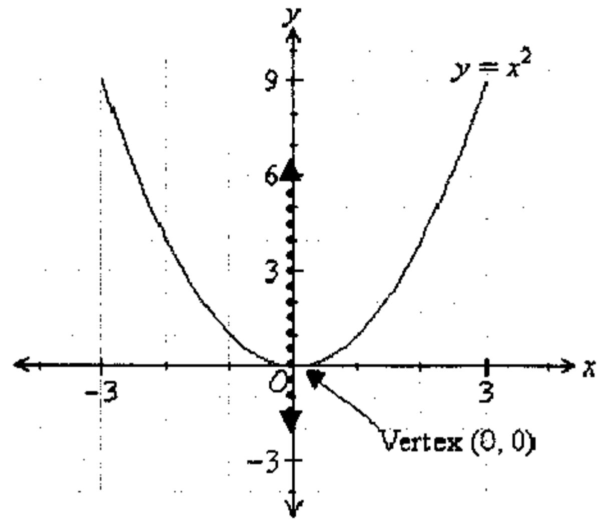
Unit 6: QUADRATICS RATIONAL

Quadratic Functions

(second degree polynomial functions)

"functions that have an exponent of 2 for the highest power of x"

- **Equation Format** : $y = ax^2 + bx + c$
  (a, b, c are coefficients AND x, y are variables).
- **Graph Shape**: PARABOLA --U shaped graph
- Quadratics are **FUNCTIONS** because each "x " value in the domain yields (produces) only ONE "y" in the range.



Axis of symmetry
$x = 0$

Parts of a Parabola

A) Minimum/Maximum (also called "TURNING POINT or "VERTEX)

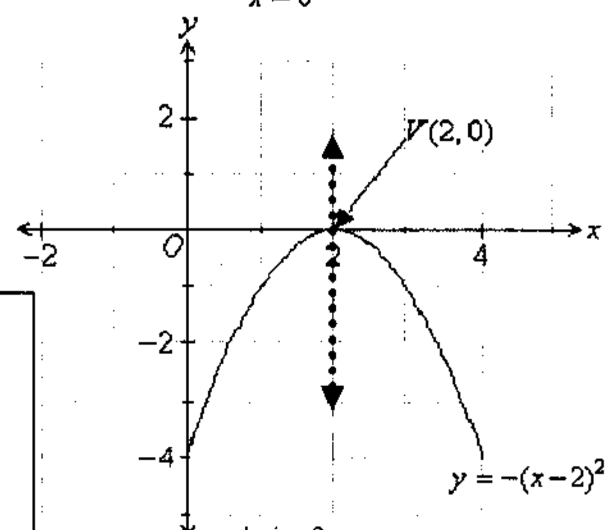
The lowest/highest **point** (ordered pair x,y) on a parabola.

B) Axis of Symmetry

A **line** (with the equation in the form x=__)
where the parabola is separated into 2 symmetrical sections.
On the graphs we will be investigating the line is a vertical line.
The access of symmetry always cuts through the vertex.

C) Roots (also called x-intercepts or solutions).

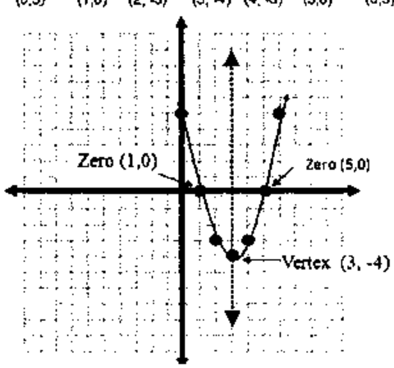
Roots are found where the parabola crosses the x-axis.
A parabola can cross 2 times, 1 time, or 0 times.
The x value on this point is called the root.
In this case (as opposed to a linear), the y coordinate is always 0.



Axis of symmetry
$x = 2$

Step 2: Graph the points from the table of values:

	0	1	2	3	4	5	6
	5	0	-3	-4	-3	0	5
	(0,5)	(1,0)	(2, -3)	(3, -4)	(4, -3)	(5,0)	(6,5)



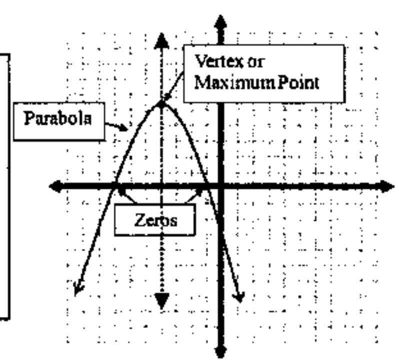
Minimum: (3, -4)
(point)

Axis of Symmetry:
x = 3 (line)

Roots: {1, 5}
(x-values at
x-intercept points.)

$f(x) = -ax^2 + bx + c$

Negative a, so the parabola opens down.



Maximum: (-4, 5)
(point)

Axis of Symmetry:
x = -4 (line)

Roots: {-1, -7}
(x-values at
x-intercept points.)

Determining Important Parts of a Quadratic Graph Using the TI-84

NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3
\Y1=2X²-16X+30
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
\Y8=

Place the equation in the Y_1.

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl

X	Y1	
-1	48	
0	30	
1	16	
2	6	
3	0	→ Root
4	-2	→ Turning point
5	0	→ Root
6	6	
7	16	
8	30	
9	48	

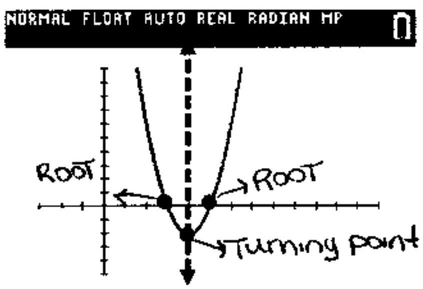
X=9

The coordinate (4,-2) is one of the most important points on the table. This would be my middle point if I was asked to make and x, y table with 7 key points. The other 3 points would be the 3 coordinates *above* (4,-2) and the 3 coordinates *below* the (4,-2).
NOTICE: Your y values duplicate on these other 6 points.

You can create the Axis of Symmetry equation from (4,-2). This would be x = -2. I have drawn in this line on the graph below.
NOTE: The axis of symmetry line will not show up on your graphing calculator graph.

The turning point is the entire coordinate (4, -2)

The roots are the x –values from the ordered pairs that have a y value of 0. The 2 coordinates on the chart with 0 y's are (3, 0) and (5, 0). So, the roots are {3, 5}



Sometimes the Axis of Symmetry, turning points, and/or roots are not integer values. When this occurs the TI-84 can give you an *estimate* as to their values.

Look at the TI-84 visuals below.

- The Axis of Symmetry in the equation below is somewhere between x = 7 and x = 8. I know this because I see that my outputs start to duplicate in this area.
- One of the roots (a.k.a solutions, x-intercepts, or zeroes) is between x = 1 and x = 2. I know this, because my outputs change from positive to negative. Therefore, I must have crossed the x-axis.

NORMAL FLOAT AUTO REAL RADIAN MP

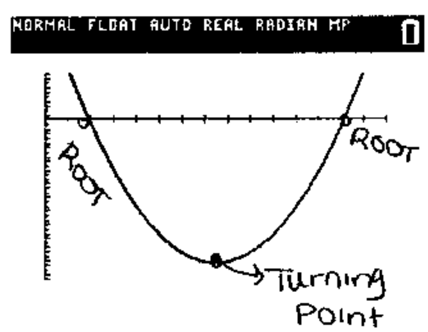
Plot1 Plot2 Plot3
\Y1=X²-15X+25
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
\Y8=

One of the Roots are between 1 and 2

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR ΔTbl

X	Y1	
1	11	
2	-1	
3	-11	
4	-19	
5	-25	
6	-29	
7	-31	Turning point Is Between 7 & 8
8	-31	
9	-29	
10	-25	
11	-19	

X=1



Name: ______________________________ **Period:**_____ **Date:**________

Prep Graphing Quadratics and Naming Key Parts **SAVE IN NOTES**

USE YOUR TI-84 for this activity

Your 1st Objective:

- Create a table of 7 coordinate that will create the U shape parabola
- Graph the ordered pairs.
  (Connect in a smooth U shape. Place arrows on the ends. Label with the Function Name.)
- Identify and label the following key parts of the quadratic.
  - **a)** Axis of symmetry (ex. x = 5 In this context x = 5 is the equation for the line.)
  - **b)** Vertex (ex. (5, 8))
  - **c)** Roots/x-Intercepts/Zeros (ex. x = -2 and x = 12) If the roots are Integers.
    Estimate the root values if they are not whole integers (ex. x ≈1.5)
    If there are no roots at all, state that too.
  - **d)** State if the Vertex is a Maximum or a Minimum

Your 2nd Objective:

The **POWER FUNCTION for a Quadratic Equation is f(x) = x^2.**

Notice the changes I made to the function from questions 1 -6.

Think about how understanding this, would help you graph the function quickly, without the TI-84.

Your 3rd Objective:

Tell me what you notice about questions 7, 8 and 9.

