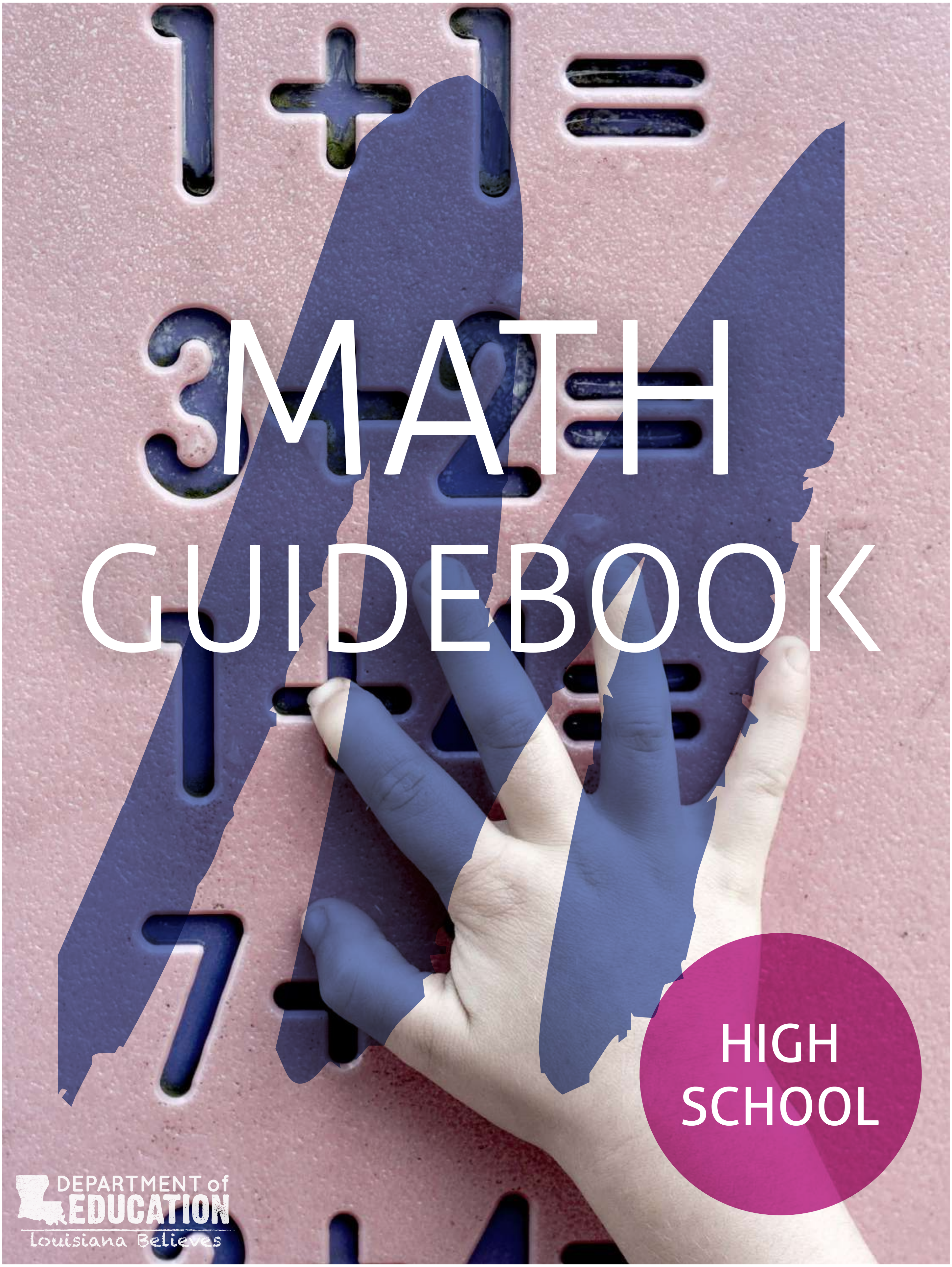


MATH GUIDEBOOK

HIGH SCHOOL

DEPARTMENT of EDUCATION
Louisiana Believes

CONTENTS

INTRODUCTION

Louisiana Believes...

Louisiana students...are **just as capable as students anywhere. They deserve high expectations** with support to reach them so that they are prepared to complete college and attain a professional career.

Louisiana teachers...will understand those expectations and work with their peers to make individual decisions to **meet their students' needs through planning and instruction.**

Louisiana principals and schools...will **create and lead meaningful structures of feedback and collaboration** to ensure teachers are able to learn and grow with support and guidance.

Louisiana districts...will **choose strong assessment and curricular** plans and **build systems that support school leaders** with goal setting, feedback, and collaboration.

Louisiana's Department of Education...will continue to shift away from prescribing local decisions and instead **provide resources, data, models, and direct teacher, principal, and district support.**

At the heart of these beliefs is good classroom teaching and learning. Effective instruction stems from the constant cycle of setting an ambitious goal, planning and teaching, and evaluating results. Our Teacher Support Toolbox in Louisiana is built to support these core actions of teachers. This instructional guidebook is a printed companion to our Teacher Support Toolbox. The guidebooks and the Teacher Support Toolbox, when used together, should support teachers and schools to make informed but independent decisions about how to provide rigorous but unique instruction in each classroom around the state.

http://www.louisianabelieves.com/resources/classroom-support-toolbox/teacher-support-toolbox

How to Use the Math Guidebook

This guide is meant to support teachers in supplementing math instruction for students. Each group of students has a unique set of needs, thus the department is not mandating that teachers use the instructional tools shared in this guide. Instead, the models are provided as a starting point for teams of teachers to use in planning for the unique needs of their students.

