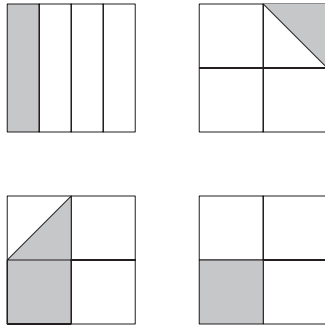


AMC 10/12 Student Practice Questions

Guide to Student Practice Questions

Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially shaded. What percent of the total area is partially shaded?



The original problem and choices from the 2011 AMC 8 contest

- (A) $12\frac{1}{2}$ (B) 20 (C) 25 (D) $33\frac{1}{3}$ (E) $37\frac{1}{2}$

2011 AMC 8, Problem #7—

“Find the shaded portion of each square separately. ”

Problem number
Hint

Solution

Answer (C): The upper left and the lower right squares are each one-fourth shaded, for a total of one-half square. The shaded portions of the upper right and lower left squares make up one-half square. So the total shaded area is one full square, which is 25% of the total area.

Solution from official solutions

Difficulty: Medium

SMP-CCSS: 2, 7

CCSS-M: 6G.1, 6RP.3C

Standards for Math Practice
Common Core State Standard

Difficulty, Percent correct

Easy	100-80%
Med Easy	80-60%
Medium	60-40%
Med Hard	40-20%
Hard	20-0%

What is $10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$?

- (A) 3 (B) 8 (C) $\frac{25}{2}$ (D) $\frac{170}{3}$ (E) 170

2014 AMC 10A, Problem #1—

“Sum up the fractions before working on multiplication”

Solution

Answer (C): Note that

$$\begin{aligned} 10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1} &= 10 \cdot \left(\frac{5}{10} + \frac{2}{10} + \frac{1}{10}\right)^{-1} \\ &= 10 \cdot \left(\frac{8}{10}\right)^{-1} \\ &= \frac{25}{2}. \end{aligned}$$

Difficulty: Medium Easy

SMP-CCSS: 1. Make Sense of Problems and Persevere in Solving Them, 2. Reason Abstractly and Quantitatively

CCSS-M: 8.EE.A. Work with radicals and integer exponents.

AMC 10/12 Student Practice Questions continued

On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and the median of the students' scores on this quiz?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

2014 AMC 10A, Problem #5—

“What is the median score if 50% of the students scored 90 or lower, and over 50% of the students scored 90 or higher?”

Solution

Answer (C): Because over 50% of the students scored 90 or lower, and over 50% of the students scored 90 or higher, the median score is 90. The mean score is

$$\frac{10}{100} \cdot 70 + \frac{35}{100} \cdot 80 + \frac{30}{100} \cdot 90 + \frac{25}{100} \cdot 100 = 87,$$

for a difference of $90 - 87 = 3$.

Difficulty: Medium

SMP-CCSS: 1. Make Sense of Problems and Persevere in Solving Them, 2. Reason Abstractly and Quantitatively.

CCSS-M: S-ID.B. Summarize, represent, and interpret data on two categorical and quantitative variables.

Which of the following numbers is a perfect square?

- (A) $\frac{14!15!}{2}$ (B) $\frac{15!16!}{2}$ (C) $\frac{16!17!}{2}$ (D) $\frac{17!18!}{2}$ (E) $\frac{18!19!}{2}$

2014 AMC 10A, Problem #8—

“What is the fraction for a perfect square?”

Solution

Answer (D): Note that $\frac{n!(n+1)!}{2} = (n!)^2 \cdot \frac{n+1}{2}$, which is a perfect square if and only if $\frac{n+1}{2}$ is a perfect square. Only choice D satisfies this condition.

Difficulty: Hard

SMP-CCSS: 1. Make Sense of Problems and Persevere in Solving Them.
CCSS-M:

AMC 10/12 Student Practice Questions continued

The two legs of a right triangle, which are altitudes, have lengths $2\sqrt{3}$ and 6. How long is the third altitude of the triangle?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

2014 AMC 10A, Problem #9—

“What is the area of the triangle and the hypotenuse length by the Pythagorean Theorem?”

Solution

Answer (C): The area of the triangle is $\frac{1}{2} \cdot 2\sqrt{3} \cdot 6 = 6\sqrt{3}$. By the Pythagorean Theorem, the hypotenuse has length $4\sqrt{3}$. The desired altitude has length $\frac{6\sqrt{3}}{\frac{1}{2} \cdot 4\sqrt{3}} = 3$.