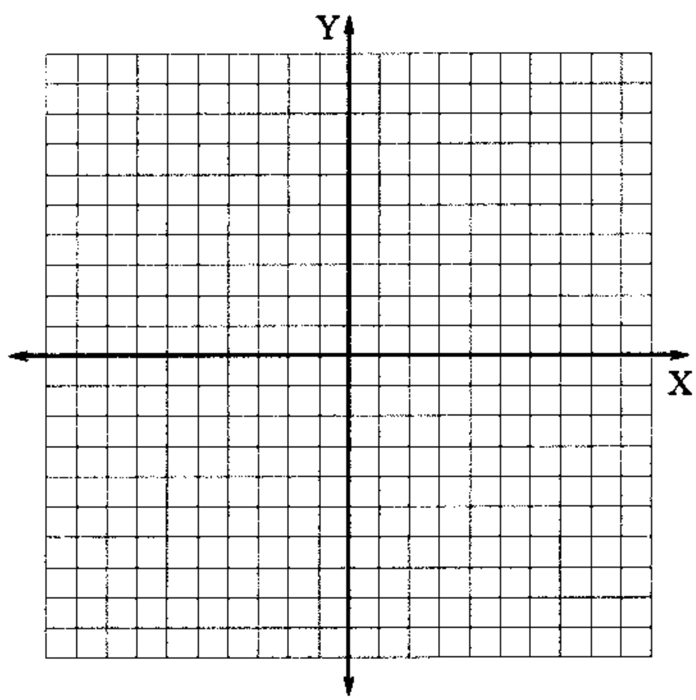
1) $y = x^2$ PARENT FUNCTION FOR THE QUADRATIC FAMILY

