



pennsylvania
DEPARTMENT OF EDUCATION

The Pennsylvania System of School Assessment

Mathematics Item and Scoring Sampler



2017–2018
Grade 8

Pennsylvania Department of Education Bureau of Curriculum, Assessment and Instruction—September 2017

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INTRODUCTION

General Introduction

The Pennsylvania Department of Education provides districts and schools with tools to assist in delivering focused instructional programs aligned with the Pennsylvania Core Standards (PCS). These tools include Academic Standards, Assessment Anchor documents, assessment handbooks, and content-based item and scoring samplers. This Item and Scoring Sampler is a useful tool for Pennsylvania educators in preparing local instructional programs. It can also be useful in preparing students for the statewide assessment.

This Item and Scoring Sampler is available in Braille format. For more information regarding Braille call (717) 901-2238.

Pennsylvania Core Standards (PCS)

This sampler contains examples of test questions designed to assess the Pennsylvania Assessment Anchors and Eligible Content aligned to the Pennsylvania Core Standards. The Mathematics, Reading, and Writing PSSA transitioned to PCS-based operational Mathematics and English Language Arts assessments starting with the spring 2015 PSSA administration.

The 2013 PCS-aligned Assessment Anchor and Eligible Content documents are posted on this portal:

- www.education.pa.gov [Hover over “K–12,” select “Assessment and Accountability,” and select “Pennsylvania System of School Assessment (PSSA).” Then select “Assessment Anchors/Eligible Content” on the right side of the screen.]

What Is Included

This sampler contains test questions (items) that have been written to align to the Assessment Anchors that are based on the Pennsylvania Core Standards (PCS). The test questions provide an idea of the types of items that will appear on an operational, PCS-based PSSA. Each sample test question has been through a rigorous review process to ensure alignment with the Assessment Anchors.

Purpose and Uses

The items in this sampler may be used as examples for creating assessment items at the classroom level, and they may also be copied and used as part of a local instructional program.¹ Classroom teachers may find it beneficial to have students respond to the open-ended item in this sampler. Educators can then use the sampler as a guide to score the responses either independently or together with colleagues within a school or district.

Item Format and Scoring Guidelines

The multiple-choice (MC) items have four answer choices. Each correct response to an MC item is worth one point.

Each open-ended (OE) item is designed to take approximately ten to fifteen minutes to complete. During the administration of the PSSA, students are given additional time as necessary to complete the test items. Each OE item in mathematics is scored using an item-specific scoring guideline based on a 0–4-point scale. In this sampler, every item-specific scoring guideline is combined with examples of student responses that represent each score point to form a practical, item-specific scoring guide.

This sampler also includes the *General Description of Scoring Guidelines for Mathematics Open-Ended Questions* that students will have access to during a PSSA mathematics administration. The general description of scoring guidelines can be distributed to students for use during local assessments and can also be used by educators when scoring local assessments.¹

¹ The permission to copy and/or use these materials does not extend to commercial purposes.

Item Alignment

All PSSA items are aligned to statements and specifications included in the *Assessment Anchors and Eligible Content Aligned to the Pennsylvania Core Standards*. The mathematics content, process skills, directives, and action statements included in the PSSA mathematics questions align with the Assessment Anchor Content Standards. The Eligible Content statements represent the limits of the content of the mathematics questions.

Testing Time and Mode of Testing Delivery for the PSSA

The PSSA is delivered in traditional paper-and-pencil format as well as in an online format. The estimated time to respond to a test question is the same for both methods of test delivery. During an official testing administration, students are given additional time as necessary to complete the test questions. The following table shows the estimated response time for each item type.

Mathematics Item Type	MC	OE
Estimated Response Time (minutes)	2	10 to 15

Mathematics Reporting Categories

The Assessment Anchors are organized into four classifications as listed below.

• A = Numbers and Operations	• C = Geometry
• B = Algebraic Concepts	• D = Data Analysis and Probability

These four classifications are used throughout the grade levels. In addition to these classifications, there are five Reporting Categories for each grade level. The first letter of each Reporting Category represents the classification; the second letter represents the Domain as stated in the Common Core State Standards for Mathematics. Listed below are the Reporting Categories for Grade 8.

- A-N = The Number System
- B-E = Expressions and Equations
- B-F = Functions
- C-G = Geometry
- D-S = Statistics and Probability

Examples of multiple-choice and open-ended items assessing these categories are included in this booklet.

General Description of Scoring Guidelines for Mathematics Open-Ended Questions

4— The response demonstrates a *thorough* understanding of the mathematical concepts and procedures required by the task.

The response provides correct answer(s) with clear and complete mathematical procedures shown and a correct explanation, as required by the task. Response may contain a minor “blemish” or omission in work or explanation that does not detract from demonstrating a *thorough* understanding.

3— The response demonstrates a *general* understanding of the mathematical concepts and procedures required by the task.

The response and explanation (as required by the task) are mostly complete and correct. The response may have minor errors or omissions that do not detract from demonstrating a *general* understanding.

2— The response demonstrates a *partial* understanding of the mathematical concepts and procedures required by the task.

The response is somewhat correct with *partial* understanding of the required mathematical concepts and/or procedures demonstrated and/or explained. The response may contain some work that is incomplete or unclear.

1— The response demonstrates a *minimal* understanding of the mathematical concepts and procedures required by the task.

0— The response has no correct answer and *insufficient* evidence to demonstrate any understanding of the mathematical concepts and procedures required by the task for that grade level.

Response may show only information copied from the question.

Special Categories within zero reported separately:

BLK (blank).....Blank, entirely erased, or written refusal to respond

OT.....Off task

LOE.....Response in a language other than English

IL.....Illegible

Item and Scoring Sampler Format

This sampler includes the test directions and scoring guidelines that appear in the PSSA Mathematics assessments. Each multiple-choice item is followed by a table that includes the alignment, the answer key, the depth of knowledge (DOK) level, the percentage² of students who chose each answer option, and a brief answer-option analysis or rationale. The open-ended item is followed by a table that includes the item alignment, DOK level, and mean student score. Additionally, each of the included item-specific scoring guidelines is combined with sample student responses representing each score point to form a practical, item-specific scoring guide. The *General Description of Scoring Guidelines for Mathematics Open-Ended Questions* used to develop the item-specific scoring guidelines should be used if any additional item-specific scoring guidelines are created for use within local instructional programs.

