Chapter 7 Review Part 1 No Calculator Name_______________

1.Find the general solution to the exact differential equation.

$$\frac{dy}{dx} = \sin x - e^{-x} + 8x^3$$

$$y = -\cos x - \frac{e^{-x}}{-1} + \frac{8x^4}{4}$$

$$\boxed{y = -\cos x + e^{-x} + 2x^4 + C}$$

2. Solve the initial value problem explicitly.

5. $\frac{dy}{dx} = 1 + x + \frac{x^2}{2}$, $\boxed{y(0) = 1}$ $(0, 1)$ x y

$$\frac{x^2}{2} = \frac{1}{2}x^2$$

$$y = x + \frac{x^2}{2} + \frac{x^3}{6} + C$$

$$1 = 0 + 0 + 0 + C$$

$$C = 1$$

$$\boxed{y = x + \frac{x^2}{2} + \frac{x^3}{6} + 1}$$

3. Find an integral equation $\boxed{y = \int_a^x f(t)dt + b}$ such that $dy/dx = \sin^3 x$ and $y = 5$ when $x = 4$.

$$\boxed{y = \int_4^x \sin^3(t)\,dt + 5}$$

$$y = 0 + 5$$

4. Evaluate the integral

$-\frac{1}{x^2} = -x^{-2}$

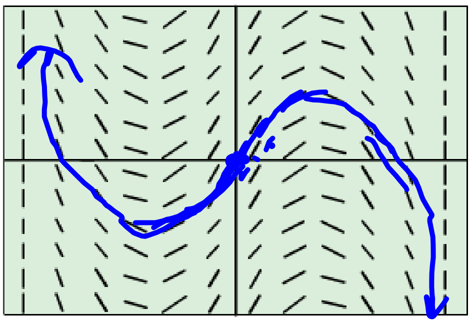
$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2} \ \boxed{(x > 0)}$$

$y = \ln|x| - \frac{x^{-1}}{-1} + c$

$\boxed{y = \ln x + \frac{1}{x} + c}$

5. ***Sketching Solutions*** Draw a possible graph for the function $y = f(x)$ with slope field given in the figure that satisfies the initial condition $\boxed{y(0) = 0.}$

$(0,0)$



[–10, 10] by [–10, 10]

9. Evaluate the integral

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt = -\frac{1}{2}\int \frac{1}{u^2}\,du \rightarrow -\frac{1}{2}\int u^{-2}\,du$$

$u = \cos(2t+1)$

$-\frac{1}{2}du = \cancel{-}\sin(2t+1)\cdot\cancel{2}\,dt \ -\frac{1}{2}$

$$-\frac{1}{2}\,\frac{u^{-1}}{-1} + C$$

$$\frac{1}{2u} + C$$

$$\boxed{\frac{1}{2\cos(2t+1)} + C}$$

In Exercises 47–52, use the given trigonometric identity to set up a u-substitution and then evaluate the indefinite integral.

10. **51.** $\int \tan^4 x\,dx, \quad \tan^2 x = \sec^2 x - 1$

11. Evaluate the definite integral by making a u-substitution and integrating from u(a) to u(b).

$$\int_0^{\pi/2} 5\sin^{3/2} x \cos x\,dx = 5\int_0^1 u^{3/2}\,du = 5\,\frac{2}{5}u^{5/2}\Big|_0^1$$

$u = \sin x$ $\left\langle \begin{array}{l}\sin\frac{\pi}{2}\\ \sin 0\end{array}\right.$

$5\,du = 5\cos x\,dx$

$$2\sqrt{u^5}\Big|_0^1$$

$$2\sqrt{1^5} - 2\sqrt{0^5}$$

$$2 - 0$$

$$\boxed{2}$$

12. Evaluate the definite integral by making a u-substitution and integrating from u(a) to u(b).

$$\int_0^1 r\sqrt{1-r^2}\,dr = -\frac{1}{2}\int_1^0 \sqrt{u}\,du = -\frac{1}{2}\int_1^0 u^{1/2}\,du$$

$u = 1-r^2$ $\quad 1-1^2,\ 1-0^2$

$-\frac{1}{2}\,du = -2r\,dr \div -\frac{1}{2}$

$-\frac{1}{2}du = r\,dr$

$$-\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2}\Big|_1^0$$

$$-\frac{1}{3}\sqrt{u^3}\Big|_1^0$$

$$\left(-\frac{1}{3}\sqrt{0}\right)-\left(-\frac{1}{3}\sqrt{1}\right)$$

$$0 - -\frac{1}{3} = \boxed{\frac{1}{3}}$$

13. Evaluate the integral (Parts)

$$\int 3t\,e^{2t}\,dt$$

$u = 3t \qquad v = \frac{e^{2t}}{2}$

$du = 3\,dt \qquad dv = e^{2t}\,dt$

$$uv - \int v\,du$$

$$\frac{3}{2}te^{2t} - 3\int \frac{e^{2t}}{2}\,dt$$

$$\frac{3}{2}te^{2t} - \frac{3}{2}\int e^{2t}\,dt$$

$$\frac{3}{2}te^{2t} - \underbrace{\frac{3}{2}\cdot\frac{e^{2t}}{2}}_{} + C$$

$$\boxed{\frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + C}$$

14. Use tabular integration to find the antiderivative.

$$\int x^3 \cos x\,dx$$

deriv		anti
x^3		$\cos x$
$3x^2$	+	$\sin x$
$6x$	−	$-\cos x$
6	+	$-\sin x$
0	−	$\cos x$

