3\sqrt{2x}}{\sqrt{3x^3}}$ ● Simplify $\frac{\left(x^{-\frac{1}{2}}y^4\right)^{\frac{1}{4}}}{x^{\frac{2}{3}}y^{\frac{3}{2}} \cdot x^{-\frac{3}{2}}y^{\frac{1}{2}}}$ ● Solve for “x” $\sqrt{2x}-\sqrt{x-3}=\frac{2}{\sqrt{x-3}}$

and *b* real.

2. Use the relation $i2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

A-SSE

B. Write expressions in equivalent forms to solve problems

3c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15t *can be rewritten as* (1.151/12)12*t* ≈1.01212t *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

A-REI

A. Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

F-IF

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value

Projects/Post Assessment

- Students will take any book and measure its height in inches. Students will then describe the height of a pile of such books, if the pile were to double each day. Students will be expected to:

  - write the pile's height as a function of time in days

  - solve for the time at which the pile will reach: the ceiling (10 feet), the Empire State Building (1,250 feet), the moon (238,857 miles).

  - Compare results to students with books of different heights

Instructional Strategies

- Interdisciplinary Connections

Exponential functions are used in the study of population, natural phenomena such as earthquakes and time of death, and finance studies such as profits and losses.

- Technology Integration

TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the calculation of more complicated expressions with rational exponents, solving exponential equations, graph analysis, and exponential regression

functions. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*	● Media Literacy Integration Students will be asked to find examples in the media of exponential growth and decay. Students will be expected to find websites, newspaper articles, video clips, and other acceptable media that discuss a real-world phenomenon or current event that is an example of exponential behavior. ● Global Perspectives Real data that describes different nations' economies and populations, as well as natural phenomenon such as earthquakes, will be modeled by linear, radical, exponential, logarithmic, polynomial, rational, and periodic functions.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triands or small groups
Pictures	Tables	In a whole group
Illustrations, diagrams & drawings	Graphs	Using cooperative group
Magazines & Newspapers	Timelines	Structures
Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations

Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

Career-Ready Practices:
CRP1: Act as a responsible and contributing citizen and employee.
CRP2: Apply appropriate academic and technical skills.
CRP3: Attend to personal health and financial well-being.
CRP4: Communicate clearly and effectively and with reason.
CRP5: Consider the environmental, social and economic impacts of decisions.
CRP6: Demonstrate creativity and innovation.
CRP7: Employ valid and reliable research strategies.
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9: Model integrity, ethical leadership and effective management.
CRP10: Plan education and career paths aligned to personal goals.
CRP11: Use technology to enhance productivity.
CRP12: Work productively in teams while using cultural global competence.

Logarithmic/Exponential Functions

Students will continue their exploration of exponential functions in their study of logarithms. Logarithms are rich with opportunities for real-world modeling.

Anchor Standard: Please see "Progress Indicators" for unit-specific standards	
Big Ideas:*Course Objectives/Content Statement(s)* ● Define logarithm ● Explore the relationship between exponential functions and logarithms ● e is a transcendental number, that, when used as a base of a log becomes what is called a natural log ● Model with logarithms	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● What is a logarithm? ● What is the relationship between exponential functions and logarithmic functions? ● How are logarithms used in real-world situations?	● A logarithm is an exponent. It answers, for example, the question, "2 to what power is 8?" ● Exponential functions and logarithmic functions are inverses of each other ● Logarithms can be used to solve problems including time of death, earthquakes and population studies
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: N-RN **A. Extend the properties of exponents to rational exponents.** 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because*	Instructional Focus: SWBAT: ● Evaluate logarithms ● Find the domain of logarithmic functions and graph using transformations ● Use properties of logarithms to expand and condense

we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-Q

A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-SSE

A. Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.*For example, interpret P*(1+*r*)n *as the product of P and a factor not depending on P*

2. Use the structure of an expression to identify ways to rewrite it. *For example, see x*4 – *y*4 *as* (*x*2)2 – (*y*2)2, *thus recognizing it as a difference of squares that can be factored as* (*x*2 –*y*2)(*x*2 + *y*2).