This guide provides:

- An explanation of strong math instruction
- Grade-level and standard specific remediation guidance
- Instructional tasks aligned to the state standards for math

This guide **does not** provide:

- A complete curriculum
- A set of plans that should be taught exactly the same in every classroom
- Daily lesson plans that all math teachers must use in their classroom

How to Read the Math Guidebook

There are two sections of this guide, which function differently.

- ***Mathematics Overview*** (page 8): This section describes how teachers approach the shifts called for in Louisiana's new math standards.
- ***Tools for Teaching*** (page 13): This section provides grade-level instructional tasks and remediation guidance. These tasks are meant to serve as a supplement to a curriculum already in use in a classroom. Teachers should collaborate to adjust these tasks to meet the needs of their students.

In addition, this guide is a companion to additional resources within the Teacher Support Toolbox.[1] Thus, throughout the guide you will see the following icons that highlight key connections.

Online Teacher Toolbox Resources: Notes a recommendation to find more available resources in the Teacher Support Toolbox.

Multimedia Components: Notes a recommendation to find a resource or video hosted on an outside Internet site.

abc ***Statewide Assessment***: Illustrates how a component of this guide connects to the statewide assessment students will take.

CC ***Compass Connections***: Illustrates the connections between instructional content and the Compass rubric.

As always, we welcome questions and feedback on these materials. If you need any support, do not hesitate to reach us at classroomsupporttoolbox@la.gov.

[1] http://www.louisianabelieves.com/resources/classroom-support-toolbox/teacher-support-toolbox

MATH OVERVIEW

MATHEMATICS OVERVIEW

STANDARDS SHIFTS

Louisiana's math standards ("APPENDIX" on page 251) help students practice and master rigorous mathematical concepts. These new standards require students to answer complex math questions correctly and also to explain their thinking on how they arrived at the answer. Because these new standards ask students to go deeper in their exploration of math content, teachers will need to shift how they plan lessons and deliver instruction.

These major shifts include:

Shift 1: Focus strongly where the standards focus.

Definition of this shift: The standards call for a greater focus in mathematics. Rather than racing to cover topics in a mile-wide, inch-deep curriculum, the standards require teachers to significantly narrow the concepts covered in a single year and deepen student understanding of those concepts. The standards focus on the major work of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom. The major work of each grade includes:

- K-2: Addition and subtraction—concepts, skills, and problem solving; place value
- 3-5: Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving
- 6: Ratios and proportional relationships; early expressions and equations
- 7: Ratios and proportional relationships; arithmetic of rational numbers
- 8: Linear algebra and linear functions

Shift 2: Coherence: Think across grades, and link to major topics within grades.

Definition of this shift: The standards progress from grade to grade in a coherent way. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new concept, but an extension of previous learning.

Illustration of this shift: The standards are written to connect between grade levels and within a grade level. The first illustration on page 9 shows a sample algebraic idea and how it is scaffolded from 6th grade to high school. The second illustration shows how standards connect to each other within one grade level.

One of several staircases to algebra.

A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression: add, subtract, multiply, and divide rational expressions.

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

8.EE.7b Solve linear equations (in one variable) with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coeffecients.

6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2+x) to produce the equivalent expression 6+3x; apply the distributive property to the expression 24x+18y to produce the equivalent expression 6(4x+3y); apply properties of operations to y+y+y to produce the equivalent expression 3y.*

Example: Geometry

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Standard HSG-SRT.C.6

Shift 3: Rigor: In major topics, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

Definition of this shift:

- ***Conceptual understanding***: The standards call for conceptual understanding of key concepts. Conceptual understanding helps students to explain how they got the correct answers and allows them to apply what they have learned to new types of problems.
- ***Procedural skill and fluency***: The standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions, such as single-digit multiplication, so that they have access to more complex concepts and procedures.
- ***Application***: The standards call for students to use math flexibly for applications in problem-solving contexts. In content areas outside of math, particularly science, students are given the opportunity to use math to make meaning of and access content.

Illustration from the Unit Plans: See the next section, Rigor, for an illustration of this shift.

To find your grade-level standards, go to the "APPENDIX" on page 251 of this document.

To find learning modules to help you better understand the standards, go to the standards page[2] in the Teacher Support Toolbox.

The new math standards are well researched.[3] Do not miss out on reviewing the research behind this approach to math instruction.

[2] http://www.louisianabelieves.com/resources/classroom-support-toolbox/teacher-support-toolbox/standards

[3] http://www.achievethecore.org/dashboard/2/search/6/2/0/1/2/3/4/5/6/7/8/9/10/11/12/page/407/mathematics-research-and-articles

ASSESSMENT AND INSTRUCTION

Instructionally, the most challenging shift comes with the focus on rigor. Rigor in a math classroom can be extremely difficult to nail down. So often, educators are tempted to practice procedures with students rather than help them master the mathematical concepts. This shift from simply practicing and assessing procedures with students to practicing and assessing concepts is challenging but critical. Even more, with the increased expectations brought on with the shift in rigor, remediation becomes critical. Teachers must work to identify which students need remediation and on which standards remediation would be most beneficial for these students. Thus, rigor and the needed remediation associated with increased rigor are two of the most important shifts teachers must be aware of as they change their instruction and assessment.

Rigor

While student fluency with math skills is critical, even more important is a student's ability to show mastery of a mathematical concept. State assessments will no longer demand that students simply perform based on memorized basic procedures. Rather, just as in real life, students are asked to solve complex problems based on their mathematical understanding.

So what does this really mean? Let's take a sample standard and consider what it would look like to teach and assess the procedure versus the concept.