Axis of symmetry:_________

Vertex:_____________

Roots:______________

Max or Min:____________



x	y

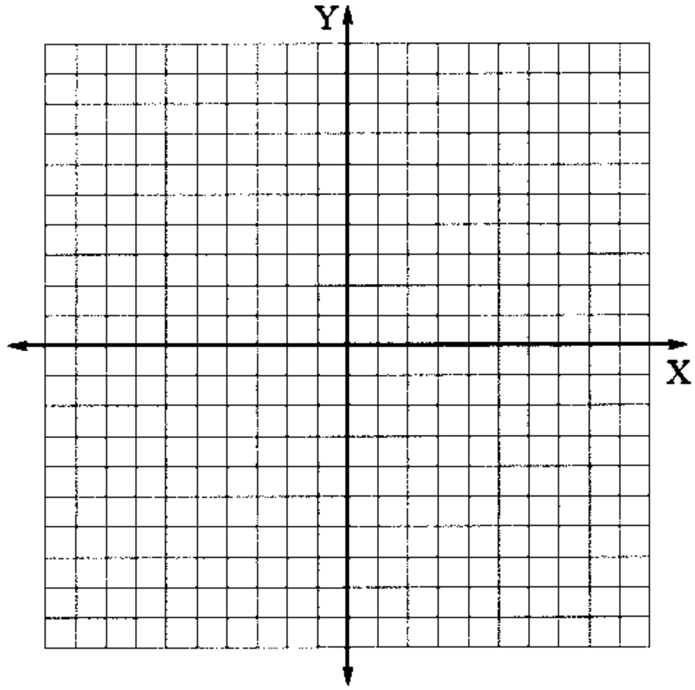
2) $y = -x^2$

Axis of symmetry:__________

Vertex:______________

Roots:_______________

Max or Min:_____________



x	y

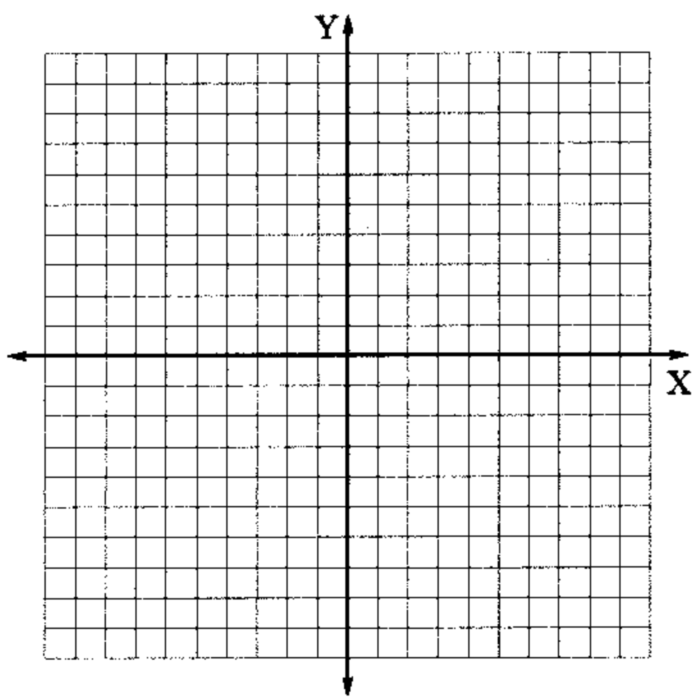
3) $y = \frac{1}{2}x^2$

Axis of symmetry:__________

Vertex:______________

Roots:_______________

Max or Min:_____________



x	y

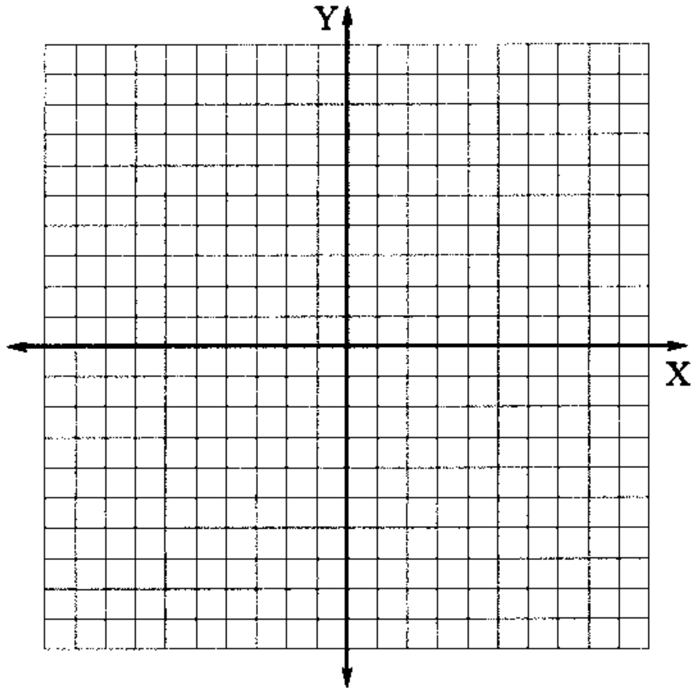
4) $y = 2x^2$

Axis of symmetry:__________

Vertex:______________

Roots:_______________

Max or Min:_____________



x	y

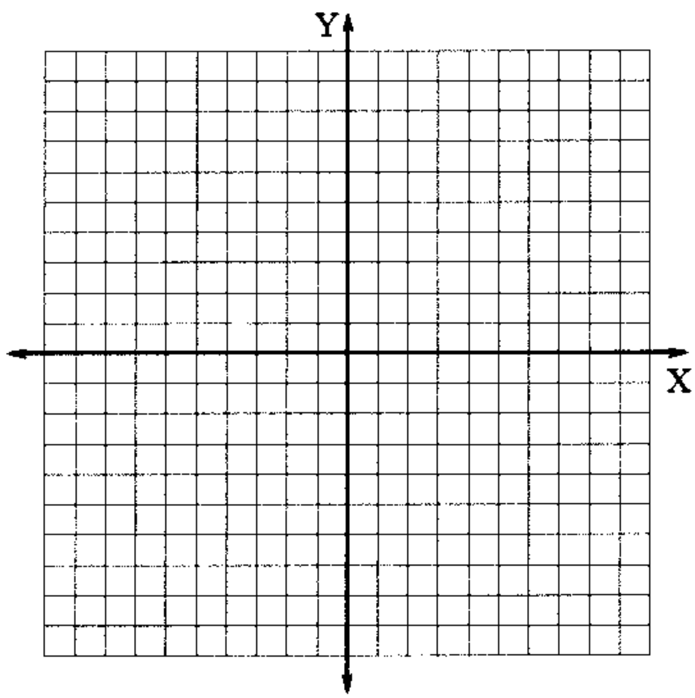
5) $y = x^2 + 1$

Axis of symmetry:__________

Vertex:______________

Roots:_______________

Max or Min:_____________



x	y

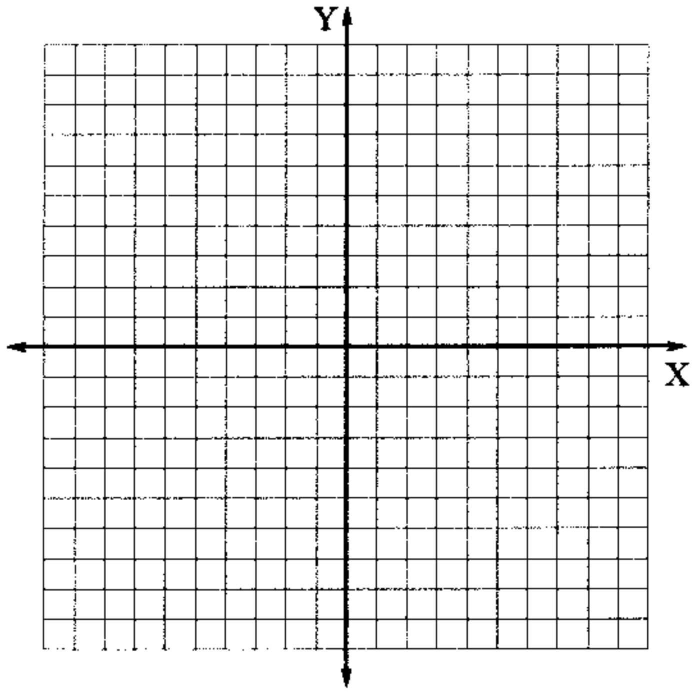
6) $y = x^2 - 5$

Axis of symmetry:__________

Vertex:______________

Roots:_______________

Max or Min:_____________



x	y

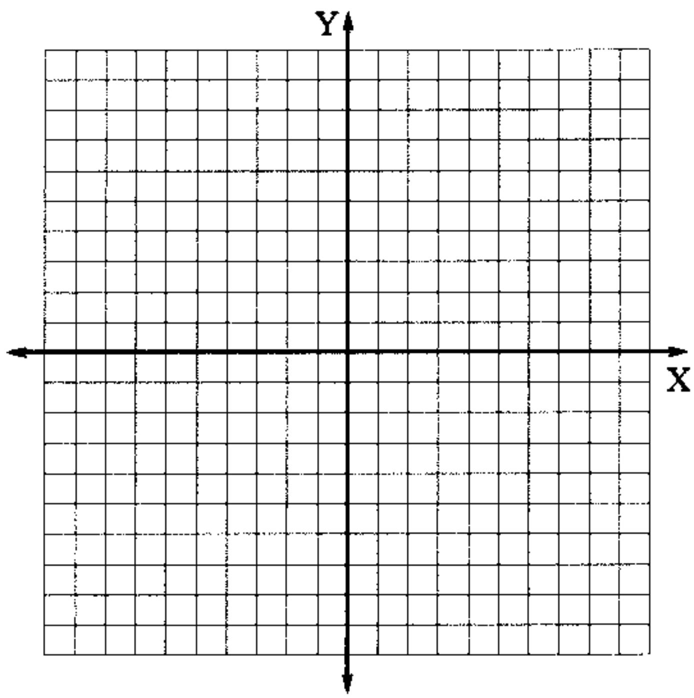
7) $y = x^2 + 2x - 8$

Axis of symmetry:__________

Vertex:______________

Roots:_______________

Max or Min:_____________



x	y

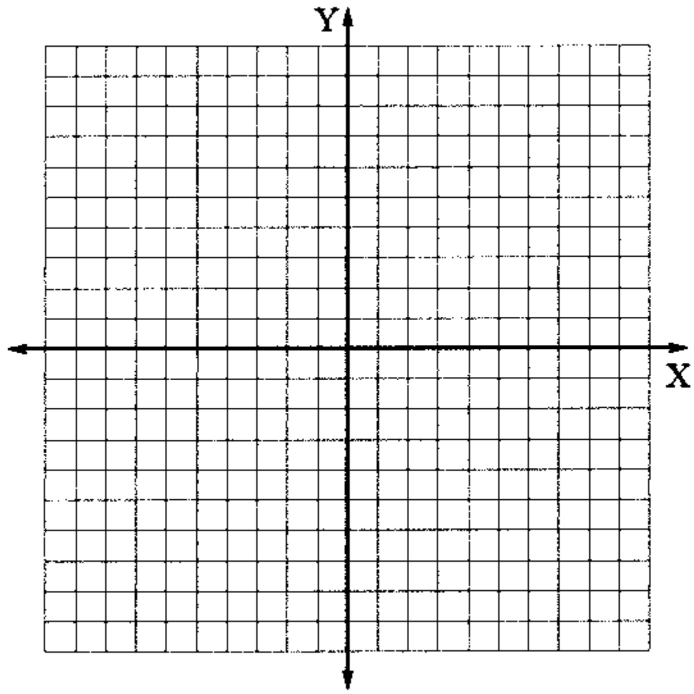
8) $y = (x - 2)(x + 4)$

Axis of symmetry:__________

Vertex:______________

Roots:_______________

Max or Min:_____________



x	y

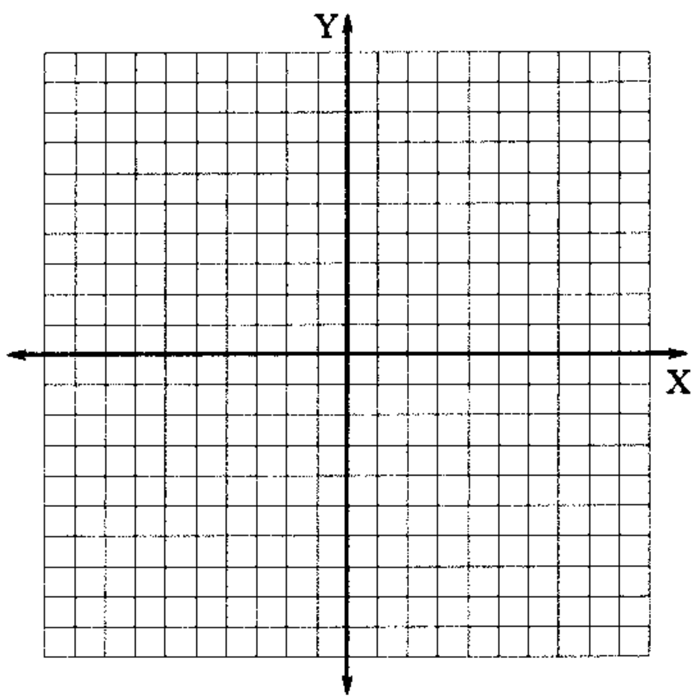
9) $y = (x + 1)^2 - 9$

Axis of symmetry:__________

Vertex:______________

Roots:_______________

Max or Min:_____________

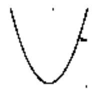


x	y

Name: ______________ Period:___ Date:________ START UP

Quadratic Graphing Questionnaire

Questions 1-6: $F(x) = x^2$ is the Parent Function (most basic) form of a quadratic function. Create a new quadratic functions that when compared to $f(x) = x^2$ will...

1) Open UP when graphed.  $\mathbf{f(x) = x^2 \rightarrow}$ __________

2) Open DOWN when graphed.  $\mathbf{f(x) = x^2 \rightarrow}$ __________

3) Have 2 terms and shift down. $\mathbf{f(x) = x^2 \rightarrow}$ __________

4) Have 2 terms and shift up. $\mathbf{f(x) = x^2 \rightarrow}$ __________

5) Has 1 term and will be wider. $\mathbf{f(x) = x^2 \rightarrow}$ __________

6) Has 1 term and will be narrower. $\mathbf{f(x) = x^2 \rightarrow}$ __________

Questions 7-9: Base your answers from you observations from question 7 – 9 on last night's prep task.

7) Write a quadratic function where the y-intercept: is visible in the equation you write. Also, state the y-intercept:

Function:______________________ Y-intercept:__________

8) Write a quadratic function where the roots (x-intercepts/zeros) are visible in the equation you write. Also, state the roots.

Function:______________________ Roots:__________

9) Write a quadratic function where the vertex is visible in the format you write. Also, list the vertex.

Function:______________________ Vertex:__________

Determining the Roots for a Quadratic Function in Standard Format

A Quadratic function represents a "**SQUARED**" **measurement**.
Two formats for expressing a quadratic function as an equation are also listed below.

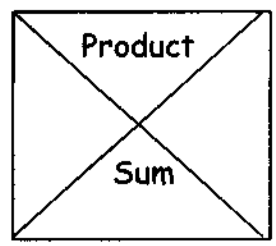
Standard Format	**Factored Format**
Represented as a SUM	Represented as a PRODUCT

$f(x)= x^2 + 7x + 12$ ← equivalent functions → $f(x)= (x + 4)(x + 3)$

From the **Factored Form** of a Quadratic Function, $f(x) = (x+4)(x+3)$,
it is **much easier to see the Zeros (Root/X-Intercepts)** are -4 & -3.

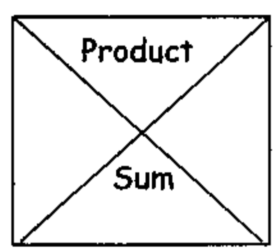
Let's Try a Few:

1) Standard Format:
$f(x) = x^2 - 6x + 8$



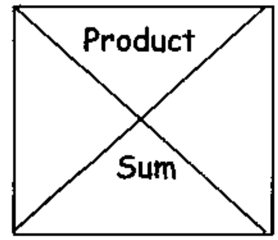
Factored Format:__________________ Roots:__________________

2) Standard Format:
$f(x) = x^2 - 4x - 12$



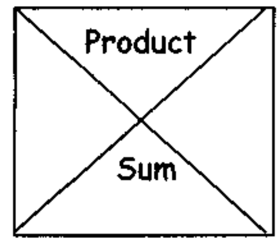
Factored Format:__________________ Roots:__________________

3) Standard Format:
$f(x) = x^2 + 4x - 12$



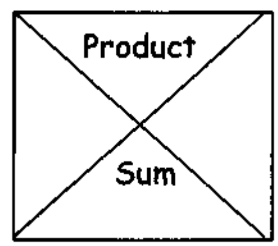
Factored Format:________________ Roots:________________

4) Standard Format:
$f(x) = 5x^2 - 35x + 50$



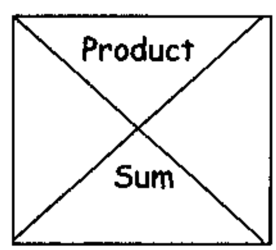
Factored Format:________________ Roots:________________

5) Standard Format:
$f(x) = -2x^2 - 10x + 72$



Factored Format:________________ Roots:________________

6) Standard Format:
$f(x) = x^2 - 49$



Factored Format:________________ Roots:________________

NOTES

Determining the Roots for a Quadratic Function in General Format

PART B

When the Lead Coefficient is Greater Than 1
AND Factoring out a GCF is NOT Possible

In order to break up the middle term of the quadratic function the lead coefficient must also be considered when determining the correct Product/Sum combination.