B. Write expressions in equivalent forms to solve

- Solve logarithmic and exponential equations
- Prove the properties of logarithms
- Use exponential regression to solve problems related to exponential growth and decay
- Solve compound interest (with continuous compounding) problems
- Use technology to solve problems involving logarithms (including natural logarithms)

Sample Assessments:

- Classwork and homework
- Exit tickets/warm-ups/short quizzes
- Long quizzes
- Unit exam

Projects/Post Assessment

- Students will take any book and measure its height in inches. Students will then describe the height of a pile of such books, if the pile were to double each day. Students will be expected to:
  - write the pile's height as a function of time in days
  - solve for the time at which the pile will reach: the ceiling (10 feet), the Empire State Building (1,250 feet), the moon (238,857 miles).
  - Compare results to students with books of different heights

Instructional Strategies

- Interdisciplinary Connections

Logarithms can be used to solve problems including time of death, earthquakes and population studies

- Technology Integration

problems 3c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15t *can be rewritten as* (1.151/12)12*t* ≈1.01212t *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* F-IF **C. Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.* F-LE **A. Construct and compare linear and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the calculation of more complicated expressions with logarithms, exponents, solving logarithmic and exponential equations, graph analysis, exponential regression, and real-world word problems involving logarithms ● Media Literacy Integration Students will be asked to find examples in the media of exponential growth and decay. Students will be expected to find websites, newspaper articles, video clips, and other acceptable media that discuss a real-world phenomenon or current event that is an example of exponential behavior. ● Global Perspectives Real data that describes different nations' economies and populations, as well as natural phenomenon such as earthquakes, will be modeled by linear, radical, exponential, logarithmic, polynomial, rational, and periodic functions.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. 4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $abct = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **B. Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context.	

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triands or small groups
Pictures	Tables	In a whole group
Illustrations, diagrams & drawings	Graphs	Using cooperative group
Magazines & Newspapers	Timelines	Structures
Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations

Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

Career-Ready Practices:
CRP1: Act as a responsible and contributing citizen and employee.
CRP2: Apply appropriate academic and technical skills.
CRP3: Attend to personal health and financial well-being.
CRP4: Communicate clearly and effectively and with reason.
CRP5: Consider the environmental, social and economic impacts of decisions.
CRP6: Demonstrate creativity and innovation.
CRP7: Employ valid and reliable research strategies.
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9: Model integrity, ethical leadership and effective management.
CRP10: Plan education and career paths aligned to personal goals.
CRP11: Use technology to enhance productivity.
CRP12: Work productively in teams while using cultural global competence.

Polynomials

This unit serves as an extension of the quadratics unit, in which students will learn more advanced factoring techniques and apply them to higher-degree polynomials. Additionally, The Fundamental Theorem of Algebra will be examined closer, and long and synthetic division of polynomials will be introduced.

Anchor Standard:
Please see "Progress Indicators" for unit-specific standards
Big Ideas:*Course Objectives/Content Statement(s)* ● Classify polynomials ● Discover and use the Remainder and Factor Theorems ● Discover and use the Fundamental Theorem of Algebra ● Find zeros and relative extrema of a polynomial function ● Model with polynomial functions

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● How are polynomials classified? ● How can the remainder and factor theorem help us find attributes of the graphs of polynomials, and why do we need to find such things? ● What is The Fundamental Theorem of Algebra and why is it so important to the study of polynomials? ● How can analyzing polynomials help us solve real-world problems?	● Polynomials can be classified based on their degree and number of terms ● The remainder theorem is an alternative for evaluating polynomials using direct substitution ● The factor theorem can identify factors of polynomials, and x-intercepts of the graphs of polynomials ● Polynomials are continuous functions ● The Fundamental Theorem of Algebra ensures that a polynomial of "nth" degree has "n" solutions (including repeated and imaginary), and "n" linear factors ● Modeling with polynomials helps us solve many real-world problems including break-even analysis, projected sales and optimization problems.