Standard: **HSG-GPE.B.4**: Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0,2).*

- The following is an item that would be used to teach and assess the procedure:
  - » A circle has its center at the origin and passes through the point (0,2). Complete the following:
    1. Write the equation of the circle.
    2. Use the equation to show that (1,$\sqrt{3}$) is a point on the circle.
- The following is an item that would be used to teach and assess the concept:
  - » Prove or disprove: The point (1,$\sqrt{3}$) lies on the circle that has its center at the origin and passes through the point (0,2). Show or explain your reasoning.

In both examples, students need to know the steps and procedures to actually solve the equation. But in the second, memorizing the procedure alone is not enough. For students to apply mathematical understanding in the future, in a variety of settings, they must know why they are using the procedures and how to adapt them to fit new settings.

The tasks included in this guidebook deliberately help students explore, practice, and show mastery of the mathematical concepts demanded in the standards. This includes students' fluent use of basic math skills but pushes them beyond simple memorization to deep understanding of the content.

Remediation

As the rigor increases for students, so do the potential gaps in their understanding. Often, the instinct is to say that if a student is not at grade level, a teacher must completely go back to previous grade levels and remediate everything before moving on. In math, like other content areas, students do need quality remediation. But that remediation must be focused just on the content needed to quickly get students to practice at grade level. By practicing content at grade level, students more quickly improve their skill and understanding.

Let's look at an example.

Let's say that Algebra I students are working on standard **HSA-SSE.B.3a**: Factor a quadratic expression to reveal the zeros of the function it defines.

If the students are struggling in Algebra I with this standard, there are a few isolated standards from previous grade levels and from within Algebra I that will prepare students for this standard.

Standards from previous grades that prepare students include:

- **6.EE.A.3:** Apply the properties of operations to generate equivalent expressions. F*or example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

- **7.EE.A.1:** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Standards from Algebra I that should be taught in advance of the above standard include:

- **HSA-SSE.A.2:** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Standards from Algebra I that should be taught at the same time as the above standard include:

- **HSA-REI.B.4b:** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Specialists created a tool[4] to help teachers more quickly determine the required previous content needed for each individual standard. This guide has taken that tool and created easy-to-access charts for teachers by grade level (in the "Tools for Teaching" section that follows). These charts will help teachers more quickly identify just the necessary remediation. This will allow students faster access to grade-level content, allowing them to grow and also practice basic skills in a more authentic setting.

Every task included in this guide includes the recommended remedial standards along with sample tasks to check on student readiness for the grade-level task. Below is an example of a chart included in every task included in this guide. The links provide sample practice problems to help teachers remediate with students in preparation for grade-level tasks.

Grade-Level Standard	The Following Standards Will Prepare Students:	Items to Check for Task Readiness:	Sample Remediation Items:
HSA-CED.A.1	• 7.EE.B.4 • 8.EE.C.7	A circle with radius *r* is cut out of a square piece of paper whose side lengths are equal to the diameter of the circle. Write an equation, in terms of r, to find the area of the square remaining once the circle is removed. $A = 4r^2 - \pi r^2$ *OR* $A = r^2 (4 - \pi)$ *OR* $A = (2r)^2 - \pi r^2$	http://www.illustrativemathematics.org/illustrations/643 http://www.illustrativemathematics.org/illustrations/583 http://www.illustrativemathematics.org/illustrations/999 http://learnzillion.com/lessonsets/120-create-equations-and-inequalities-in-one-variable-and-use-them-to-solve-problems

[4] http://www.edutron.com/0/Math/ccssmgraph.htm

TOOLS FOR TEACHING

ALGEBRA I TOOLS

Algebra I Remediation Guide

As noted in "Remediation" on page 11 isolated remediation helps target the skills students need to more quickly access and practice on-grade level content. This chart is a reference guide for teachers to help them more quickly identify the specific remedial standards necessary for every Algebra I math standard[5].

Algebra I Standards	Previous Grade Standards	Algebra I Standards to be Taught before (Scaffolded)	Algebra I Standards to be Taught Concurrently
HSN-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	• 8.NS.A.1		
HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	• 6.RP.A.3d		• HSN-Q.A.2
HSN-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.			• HSN-Q.A.1
HSN-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	• 8.EE.A.4		
HSA-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* P *and a factor not depending on* P.	• 6.EE.A.2b • 7.EE.A.2		
HSA-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	• 6.EE.A.3 • 7.EE.A.1	• HSA-SSE.A.1	
HSA-SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.	• 6.EE.A.3 • 7.EE.A.1	• HSA-SSE.A.2	• HSA-REI.B.4b
HSA-SSE.B.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.		• HSA-SSE.A.2 • HSA-REI.B.4a	

[5] This content comes from research found here: http://www.edutron.com/0/Math/ccssmgraph.htm

Algebra I Standards	Previous Grade Standards	Algebra I Standards to be Taught before (Scaffolded)	Algebra I Standards to be Taught Concurrently
HSA-SSE.B.3c Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	• 8.EE.A.1	• HSA-SSE.A.1 • HSA-SSE.A.2	
HSA-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	• 6.EE.A.3 • 6.EE.A.4 • 7.EE.A.1 • 8.EE.A.1		
HSA-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	• 7.EE.A.1	• HSA-SSE.A.2 • HSA-SSE.B.3a	
HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	• 7.EE.B.4 • 8.EE.C.7		• HSA-REI.A.1 • HSA-REI.B.3
HSA-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	• 8.EE.C.8 • 8.F.A.3 • 8.F.B.4	• HSA-CED.A.1	• HSA-REI.D.10
HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*		• HSA-CED.A.1 • HSA-CED.A.2 • HSA-REI.D.10	
HSA-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*		• HSA-CED.A.2	• HSA-REI.A.1 • HSA-REI.B.3
HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	• 7.EE.B.4a • 8.EE.C.7		• HSA-CED.A.1 • HSA-CED.A.4 • HSA-REI.B.3 • HSA-REI.B.4a
HSA-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	• 7.EE.B.4a • 8.EE.C.7		• HSA-CED.A.1 • HSA-CED.A.4 • HSA-REI.A.1 • HSA-REI.B.4a