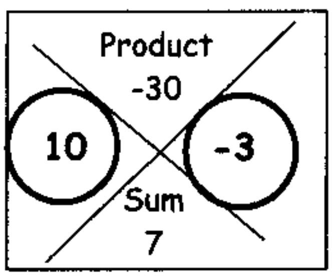
Standard Format

$2x^2 + 7x - 15$

Product
-30

$2x^2 + 7x - 15$

Sum
7



Split the Middle Term using 10 and -3.

$2x^2 + 10x - 3x - 15$

GCF the 1st Half and Last Half

$2x(x + 5) - 3(x - 5)$

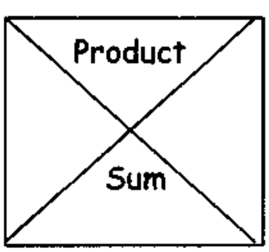
Factored Format

$(2x - 3)(x + 5)$

From the Factored Form of a Quadratic Function, $f(x) = (2x - 3)(x + 5)$, it is much easier to see the Zeros (Root/X-Intercepts) are 3/2 & - 5.

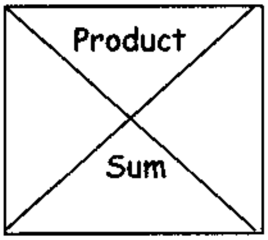
Let's Try a Few:

1) Standard Format:
$f(x) = 35x^2 - 3x - 2$



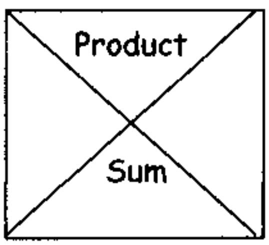
Factored Format:________________ Roots:________________

2) Standard Format:
$f(x) = 6x^2 + 23x + 20$



Factored Format:__________________ Roots:__________________

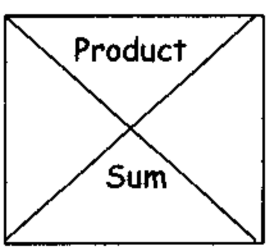
3) Standard Format:
$f(x) = 2x^2 + 11x - 21$



Factored Format:__________________ Roots:__________________

4) Standard Form (Special Case) "Difference of Perfect Squares

$f(x) = 9x^2 - 49$



Factored Format:__________________ Roots:__________________

Factoring Completely
(Quadratics and Beyond!!!)
In Order to Find Zeros
(A.K.A Solutions, A.K.A Roots, A.K.A X-Intercepts)

BASIC GUIDELINES that make factoring faster and more efficient:

1st: ***FACTOR out the GREATEST COMMON FACTOR (if there is one).*** $2x^2 - 6x + 10$

- GCF's are any factors other than +1 that are present in all the terms
- They can be numbers, variables, or a combination of both.
- If the term with the highest power has a negative coefficient, the NEGATIVE is part of the GCF

2nd: ***FACTOR USING the PRODUCT/SUM METHOD (if the Quadratic is a trinomial).***
Once you have found the correct combination for splitting up the middle term, you can use the **SPLIT** the middle technique.
$x^2 - 5x + 6$

2nd: ***SPECIAL CASE:(if Difference of PERFECT SQUARES)***
The "difference (subtraction)" of two monomials made up of ALL perfect squares. $4x^2 - 25$

Create a trinomial with a middle term and use one of the Product/Sum techniques:

$4x^2 - 25$

$4x^2 + 0x - 25$
Now do product/sum

OR

Recognize the problem as a DOPS problem (difference of perfect squares) and go straight to the shortcut for factoring it by taking the square root of all terms.
$4x^2 - 25$
DONE! $(2x - 5)(2x + 5)$

3^2d: ***Check each factor to see if you missed a GCF.*************

If you did, GCF again to finish factoring.
$(2x - 6)(x - 5)$ oops! $2x - 6$ has a GCF

AND

Check each factor to see if it can be factored again by product/sum.
Sometimes binomials fall under a special case called Difference of Perfect Squares and can be factored several times, until they are done. $(x^2 + 1)(x^2 - 1)$ oops! $x^2 - 1$ can be factored again.

Name:____________________ Period:_________ Date:________

PRACTICE/PREP: More Factoring

Directions: You **must** show all work to solve.
You **should** check your answers using a TI-84 strategy.

1) Create an equation (in factored form) that has the same solutions as $2x^2 + x - 3 = 0$

2) Solve the equation $4x^2 - 12x = 7$ algebraically for x.

3) Find algebraically all possible values for *b*, in the equation $x^2 + 10x + 24 = (x + a)(x + b)$, where b is an integer.

4) Factor the expression $x^4 + 6x^2 - 7$ completely.

5) Factor the expression $p^4 - 81$ completely.
Make sure you examine each new factor you create to determine if the factor can be factored again.

6) Factor $4x^2 - 100 = 0$ completely and write the equivalent factored form.
Also, determine the roots.

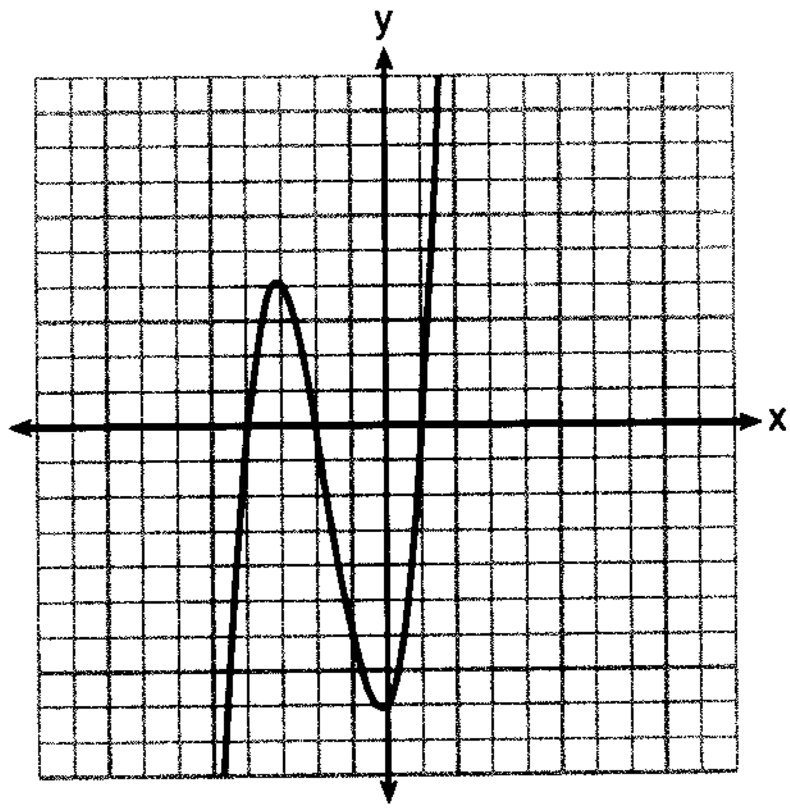
7) Factor the expression $x^4 - 12x^2 + 36$.

8) If the area of a rectangle is expressed as $x^4 - 9y^2$. This expression can be written as a product of the length and the width of the rectangle. Determine the factors that would represent the length and width.

9) Write an equation that defines $m(x)$ as a trinomial where
$m(x) = (3x - 1)(3 - x) + 4x^2 + 19$

Solve for x, when $m(x) = 0$

10) The graph of $f(x)$ is shown below.



Which function could represent the graph of $f(x)$?

(1) $f(x) = (x + 2)(x^2 + 3x - 4)$

(2) $f(x) = (x - 2)(x^2 + 3x - 4)$

(3) $f(x) = (x + 2)(x^2 + 3x + 4)$

(4) $f(x) = (x - 2)(x^2 + 3x + 4)$

11) **Combine a Solving Systems Technique and Factoring To Solve:**
John and Sarah are each saving money for a car. The total amount of money John will save is given by the function $f(x) = 60 + 5x$. The total amount of money Sarah will save is given by the function $g(x) = x^2 + 46$. After how many week, x, will they have the same amount of money saved?

12) Use your Graphing Calculator to Answer:
How many real solutions does the equation $x^2 - 2x + 5 = 0$ have? Justify your answer.

Name:________________ Period:_______ Date:_____________ PREP

Factoring Reinforcement

Factor each polynomial completely. Use the notes provided to ensure you have factored "COMPLETELY".

1) $x^2 + 2x - 15$

2) $2x^2 - 10x - 48$

3) $6x^2 - 7x - 20$

4) $9x^2 - 25$

5) $-2x^4 - 4x^3 + 30x^2$

6) $16x^2 - 36$

There is a back, if you dare ☺ !

7) $\frac{4}{25}x^2 - \frac{9}{16}$

8) $x^2 + \frac{1}{4}x - \frac{3}{8}$

9) $x^4 - 2x^2 - 3$

10) Determine ALL of the roots. Show work to justify your answers.

$f(x) = 5x(x - 2)(x + 7)(x - 15)(2x + 5)(3x - 1)$

Name:_______________ **Period**______ **Date:**______

Linear VS Quadratic Translations

Directions: For each word problem, translate ONLY.
State whether your expression is Linear or Quadratic.
How many solutions will there be?