Areas of Focus: Proficiencies (Progress Indicators)	Examples, Outcomes, Assessments
Students will: N-Q **A. Reason quantitatively and use units to solve problems.** 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N-CN **A. Perform arithmetic operations with complex numbers.** 1. Know there is a complex number *i* such that *i*2 = –1, and every complex number has the form*a* + *bi* with *a* and *b* real. 2. Use the relation *i*2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. 3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **C. Use complex numbers in polynomial identities and equations.** 7. Solve quadratic equations with real coefficients that have complex solutions. 8. (+) Extend polynomial identities to the complex	Instructional Focus: SWBAT: ● Classify polynomials by their degree and number of terms ● Add, subtract, multiply and divide (long and synthetically) polynomials ● Factor and solve expressions more complex than in the quadratics unit, and with higher degrees ● Use The Fundamental Theorem of Algebra (occasionally with the help of technology) to find all linear factors and zeros of a function of any degree ● Factor sum and difference of cubes ● Write a polynomial given its roots, through a point ● Locate the relative extrema of a polynomial function by hand and using technology, as well as the intervals of increase and decrease ● Solve profit and optimization problems involving polynomial models

numbers. *For example, rewrite* $x2 + 4$ *as* $(x + 2i)(x – 2i)$.

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A-APR

A. Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity*$(x2 + y2)2 = (x2 – y2)2 + (2xy)2$ *can be used to generate Pythagorean triples.*

F-IF

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end

Sample Assessments:

- Classwork and homework
- Exit tickets/warm-ups/short quizzes
- Long quizzes
- Unit exam

Sample items:

- An open box is to be made from a 10-in by 12-in piece of cardboard by cutting x-in squares from each corner and folding up the sides. Write a function giving the volume of the box in terms of x. Sketch the graph of the function in an appropriate domain and use it to approximate the value of x that produces the greatest volume.

- Solve for all values of x
$27x^5-54x^3– x^2+ 2 = 0$

- Find the value of "k" such that (x + 1) is a factor of $kx^3+ 8x^2– 15kx - 50$

Projects/Post Assessment

- The Ultimate Box: Students will be asked to create an open box out of cardboard by cutting out squares from each corner of the cardboard and folding up the resulting flaps. Given that the cut out squares are x- by-x inches:

  -Write polynomial functions to model the outside surface area and the volume of the resulting box.

  -Graph the function for the volume of the box over a reasonable domain. -Use technology to determine the maximum value of the volume function. What sized squares should be cut out to maximize volume?

behavior. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	Instructional Strategies ● Interdisciplinary Connections Profits and losses can be approximated by polynomial models. Students will use data to create a regression in which they can analyze connections to business and science. ● Technology Integration TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the calculation roots, relative and absolute extrema, intersections, and real-world problems. ● Media Literacy Integration Students will use technology to investigate interdisciplinary connections (above) and global perspectives (below). Students will have the opportunity to express results on Google Classroom, Ed Puzzle, Google Docs, interactive presentations, and self-made videos. ● Global Perspectives Real data that describes different nations' economies and populations, as well as natural phenomenon such as earthquakes, will be modeled by linear, radical, exponential, logarithmic, polynomial, rational, and periodic functions.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triands or small groups
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Illustrations, diagrams & drawings	Graphs	Using cooperative group
Magazines & Newspapers	Timelines	Structures
Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations

Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

Career-Ready Practices:
CRP1: Act as a responsible and contributing citizen and employee.
CRP2: Apply appropriate academic and technical skills.
CRP3: Attend to personal health and financial well-being.
CRP4: Communicate clearly and effectively and with reason.
CRP5: Consider the environmental, social and economic impacts of decisions.
CRP6: Demonstrate creativity and innovation.
CRP7: Employ valid and reliable research strategies.
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9: Model integrity, ethical leadership and effective management.
CRP10: Plan education and career paths aligned to personal goals.
CRP11: Use technology to enhance productivity.
CRP12: Work productively in teams while using cultural global competence.

Rational Functions

In this unit, students will perform algebraic operations on rational expressions, as well as be exposed to pre-calculus and calculus concepts such as types of discontinuities and end behavior (connection to limits). An analysis of the graphs of rational functions, and real-world applications round out the unit.

