
MATHCOUNTS®

2017
■ Chapter Competition ■
Countdown Round
Problems 1–80

**This booklet contains problems to be used
in the Countdown Round.**

Raytheon

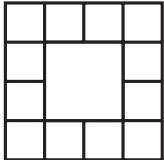
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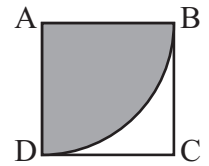
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1. _____ If $f(x) = x^2 - 3$ and $g(x) = 2x + 1$, what is $f(g(3))$?
2. _____ What is the value of $\frac{(2^4)^9}{(4^8)^2}$?
3. _____ Robin rolls two fair octahedral dice, each with faces numbered 1 through 8. What is the probability that the sum of the numbers she rolls is 8? Express your answer as a common fraction.
4. _____ What is 120% of 45% of 1200?
5. _____ What is the greatest two-digit prime number whose digits are both prime numbers?
6. _____ What is the value of $212^2 - 211^2$?
7. _____ (inches) A stalk of corn is 32 inches tall. It grows by 25% of its height each month for the next three months. What is the height of the stalk, in inches, at the end of the 3-month period? Express your answer as a decimal to the nearest tenth.
8. _____ Three-fourths of the sum $x + 3$ is four less than twice x . What is the value of x ?
9. _____ (inches) A rhombus has an area of 84 in^2 and a diagonal of length 14 inches. What is the length, in inches, of the other diagonal?
10. _____ (squares) How many squares of any size are in this figure consisting of adjacent unit squares surrounding a larger square? 
11. _____ If a , b and c are positive integers such that $a + b + c = 7$, what is the least possible value of $a! + b! + c!$?
12. _____ What is the value of the sum $\frac{1}{9} + \frac{2}{9} + \frac{3}{9} + \frac{4}{9} + \frac{5}{9} + \frac{6}{9} + \frac{7}{9} + \frac{8}{9}$?

13. _____

If the gray region of the figure shown is a quarter-circle centered at A, what is the probability that a tiny drop of ink that falls at random onto square ABCD lands on the gray region? Express your answer as a common fraction in terms of π .



14. _____

The mean of seven of Charlotte's scores is 80. The mean of three of those scores is 60. What is the mean of the other four scores?

15. _____

What common fraction has a value that is halfway between $\frac{13}{18}$ and $\frac{5}{6}$?

16. _____

If $16^{x+3} = 2^{5x}$, what is the value of x ?

17. _____

What is the value of $1 + 2 - 4 + 8 + 16 - 32$?

18. _____

A person who has a quarters and b nickels has \$5.60 more than a person who has a nickels and b quarters. What is the value of $a - b$?

19. _____ (integers)

For how many positive integers m does the line given by $y = mx$ intersect the segment with endpoints $(20, 17)$ and $(17, 20)$?

20. _____ (times)

An earthquake that measures $x + 1$ on the Richter scale is 10 times as strong as an earthquake that measures x on the same scale for any positive number x . How many times stronger is an earthquake that measures 6.9 than one measuring 4.9 on the Richter scale?

21. _____ (minutes)

At the time 2:58 p.m., the digits displayed on a clock form an arithmetic sequence in the order in which they appear. How many minutes later will the digits displayed next form an arithmetic sequence?

22. _____

What is the value of $(1 + 3 + 5 + \dots + 2017) - (2 + 4 + 6 + \dots + 2016)$?

23. _____ (cents)

Rebecca uses a \$20 bill to pay for five notebooks that cost \$3.97 each including tax. How many cents should Rebecca receive in change?

24. _____ (permutations)

How many permutations of three different letters can be made from the letters of LINES?

25. _____ Jess selects a 3-digit positive integer at random. What is the probability that she selects a number with at least one odd digit? Express your answer as a common fraction.

26. _____ What is the value of $(5^{-1} + 6^{-1})^{-1}$? Express your answer as a common fraction.

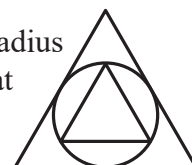
27. _____ (packages) A 120-pound box contains packages of candy, each with a total weight of 6 ounces. Ignoring the weight of the packaging, how many packages of candy are in the box?