$$x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C$$

15. Use separation of variables to solve the initial value problem.

$$\frac{dy}{dx} = e^{x-y} \quad \text{and} \quad y = 2 \text{ when } x = 0$$

$$\frac{dy}{dx} = \frac{e^x}{e^y}$$

$$e^y\,dy = e^x\,dx$$

$$e^y = e^x + C$$

$$\boxed{(0,2)}$$

$$e^2 = e^0 + C$$

$$e^2 = 1 + C$$

$$-1 \qquad -1$$

$$C = e^2 - 1$$

$$e^y = e^x + e^2 - 1$$

$$\log_e(e^x + e^2 - 1) = y$$

$$\boxed{y = \ln(e^x + e^2 - 1)}$$

Chapter 7 Review Part 2 Calculator Name_______________

16. Find the solution to the differential equation dy/dt = ky, k is a constant, that satisfies the given conditions.

$r = k$

$y(0) = 60, \quad y(10) = 30$

$P = 60 \quad (10, 30)$

$y = Pe^{rt}$

$30 = 60e^{r10}$

$\boxed{y = 60e^{-.069t}}$

$\frac{1}{2} = e^{10r}$

$\log_e \frac{1}{2} = 10r$

$\frac{\ln \frac{1}{2}}{10} = \frac{10r}{10}$ $r = -.069$

17. Solve the problem.

In Exercises 15–18, complete the table for an investment if interest is compounded continuously.

	Initial Deposit ($)	Annual Rate (%)	Doubling Time (yr)	Amount in 30 yr ($)
15.	1000	8.6		
16.	2000		15	
17.	600	5.25		2898.44
18.	1200			10,405.37

$y = Pe^{rt}$

$\frac{2898.44}{e^{.0525(30)}} = \frac{Pe^{.0525(30)}}{e^{.0525(30)}}$

$P = \$600$

$r = 5.25\% = .0525$

$(30, 2898.44)$

$t = \frac{\ln 2}{k}$

$t = \frac{\ln 2}{.0525}$

$t = 13.203\text{yrs}$

$\boxed{14\text{yrs}}$

6. Evaluate the integral

$$\int \frac{dx}{\sqrt[3]{3x+4}} = \int \frac{1}{\sqrt[3]{3x+4}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt[3]{u}}\,du = \frac{1}{3}\int u^{-1/3}\,du$$

$u = 3x+4$

$\frac{1}{3}du = 3\,dx\,\frac{1}{3}$

$\frac{1}{3}du = 1dx$

$\frac{1}{3}\cdot\frac{3}{2}u^{2/3}$

$\frac{1}{2}u^{2/3}+C$

$\boxed{\frac{1}{2}(3x+4)^{2/3}+C}$

7. Evaluate the integral

$$\int \frac{x\,dx}{x^2+1} \longrightarrow \frac{1}{2}\int \frac{1}{u}\,du$$

$u = x^2+1$

$\frac{1}{2}du = \frac{1}{2}2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\ln|u|+c$

$\frac{1}{2}\ln|x^2+1|+c$

$\boxed{\frac{1}{2}\ln(x^2+1)+c}$

8. Evaluate the integral

$$\int \sqrt{\cot x}\,\csc^2 x\,dx = -\int \sqrt{u}\,du = -\int u^{1/2}\,du$$

$u = \cot x$

$-du = -\csc^2 x\,dx$

$-\frac{2u^{3/2}}{3}+C$

$\boxed{-\frac{2}{3}(\cot x)^{3/2}+C}$

18. ***Half-Life*** The radioactive decay of Sm-151 (an isotope of samarium) can be modeled by the differential equation $dy/dt = -0.0077y$, where t is measured in years. Find the half-life of Sm-151.

$k = .0077$

$t = \frac{\ln 2}{k}$

$t = \frac{\ln 2}{.0077}$

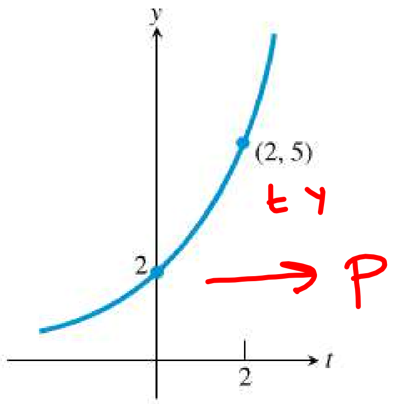
$\boxed{t = 90.019\text{yrs}}$

19. In Exercises 15–18, complete the table for an investment if interest is compounded continuously.

	Initial Deposit ($)	Annual Rate (%)	Doubling Time (yr)	Amount in 30 yr ($)
15.	1000	8.6		
16.	2000		15	
17.		5.25		2898.44
18.	1200			10,405.37

20. In Exercises 27 and 28, find the exponential function $y = y_0e^{kt}$ whose graph passes through the two points.

27.



$y = Pe^{rt}$

$5 = 2e^{r2}$

$2.5 = e^{2r}$

$\frac{\ln 2.5}{2} = \frac{2r}{2}$

$r = .458$

$y = 2e^{.458t}$

21. ***Cooling a Pie*** A deep-dish apple pie, whose internal temperature was 220°F when removed from the oven, was set out on a 40°F breezy porch to cool. Fifteen minutes later, the pie's internal temperature was 180°F. How long did it take the pie to cool from there to 70°F?