<table>
<tr><td colspan="2">Anchor Standard:

Please see “Progress Indicators” for unit-specific standards</td></tr>
<tr><td colspan="2">Big Ideas:Course Objectives/Content Statement(s)
● Using direct, inverse and joint variation can help solve real-world problems
● Analyze the characteristics and attributes of the graphs of rational functions</td></tr>
<tr><td>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</td><td>Enduring Understandings
What will students understand about the big ideas?</td></tr>
<tr><td>● How can rational functions be used to solve real-world problems?

● What are the important characteristics of the graphs of rational functions, and how can we find them?

● How are polynomials and techniques to factor relevant to the study of rational functions?

● Why must the domain be considered when solving rational equations?</td><td>● Factoring and simplifying rational functions can reveal characteristics of the graph of that function

● Rational functions can have vertical, horizontal, and slant asymptotes. Additionally, they can contain removable discontinuities. A rational function should be factored and simplified to determine which, if any of these, the graph of the function contains.

● Quantities in the real world can vary directly, inversely, and jointly. Using techniques from this unit, real-world problems can be solved depending on the type of variation. Inverse variation can be linked to a rational function, while direct can be linked to a linear function.

● The process by which we solve rational equations may result in an extraneous solution. Because rational equations have a denominator, it is possible that a proposed solution creates a zero in the denominator, and therefore must be excluded from the domain, as it will not produce a valid output.</td></tr>
</table>

Areas of Focus: Proficiencies (Progress Indicators)	Examples, Outcomes, Assessments
Students will: N-Q **A. Reason quantitatively and use units to solve problems.** 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. A-APR **D. Rewrite rational expressions** 6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. 7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. A-REI **A. Understand solving equations as a process of reasoning and explain the reasoning** 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the	Instructional Focus: SWBAT: ● Simplify rational expressions ● Perform operations (add, subtract, multiply and divide) on rational expressions ● Solve rational equations, and complex fraction equations, and check for extraneous solutions ● Find the domain and end behavior of rational functions ● Solve rational inequalities algebraically Sample Assessments: ● Classwork and homework ● Exit tickets/warm-ups/short quizzes ● Long quizzes ● Unit exam Sample items: ● Solve $\frac{\left(\frac{4}{x}+\frac{6}{x+1}\right)}{\left(\frac{3}{x+1}-5\right)}=-1$ ● Simplify $\frac{\frac{1}{x}-\frac{2}{x^2}-\frac{3}{x^3}}{\frac{9}{x}-x}$ ● Graph $f(x)=\frac{x^2-9}{3-x}$

previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. F-IF **C. Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	Projects/Post Assessment ● Students will work together to create expressions with given specificities. For example: Create a rational expression that has a removable discontinuity at x=3 and an infinite discontinuity at x=1. These will be shared with other groups who have to simplify the expression, and check with the original group to see if they are correct. Instructional Strategies ● Interdisciplinary Connections Gas consumption, biology (connections between height and weight), and the connection between a thunder clap and a lightning strike can all be modeled by direct, inverse, or joint variation. ● Technology Integration TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the analysis of the graphs of rational functions, as well as real-world problems involving direct, inverse, and joint variation. ● Media Literacy Integration Students will use technology to investigate interdisciplinary connections (above) and global perspectives (below). Students will have the opportunity to express results on Google Classroom, Ed Puzzle, Google Docs, interactive presentations, and self-made videos. ● Global Perspectives Real data that describes different nations' economies and populations, as well as natural phenomenon such as earthquakes, will be modeled by linear, radical, exponential, logarithmic, polynomial, rational, and periodic functions.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
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Career-Ready Practices:
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CRP12: Work productively in teams while using cultural global competence.

Trigonometry

In this unit, the longest of the year, students will start with a basic overview of trigonometry, and work their way up to advanced topics such as the sum and difference properties and double and half angle formulas. Additionally, inverse trigonometric functions, solving trigonometric equations and applying the law of sines and cosines will be seen in depth.