Example Multiple-Choice Item Information Table

Item Information	
Alignment	Assigned AAEC
Answer Key	Correct Answer
Depth of Knowledge	Assigned DOK
p-value A	Percentage of students who selected each option
p-value B	Percentage of students who selected each option
p-value C	Percentage of students who selected each option
p-value D	Percentage of students who selected each option
Option Annotations	Brief answer-option analysis or rationale

Example Open-Ended Item Information Table

Alignment	Assigned AAEC	Depth of Knowledge	Assigned DOK	Mean Score	
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² All p-value percentages listed in the item information tables have been rounded.

Grade 8 Formula Sheet

Formulas that you may need to work questions on this test are found below.

You may refer back to this page at any time during the mathematics test.

You may use calculator π or the number 3.14.

2017

Grade 8

Exponential Properties

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

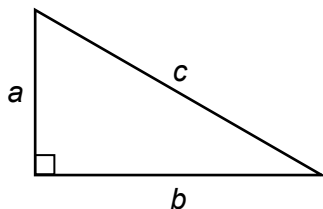
$$a^{-1} = \frac{1}{a}$$

Algebraic Equations

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

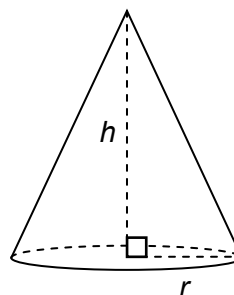
Slope-Intercept Form: $y = mx + b$

Pythagorean Theorem



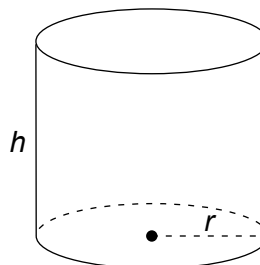
$$a^2 + b^2 = c^2$$

Cone



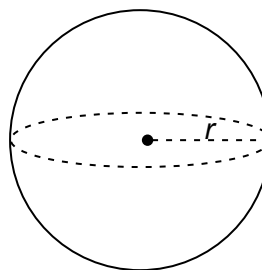
$$V = \frac{1}{3} \pi r^2 h$$

Cylinder



$$V = \pi r^2 h$$

Sphere



$$V = \frac{4}{3} \pi r^3$$

Mathematics Test Directions

On the following pages are the mathematics questions.

- You may not use a calculator for question 1. You may use a calculator for all other questions on this test.

Directions for Multiple-Choice Questions:

Some questions will ask you to select an answer from among four choices.

For the multiple-choice questions:

- First solve the problem on scratch paper.
- Choose the correct answer and record your choice in the answer booklet.
- If none of the choices matches your answer, go back and check your work for possible errors.
- Only one of the answers provided is the correct response.

Directions for Open-Ended Questions:

Some questions will require you to write your response.

For the open-ended questions:

- These questions have more than one part. Be sure to read the directions carefully.
- You cannot receive the highest score for an open-ended question without completing all tasks in the question. For example, if the question asks you to show your work or explain your reasoning, be sure to show your work or explain your reasoning in the space provided.
- If the question does **not** ask you to show your work or explain your reasoning, you may use the space provided, but only those parts of your response that the question specifically asks for will be scored.
- Write your response in the appropriate location within the response box in the answer booklet. Some answers may require graphing, plotting, labeling, drawing, or shading. If you use scratch paper, be sure to transfer your final response and any needed work or reasoning to the answer booklet.

Question 1 in this sampler is to be solved without the use of a calculator.

MULTIPLE-CHOICE ITEMS

1. The volume of Jupiter is approximately 10^{14} cubic kilometers. The volume of Earth is approximately 10^{11} cubic kilometers. How many planets the size of Earth does it take to equal the volume of Jupiter?
- A. 10^{-3}
 - B. 10^3
 - C. 10^{25}
 - D. 10^{154}

Item Information	
Alignment	B-E.1.1.1
Answer Key	B
Depth of Knowledge	2
p-value A	5%
p-value B	71% (correct answer)
p-value C	20%
p-value D	4%
Option Annotations	A. divides exponents and rounds B. correct C. adds exponents D. multiplies exponents

A calculator is permitted for use in solving questions 2–17 in this sampler.

2. The Blue Ridge Mountains are rising at a rate of 1×10^{-4} feet per year. To determine how many feet the mountains will rise over the next 1×10^6 years, a scientist performs the operation shown below.

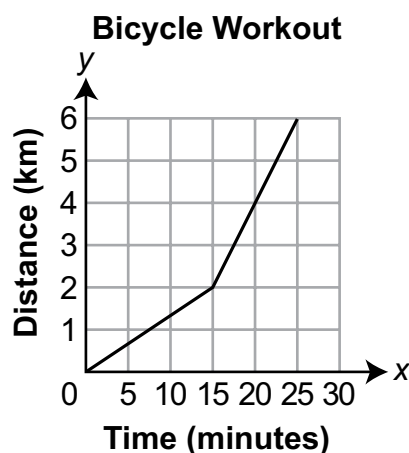
$$(1 \times 10^{-4}) \times (1 \times 10^6)$$

How many feet will the Blue Ridge Mountains rise over the next 1×10^6 years?

- A. 0.001
- B. 0.01
- C. 10
- D. 100

Item Information	
Alignment	B-E.1.1.4
Answer Key	D
Depth of Knowledge	1
p-value A	8%
p-value B	9%
p-value C	11%
p-value D	72% (correct answer)
Option Annotations	<p>A. incorrectly simplifies expression as 1×10^{-2} and incorrectly applies the -2 in the exponent by placing two 0s between the decimal point and the 1</p> <p>B. incorrectly simplifies expression as 1×10^{-2}</p> <p>C. incorrectly applies the 2 in the exponent by making the value of the expression a 2-digit number</p> <p>D. correct</p>

3. Trevor is riding his bicycle for a workout. The graph below shows the relationship between the amount of time, in minutes, he rides and the distance, in kilometers, he rides during his 25-minute workout.