Algebra I Standards	Previous Grade Standards	Algebra I Standards to be Taught before (Scaffolded)	Algebra I Standards to be Taught Concurrently
HSA-REI.B.4a Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	• 8.EE.A.2		• HSA-CED.A.1 • HSA-REI.A.1 • HSA-REI.B.3
HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.	• 7.EE.A.1 • 8.EE.A.2	• HSA-REI.B.4a	• HSA-SSE.B.3a
HSA-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	• 8.EE.C.8	• HSA-CED.A.2 • HSA-CED.A.3	• HSA-REI.C.6
HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	• 8.EE.C.8	• HSA-CED.A.2 • HSA-CED.A.3	• HSA-REI.C.5
HSA-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	• 8.EE.B.5		• HSA-CED.A.2
HSA-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★	• 8.EE.C.8a • 8.EE.C.8b	• HSA-REI.C.5 • HSA-REI.C.6	• HSA-REI.D.12
HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.		• HSA-REI.C.5 • HSA-REI.C.6 • HSA-REI.D.11	•
HSF-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of f corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.	• 8.F.A.1 • 8.F.A.2 • 8.F.A.3		
HSF-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	• 6.EE.A.2c	• HSF-IF.A.1	

Algebra I Standards	Previous Grade Standards	Algebra I Standards to be Taught before (Scaffolded)	Algebra I Standards to be Taught Concurrently
HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*		• HSF-IF.A.1 • HSF-IF.A.2	• HSF-BF.A.1a
HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★	• 8.F.B.5	• HSF-IF.A.1 • HSN-Q.A.1	
HSF-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★		• HSF-IF.A.1 • HSF-IF.B.4	
HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	• 8.F.B.4	• HSF-IF.A.2	
HSF-IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.	• 8.EE.B.5 • 8.F.A.3	• HSF-IF.A.1	• HSF-IF.C.8a • HSF-LE.A.1b
HSF-IF.C.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.		• HSF-IF.A.1	• HSF-BF.B.3
HSF-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	• 7.EE.A.1		• HSF-IF.C.7a
HSF-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*		• HSF-IF.B.4 • HSF-IF.C.8a	
HSF-BF.A.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	• 8.F.B.4		• HSF-IF.A.3

Algebra I Standards	Previous Grade Standards	Algebra I Standards to be Taught before (Scaffolded)	Algebra I Standards to be Taught Concurrently
HSF-BF.B.3 Identify the effect on the graph of replacing *f(x)* by *f(x)* + *k*, *k f(x)*, *f(kx)*, and *f*(*x* + *k*) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.			• HSF-IF.C.7b
HSF-LE.A.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	• 8.F.A.3	• HSF-LE.A.1b • HSF-LE.A.1c	
HSF-LE.A.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	• 8.F.B.4		• HSF-LE.A.1c • HSF-IF.C.7a
HSF-LE.A.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.			• HSF-LE.A.1b
HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	• 8.F.B.4	• HSF-LE.A.1b • HSF-LE.A.1c • HSF-BF.A.1a	• HSS-ID.B.6a
HSF-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.		• HSF-LE.A.1c	
HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.		• HSF-LE.A.2 • HSF-BF.B.3	
HSS-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).	• 6.SP.B.4		
HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	• 6.SP.A.2 • 6.SP.B.5	• HSS-ID.A.1	• HSS-ID.A.3
HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	• 6.SP.B.5	• HSS-ID.A.1	• HSS-ID.A.2

Algebra I Standards	Previous Grade Standards	Algebra I Standards to be Taught before (Scaffolded)	Algebra I Standards to be Taught Concurrently
HSS-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	• 8.SP.A.4		
HSS-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	• 8.SP.A.3		
HSS-ID.B.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.	• 8.SP.A.2 • 8.SP.A.3	• HSS-ID.B.5	• HSF-LE.A.2
HSS-ID.B.6b Informally assess the fit of a function by plotting and analyzing residuals.	• 8.SP.A.2	• HSS-ID.B.6a • HSS-ID.B.6c	
HSS-ID.B.6c Fit a linear function for a scatter plot that suggests a linear association.	• 8.SP.A.1 • 8.SP.A.2 • 8.SP.A.3	• HSS-ID.B.6a	
HSS-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• 8.SP.A.3	• HSS-ID.B.6c	
HSS-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.		• HSS-ID.B.6c	• HSS-ID.C.9
HSS-ID.C.9 Distinguish between correlation and causation.		• HSS-ID.B.6c	• HSS-ID.C.8

Algebra I Tasks at a Glance

There are 10 sample tasks included in this guidebook that can be used to supplement any curriculum.

The tasks for Algebra I include:

- **5 Extended Constructed Response (ECR):** These short tasks, aligned to the standards, mirror the extended constructed response items students will see on their end of year state assessments.
- **5 Instructional Tasks (IT):** These complex tasks are meant to be used for instruction and assessment. They will likely take multiple days for students to complete. They can be used to help students explore and master the full level of rigor demanded by the standards. Teachers can use the table below to find standards associated with current instruction and add in these practice items to supplement any curriculum. These tasks should be used after students have some initial understanding of the standard. They will help students solidify and deepen their understanding of the associated content.

This is an overview of the Algebra I tasks included on the following pages.

