
MATHCOUNTS®

2018

■ **School Competition** ■
Sprint Round
Problems 1–30

Name _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. _____ The average of Jerrold's vocabulary quizzes is 88.75. What is Jerrold's quiz average expressed to the nearest whole number?

2. _____ What is the value of $2^5 - 5 \times 6$?

3. \$ _____ A particular store sells items for only \$1, \$3 and \$5 and charges no sales tax. If Georgia buys nine \$1 items, seven \$3 items and three \$5 items from this store, what is the total amount she will be charged?



4. _____ °F Water boils at 212 °F and freezes at 32 °F. What is the difference between the boiling point and freezing point of water?



Boiling Point
212 °F

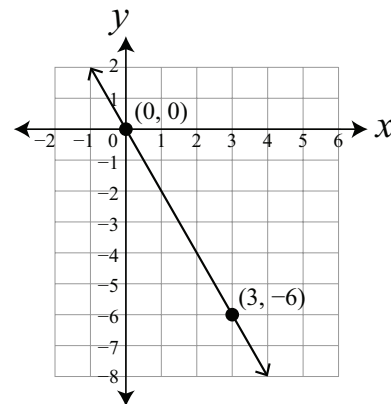


Freezing Point
32 °F

5. _____ If the first five terms of an arithmetic sequence are 26, 16, 6, -4, -14, what is the sixth term of this sequence?

6. _____ If $\frac{2^{12}}{2^n} = 2^3$, what is the value of n ?

7. _____ What is the slope of the line containing the points $(0, 0)$ and $(3, -6)$ as shown?



8. _____ The least common multiple of 15 and 20 is k . What is the least common multiple of 12 and k ?

9. _____ pounds When Allen Iverson and Shaquille O'Neal faced off in the 2001 NBA Finals, O'Neal weighed exactly twice as much as Iverson, and their weights totaled 495 pounds. How many pounds did Iverson weigh?

10. _____ If $3x + 7 = 22$, what is the value of $\frac{1}{3x+11}$. Express your answer as a common fraction.

11. _____ The table shows the results of Jake's last twenty times at bat. Based on this data, what is the probability that Jake hits a double on his next time at bat? Express your answer as a decimal to the nearest tenth.

Batting Results

Result	Number
Home Run	3
Triple	1
Double	2
Single	8
Walk	3
Out	3

12. _____ percent A square pen that is surrounded by a fence is divided into two rectangular regions by adding a new fence that connects the midpoints of two opposite sides. By what percent has the total length of fencing been increased?

13. _____ What is the sum of the greatest common factor of 4 and 18 and the least common multiple of 4 and 18?

14. _____ years Claire's three sons, from oldest to youngest, are Evan, Joel and Alex. The difference between the ages of Evan and Joel is the same as the difference between the ages of Joel and Alex. If the sum of the three ages is 39, how old is Joel?

15. $\frac{\text{mile}}{\text{marker}}$ Danica started her trip at mile marker 66 and ended her trip at mile marker 194. She drove at a constant speed the entire trip. Which mile marker had she reached by 75% of her trip?

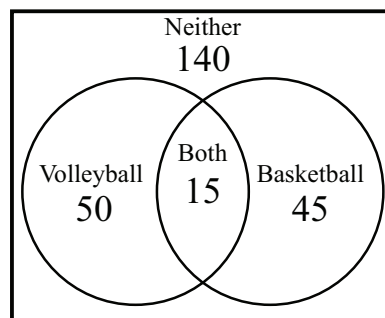
16. _____ What is the value of $(20 + 12)^2 - (20 - 12)^2$?

17. _____ visitors The table shows the number of visitors over a 4-week period to a new website. What was the average number of visitors to the site per week?

Website Visitors

Week	Visitors
1	1028
2	1100
3	1060
4	1056

18. _____ percent The Venn diagram shows the number of students at Ramanujan Middle School who play both volleyball and basketball, the number who play one of these sports but not the other, and the number who play neither of these sports. All students at the school are represented in the diagram. What percent of the students at the school play basketball?

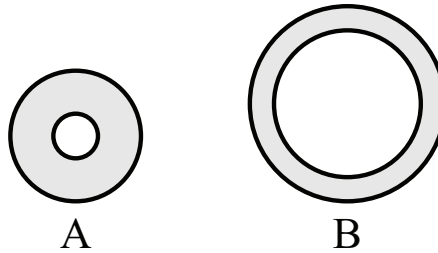


19. _____ Josie selected a number n . She divided n by 2 and then subtracted $\frac{1}{2}$ from the result. She took half of that result and then subtracted $\frac{1}{2}$ to get the final result of 10. What is the value of n ?

20. _____ times Scott repeatedly rolls a pair of standard six-sided dice and keeps track of the sum of the two numbers rolled each time. So far he has not rolled any sum twice. What is the maximum possible number of times he has thrown the dice?