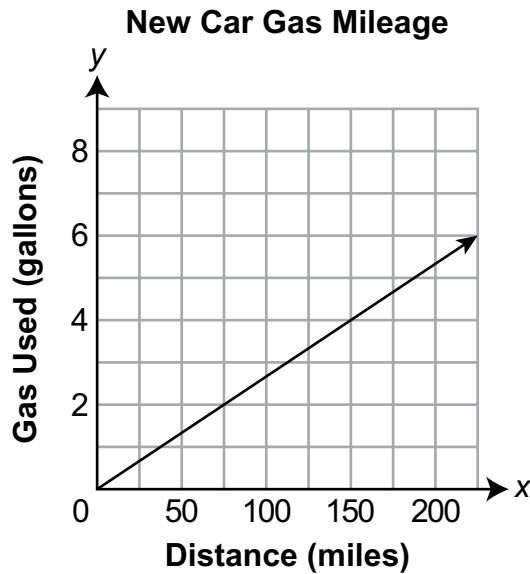


Based on the graph, which statement about the rate at which Trevor rides during his workout is true?

- A. Trevor's rate changes from 7.5 kilometers per minute to 2.5 kilometers per minute.
- B. Trevor maintains a steady rate of 2 kilometers per 5 minutes during the workout.
- C. Trevor maintains a steady rate of 2.4 kilometers per 10 minutes during the workout.
- D. Trevor's rate changes from 1 kilometer every 7.5 minutes to 1 kilometer every 2.5 minutes.

Item Information	
Alignment	B-E.2.1
Answer Key	D
Depth of Knowledge	2
p-value A	12%
p-value B	14%
p-value C	10%
p-value D	64% (correct answer)
Option Annotations	A. inverts the slopes of the two line segments B. identifies the rate of change only from 15 minutes to 25 minutes C. uses the "average" rate of change from (0, 0) to (25, 6) D. correct

4. Davis is comparing the gas mileage for his old car with the gas mileage for his new car. His old car used 19 gallons of gas to travel 323 miles. The graph shown below represents the gas mileage for his new car.



What is the difference in the gas mileages for the two cars?

- A. 8 miles per gallon
- B. 11.2 miles per gallon
- C. 17 miles per gallon
- D. 20.5 miles per gallon

Item Information	
Alignment	B-E.2.1.1
Answer Key	D
Depth of Knowledge	2
p-value A	14%
p-value B	17%
p-value C	20%
p-value D	49% (correct answer)
Option Annotations	<p>A. for the new car, divides the x-scale by the y-scale ($50 \div 2 = 25$) and then subtracts 17 from 25</p> <p>B. determines the difference in distance by subtracting the greatest value on the x-axis from 323, determines the difference in gas used by subtracting the greatest value on the y-axis from 19, then divides the difference of the distances by the difference of the gas used ($123 \div 11$)</p> <p>C. calculates the gas mileage of only the old car</p> <p>D. correct</p>

5. Line j and line k are graphed on the same coordinate grid.
- Line j has a slope of $-\frac{2}{3}$ and intersects the y -axis at -4 .
 - Line k has a different negative slope than line j and intersects the y -axis above line j .

Which equation could describe line k ?

- A. $y = -\frac{1}{6}x$
- B. $y = -\frac{2}{3}x - 3$
- C. $y = -\frac{1}{2}x - 5$
- D. $y = x + 1$

Item Information	
Alignment	B-E.2.1.3
Answer Key	A
Depth of Knowledge	1
p-value A	35% (correct answer)
p-value B	33%
p-value C	24%
p-value D	8%
Option Annotations	<p>A. correct</p> <p>B. identifies a line that intersects the y-axis above line j, but the line has the same slope as line j</p> <p>C. identifies a line with a different negative slope than line j, but the line intersects the y-axis below line j</p> <p>D. identifies a line with a different slope than line j and intersects the y-axis above line j, but the line has a positive slope</p>

6. Luisa is solving an equation on the bottom of a page. The corner of the page where the equation is written is torn off as shown below.

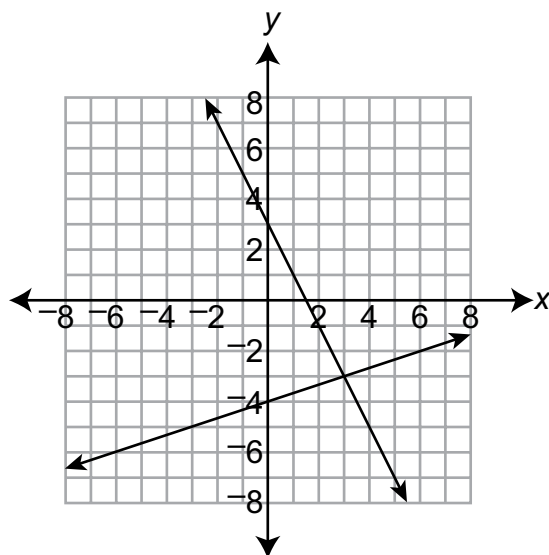
$$7(x + 3) = 7(x + 5) -$$

Luisa knows only one number was torn off, and she knows that the equation has an infinite number of solutions. What **must** be the missing number?

- A. 2
- B. 14
- C. 21
- D. 35

Item Information	
Alignment	B-E.3.1.1
Answer Key	B
Depth of Knowledge	2
p-value A	27%
p-value B	58% (correct answer)
p-value C	9%
p-value D	6%
Option Annotations	<p>A. does not apply the distributive property when solving the equation</p> <p>B. correct</p> <p>C. correctly sets up the left side of the equation as $7x + 21$, then misidentifies the missing number as 21 to “cancel out” the 21</p> <p>D. correctly sets up the right side of the equation as $7x + 35$, then misidentifies the missing number as 35 to “cancel out” the 35 without considering the $7 \cdot 3$ value on the left side of the equation</p>

7. A system of two linear equations is graphed on the coordinate grid shown below.



What ordered pair is the solution of the system of equations?