Anchor Standard: Please see "Progress Indicators" for unit-specific standards	
Big Ideas:*Course Objectives/Content Statement(s)* ● Develop and analyze the unit circle ● Analyze the graphs of the six trigonometric functions ● Verify trigonometric identities ● Explore and analyze inverse trigonometric functions ● Use trigonometric functions to model and solve real-world problems ● Use technology to aid in the use of trigonometric functions to solve problems ● Solve trigonometric equations given restricted and unrestricted domains	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● How can special right triangles be used to create and make sense of the unit circle? ● How does the unit circle help evaluate trig functions for any angle in the domain? ● What does it mean to be periodic? ● How are even and odd functions relevant to trigonometric functions? ● How can we use proof techniques learned in geometry to verify trigonometric identities? ● Can trigonometric functions be used to solve real-world problems? ● Do we have to consider domain restrictions when working with trigonometric functions and equations?	● Given a radius of one on a circle centered at the origin, reference angles of 30, 45 and 60 degrees will give you (x,y) coordinates that correspond to the (cos,sin) of an angle. ● Periodic functions have the same value at f(x) and f(x+c) for all x in the domain of the function, for the same constant "c" ● All trig functions can be classified as either even or odd. This not only helps us graph, but helps with the verification of identities ● Trigonometric functions can be used to solve many problems including ocean waves and satellite orbits. ● Because of the nature of reciprocal identities, it is possible that, at certain values, there will be a division by zero. Therefore, domain has to be considered when working with trigonometric

	functions and equations ● The law of sines and cosines can be used to solve triangles that have no right angle, and this can be applied to real-world situations.
Areas of Focus: Proficiencies (Progress Indicators)	**Examples, Outcomes, Assessments**
Students will: N-Q **A. Reason quantitatively and use units to solve problems.** 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. F-IF **C. Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F-TF **A. Extend the domain of trigonometric functions using the unit circle** 1. Understand radian measure of an angle as the length	Instructional Focus: SWBAT: ● Evaluate the six trig functions at special angles ● Use ratios of special right triangles to find the length of the legs and hypotenuse ● Sketch angles in standard position, in radians and degrees ● Convert from radians to degrees, and vice versa ● Find coterminal, reference, complementary and supplementary angles ● Evaluate the six trigonometric functions given a rectangular point in the coordinate plane ● Create the unit circle ● Convert back and forth from Degree Minute Second form ● Verify trigonometric identities ● Graph all six trigonometric functions ● Write sine and cosine functions from graphs or from key information ● Solve word problems using sine and cosine ● Evaluate inverse trigonometric expressions

of the arc on the unit circle subtended by the angle.

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for$\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx, $\pi+x$, and $2\pi–x$ in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

B. Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

C. Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

- Use the sum and difference properties, and double and half angle formulas to evaluate trig functions

- Solve trigonometric equations in both specific and unrestricted domains

- Use the law of sines and cosines to solve oblique triangles, and to solve word problems

- Use sinusoidal regression to solve word problems

Sample Assessments:

- Classwork and homework
- Exit tickets/warm-ups/short quizzes
- Long quizzes
- Unit exam

Sample items:

- Are the angles coterminal? $\frac{17\pi}{3}, -\frac{13\pi}{3}$
- Evaluate the expression leaving the answer in simplest radical form where necessary
  $\sec\frac{\pi}{4}\sin\frac{\pi}{4} - \tan\frac{3\pi}{4}\csc\frac{\pi}{3}$
- Graph two periods. Provide the domain and range

$y = \frac{1}{10}\cot 4\left(x - \frac{\pi}{3}\right)$

Projects/Post Assessment

- Students will be asked to research phenomena that occur periodically in nature (daily average temperature for a city over a given year, or time of high and low tides are recommended topics). Students will then:
  -find data for a particular city on the internet (http://www.weather.com) -graph the data points using Geogebra (http://www.geogebra.com) or Microsoft Excel