Difficulty: Medium Hard

SMP-CCSS: 1. Make Sense of Problems and Persevere in Solving Them, 7. Look For and Make Use of Structure.

CCSS-M: G-SRT. Understand similarity in terms of similarity transformations.

AMC 10/12 Student Practice Questions continued

David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

- (A) 140 (B) 175 (C) 210 (D) 245 (E) 280

2014 AMC 10A, Problem #15—

“What is the remaining distance after one hour of driving, and the remaining time until his flight?”

Solution

Answer (C): Let d be the remaining distance after one hour of driving, and let t be the remaining time until his flight. Then $d = 35(t + 1)$, and $d = 50(t - 0.5)$. Solving gives $t = 4$ and $d = 175$. The total distance from home to the airport is $175 + 35 = 210$ miles.

OR

Let d be the distance between David's home and the airport. The time required to drive the entire distance at 35 MPH is $\frac{d}{35}$ hours. The time required to drive at 35 MPH for the first 35 miles and 50 MPH for the remaining $d - 35$ miles is $1 + \frac{d-35}{50}$. The second trip is 1.5 hours quicker than the first, so

$$\frac{d}{35} - \left(1 + \frac{d-35}{50}\right) = 1.5.$$

Solving yields $d = 210$ miles.

Difficulty: Medium Hard

SMP-CCSS: 1. Make Sense of Problems and Persevere in Solving Them, 2. Reason Abstractly and Quantitatively.

CCSS-M: A-CED. Create equations that describe numbers or relationships.

AMC 10/12 Student Practice Questions continued

Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

- (A) $\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{7}{4}$ (D) 2 (E) $\frac{13}{4}$

2014 AMC 10B, Problem #4—

“What is the cost of muffin and a banana?”

Solution

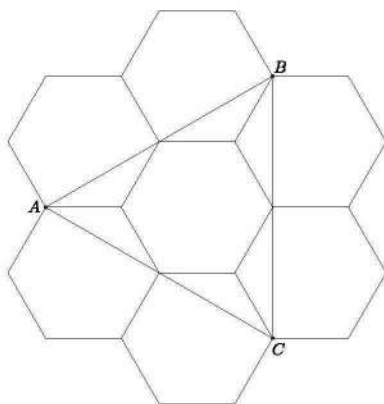
Answer (B): Let a muffin cost m dollars and a banana cost b dollars. Then $2(4m + 3b) = 2m + 16b$, and simplifying gives $m = \frac{5}{3}b$.

Difficulty: Medium Easy

SMP-CCSS: 1. Make Sense of Problems and Persevere in Solving Them, 2. Reason Abstractly and Quantitatively.

CCSS-M: A.SSE.B. Write expressions in equivalent forms to solve problems.

Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?



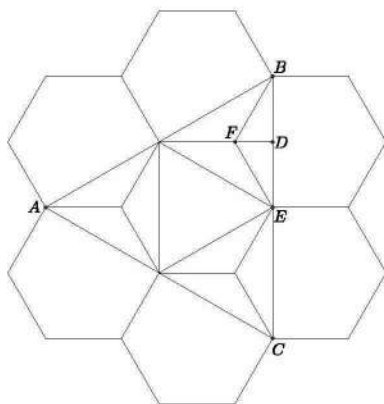
- (A) $2\sqrt{3}$ (B) $3\sqrt{3}$ (C) $1 + 3\sqrt{2}$ (D) $2 + 2\sqrt{3}$ (E) $3 + 2\sqrt{3}$

2014 AMC 10B, Problem #13—

“What is the midpoint of $\overline{BE}$?”

Solution

Answer (B): Label points E and F as shown in the figure, and let D be the midpoint of $\overline{BE}$. Because $\triangle BFD$ is a 30° – 60° – 90° triangle with hypotenuse 1, the length of $\overline{BD}$ is $\frac{\sqrt{3}}{2}$, and therefore $BC = 2\sqrt{3}$. It follows that the area of $\triangle ABC$ is $\frac{\sqrt{3}}{4} \cdot (2\sqrt{3})^2 = 3\sqrt{3}$.



OR

Notice that $AE = 3$ since AE is composed of a hexagon side (length 1) and the longest diagonal of a hexagon (length 2). Triangle ABE is 30° – 60° – 90° , so $BE = \frac{3}{\sqrt{3}} = \sqrt{3}$. The area of $\triangle ABC$ is $AE \cdot BE = 3\sqrt{3}$.

Difficulty: Medium

SMP-CCSS: 1. Make Sense of Problems and Persevere in Solving Them.
CCSS-M: G.SRT, Similarity, Right Triangles, and Trigonometry.