- A. $(0, -4)$
- B. $(0, 3)$
- C. $(3, -3)$
- D. $(3, -1)$

Item Information	
Alignment	B-E.3.1.3
Answer Key	C
Depth of Knowledge	1
p-value A	8%
p-value B	6%
p-value C	81% (correct answer)
p-value D	5%
Option Annotations	<p>A. identifies the y-intercept of the “lower” line</p> <p>B. identifies the y-intercept of the “upper” line</p> <p>C. correct</p> <p>D. identifies the correct x-coordinate but adds the two y-intercepts to determine the y-coordinate</p>

8. Which relation is a linear function with the **greatest** slope?

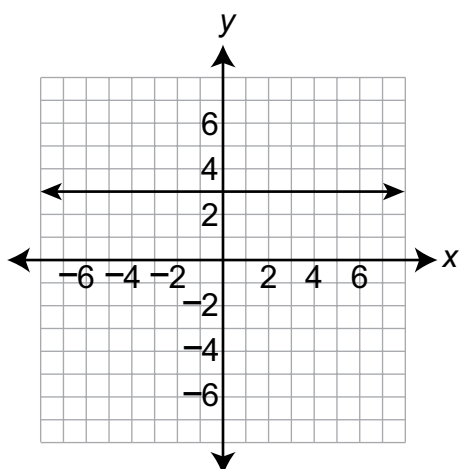
A.

x	y
-3	1
0	3
3	5

B.

x	y
4	0
4	10
4	20
4	30

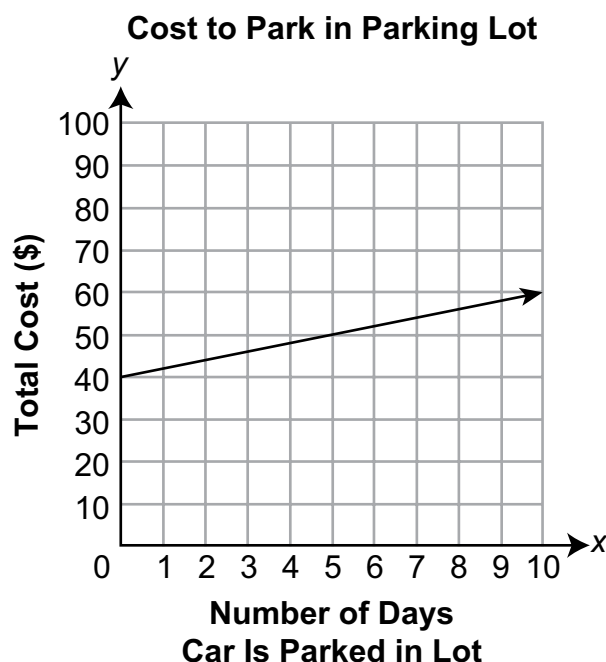
C.



D. $y = 5x + 1$

Item Information	
Alignment	B-F.1.1.2 B-F.1.1.1
Answer Key	D
Depth of Knowledge	2
p-value A	14%
p-value B	27%
p-value C	9%
p-value D	50% (correct answer)
Option Annotations	<p>A. recognizes the table represents a function but incorrectly determines slope by subtracting the least value in the table from the greatest value</p> <p>B. incorrectly identifies the table as representing a function and uses the change of 10 in y-values as the slope</p> <p>C. recognizes the line represents a function but incorrectly determines slope by subtracting the least value on the x-axis from the greatest value</p> <p>D. correct</p>

9. A parking lot charges a monthly fee plus a charge per day for a car parked in the lot. The graph below shows the relationship between the total cost (y), in dollars, and the number of days (x) the car is parked in the lot.



Which equation could represent the relationship between the total cost (y), in dollars, and the number of days (x) the car is parked in the lot?

- A. $y = mx$
- B. $y = mx + b$
- C. $y = x^2$
- D. $y = x^2 + b$

Item Information	
Alignment	B-F.1.1.3
Answer Key	B
Depth of Knowledge	1
p-value A	12%
p-value B	73% (correct answer)
p-value C	8%
p-value D	7%
Option Annotations	A. omits the y-intercept B. correct C. selects a nonlinear equation and does not consider the y-intercept D. selects a nonlinear equation

10. Which situation can be modeled by the equation $y = mx + b$?

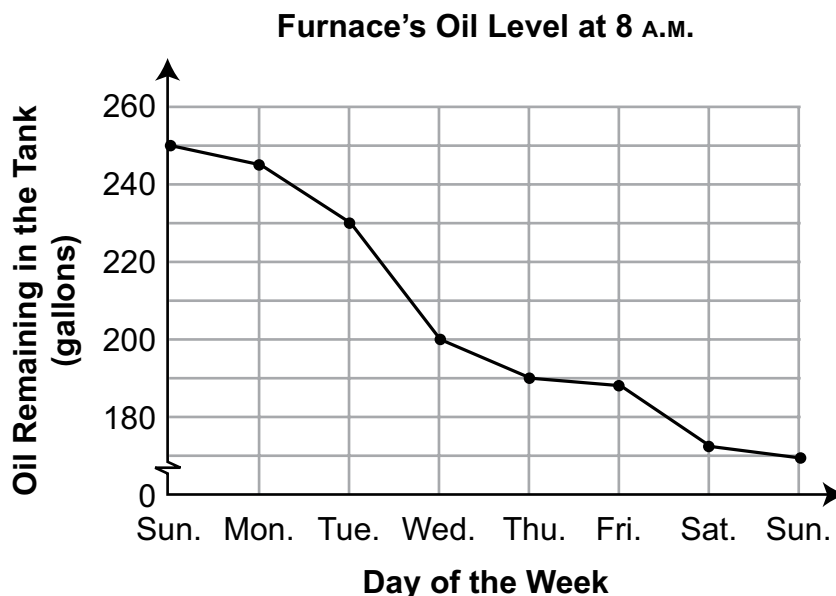
- A. The number of butterflies (y) in a population doubles every month (x).
- B. The time (y) it takes a car to travel 100 miles depends on the speed (x) of the car.
- C. The volume (y) of a cylindrical tank with a height of 2 feet depends on the radius (x) of the tank.
- D. The total income (y) of a worker who earns 8 dollars per hour depends on the number of hours (x) worked.