Column A	Column B	
1a) The difference of two numbers is 3. Their sum is 7. What are the numbers?	1b) The difference of two numbers is 3. Their product is 10. What are the numbers?	
2a) Find two consecutive odd integers such that their sum is -8.	2b) Find two consecutive odd integers such that the sum of their squares 34.	2c) Find two consecutive odd integers such that their product is 63.
3a) The length of a rectangle is 2 inches more than its width. The perimeter of the rectangle is 20 inches. Find the length and the width.	3b) The length of a rectangle is 2 inches more than its width. The area of the rectangle is 24 inches squared. Find the length and the width.	

Name ______________________________ Date ______________ NOTES
Math 8 ACC

NOTES - Word Problems Solved by Factoring : *Three Types*

Number Problems:

Example 1 : The difference of two numbers is 3. Their product is 10. What are the two numbers?

Solution:

Let x = smaller number
$x+3$ = larger number
Given: Their product is 10

$x(x+3)=10$

$x^2+3x=10$

$-10 \quad -10$

$x^2+3x-10=0$

$(x-2)(x+5)=0$

$x-2=0$	$x+5=0$
$x=2$	$x=-5$
$x+3$	$x+3$
$2+3$	$-5+3$
5	-2

There are two solutions:
$2,5$ and $-5,-2$

Consecutive Integer Problems:

Example 2 : Find two consecutive odd integers such that the sum of their squares is 34.

Solution:

Let x = first consecutive odd
$x+2$ = second consecutive odd
Given: Sum of their squares is 34

$x^2+(x+2)^2=34$

$x^2+x^2+4x+4=34$

$2x^2+4x-30=0$

$2(x^2+2x-15)=0$

$2(x-3)(x+5)=0$

$2\neq 0$	$x-3=0$	$x+5=0$
	$x=3$	$x=-5$
	$x+2$	$x+2$
	$3+2$	$-5+2$
	5	-3

There are two solutions:
$3,5$ and $-5,-3$

Geometry Problems:

Example 3 : The length of a rectangle is 2 inches more than its width. The area of the rectangle is 24 square inches. Find the length and width.

Solution:

Let w = width

$w+2$ = length

Given: Area is 24 sq. in

$A = l \bullet w$

$24 = (w+2)w$

$24 = w^2 + 2w$

$0 = w^2 + 2w - 24$

$0 = (w-4)(w+6)$

$w - 4 = 0$

$w = 4$

length is $w + 2$

$4 + 2$

6

~~$w + 6 = 0$~~

~~$w = -6$~~

****Not a solution because lengths cannot be negative***

The length of the rectangle is 6 inches and the width is 4 inches.

Name:______________ Period:___Date:_____
PRACTICE Linear Word vs Quadratic Word

Solve each function you created to determine the answers to the questions.

Linear	Quadratic
1a) The difference of two numbers is 3. Their sum is 7. What are the numbers?	1b) The difference of two numbers is 3. Their product is 10. What are the numbers?

2a) Find two consecutive odd integers such that their sum is -8.	2b) Find two consecutive odd integers such that the sum of their squares 34.

3a) The length of a rectangle is 2 inches more than its width. The perimeter of the rectangle is 20 inches. Find the length and the width.	3b) The length of a rectangle is 2 inches more than its width. The area of the rectangle is 24 inches squared. Find the length and the width.

Prep Problems: Word Problems Requiring Factoring

1) The difference of two numbers is 5. Their product is 14. Find the numbers.

2) Find 3 consecutive integers such that the product of the first two is 7 more than the third.

3) One number is three times another number. The sum of their squares is 40. Find the numbers.

4) The length of a rectangle is 5 more feet than its width. Its area is 84 sq. ft. Find the length and width.

Name ____________________________ Per # ______ **PREP TASK**

Math 8 ACC Date _______________

Word Problems That Require Factoring: ***Practice 3 Types***

Directions: You will find that these questions are very similar to the problems you completed the previous night for your prep.

- Set up the problems and solve them exactly the way you did on the prior assignment.
- When you are done, go back and read the question. THE ONLY CHANGE will be which solutions you keep in your final answer and which ones you will discard based on the subtle change in wording.

1) The length of a rectangle is represented by (x + 4) and the width by (x - 2). Write an *expression* in standard form to represent the area.

2) Given two positive consecutive odd integers, it turns out that the square of the smaller minus 5 times the larger is 26. What is the sum of the integers?

3) Find three consecutive positive integers such that the square of the first increased by twice the second is 3 less than four times the third.

4) If the square of a positive number is decreased by 5 times the number, the result is 14. Find the number.

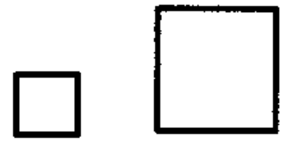
5) The area of a rectangular pool is 192 square meters. The length of the pool is 4 meters more than its width. Find the length and the width.

6) Twice the square of a certain positive number is 144 more than twice the number. What is the number?

Name:_____________________________ Period:_____ Date:_______

Yet Even More Quadratic Word Applications

1) **The measure of a side of a square is x units. A new square is formed with each side 6 units longer that the original square's side.**



a) **Write an expression to represent the area of the original square.**

b) **Write an expression to represent the area of the larger square.**

c) **Write an expression that represents the difference between the areas of the squares.**

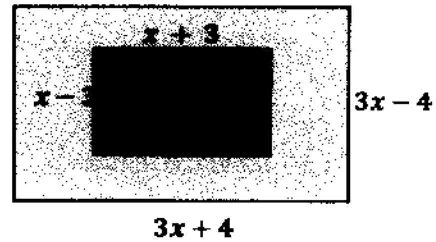
d) **If the Area of the larger square is 100 square units, what is the value of x?**

e) **What is the area of the smaller square if the larger square is 100 square units?**

f) **Use the expression from c) and your value of x from d) to find the difference between the areas?**

g) **Take the area of the larger square (100 square units) and subtract the area of the smaller square you found in e. Does it match the answer in f? Why or why not?**

2) In the accompanying diagram, the width of the inner rectangle is represented by $x - 3$ and its length by $x + 3$. The width of the outer rectangle is represented by $3x + 4$ and its length by $3x - 4$.



a) Write an expression to represent the area of the larger rectangle.

b) Write an expression to represent the area of the smaller rectangle.

c) Express the area of the lightly shaded region as a polynomial in terms of x.

d) If the area of the smaller rectangle is 72 cm squared, then what is the area of the lightly shaded region.

3) The formula for the Area of a trapezoid is A = $\frac{(b_1 + b_2)h}{2}$. The area of this trapezoid is 26 units squared.

a) Determine the value of x.

b) Using the value of x determine the actual lengths of b_1, b_2, and h.

Base 1=________________ Base 2= ________________ Height=___________

Determining Important Parts of a Quadratic Graph By Hand

If you are asked to find the key values for a quadratic equation, but the numbers are not integer values OR if you are asked to show calculations by hand follow the process below.

Standard form of a quadratic equation:

$$y = ax^2 + bx + c$$

(a, b, c are coefficients AND x, y are variables).

A) **Finding the Axis of Symmetry Algebraically**: use the following formula

Axis of Symmetry Formula: $x = \dfrac{-b}{2a}$

Example: $y = 2x^2 - 16x + 30$

$a = 2$ $b = -16$ $c = 30$

a) State the formula: $x = \dfrac{-b}{2a}$

b) Substitute the a and b values: $x = \dfrac{-(-16)}{2(2)}$

c) Solve to determine the value of x: $x = \dfrac{16}{4}$

Final answer: axis of symmetry is $x = 4$.

Line of symmetry for the parabola AND the x-value of your vertex coordinate.

B) **Finding the Turning Point Algebraically:**

1) After you find the axis of symmetry, substitute this x-value back into the original quadratic equation $y = ax^2 + bx + c$ equation.
2) Solve to determine the y-value.
3) Create an (x, y) coordinate using the x-value from the axis of symmetry and the y-value you found in step 2. This is your turning point.

Now use x = 2 from finding the axis of symmetry and $y = x^2 - 4x + 3$ to find the value of y.

a) State the formula: $y = 2x^2 - 16x + 30$

b) Substitute the of x values: $y = 2(4)^2 - 16(4) + 30$

c) Solve to determine the value of y: $y = 32 - 64 + 30$

$y = -2$

Final Answer: Turning Point is (4, -2)

C) **Creating a 7 Point Table:**

1) Use the Axis of Symmetry as your middle value in the table.
2) List 3 x values that are less than the x value in your ordered pair and state 3 values greater than that x value.
3) State, Substitute, and Solve each of those values into the quadratic equation to create a list of 7 (x,y) pairs.
4) These are the 7 points that help to create the U-shape.

Graph the 7 points if requires.
Place Arrows on each end.
Label with the equation name.

x	$2x^2 - 16x + 30$	y
1		
2		
3		
4	$2x^2 - 16x + 30$ $2(4)^2 - 16(4) + 30$ $32 - 64 + 30$ -2	-2
5		
6		
7		

D) <u>Finding the roots of a Quadratic using Algebra</u>

Standard Form $ax^2 + bx + c = 0$ **(where a, b, and c are real number and $a \neq 0$)**

$$y = 2x^2 - 16x + 30$$

The "ROOTS" are the x-values that will make the y-value equal "zero" when substituted in the equation.
We use this knowledge to help us find the roots algebraically, instead of using a graph.

$$y = 2x^2 - 16x + 30$$

$$0 = 2x^2 - 16x + 30$$

$$0 = 2(x^2 - 8x + 15)$$

$$0 = 2(x - 3)(x - 5)$$

1) We need the values of x that will make y = 0, so substitute 0 in for y.
2) Factor the quadratic expression. (GFC, UNFOIL, and/or DOPS)
3) Since, the product of (2), (x - 3) and (x -5) are 0, at least one of the these binomials would have to be a value of zero. I need to find the values of x that could make this happen.
4) Set each factor equal to zero and solve to find the roots (a.k.a. x-intercepts, solutions, or zeroes.)