28. _____ For what positive real number x , is $(x + 3)$ the reciprocal of $(x - 3)$? Express your answer in simplest radical form.

29. _____ What is the least integer x for which $x^{(x^2 - 9)} = 1$?

30. _____ The arithmetic mean of ten numbers is 37. What number can be added to the set so that the arithmetic mean of the eleven numbers is 41?

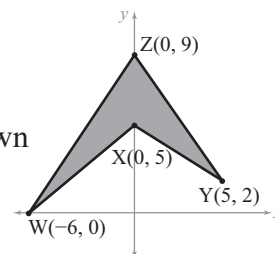
31. _____ This figure shows an equilateral triangle with an inscribed circle of radius 5 cm that is circumscribed around a smaller equilateral triangle. What is the ratio of the area of the smaller triangle to the area of the larger triangle? Express your answer as a common fraction.



32. _____ What is the least positive integer that contains each of the digits from 1 to 3 at least once and is divisible by 9?

33. _____ If $2x + 3 = 1000$, what is the value of $4x^2 - 9$?

34. _____ (units²) What is the area, in square units, of quadrilateral WXYZ, shown here, with coordinates $W(-6, 0)$, $X(0, 5)$, $Y(5, 2)$ and $Z(0, 9)$?



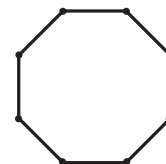
35. _____ (in²) A diagonal on one face of a cube measures $4\sqrt{3}$ inches. What is the total surface area of the cube, in square inches?

36. _____ The lines given by the equations $2y + ax + 6 = 0$ and $4y + x - 30 = 0$ are perpendicular. What is the value of a ?

37. _____ (outcomes) Six identical cupcakes are distributed to two friends. If each friend gets at least one cupcake, how many possible outcomes are there?
38. _____ What is the value of the sum $2 + 2(2) + 2(2)(2) + 2(2)(2)(2)$?
39. _____ (arrange-ments) Bill, Melissa and three friends are seated at a round table. Melissa does not sit next to Bill. Assume that two arrangements are considered the same if they are rotations of each other. How many distinct seating arrangements are possible?
40. _____ Given that $x + \frac{1}{x} = 4$, what is the value of $x^4 + \frac{1}{x^4}$?
41. _____ In the arithmetic sequence 12, w , x , y , z , 47, what is the value of y ?
42. _____ What is the 2017th odd positive integer?
43. _____ If $(x + y)^2 = 31$ and $(x - y)^2 = 29$, what is the value of xy ? Express your answer as a common fraction.
44. _____ (pounds) Madeline has eight dogs. The mean weight of the lightest six is 40 pounds. The mean weight of the heaviest six is 55 pounds. What is the least possible range of weights of her dogs?
45. _____ (questions) The Trivia Nite Quiz has 25 questions. Each correct answer adds four points, each wrong answer subtracts one point, and skipped questions are ignored. Liz answered all of the questions and had 12 wrong answers. Malaika got the same score without any wrong answers. How many questions did Malaika skip?
46. _____ (points) How many points on the circle given by $x^2 + y^2 = 169$ have a pair of integer coordinates?
47. _____ What is the value of the arithmetic series $26 + 39 + 52 + \cdots + 143$?
48. _____ The two distinct solutions of the equation $x^2 + bx + c = 0$ are reciprocals. What is the value of c ?

49. _____

Three vertices of a regular octagon are chosen at random to form the vertices of a triangle. What is the probability that such a triangle is isosceles? Express your answer as a common fraction.



50. _____ (cm²)

Nine different rectangles can be formed by choosing a width from the set {1 cm, 3 cm, 4 cm} and a length from the set {6 cm, 10 cm, 11 cm}. What is the mean of the areas of these nine rectangles, in square centimeters?

51. _____

The mean and unique mode of the list 3, 4, 5, x are the same integer. What is the value of x ?

52. _____ (units)

What is the length, in units, of a tangent from the point (10, 0) to the circle given by $x^2 + y^2 = 25$? Express your answer in simplest radical form.

53. _____

Let A be the set {3, 6, 9, 12} and B be the set of prime numbers between 30 and 50. One value from A and one value from B are chosen at random. What is the probability that their sum contains at least one 5 as a digit? Express your answer as a common fraction.