Item Information	
Alignment	B-F.1.1.3
Answer Key	D
Depth of Knowledge	2
p-value A	15%
p-value B	18%
p-value C	14%
p-value D	53% (correct answer)
Option Annotations	<p>A. misidentifies “doubles every month” as $y = 2x$ instead of as $y = 2^x$</p> <p>B. misidentifies situation as being represented by $d = rt$ instead of as $t = d/r$</p> <p>C. uses $V = \pi (2)^2x = 12.56x$ instead of $V = \pi x^2 (2) = 6.28x^2$</p> <p>D. correct</p>

11. Ben currently has \$70 in his trip fund. He makes \$45 per week doing yard work for his neighbors. He keeps \$18 of the \$45 for weekly expenses. The remaining amount goes into his trip fund. Which function can be used to determine the amount of money (y), in dollars, that Ben has in his trip fund at the end of x weeks?
- A. $y = 70 + 27x$
 - B. $y = 52 + 45x$
 - C. $y = 45 + 52x$
 - D. $y = 27 + 70x$

Item Information	
Alignment	B-F.2.1.1
Answer Key	A
Depth of Knowledge	2
p-value A	73% (correct answer)
p-value B	9%
p-value C	7%
p-value D	11%
Option Annotations	A. correct B. subtracts 18 from 70 instead of from 45 C. subtracts 18 from 70 instead of from 45 and switches the slope and y-intercept D. switches the slope and y-intercept

12. Zach is studying the school furnace's oil usage. He records the amount of oil remaining in the tank at 8 A.M. every day for a week. No additional oil is added to the tank during the week. The graph below shows his data.

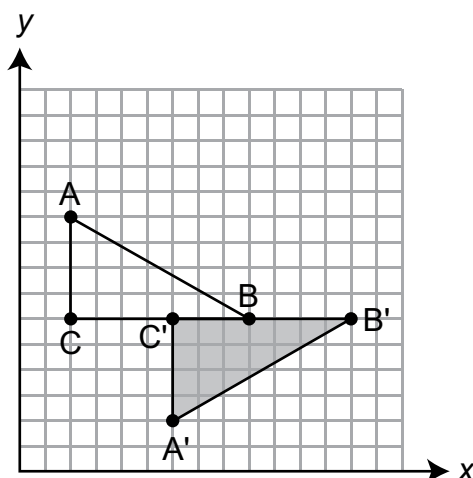


As the outdoor temperature gets colder, more oil is used to keep the indoor temperature constant. Based on the graph, which 24-hour time interval indicates the coldest period for the week?

- A. Sunday to Monday
- B. Tuesday to Wednesday
- C. Friday to Saturday
- D. Saturday to Sunday

Item Information	
Alignment	B-F.2.1.2
Answer Key	B
Depth of Knowledge	2
p-value A	17%
p-value B	47% (correct answer)
p-value C	5%
p-value D	31%
Option Annotations	A. selects the first decrease B. correct C. selects second-largest decrease D. uses the interval with the lowest levels in the tank for the week

13. A triangular sign was moved. The new location of the sign is represented by the shaded triangle.



Which transformations could have been used to move the sign to the new location?

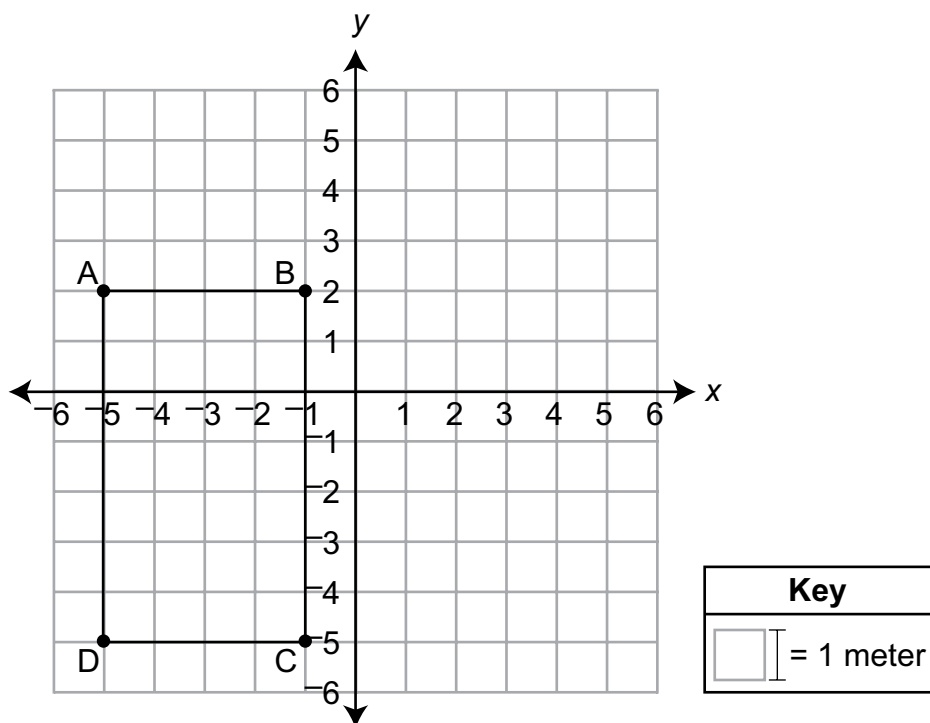
- A. reflection across \overline{BC} and reflection across \overline{AC}
- B. reflection across \overline{BC} and translation 4 units right
- C. translation 4 units right and translation 4 units down
- D. rotation 180 degrees clockwise about C and translation 4 units right