Title	Type	Task Standards	Task Remedial Standards
Government Purchase Page 25	ECR	• HSA-CED.A.1 • HSA-REI.B.3	• 7.EE.B.4a • 7.EE.B.4 • 8.EE.C.7
Technical Support Center Page 30	ECR	• HSA-CED.A.3 • HSA-REI.C.6 • HSA-REI.D.12	• 8.EE.C.8 • HSA-CED.A.1 • HSA-CED.A.2 • HAS-REI.D.10 • HSA-REI.D.11 • HSA-REI.C.5
Pool Dimensions Page 38	ECR	• HSA-CED.A.1 • HSA-REI.B.4	• 7.EE.B.4 • 8.EE.A.2 • 8.EE.C.7
Trophy Boxes Page 44	ECR	• HSA-SSE.A.1 • HSA-APR.A.1	• 6.EE.A.2b • 6.EE.A.3 • 6.EE.A.4 • 7.EE.A.1 • 7.EE.A.2 • 8.EE.A.1
Rate of Change Page 51	ECR	• HSA-REI.B.4b • HSF-IF.B.4 • HSF-IF.B.6 • HSF-BF.B.3	• 7.EE.A.1 • 8.EE.A.2 • HSA-REI.B.4a • 8.F.B.5 • HSN-Q.A.1 • HSF-IF.A.1 • 8.F.B.4 • HSF-IF.A.2
Ice Cream Cones Page 57	IT	• HSA-CED.A.1	• 7.EE.B.4 • 8.EE.C.7
Lemonade Stand Page 65	IT	• HSA-CED.A.3 • HSA-REI.D.12	• HSA-CED.A.1 • HSA-CED.A.2 • HSA-REI.C.5 • HSA-REI.C.6 • HSA-REI.D.10 • HSA-REI.D.11

Title	Type	Task Standards	Task Remedial Standards
Solving Quadratic Equations Page 78	IT	• HSA-REI.A.1 • HSA-REI.B.4	• 7.EE.B.4a • 8.EE.A.2 • 8.EE.C.7
Investment Opportunity Page 83	IT	• HSF-IF.B.6	• 8.F.B.4 • HSF-IF.A.2
M&M's® Data Analysis Page 91	IT	• HSS-ID.B.6a • HSS-ID.C.7 • HSS-ID.C.8 • HSS-ID.C.9	• 8.SP.A.2 • 8.SP.A.3 • HSS-ID.B.5 • HSS-ID.B.6c

Government Purchase (ECR)

Overview

Students will create equations for three different scenarios using the same variables defined in the stem of the item. Then students will solve the equations they have written.

Standards

Create equations that describe numbers or relationships.

HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Solve equations and inequalities in one variable.

HSA-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.1	• 7.EE.B.4 • 8.EE.C.7	1. Jaquel bought j pairs of jeans at \$24.99 each and s shirts at \$14.99 each for a total of \$104.95 before taxes. Write an equation to represent this situation. a. $24.99j + 14.99s = 104.95$ 2. http://www.illustrativemathematics.org/illustrations/581 3. http://www.illustrativemathematics.org/illustrations/582	• http://www.illustrativemathematics.org/illustrations/643 • http://www.illustrativemathematics.org/illustrations/583 • http://www.illustrativemathematics.org/illustrations/999 • http://learnzillion.com/lessonsets/120-create-equations-and-inequalities-in-one-variable-and-use-them-to-solve-problems
HSA-REI.B.3	• 7.EE.B.4a • 8.EE.C.7	1. Solve the following: a. $\frac{3}{4}x + \frac{5}{6} = 5x - \frac{125}{3}$ i. 10 2. Find s if $24.99j + 14.99s = 104.95$ and $j = 3$. a. $s = 29.98$	• http://www.illustrativemathematics.org/illustrations/550 • http://www.illustrativemathematics.org/illustrations/392 • http://learnzillion.com/lessonsets/741-solve-linear-equations-and-inequalities-in-one-variable • http://learnzillion.com/lessonsets/180-solve-linear-equations-and-inequalities-in-one-variable

After the Task

Students may struggle with writing the equation for each situation since each scenario has a different unknown quantity. Encourage students to write an equation with all of the variables first, as if they were creating a formula. Relate this equation to finding the cost of items at a store or tickets at a concert. If students can write the equation $B = zx + wy$ based on the description in the stem, then students can substitute given values for parts a, b, and c to find the solution.

For part c, students must write the value of z in terms of w. Students who struggle with this may benefit from listing the values of all the variables prior to writing the equation. Have students write the verbal expression first, then translate the verbal expression into an algebraic expression.

Provide students who struggle with this task additional practice writing and solving equations based on real-world scenarios.

Student Extended Constructed Response

The government buys *x* fighter planes at *z* dollars each and *y* tons of wheat at *w* dollars each. It spends *B* dollars. In each of the following, write and solve an equation to find the unknown quantity.

1. Write and solve an equation to find the number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each. Show your work and state your answer in the context of the given situation.

2. Write and solve an equation to find the price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 per ton, for a total of $50 million. Show your work and state your answer in the context of the given situation.

3. Write and solve an equation to find the price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million. Show your work and state your answer in the context of the given situation.

Task adapted from http://www.illustrativemathematics.org/illustrations/580.

Extended Constructed Response Exemplar Response

The government buys *x* fighter planes at *z* dollars each and *y* tons of wheat at *w* dollars each. It spends *B* dollars. In each of the following, write an equation in terms of the unknown quantity.

1. Write and solve an equation to find the number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each. Show your work and state your answer in the context of the given situation.