54. _____

What is the slope of the line containing the points (−2, 3) and (1, 9)?

55. _____ (in²)

A wooden cube, 4 inches on each edge, is cut into eight 2-inch cubes. What is the sum of the surface areas of the eight cubes, in square inches?

56. _____

Three tennis balls are stacked tightly in a cylindrical can. What fraction of the volume of the can is filled by the balls? Express your answer as a common fraction.



57. _____

What is the product of all real numbers x such that $x^3 + 2x^2 - 6x - 12 = 0$?

58. _____

What is the value of $25^2 - (20)(30)$?

59. _____

What is the sum of all four-digit positive integers that contain each of the digits 6, 7, 8 and 9?

60. _____ What is the geometric mean of $\sqrt{7}$ and $7\sqrt{7}$?
61. _____ In a sequence of numbers, each term after the second is the sum of the two previous terms. The first three terms are 3, 3, 6. What is the value of the first term greater than 200?
62. _____ (arrange-
ments) How many more unique arrangements of the letters in the name JESSE are there than in the name ALANA?
63. _____ What is the greatest value of x such that $x^2 - 11x + 30 = 0$?
64. _____ What is the value of the expression:
 $1 \times 9 + 2 \times 99 + 3 \times 999 + 4 \times 9999 + 5 \times 99999$?
65. _____ (in²) Two 8.5-inch by 11-inch sheets of paper are lying flat on an otherwise unoccupied 2-foot by 3-foot tabletop. Exactly 700 in² of the table are not covered by the sheets of paper. What is the area of the overlap of the two sheets, in square inches?
66. _____ Square WXYZ in the coordinate plane has vertices W(2, -7), X(-6, 8), Y(9, 16) and Z(a , b). What is the value of $a + b$?
67. _____ What is the least value of x that is a solution of $5 + \frac{3}{x} = \sqrt{5 + \frac{3}{x}}$? Express your answer as a common fraction.
68. _____ (feet) On a scale model of a building, one room measures $2\frac{3}{8}$ inches long by $1\frac{7}{8}$ inches wide. If the actual room is 19 feet long, how many feet wide is it?
69. _____ (units) Two legs of a right triangle have lengths 10 and 15. What is the length of the hypotenuse, in units? Express your answer in simplest radical form.
70. _____ Jim eats two-fifths of a chocolate bar and leaves the remaining three-fifths in the kitchen. Marie comes in and eats one-third of the remaining bar, and then Sage finds it and eats one-fourth of what's left. What fraction of the original chocolate bar remains? Express your answer as a common fraction.

71. _____ (frapts) On Mercury, 13 farfels equal 14 freks, 7 freks equal 16 fremps, 8 fremps equal 11 frindels, 12 frindels equal 9 frapts. How many frapts is 26 farfels?
72. _____ (cubes) How many cubes of edge length 0.5 inches are needed to fill a cube with edge length 2 inches?
73. _____ What is the slope of the perpendicular bisector of the segment connecting points (20, 15) and (19, 84)? Express your answer as a common fraction.
74. _____ What is the arithmetic mean of $27^{-\frac{1}{3}}$, $27^{-\frac{2}{3}}$ and 27^{-1} ? Express your answer as a common fraction.
75. _____ (cm) Larry needs three pieces of ribbon totaling 30 meters in length for an art project. He needs one piece of ribbon that is 15 cm longer than twice the length of the shortest piece, and he needs another piece that is 30 cm shorter than three times the length of the second piece. What is the length of the shortest piece of ribbon that Larry needs, in centimeters?
76. _____ (hours) How many hours are in $\frac{1}{28}$ of one week?
77. _____ What is the sum of the odd integers from 1 through 17, inclusive?
78. _____ If $\frac{48}{2x+5y} = \frac{24}{3x-2y}$, what is the ratio of x to y ? Express your answer as a common fraction.
79. _____ (dollars) Rich works for Wall Smart earning \$20 per hour. By mistake his wage was reduced by 50%. To attempt to fix the mistake, his reduced wage is increased by 50%. How many dollars does Rich currently earn per hour?
80. _____ For what value of c does the parabola given by $y = x^2 - 10x + c$ have exactly one x -intercept?