21. _____ cm

An *annulus* is a ring bounded by two concentric circles. In the figure, annulus A has an inner circle of radius 2 cm and an outer circle of radius 6 cm. Annulus B has an outer circle of radius 9 cm. If annulus B has area equal to that of annulus A, what is the radius of the inner circle of annulus B?



22. _____

Let $Q = \{1.7, 1.1, 1.4, 2.1, 2.3, s\}$. What is the absolute difference between the greatest and least possible values of the median of set Q ? Express your answer as a decimal to the nearest hundredth.

23. _____ tickets

A bag contains 25 tickets, each colored either red or yellow. Red tickets are worth \$0.50, and yellow tickets are worth \$5.00. If the expected value of a ticket drawn at random from this bag is \$3.20, how many of the tickets are red?

24. _____

If a and b are integers such that $a^2 - b^2 = 100$, what is the greatest possible value of a ?

25. _____ cents

In cents, what is the least total amount that cannot be obtained by using a combination of fewer than eight coins from a collection of pennies, nickels, dimes and quarters?

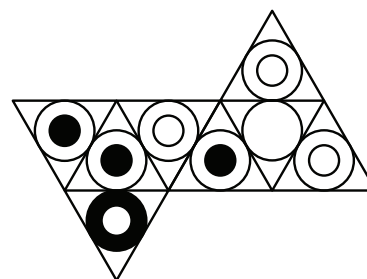
26. fourth
powers How many of the first one thousand perfect fourth powers have either 1 or 6 as their units digit?

27. _____ Seven jars hold a total of 250 candies. They are lined up left to right from fullest to emptiest. The left-most jar has 72 candies, and the right-most jar has 13. No two jars hold the same number of candies. What is the positive difference between the greatest number of candies that could be in the second jar from the left, and the least number of candies that could be in that jar?

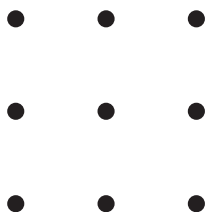


28. _____ What is the greatest integer k such that 2^k is a factor of $67!$?

29. nets How many distinct tetrahedron nets, formed from four connected equilateral triangles, can be cut from the figure shown?



30. triangles The rows and columns of lattice points in a three-by-three square array are evenly spaced one unit apart. How many distinct triangles with at least one side of length $\sqrt{2}$ units can be drawn using three lattice points for the vertices?



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■ **School Competition** ■
Target Round
Problems 1 & 2

Name _____

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This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the problem sheets. If you complete the problems before time is called, use the time remaining to check your answers.

Problem 1	Problem 2	Scorer's Initials

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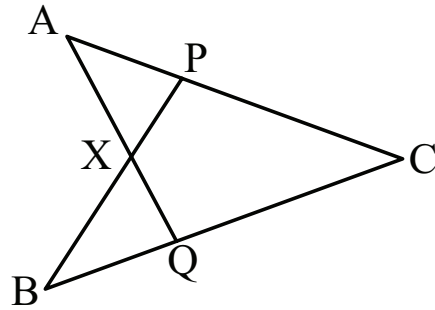
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1. _____ degrees

In the figure, points P and Q lie on segments AC and BC, respectively. Segments AQ and BP intersect at X. The measures of angles PAQ and PBQ are 42 degrees and 37 degrees, respectively. The measure of angle PXQ is 118 degrees. What is the measure of angle ACB?



2. _____

What is the least common multiple of 143 and 1001?

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Target Round
Problems 3 & 4

Name _____

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Problem 3	Problem 4	Scorer's Initials

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3. _____ If n is a positive integer such that n^3 is a four-digit number with thousands digit 9, what is the value of n ?

4. _____ degrees Latoya created this table showing how she spent her time yesterday. She wants to make a pie chart (circle graph) of this data. What is the sum, in degrees, of the central angles of the regions representing the hours she spent at school and the hours she spent doing homework?

Latoya's Activities

Activity	School	Meals	Homework	Leisure	Sleep	Other
Hours	7	$1\frac{1}{2}$	$1\frac{1}{3}$	$1\frac{1}{2}$	$8\frac{1}{2}$	$4\frac{1}{6}$

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■ School Competition ■
Target Round
Problems 5 & 6

Name _____

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Problem 5	Problem 6	Scorer's Initials

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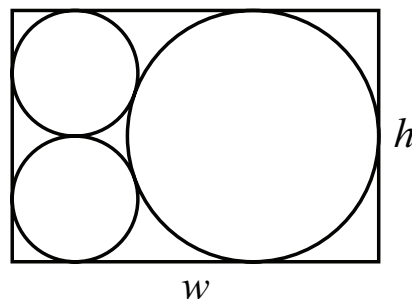
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5. _____ Three dogs arrived at the dog park at three different times, with their three owners, using three different leash colors. The Dalmatian was next to arrive after the Schnauzer. The dog with the red leash is owned by the 7th grader. The Poodle was not the first to arrive. The Schnauzer does not use a green leash. If the 6th grader owns the Dalmatian, then the Poodle's leash is blue. The 8th grader arrived first. What is the product of the grade levels of the student who arrived second, the student who owns the Poodle and the student who uses the green leash?