Item Information	
Alignment	C-G.1.1.1
Answer Key	B
Depth of Knowledge	2
p-value A	10%
p-value B	58% (correct answer)
p-value C	11%
p-value D	21%
Option Annotations	<p>A. identifies a potentially correct first transformation but second transformation would move point B to quadrant II and points A and C would be 4 units from points A' and C', respectively</p> <p>B. correct</p> <p>C. identifies a potentially correct first transformation but second transformation would move point A to point C', point C to point A', and point B 4 units below point B'</p> <p>D. identifies a set of transformations that would move point A to point A' and point C to point C', but point B would move to quadrant II</p>

14. The diagonal of Andre's rectangular TV screen is 39 inches. The width of his TV screen is 34 inches. Which measurement is **closest** to the height of Andre's TV screen?
- A. 8.5 inches
 - B. 19.1 inches
 - C. 36.5 inches
 - D. 51.7 inches

Item Information	
Alignment	C-G.2.1.2
Answer Key	B
Depth of Knowledge	2
p-value A	12%
p-value B	49% (correct answer)
p-value C	28%
p-value D	11%
Option Annotations	A. determines the square root of the sum $39 + 34$ B. correct C. divides the sum $39 + 34$ by 2 D. determines the square root of the sum $39^2 + 34^2$ instead of the difference $39^2 - 34^2$

15. In the grid shown below, rectangle ABCD represents the location of a pool at a city park.



Anna swims in a straight line from corner A to corner C of the swimming pool. What is the distance, in meters, Anna swims?

- A. $\sqrt{22}$
- B. 7
- C. $\sqrt{65}$
- D. 11

Item Information	
Alignment	C-G.2.1.3
Answer Key	C
Depth of Knowledge	2
p-value A	10%
p-value B	35%
p-value C	39% (correct answer)
p-value D	16%
Option Annotations	A. selects the square root of the sum $2(7) + 2(4)$ instead of the sum $7^2 + 4^2$ B. determines the length of the pool and does not consider the width C. correct D. adds the length and the width or divides the sum $2(7) + 2(4)$ by 2

16. A cone-shaped dispenser is filled with cake frosting. The cone has a radius of 1.5 inches and a height of 5 inches. Which measurement is **closest** to the volume of cake frosting that the cone-shaped dispenser holds?
- A. 3.75 cubic inches
 - B. 11.78 cubic inches
 - C. 47.12 cubic inches
 - D. 141.37 cubic inches

Item Information	
Alignment	C-G.3.1.1
Answer Key	B
Depth of Knowledge	1
p-value A	15%
p-value B	73% (correct answer)
p-value C	9%
p-value D	3%
Option Annotations	<p>A. omits π (i.e., uses $V = \frac{1}{3} r^2 h$ instead of $V = \frac{1}{3} \pi r^2 h$)</p> <p>B. correct</p> <p>C. uses $r = 3$</p> <p>D. uses $r = 3$ and omits $\frac{1}{3}$ (i.e., uses $V = \pi r^2 h$ instead of $V = \frac{1}{3} \pi r^2 h$)</p>

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OPEN-ENDED QUESTION

17. The locations of four points on a number line are described by the numbers listed below.

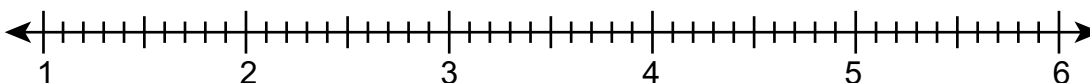
point J: $\sqrt{12}$

point K: $\frac{15}{7}$

point L: $\sqrt{16}$

point M: $\frac{3\pi}{2}$

- A. For **only** the **irrational** numbers that are listed, write the approximate value of each irrational number as a decimal rounded to the nearest tenth. Graph these approximate values on the number line below to show the locations of the irrational numbers. Label each graphed point with the correct letter.



Go to the next page to finish question 17.

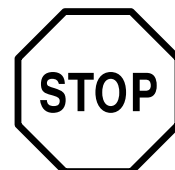


17. Continued. Please refer to the previous page for task explanation.

An integer is represented by x . A rational number is represented by \sqrt{x} . The value of \sqrt{x} is between $\frac{84}{18}$ and $\sqrt{41}$.

- B.** Explain why there are only two possible values of x . As part of the explanation, state the two possible values of x .

After you have checked your work, close your answer booklet and test booklet so your teacher will know you are finished.



Item-Specific Scoring Guideline

#17 Item Information

Alignment	A-N.1	Depth of Knowledge	3	Mean Score	0.98
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Assessment Anchor this item will be reported under:

M08.A-N.1 — Demonstrate an understanding of rational and irrational numbers.

Specific Anchor Descriptor addressed by this item:

M08.A-N.1.1 — Apply concepts of rational and irrational numbers.

Scoring Guide

Score	In this item, the student . . .
4	Demonstrates a thorough understanding of rational and irrational numbers by correctly solving problems and clearly explaining procedures.
3	Demonstrates a general understanding of rational and irrational numbers by correctly solving problems and clearly explaining procedures with only minor errors or omissions.
2	Demonstrates a partial understanding of rational and irrational numbers by correctly performing a significant portion of the required task.
1	Demonstrates minimal understanding of rational and irrational numbers.
0	The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures as required by the task. Response may show only information copied from the question.

Top-Scoring Student Response and Training Notes

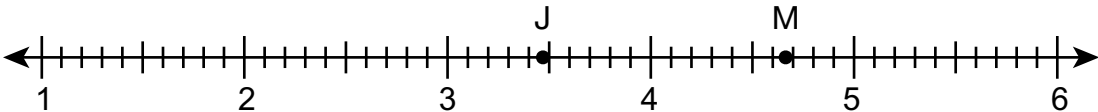
Score	Description
4	Student earns 4 points.
3	Student earns 3.0–3.5 points.
2	Student earns 2.0–2.5 points.
1	Student earns 0.5–1.5 points. OR Student demonstrates minimal understanding of rational and irrational numbers.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.