   Given B = 100,000,000, w = 300, x = 5, and z = 15,000,000; y = ?

$$100{,}000{,}000 = 15{,}000{,}000(5) + 300y$$

$$100{,}000{,}000 = 75{,}000{,}000 + 300y$$

$$25{,}000{,}000 = 300y$$

$$\frac{25{,}000{,}000}{300} = \frac{300y}{300}$$

$$83{,}333\frac{1}{3} = y$$

   The government can buy 83,333 1/3 tons of wheat.

2. Write and solve an equation to find the price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 per ton, for a total of $50 million. Show your work and state your answer in the context of the given situation.

   Given x = 3, y = 10,000, w = 500, and B = 50,000,000; z = ?

$$50{,}000{,}000 = 3z + 10{,}000(500)$$

$$50{,}000{,}000 = 3z + 5{,}000{,}000$$

$$45{,}000{,}000 = 3z$$

$$\frac{45{,}000{,}000}{3} = \frac{3z}{3}$$

$$15{,}000{,}000 = z$$

   The price of a fighter plane is $15 million.

3. Write and solve an equation to find the price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million. Show your work and state your answer in the context of the given situation.

   Solution: Given z = 100,000w, x = 20, y = 15,000, B = 90,000,000; w = ?

$$90{,}000{,}000 = 20(100{,}000w) + 15{,}000w$$

$$90{,}000{,}000 = 2{,}000{,}000w + 15{,}000w$$

$$90{,}000{,}000 = 2{,}015{,}000w$$

$$\frac{90{,}000{,}000}{2{,}015{,}000} = \frac{2{,}015{,}000w}{2{,}015{,}000}$$

$$44.67 \approx w$$

   The price of a ton of wheat is approximately $44.67

Technical Support Center (ECR)

Overview

Students will solve systems of inequalities and equations to determine the number of cubicles to be built and the number of operators to be employed in a technical support center.

Standards

Create equations that describe numbers or relationships.

HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Solve systems of equations.

HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.3	• HSA-CED.A.1 • HSA-CED.A.2 • HAS-REI.D.10	1. D'Andre earns some money by helping people with their yards. It takes him an average of 2 hours to weed a flowerbed and an average of 1.5 hours to mow and edge a lawn. He can work no more than 6 hours a day. He charges $60 to weed a flowerbed and $75 to mow and edge a lawn. He needs to cover his expenses of $150 to make a profit. Write a system of inequalities that would help determine how many flowerbeds and lawns he could complete on a given day.	• http://www.illustrativemathematics.org/illustrations/582 • http://www.illustrativemathematics.org/illustrations/1010 • http://www.illustrativemathematics.org/illustrations/1351 • http://www.illustrativemathematics.org/illustrations/1066 • http://learnzillion.com/lessonsets/516-represent-constraints-by-linear-equations-inequalities-and-systems-interpret-solutions-as-viable-and-nonviable

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		a. $2f + 1.5m \leq 6$ $60f + 75m \geq 150$ Where f = # of flowerbeds and m = # of lawns to be mowed and edged 2. http://www.illustrativemathematics.org/illustrations/610 3. http://www.illustrativemathematics.org/illustrations/1351	
HSA-REI.C.6	• 8.EE.C.8 • HSA-CED.A.2	1. Solve the system: $-2x = 6y + 36$ $5x - 18y = 141$ a. (3, 7) 2. http://www.illustrativemathematics.org/illustrations/462	• http://www.illustrativemathematics.org/illustrations/469 • http://www.illustrativemathematics.org/illustrations/472 • http://www.illustrativemathematics.org/illustrations/1003 • http://learnzillion.com/lessonsets/486-solve-systems-of-two-linear-equations-in-two-variables • http://learnzillion.com/lessonsets/247-solve-systems-of-linear-equations-exactly-and-approximately
HSA-REI.D.12	• HSA-REI.C.5 • HSA-REI.D.11	1. http://www.illustrativemathematics.org/illustrations/644 2. http://www.illustrativemathematics.org/illustrations/1205	• http://learnzillion.com/lessonsets/678-graph-the-solutions-to-a-linear-inequality-as-a-halfplane-graph-the-solutions-to-a-system-of-linear-inequalities-as-an-intersection-of-halfplanes • http://learnzillion.com/lessonsets/624-graph-the-solutions-to-a-linear-inequality-as-a-halfplane-and-the-solutions-to-a-system-of-linear-inequalities-as-an-intersection-of-halfplanes

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- **What is a technical support center?** Companies typically have technical support centers to answer questions that customers might have about products they have purchased. In some cases, operators who work in the center could provide support to customers by helping them troubleshoot problems they may be having with the products.
- **What is a cubicle?** A cubicle is a workspace provided to employees in order for them to have some privacy, but it is not an office. Typically, cubicles do not have doors and are created using portable wall dividers.
- **What is an operator?** In the case of this task, an operator would be a person answering phone calls and emails from customers.
- **What are employee benefits?** Employers typically pay for benefits such as health insurance, vacation time, and sick-leave time. The cost of these benefits is used to determine the most accurate amount an employee is paid.
- **What is an open floor plan?** *Open floor plan* is a term used to describe any floor plan that makes use of large, open spaces and minimizes the use of small rooms, such as private offices. This allows for the use of "movable" walls to create different arrangements to fit a company's needs.

After the Task

Students may struggle with writing the inequalities for question one as they may try to incorporate the square footage. Remind students to use only the information they know for sure—total number of cubicles and total number of employees—when creating the system of inequalities.

For question 2, students may forget that the axes represent additional constraints. Have students explain why the solutions are confined to quadrant one and determine the best way to state the constraints that would be graphed on the coordinate system.

Students who understand what each of the boundary lines and the shaded region represent will be able to determine the answer to question 3 with little work. Other students will test many points to find the maximum and minimum values for the operators and cubicles, respectively. Some students may find multiple points rather than the one point that satisfies both conditions. Have students discuss what the boundary lines represent and what their intersection means.