$2 \neq 0$	$x - 3 = 0$	$x - 5 = 0$
	$+3 \quad +3$	$+5 \quad +5$
	$x = 3$	$x = 5$

The Roots are {3, 5}

This can be confirmed by looking at the graph on the first page of this packet.

Depending on the equations and where it is graphed there could be:

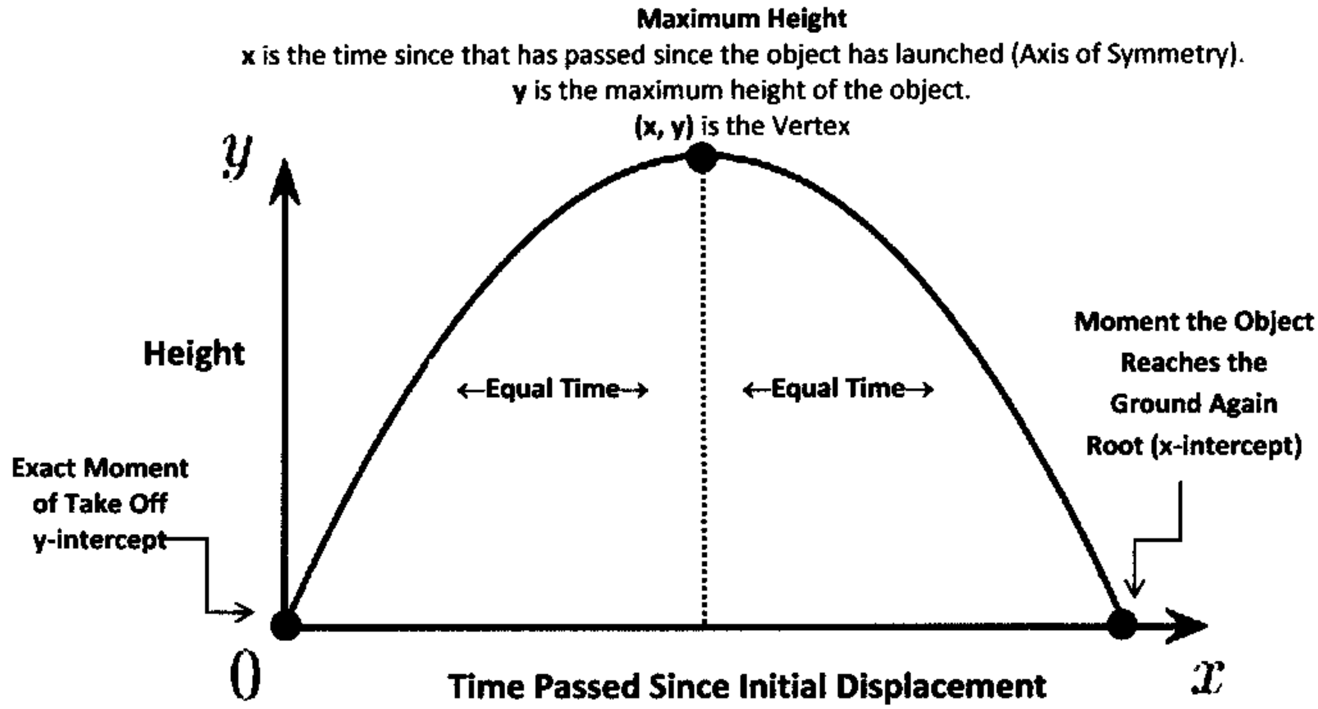
- 2 real roots (The graph crosses the x-axis in two places).
- 1 real root (The graph crosses the x-axis in one place).
- 0 real roots (The graph does not cross the x-axis at all. This means there are "imaginary roots" which will be discussed in the future).

Name:____________________ Period:______________ Date:_____________

Quadratic Applications (Trajectory)

PROJECTILES

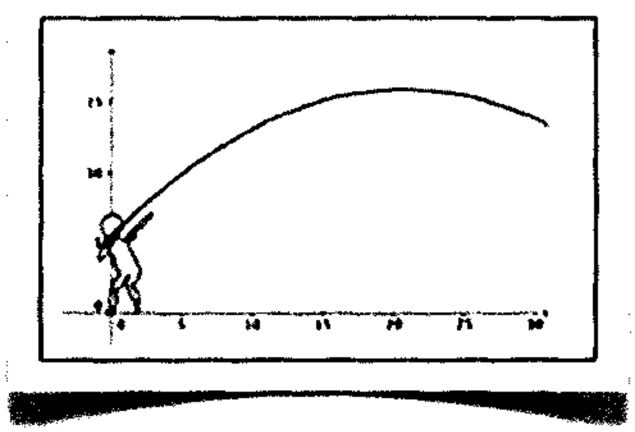
A parabola can be used to plot the time (x) that has passed after the initial launch of a projectile, as well as the height (y) of a projected image at that given time.



$$y = ax^2 + bx + c$$

$$s(t) = -16t^2 + v_0 t + s_0$$

- $-16t^2$: Due to -32ft./sec.^{2} gravity
- $v_0 t$: Initial Velocity
- s_0: Initial Displacement



********Please NOTE: The moment of take off <u>IS NOT ALWAYS at a HEIGHT OF 0</u>.*********

See the diagram above where someone releases a ball at the height of 5 feet.

EXAMPLES ARE ON BACK→

EXAMPLE: A baseball player throws a ball from the outfield toward home plate.
The ball's height above the ground is modeled by the equation $y = -16x^2 + 48x + 6$, where y represents height, in feet, and x represents time, in seconds.
The ball is initially thrown from a height of 6 feet.

<u>Question 1) At what time does the ball reach it's maximum height?</u>

Axis of Symmetry formula for a Quadratic in General (Standard) Form is $x = \frac{-b}{2a}$

This will give you the <u>time</u> that the ball reaches the maximum height.

$y = -16x^2 + 48x + 6$
a b c

ANSWER: $x = \frac{-b}{2a}$

$x = \frac{-48}{2(-16)}$ → **x=1.5** **The ball will reach maximum height in 1.5 seconds.**

<u>Question 2) What is the maximum height that the ball reaches?</u>

ANSWER: Substitute in the time in x to find out the maximum height.

$y = -16x^2 + 48x + 6$

$y = -16(1.5)^2 + 48(1.5) + 6$ → **y = 42** **The maximum height is 42 feet.**

Question 3) At how many second(s) after the baseball player releases the ball, is the ball <u>38 feet above the ground? (Remember: y represents the height.)</u>

ANSWER: Substitute the height in y to determine the time(s).

$y = -16x^2 + 48x + 6$

$38 = -16x^2 + 48x + 6$
$-38 \qquad\qquad -38$

$0 = -16x^2 + 48x - 32$ → **Now factor!** → $0 = -16(x-2)(x-1)$

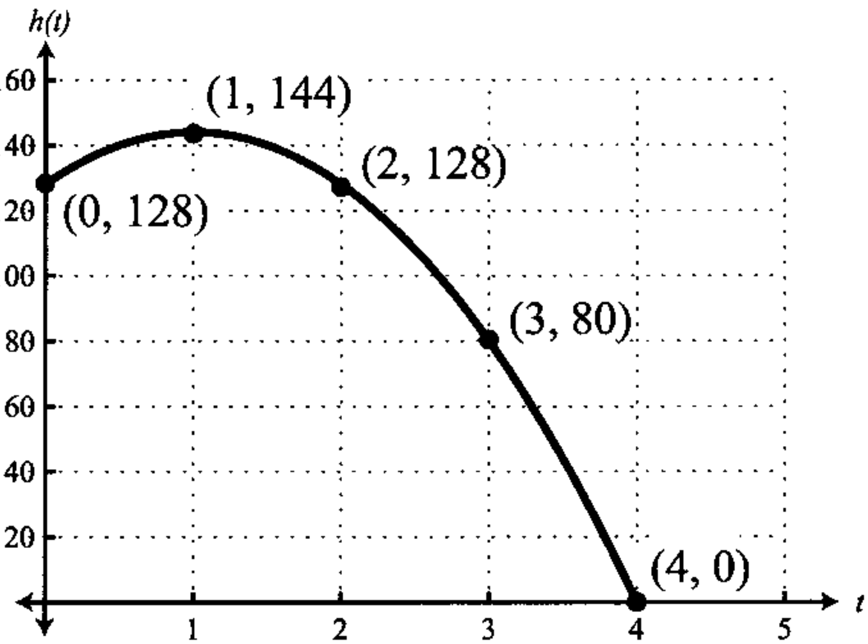
$x - 2 = 0$ or $x - 1 = 0$
x = 2 or x = 1

The ball will be at 38 feet 1 second and 2 seconds after the pitcher releases the ball.

Name:________________________ Period:________ Date:_______________

Practice: Quadratic Projectiles

Function Rule:
$h(t) = -16t^2 + 32t + 128$



INTERPRETING THE GRAPH

A projectile is launched from a certain height(in feet) and tracked over time (in seconds).