Top-Scoring Response

Part A (2 points):

$\frac{1}{2}$ point for each correct decimal (ignore decimals for K and L)

$\frac{1}{2}$ point for each correctly plotted and labeled point (lose $\frac{1}{2}$ point for plotted K and/or L)

What?	Why?
<p>point J: 3.5 point M: 4.7</p> 	

Part B (2 points):

$\frac{1}{2}$ point for each correct answer

1 point for correct and complete explanation

OR $\frac{1}{2}$ point for correct but incomplete explanation

What?	Why?
<p>25</p> <p>AND</p> <p>36</p>	<p>Sample Explanation:</p> <p>The value of $\frac{84}{18}$ is approximately 4.7. The value of $\sqrt{41}$ is between 6.4 and 6.5. So \sqrt{x} is between 4.7 and 6.5. That means x must be between 4.7^2, or 22.09, and 42.25. There are only two perfect squares between 22.09 and 42.25, and they are 25 and 36.</p>

STUDENT RESPONSE

Response Score: 4 points



PART A

Question 17
Page 1 of 2

Item ID ?

Line Guide

The locations of four points on a number line are described by the numbers listed below.

point J: $\sqrt{12}$ point K: $\frac{15}{7}$ point L: $\sqrt{16}$ point M: $\frac{3\pi}{2}$

A. For **only** the **irrational** numbers that are listed, write the approximate value of each irrational number as a decimal rounded to the nearest tenth. Graph these approximate values on the number line below to show the locations of the irrational numbers. Label each graphed point with the correct letter.

Point J = $\sqrt{12} = 3.5$
Point M = $\frac{3\pi}{2} = 4.7$

34 / 1000

?

J M

1 2 3 4 5 6

The student has given 2 correct decimal approximations and plotted and labeled both correctly.

Review/End Test Pause Flag Options Next

PART B

Question 17
Page 2 of 2

Item ID

?

Line Guide

Calculator

Flag

Pause

Options

Review/End Test

Back

Next

An integer is represented by x . A rational number is represented by \sqrt{x} . The value of \sqrt{x} is between $\frac{84}{18}$ and $\sqrt{41}$.

B. Explain why there are only two possible values of x . As part of the explanation, state the two possible values of x .

Eq

There are only 2 possible values of x , which are 25 and 36 because, since x has to be an integer, any numbers that are not perfect square integers will be come irrational when square rooted. since the value of \sqrt{x} is between $\frac{84}{18}$, or $4\frac{2}{3}$, and $\sqrt{41}$, or 6.4, in order for x to be an integer, \sqrt{x} must be a positive integer as well. So, \sqrt{x} must be 5 or 6. 5 squared is 25, and 6 squared is 36. So, x must be either 25 or 36.

413 / 1000

The student has given 2 correct answers and a correct and complete explanation.

STUDENT RESPONSE

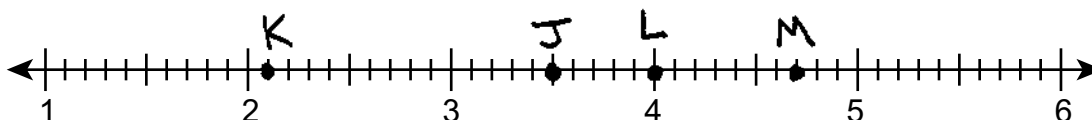
Response Score: 3 points

17. The locations of four points on a number line are described by the numbers listed below.

point J: $\sqrt{12}$ point K: $\frac{15}{7}$ point L: $\sqrt{16}$ point M: $\frac{3\pi}{2}$

- A. For **only** the **irrational** numbers that are listed, write the approximate value of each irrational number as a decimal rounded to the nearest tenth. Graph these approximate values on the number line below to show the locations of the irrational numbers. Label each graphed point with the correct letter.

point J: $\sqrt{12} = 3.5$
 point K: $15/7 = 2.1$
 point L: $\sqrt{16} = 4$
 point M: $\frac{3\pi}{2} = 4.7$



The student has given 2 correct decimal approximations and plotted and labeled both correctly. However, the rational numbers are plotted as well.

Go to the next page to finish question 17.



17. **Continued.** Please refer to the previous page for task explanation.

An integer is represented by x . A rational number is represented by \sqrt{x} . The value of \sqrt{x} is between $\frac{84}{18}$ and $\sqrt{41}$.

- B. Explain why there are only two possible values of x . As part of the explanation, state the two possible values of x .

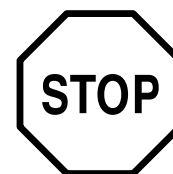
$$84 \div 18 = 4.7$$

$$\sqrt{41} = 6.4$$

The two possible values of x are 25 and 36. There are only two possible values because 25 and 36 are the only numbers that have a rational square root that is in between 4.7 and 6.4. Any other numbers that would have a square root between 4.7 and 6.4 would also have an irrational square root. For example, $\sqrt{35} = 5.91607...$ a non-terminating decimal. Therefore it is not rational.

The student has given 2 correct answers and a correct and complete explanation.

After you have checked your work, close your answer booklet and test booklet so your teacher will know you are finished.



STUDENT RESPONSE

Response Score: 2 points



PART A

Question 17
Page 1 of 2

Item ID ?

Line Guide

The locations of four points on a number line are described by the numbers listed below.

point J: $\sqrt{12}$ point K: $\frac{15}{7}$ point L: $\sqrt{16}$ point M: $\frac{3\pi}{2}$

A. For **only** the **irrational** numbers that are listed, write the approximate value of each irrational number as a decimal rounded to the nearest tenth. Graph these approximate values on the number line below to show the locations of the irrational numbers. Label each graphed point with the correct letter.

J $\sqrt{12} = 3.5$
M $\frac{3\pi}{2} = 4.7$

20 / 1000

?

The student has given 2 correct decimal approximations and plotted and labeled both correctly.