Students may overlook the word "**both"** in the question and provide the answer (0, 10) or 10 large cubicles. Remind students to read the question closely. Since the question requires the use of both types of cubicles, there must be at least one small cubicle. If this is the case, there will only be 19 employees.

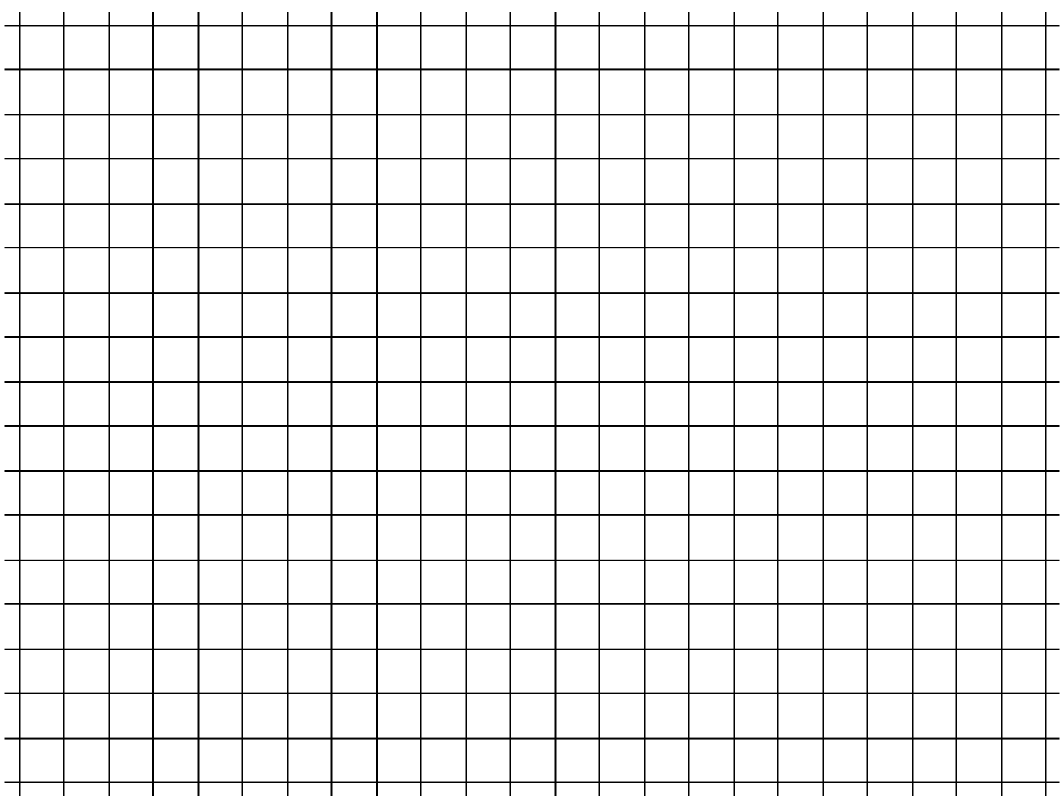
For question 5, students may try to use the inequality $s + 2l \leq 20$ rather than the expression $s + 2l$. Remind students that there is no longer a maximum value placed on the number of operators and therefore students only need to know how many cubicles of each type they will need. If necessary, provide students with additional optimization problems to help them see the use of these expressions.

Student Extended Constructed Response

A new home security company is planning to open a technical support center to answer customer phone calls and emails. The company plans to purchase a large building with an open floor plan to house cubicles where operators will sit while they work. There are two options for cubicles: a small cubicle for one operator that is approximately 36 square feet and a larger cubicle for two operators that is approximately 60 square feet. The company knows it wants a minimum of 15 cubicles in the technical support center; however, because of the rising costs of employee benefits, the company wants to keep the total number of operators at a maximum of 20.

1. Write a system of inequalities to model the requirements for the new technical support center that would help the company determine the number of cubicles to build and the number of operators that could be employed. *Be sure to define any variables used.*

2. Graph the solution set of the system you created. State any additional constraints the model may have. *Be sure to label the graph.*



3. Which combination of small and large cubicles will allow the company to employ the maximum number of operators with the minimum number of cubicles? *Provide evidence to support your response.*

4. The first location the company finds is smaller than it would like. The space will only hold 10 cubicles total. If the company still wants to employ up to 20 operators, is it possible in this space to have the maximum number of cubicles and operators with a combination of **both** small and large cubicles? *Provide evidence to support your response.*

5. The location the company decides to use has an 1,800-square-foot open floor plan. The owner of the company also decides to remove the maximum limit from the number of operators he is willing to employ. He wants six fewer small cubicles than large cubicles. Calculate the maximum number of operators that can be employed in the 1,800-square-foot space using six fewer small cubicles than large cubicles. *Provide evidence to support your response.*

Task adapted with permission from Universal Achievement, LLC.

Extended Constructed Response Exemplar Response

A new home security company is planning to open a technical support center to answer customer phone calls and emails. The company plans to purchase a large building with an open floor plan to house cubicles where operators will sit while they work. There are two options for cubicles: a small cubicle that is approximately 36 square feet that will house one operator and a larger cubicle of approximately 60 square feet that will house two operators. The company knows it wants a minimum of 15 cubicles in the technical support center; however, because of the rising costs of employee benefits, the company wants to keep the total number of operators at a maximum of 20.