Answer the following questions about the projectile. Also, on the questions with a ☺ provide the name for the part of the parabola that you used to determine your answer (using an appropriate math vocabulary term),

1) How high was the object at the moment it was launched? ☺

2) How long did it take to reach the apex of it's path? ☺

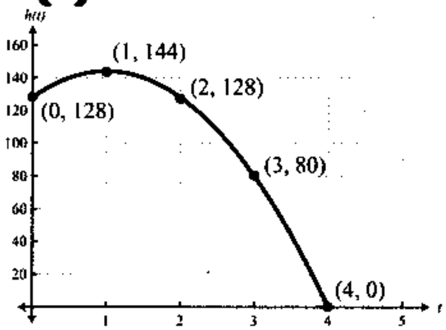
3) How high was the object when it reached it's apex? ☺

4) How long did it take to reach the ground again? ☺

5) If the object has been in motion for 3 seconds, how high off the ground is it?

6) Why don't we see the other x-intercept on this graph?

Function Rule:
$h(t) = -16t^2 + 32t + 128$



USING THE FUNCTION RULE TO DETERMINE KEY VALUES
"Solving By Hand"

Answer the following questions about the projectile, by hand.

1) How high was the object at the moment it was launched? ☺

2) How long did it take to reach the apex of it's path? ☺

3) How high was the object when it reached it's apex? ☺

4) How long did it take to reach the ground again? ☺

5) If the object has been in motion for 3 seconds, how high off the ground is it?

Standard Form of a Quadratic: $ax^2 + bx + c$ **Quadratic Formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Found on the Regents Reference Sheet.

Name:____________________Period:_____ Date:_______

Regents Quadratics Questions

1) Let $h(t) = -16t^2 + 64t + 80$ represent the height of an object above the ground after *t* seconds.

a) Determine the number of seconds it takes to achieve its maximum height. Justify your answer.

b) What is the object's maximum height (in feet)?

c) What was the height of the object at the moment it was projected?

d) How many seconds did it take to reach the ground again?

e) State the time interval, in seconds, during which the height of the object *decreases.* Explain your reasoning.

2) A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented as x, and the area of the garden is 108 square meters.

Determine, algebraically, the dimensions of the garden in meters.

Name:______________________ Period: _________ Date:________

ALGEBRAIC PROPORTIONS IN ***QUADRATIC FORM***

Solve each proportion.

- Set the product of the means equal to the product of the extremes.
- Set each resulting quadratic equal to zero, so it is in $0=ax^2+bx+x$ form.
- Find the roots (A.K.A "solutions", A.K.A "x-intercepts")

1) $\frac{8}{x}=\frac{x}{2}$

2) $x=\frac{10}{x-3}$

3) $\frac{x}{3}=\frac{-2}{x+7}$

4) $\frac{4}{y-8}=\frac{y}{5}$

5) $\frac{4}{x-10}=\frac{x}{-4}$

6) $\frac{2a}{3}=\frac{6}{a}$

7) $\frac{5x}{8}=\frac{2}{5x}$

8) $\frac{2x}{4}=\frac{7}{x+5}$

9) $\frac{x}{x+1}=\frac{x+2}{6x}$

Name ______________________________ Date __________ **Notes/Practice/PREP**
Math 8 ACC

Solutions to Equations Set Equal To Zero

Zero-Product Property

If ab = 0, than either a = 0 or b = 0

- *At least one of the factors must be zero if the product of two numbers is zero.*
- *If any product of numbers is zero, at least one of the factors in that product is zero.*

Example 1) $(x-10)(\frac{x}{2}+20) = 0$

Set each side equal to zero and find the solutions set:

$$x - 10 = 0$$
$$\underline{+10} \quad \underline{+10}$$
$$x = 10$$

$$\frac{x}{2} + 20 = 0$$
$$\underline{-20} \quad \underline{-20}$$
$$(2)(\frac{x}{2}) = (-20)(2)$$
$$x = -40$$

The solution set contains two values: { -40, 10}
This means that...
x = -40 **OR** when x = 10
both will satisfy the equation.

We can check each by substituting the values of x back into the original equation *one at a time*.

When x = -40:

$$(x-10)(\frac{x}{2}+20) = 0$$
$$((-40)-10)(-\frac{40}{2}+20) = 0$$
$$(-50)(-20+20) = 0$$
$$(-50)(0) = 0$$
$$0 = 0$$

When x = 10:

$$(x-10)(\frac{x}{2}+20) = 0$$
$$((10)-10)(\frac{10}{2}+20) = 0$$
$$(0)(5+20) = 0$$
$$(0)(25) = 0$$
$$0 = 0$$

Practice:

- Set each factor equal to zero (which create a compound statement using **OR**).
- Solve each equation to find the solution set.
- Check each solution using your TI-84.

1) $(x-4)(x+3)=0$

2) $(x+4)(x-6)(x-10)=0$

3) $(x-3)(x-3)=0$

4) $(3x-2)(x+12)=0$

5) $(x-1)(x-2)(x-3)=0$

6) $(x-\frac{1}{2})(2\frac{3}{5}x+10)=0$

Hint: On #7 and #8 finish the split the middle process "before" determining the zeroes.

7) $x(x-3)+5(x-3)=0$

8) $x(x+7)+5(x+7)=0$

There is a back!

Hint: On #'s 9-11 factor by GCF "before" determining the zeroes.

9) $2x^2 - 10x = 0$

10) $x^2 - 11x = 0$

11) $3x^2 + 27x = 0$

12) **Work backwards:** Create an equation that has {15, -10} as its only solutions.

13) $(x - 10)(2x + 6)(x^2 - 36)(x^2 - 100)\left(\frac{x}{2} + 20\right) = 0$

(No work required. Just state "all" answers. There are 7!)

Determining Zeroes with Common Factors on Both Sides — NOTES

BASIC

Factor • Factor = Factor • Factor

$$3a = 3b$$

Since we know 3 = 3,
then in 3a = 3b
it must also be true that
$a = b$

A bit more to look at but the same concept

Factor • Factor = Factor • Factor

$$(x - 2)(2x - 3) = (x - 2)(x + 5)$$

Using the same logic:

Since we know that $(x - 2) = (x - 2)$,
then in $(x - 2)(2x - 3) = (x - 2)(x + 5)$
it must also be true that
$2x - 3 = x + 5$

When set equal to zero would be: →

$$2x - 3 = x + 5$$
$$\underline{-x}\ \ \underline{-5}\ \ \underline{-x}\ \ \underline{-5}$$
$$x - 8 = 0$$

$$x = 8$$

We also need to set $x - 2 = 0$ and solve to find the other solution
$$\underline{+2}\ \ \underline{+2}$$
$$x = 2$$

Name ______________________________ Date ___________ **Practice**

Math 8 ACC

Solutions to Equations NOT Set Equal To Zero (BUT in Factored Form)

Directions: Determine the Zeroes of Each Equation

1) $(x - 2)(2x - 3) = (x - 2)(x + 5)$

2) $(x - 10)(x + 8) = (6x + 5)(x + 8)$

3) $(x - 7)(2x + 5) = (x - 7)(3x + 7)$

4) $5(x + 8) = (-3x + 12)(x + 8)$

Hint: Since there are no common factors, set the equation equal to zero and factor to find the zeroes.

5) $x^2 + 2x = 5x$

6) $x^2 - 5x = -6$

Hint: Finish Split the Middle Process on the left, then use the technique you did in #'s 1- 4.

7) $x(5x - 20) + 2(5x - 20) = 5(5x - 20)$

8) $(4x - 8)(2x + 10) + 5(2x + 10) = (x + 7))(2x + 10)$

Name ______________________________ **PREP**

Math 8 ACC Date ____________________

Determining Zeroes Final Practice.

Apply what you know about the Zero-Product Property to solve each of the following:

1) $(x - 7)(x - 8)(x + 15) = 0$

2) $x(x + 9) - 6(x + 9) = 0$

3) $5x^2 + 25x = 0$

4) $(x + 7)(5x - 6) = (x + 7)(3x + 4)$

5) $(5x - 6)(x + 7) = (3x + 4)(x + 7)$

6) $(x + 4)(x + 2) = (3x + 2)(x + 2)$

7) Keith determines the zeros of the function $f(x)$ to be -6 and 5. What could be Keith's function?

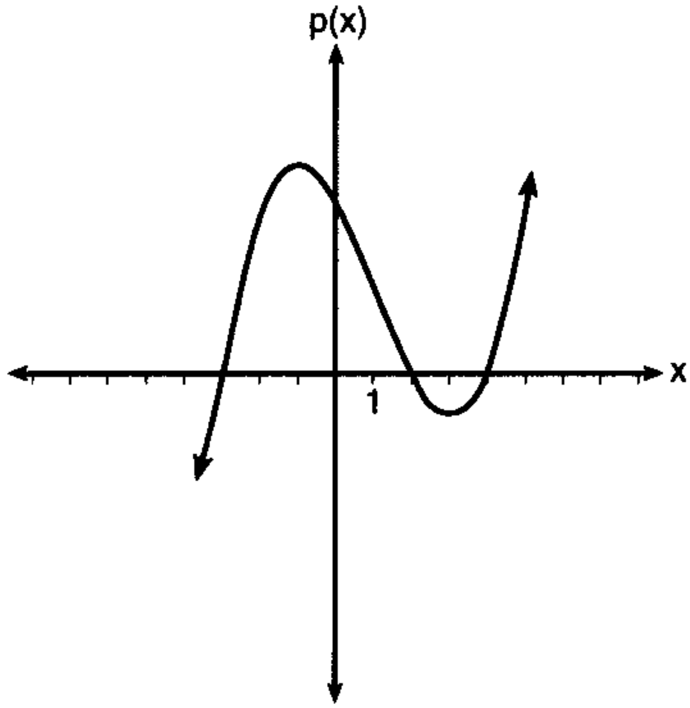
(1) $f(x) = (x + 5)(x + 6)$

(2) $f(x) = (x + 5)(x - 6)$

(3) $f(x) = (x - 5)(x + 6)$

(4) $f(x) = (x - 5)(x - 6)$

8) Based on the graph below, which expression is a possible factorization of $p(x)$?