6. _____ Three circles are inscribed in a rectangle of width w and height h as shown. Two of the circles are congruent and are each tangent to two adjacent sides of the rectangle and to each other. The other circle is larger and is tangent to three sides of the rectangle and to the two smaller circles. What the ratio of h to w ? Express your answer as a decimal to the nearest hundredth.



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■ **School Competition** ■
Target Round
Problems 7 & 8

Name _____

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Problem 7	Problem 8	Scorer's Initials

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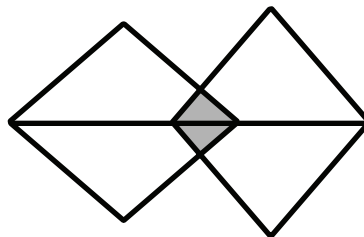
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7. _____ A box contains 4 chocolates and 1 fruit chew. Clark and Chloe take turns drawing a treat out of the box without replacement. Whoever draws the fruit chew wins. Clark draws first. What is the probability that Chloe wins? Express your answer as a common fraction.

8. _____ units^2 A rhombus has diagonals of length 12 units and 16 units. It is rotated 90 degrees and positioned, as shown, such that the shorter diagonal of the rotated figure is collinear with the longer diagonal of the original with an overlap of 4 units. What is the area of the overlapping shaded region? Express your answer as a decimal to the nearest hundredth.



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■ School Competition ■
Team Round
Problems 1–10

Team
Members _____, Captain

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This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

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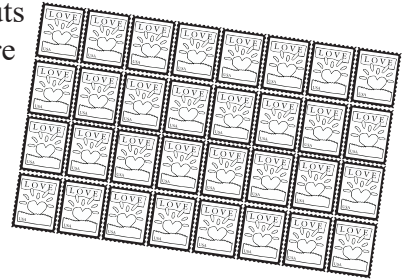
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1. _____ If $a \odot b = a^2 + ab + b^2$, what is the value of $(3 \odot 4) - 12$?

2. _____ Sammi has a sheet of stamps containing 4 rows of stamps, with 8 stamps in each row. Sammi wants to separate all of the stamps by tearing the sheet in as few tears as possible. So, after she makes one tear, she puts the two pieces of the sheet on top of each other before making the next tear. What is the fewest number of tears she can use to get all the stamps separated?



3. _____
$$\begin{array}{r} \text{ONE} \\ + \text{ONE} \\ \hline \text{TWO} \end{array}$$
 In the addition problem shown, each letter represents a different digit between 1 and 9, inclusive. If E and T represent 6 and 5, respectively, what digit does N represent?

4. _____ fluffy friends



All the kittens in a certain litter, which includes Billy and Peaches, are either fluffy or sleek and are friends with one another. Billy, a fluffy kitten, has 7 more fluffy kitten friends than sleek kitten friends. How many more fluffy friends than sleek friends does sleek kitten Peaches have?



5. _____ pounds

Marcia wants to estimate the total weight of the sand in her sandbox. She has a drinking glass whose interior is a cylinder with diameter 2 inches and height 4 inches; the glass weighs 4.1 ounces when empty. After she fills the glass with sand, the weight of the glass is 14.8 ounces. If the sandbox is a rectangle measuring 6 feet by 4 feet, and the average depth of the sand is 6 inches, what is the total weight of the sand in the sandbox? Express your answer as a whole number to the nearest hundred. (1 foot = 12 inches; 1 pound = 16 ounces.)

6. _____ There are three integer values of x that make the equation $x^3 + 6x^2 + 11x + 6 = 0$ true. What is the least of these values?

7. _____ A dorm dining hall has three tables. Currently one student is seated at one table, two are seated at another and three are seated at the remaining table. Each student who enters the dining hall picks a table to join with probability proportional to the number of students already seated at the table. Terri enters the dining hall and sits at a table. Ursula then enters the dining hall and sits at a table. What is the probability that there is now a table at which at least four students are seated? Express your answer as a common fraction.

8. _____ A list of numbers of the form $\frac{x}{2}$, for positive integers x , has a sum of 7. What is maximum product of the list of numbers? Express your answer as a common fraction.

9. _____ The figure on the left is a portion of a standard multiplication table that contains the products of all pairs of positive integers. The figure on the right is a contiguous 2-by-2 section of the standard multiplication table, with some digits replaced with question marks. What is the sum of the four numbers in this section?

	1	2	3	4	5	...
1	1	2	3	4	5	
2	2	4	6	8	10	
3	3	6	9	12	15	
4	4	8	12	16	20	
5	5	10	15	20	25	
⋮						

7?2	??9
?2?	8??

10. _____ ways Prentice has five daughters and ten identical pens. In how many ways can the pens be distributed among his daughters if two of them, Charlotte and Emily, must get the same number of pens, and every daughter is not required to get a pen?