Review/End Test Pause Flag Options Next

PART B

Question 17
Page 2 of 2



An integer is represented by x . A rational number is represented by \sqrt{x} . The value of \sqrt{x} is between $\frac{84}{18}$ and $\sqrt{41}$.

B. Explain why there are only two possible values of x . As part of the explanation, state the two possible values of x .

Eq

$$\frac{84}{18} \text{ and } \sqrt{41}$$

$$\frac{84}{18} = \frac{42}{9} = \frac{14}{3} \quad \sqrt{41} = 6.4$$

These are the only 2 possible values of x , because irrational numbers are ones you cannot make into fractions. As a result $\frac{84}{18}$ and $\sqrt{41}$ are the only possible x .

The student has given no correct answers
and an incorrect explanation.

180 / 1000

Review/End Test

Pause

Flag

Options

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STUDENT RESPONSE

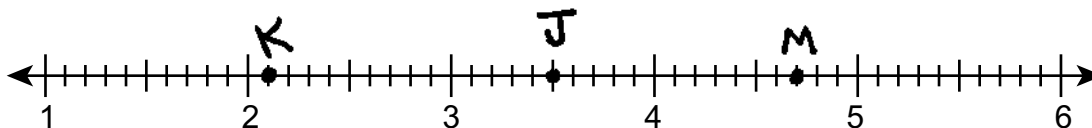
Response Score: 1 point

17. The locations of four points on a number line are described by the numbers listed below.

point J: $\sqrt{12}$ point K: $\frac{15}{7}$ point L: $\sqrt{16}$ point M: $\frac{3\pi}{2}$

- A. For **only** the **irrational** numbers that are listed, write the approximate value of each irrational number as a decimal rounded to the nearest tenth. Graph these approximate values on the number line below to show the locations of the irrational numbers. Label each graphed point with the correct letter.

$$\begin{aligned} J &= 3.5 \\ K &= 2.1 \\ L &= 4 \text{ (rational)} \\ M &= 4.7 \end{aligned}$$



The student has given 2 correct decimal approximations and plotted and labeled both correctly. However, one of the rational numbers is also plotted.

Go to the next page to finish question 17.



17. **Continued.** Please refer to the previous page for task explanation.

An integer is represented by x . A rational number is represented by \sqrt{x} . The value of \sqrt{x} is between $\frac{84}{18}$ and $\sqrt{41}$.

- B. Explain why there are only two possible values of x . As part of the explanation, state the two possible values of x .

$$\begin{array}{rcl} x & & \\ \sqrt{x} & = & \frac{84}{18} \quad \sqrt{41} \\ & & 4.7 \quad \overline{5.6} \quad 6.4 \\ & & \quad 5.4 \end{array}$$

There are only 2 possible values of x because when you divide 84 by 18 you get 4.7 and when you solve for the square root of 41 you get 6.4. When you add these 2 numbers and divide by 2 to get the middle you get 5.55. This means that one value could round to an irrational number of 5.6 but one could round to a rational number of 6. This could allow x to be rational.

The student has given no correct answers and an incorrect explanation.

After you have checked your work, close your answer booklet and test booklet so your teacher will know you are finished.



STUDENT RESPONSE

Response Score: 0 point



PART A

Question 17
Page 1 of 2

Item ID

Line Guide

The locations of four points on a number line are described by the numbers listed below.

point J: $\sqrt{12}$ point K: $\frac{15}{7}$ point L: $\sqrt{16}$ point M: $\frac{3\pi}{2}$

A. For **only** the **irrational** numbers that are listed, write the approximate value of each irrational number as a decimal rounded to the nearest tenth. Graph these approximate values on the number line below to show the locations of the irrational numbers. Label each graphed point with the correct letter.

0 / 1000

?

1 2 3 4 5 6

J L K M

The student has given no decimal approximations, and none of the plotted points are correct.

Next

Options

Flag

Pause

Review/End Test

PART B

Question 17
Page 2 of 2

Item ID

?

Calculator

Line Guide

Eraser

Highlighter

Selection

Eq

An integer is represented by x . A rational number is represented by \sqrt{x} . The value of \sqrt{x} is between $\frac{84}{18}$ and $\sqrt{41}$.

B. Explain why there are only two possible values of x . As part of the explanation, state the two possible values of x .

The reason that there is only two possible values of x is because $x = 4\frac{2}{3}$ and there are only two of them that are close to that fraction.

The student has given no correct answers and an incorrect explanation.

133 / 1000

Review/End Test

Pause

Flag

Options

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MATHEMATICS—SUMMARY DATA

MULTIPLE-CHOICE

Sample Number	Alignment	Answer Key	Depth of Knowledge	p-values A	p-values B	p-values C	p-values D
1	B-E.1.1.1	B	2	5%	71%	20%	4%
2	B-E.1.1.4	D	1	8%	9%	11%	72%
3	B-E.2.1	D	2	12%	14%	10%	64%
4	B-E.2.1.1	D	2	14%	17%	20%	49%
5	B-E.2.1.3	A	1	35%	33%	24%	8%
6	B-E.3.1.1	B	2	27%	58%	9%	6%
7	B-E.3.1.3	C	1	8%	6%	81%	5%
8	B-F.1.1.2 B-F.1.1.1	D	2	14%	27%	9%	50%
9	B-F.1.1.3	B	1	12%	73%	8%	7%
10	B-F.1.1.3	D	2	15%	18%	14%	53%
11	B-F.2.1.1	A	2	73%	9%	7%	11%
12	B-F.2.1.2	B	2	17%	47%	5%	31%
13	C-G.1.1.1	B	2	10%	58%	11%	21%
14	C-G.2.1.2	B	2	12%	49%	28%	11%
15	C-G.2.1.3	C	2	10%	35%	39%	16%
16	C-G.3.1.1	B	1	15%	73%	9%	3%

OPEN-ENDED

Sample Number	Alignment	Points	Depth of Knowledge	Mean Score
17	A-N.1	4	3	0.98

PSSA Grade 8 Mathematics Item and Scoring Sampler

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