(1) $(x + 3)(x - 2)(x - 4)$

(2) $(x - 3)(x + 2)(x + 4)$

(3) $(x + 3)(x - 5)(x - 2)(x - 4)$

(4) $(x - 3)(x + 5)(x + 2)(x + 4)$

<u>PREVIEW Question for our next lesson.</u>

Place the expression in the Ti-84 $\frac{(x-2)(x-5)}{(x^2-16)(x+7)}$ and fill in y values for...

x	y
-7	
-4	
4	

Explain why these y values make sense:

Name ______________________________ **Notes/Practice**

Math 8 ACC Date ______________

Equations Involving a Variable Expression in the Denominator

(Causing values that are NOT PERMISSIBLE in the Domain AND Solution Set).

Multiplication and Division are ***inverse operations***, which means that they ***undo*** each other.

We know that if $\frac{8}{4} = 2$ then it holds true that $4 \cdot 2 = 8$

Consider $\frac{8}{0} = x$ What value of x would make the inverse $x \cdot 0 = 8$???

- No value of x would exist that could make this a true statement.
- Therefore having ***zero in the denominator is not allowed***.
- This is referred to as being ***impermissible*** or ***undefined***.

- IF THE POSSIBILITY EXISTS THAT AN INPUT COULD RESULT IN A VALUE OF ZERO IN THE DENOMINATOR, IT IS YOUR JOB TO IDENTIFY THOSE VALUES.
- CAREFUL THOUGH !!!
  Restricting the domain, could result in a restriction in the solution set.

DIRECTIONS:

For each of the following expression, write a compound statement that would restrict the domain values that would result in a value of zero in the denominator (impermissible domain values).

YOUR ONLY FOCUS SHOULD BE THE DENOMINATOR:

Example 1) $\frac{5}{x+2}$ AND ______________

Example 2) $\frac{1}{x} = \frac{(x+5)(x-8)}{x-2}$ AND ______________

Example 3) $\frac{x^2-25}{(x^2-9)(x+4)}$ AND ______________

Explain why 5 and -5 ARE NOT considered to be impermissible values.

____

Example 4) In the following equation $\frac{x+3}{x-2} = \frac{5}{x-2}$, the domain restriction is x ≠ 2.

What affect does this restriction have on the solution set?

Example 5) EXTENSION QUESTION

$y = \frac{5}{x+2}$ is an equation with a domain restriction of x ≠ - 2.

But, sometimes the range is restricted as well.
In this case, there is a value that y can never be. What is that value?

Name:________________ Period:____ Date:_____

Practice

Zeroes and Impermissible Values of Rational Equations

Rational Equations Set up the product of the extremes and the product of the means. Do not distribute and do not finish solving.	What degree will the resulting equation be if you were to distribute?	What are the impermissible values for x?	What are the solutions?	If the restrictions affected the solution set, write the revised solution set.
$\frac{x-3}{x-9}=\frac{4}{10}$				
$\frac{2x-8}{x-5}=\frac{3x+7}{x-5}$				

Rational Equations Set up the product of the extremes and the product of the means. Do not distribute and do not finish solving.	What degree will the resulting equation be if you were to distribute?	What are the impermissible values for x?	What are the solutions?	If the restrictions affected the solution set, write the revised solution set.
$\frac{x-8}{2x}=\frac{-6}{x}$				

Name ________________________________ Per # _____ **NOTES/PRACTICE**
Math 8 ACC Date ______________

Domain (Input) Restrictions

Domain (Input) values such are only allowed if, the output values (range) will result in values that belong to the Real Number set.

Therefore, we must always start by determining an appropriate domain, so that the range (outputs) will be appropriate (Real Numbers) as well.

IF THERE ARE VARIABLES IN THE EXPRESSION, THERE IS A POTENTIAL FOR A RESTRICTION IN THE DOMAIN!!!!!! CHECK FOR THE FOLLOWING ...

1) Total Value of 0 in the denominator :

Ex: $f(x) = \frac{7}{x-5}$ **:**

In this expression, x cannot equal 5, since it will result in an "undefined fraction". The domain would then be all Real Numbers except 5.

2) Total Value that is Negative Under Radical Symbol

Ex: $f(x) = \sqrt{x-2}$

In this expression, x cannot be less than 2, since it will result in a negative number under the radical symbol. This answer will not be in the Real Number System. The Domain would then be all Real Number, such that $x \geq 2$.

PRACTICE:
Directions:
a) Determine the restrictions for each Domain.
b) Write the appropriate domain that reflects the restrictions you have identified.

1) $f(x) = \frac{3}{2x-7}$

Restrictions Domain

2) $h(x) = \frac{3x^2-5}{3(x-1)+x}$

Restrictions Domain

3) $f(x) = \sqrt{2x-6}$

Restrictions Domain

4) $d(x) = \sqrt{6-3x}$

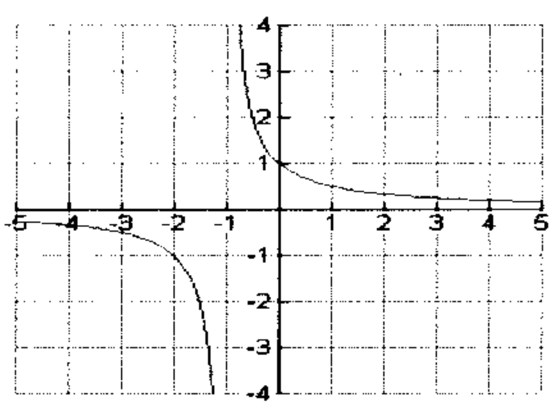
Restrictions Domain

5) $f(x) = \frac{3}{\sqrt{x+4(x-2)}}$

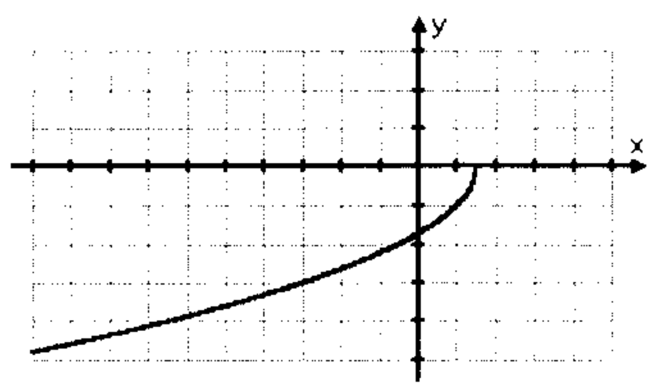
Restrictions Domain

Determine the domain and range of each function shown below by examining the graph of the function:

6) $f(x) = \frac{1}{x+1}$



7) $f(x) = -\sqrt{-2x+3}$



Name:________________ Period:__________ Date:_________ **REVIEW**

Zeroes, Impermissible Values, and Domain

Directions: For questions 1-3, determine the Impermissible Values of x for each expression. Show Work to justify how you determined your answers.

1) $\frac{x+4}{x+6}$

Impermissible Value(s) for x:________________

2) $\frac{x+4}{x-8} = \frac{7}{x}$

Impermissible Value(s) for x:________________

3) $\frac{(x+4)(2x+5)}{(x^2-64)(x+20)}$

Impermissible Value(s) for x:________________

Multiple Choice: For Questions 1-4 determine the correct domain for each function.

4) $h(x) = \frac{x-5}{3(x-1)+x}$

a) $x \in \mathbb{R}$, where $x = \frac{3}{4}$

b) $x \in \mathbb{R}$, where $x \neq \frac{3}{4}$

c) $x \in \mathbb{R}$, where $x \geq \frac{3}{4}$

d) $x \in \mathbb{R}$, where $x > \frac{3}{4}$

5) $g(x) = \sqrt{2x-6}$

a) $x \in \mathbb{R}$, where $x = 3$

b) $x \in \mathbb{R}$, where $x \neq 3$

c) $x \in \mathbb{R}$, where $x \geq 3$

d) $x \in \mathbb{R}$, where $x > 3$

6) $f(x) = \dfrac{1}{\sqrt{2x-6}}$

a) x ∈ ℝ, where x = 3

b) x ∈ ℝ, where x ≠ 3

c) x ∈ ℝ, where x ≥ 3

d) x ∈ ℝ, where x > 3

7) $k(x) = \dfrac{x-8}{x^2-25}$

a) x ∈ ℝ, where x ≠ 5

b) x ∈ ℝ, where x ≠ 5, x ≠ -5

c) x ∈ ℝ, where x ≠ 8

d) x ∈ ℝ, where x ≠ 5, x ≠ -5, x ≠ 8

8) Directions: Write True or False for each statement.

______a) Domain are all possible output values for a given function.

______b) A zero numerator value *is impermissible.*

______c) Domain values are assumed to be all Real numbers unless a restriction is identified.

______d) Domain restrictions can result in restrictions to the solution set.

______e) Quantities under a radical sign $\sqrt{\;}$ must be greater than or equal to zero.

9) Use the following rational equation to answer the questions that follow.

$$\frac{2x-8}{x-4} = \frac{6x-32}{x-4}$$

A) Determine ALL the potential solutions of the equation. Show Work.

B) Determine the impermissible values of x for the equation. Show Work

C) Write the FINAL solution set for your equation, based upon your information in Parts A and B.