Identify (approximately): the range of the data, the

	period of the data, the amplitude of the data Use the values found above to write two regression equations for the data (one must be a sine function, the other a cosine function) Use the regression equations (and algebraic techniques) to make predictions. Instructional Strategies ● Interdisciplinary Connections Connections to astronomy and other sciences will be made, as planetary and satellite orbits can be modeled periodically, as well as ocean waves. ● Technology Integration TI 83+, Desmos, Geogebra, Wolfram Alpha will assist in the calculation of complicated trigonometric expressions, the analysis of trigonometric functions, solving trigonometric equations, evaluating inverse trigonometric expressions, law of sines and cosines, and real-world problems involving trigonometry. ● Media Literacy Integration Students will use technology to investigate interdisciplinary connections (above) and global perspectives (below). Students will have the opportunity to express results on Google Classroom, Ed Puzzle, Google Docs, interactive presentations, and self-made videos. ● Global Perspectives Real data that describes different nations' economies and populations, as well as natural phenomenon such as earthquakes, will be modeled by linear, radical, exponential, logarithmic, polynomial, rational, and periodic functions.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triands or small groups
Pictures	Tables	In a whole group
Illustrations, diagrams & drawings	Graphs	Using cooperative group
Magazines & Newspapers	Timelines	Structures
Physical activities	Number lines	With the Internet / Software
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		

Intervention Strategies		
Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations

Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading

Career-Ready Practices:
CRP1: Act as a responsible and contributing citizen and employee.
CRP2: Apply appropriate academic and technical skills.
CRP3: Attend to personal health and financial well-being.
CRP4: Communicate clearly and effectively and with reason.
CRP5: Consider the environmental, social and economic impacts of decisions.
CRP6: Demonstrate creativity and innovation.
CRP7: Employ valid and reliable research strategies.
CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9: Model integrity, ethical leadership and effective management.
CRP10: Plan education and career paths aligned to personal goals.
CRP11: Use technology to enhance productivity.
CRP12: Work productively in teams while using cultural global competence.

Summit Public Schools

Summit, New Jersey

Curricular Addendum

Career-Ready Practices **CRP1**: Act as a responsible and contributing citizen and employee. **CRP2**: Apply appropriate academic and technical skills. **CRP3**: Attend to personal health and financial well-being. **CRP4**: Communicate clearly and effectively and with reason. **CRP5**: Consider the environmental, social and economic impacts of decisions. **CRP6**: Demonstrate creativity and innovation. **CRP7**: Employ valid and reliable research strategies. **CRP8**: Utilize critical thinking to make sense of problems and persevere in solving them. **CRP9**: Model integrity, ethical leadership and effective management. **CRP10**: Plan education and career paths aligned to personal goals. **CRP11**:. Use technology to enhance productivity. **CRP12**: Work productively in teams while using cultural global competence.	**Interdisciplinary Connections** ● Close Reading of works of art, music lyrics, videos, and advertisements ● Use Standards for Mathematical Practice and Cross-Cutting Concepts in science to support debate/inquiry across thinking processes **Technology Integration** Ongoing: ● Listen to books on CDs, Playaways, videos, or podcasts if available. ● Use document camera or overhead projector for shared reading of texts. Other: ● Use Microsoft Word, Inspiration, or SmartBoard Notebook software to write the words from their word sorts. ● Use available technology to create concept maps of unit learning.

Instructional Strategies:
Supports for English Language Learners:

Sensory Supports	Graphic Supports	Interactive Supports
Real-life objects (realia)	Charts	In pairs or partners
Manipulatives	Graphic organizers	In triads or small groups
Pictures & photographs	Tables	In a whole group
Illustrations, diagrams, & drawings	Graphs	Using cooperative group
Magazines & newspapers	Timelines	structures
Physical activities	Number lines	With the Internet (websites) or software programs
Videos & films		In the home language
Broadcasts		With mentors
Models & figures		

from https://wida.wisc.edu

Media Literacy Integration

- Use multiple forms of print media (including books, illustrations/photographs/artwork, video clips, commercials, podcasts, audiobooks, Playaways, newspapers, magazines) to practice reading and comprehension skills.

Global Perspectives

- The Global Learning Resource Library

Differentiation Strategies:

Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/ expectations
Repeat/confirm directions	Increase task structure (e.g., directions, checks for understanding, feedback)	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding (e.g., writing, reading aloud, answering questions in class)	Individualized assessment tools based on student need
Audio Books	Utilize prereading strategies and activities: previews, anticipatory guides, and semantic mapping	Modified assessment grading