1. Write a system of inequalities to model the requirements for the new technical support center that would help the company determine the number of cubicles to build and the number of operators that could be employed. *Be sure to define any variables used.*

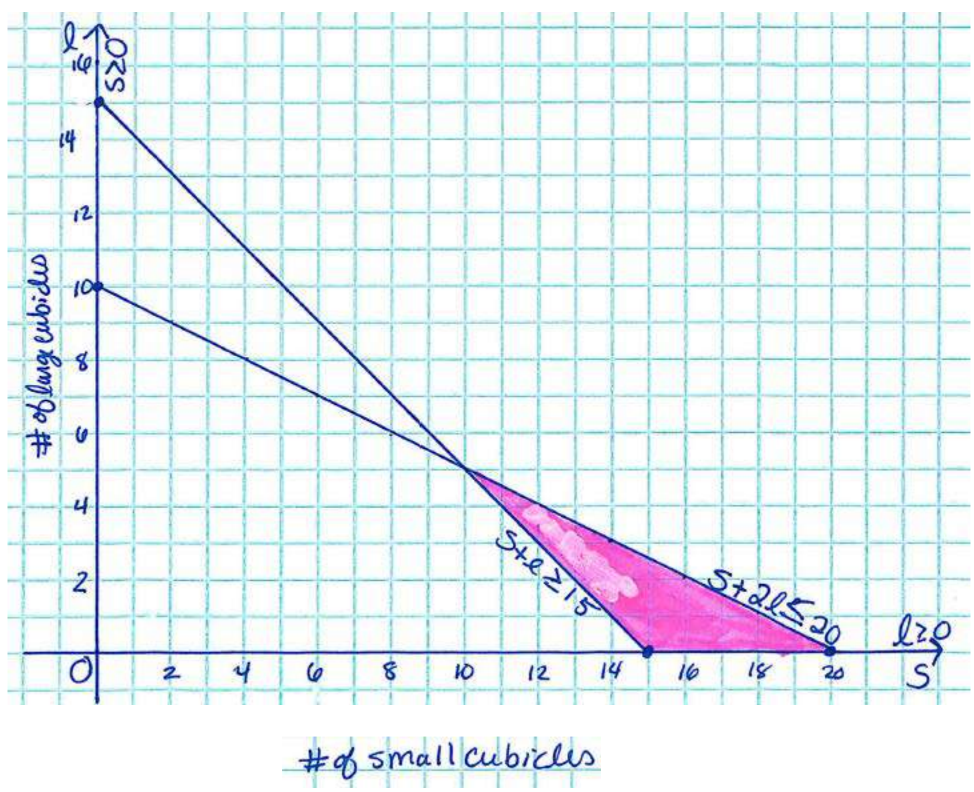
   s = # of small cubicles   *Total number of cubicles:* $s + l \geq 15$

   l = # of large cubicles   *Total number of operators:* $s + 2l \leq 20$

2. Graph the solution set of the system you created. State any additional constraints the model may have.

   Other constraints on the model are that $s \geq 0$ *and* $l \geq 0$ *because there cannot be a negative number of cubicles. The region shaded in pink, including solutions on the boundaries of the region, would represent all of the possible solutions to this system. Also, the solutions can only be whole numbers as there cannot be a fraction of a cubicle.*

   ***Note: Students' responses are dependent upon their response to question 1. Therefore, student work in this section will need to be checked for accuracy based on the inequalities they wrote.*



3. Which combination of small and large cubicles will allow the company to employ the maximum number of operators with the minimum number of cubicles? *Provide evidence to support your response.*

   ***Note: Student responses are dependent upon the system of inequalities in question 1. If those are incorrect, student approaches to answer this question will need to be assessed based on their previous incorrect responses.*

   From the information in the problem, the minimum number of cubicles is 15. The maximum number of operators is 20. The only point in the solution set that satisfies these conditions is (10, 5), which is the point where the boundary lines intersect. This ordered pair represents 10 small cubicles and 5 large cubicles.

   $s + l \geq 15$ $\qquad\qquad$ $s + 2l \leq 20$

   $10 + 5 \geq 15$ $\qquad\qquad$ $10 + 2(5) \leq 20$

   $15 \geq 15$ $\qquad\qquad$ $20 \leq 20$

   All other points in the region would be combinations that provide for:

   - *exactly 15 cubicles but fewer than 20 operators; or*
   - *exactly 20 operators but more than 15 cubicles; or*
   - *more than 15 cubicles and fewer than 20 operators.*

4. The first location the company finds is smaller than it would like. The space will only hold 10 cubicles total. If the company still wants to employ up to 20 operators, is it possible in this space to have the maximum number of cubicles and operators with a combination of **both** small and large cubicles? *Provide evidence to support your response.*

   s = # of small cubicles $\qquad\qquad$ *Total of 10 cubicles:* $s + l = 10$

   l = # of large cubicles $\qquad\qquad$ *Total of 20 operators:* $s + 2l = 20$

   $-1(s + l) = -1(10) \quad \rightarrow \qquad -s - l = -10$

   $$s + 2l = 20$$

   By combining these equations: $\qquad l = 10$

   So there could be 10 large cubicles that would house 20 operators. This means there would be no small cubicles $(s + 10 = 10; s = 0)$. *Therefore, this space will not work because it cannot have the maximum number of cubicles and the maximum number of operators with a combination of **both** large and small cubes.*

5. The location the company decides to use has an 1,800-square-foot open floor plan. The owner of the company also decides to remove the maximum limit from the number of operators he is willing to employ. He wants six fewer small cubicles than large cubicles. Calculate the maximum number of operators that can be employed in the 1,800-square-foot space using six fewer small cubicles than large cubicles. *Provide evidence to support your response.*

s = # of small cubicles *Total number of cubicles:* $s = l - 6$

l = # of large cubicles *Total area of cubicles:* $36s + 60l = 1800$

$36(l - 6) + 60l = 1800$

$36l - 216 + 60l = 1800$

$96l - 216 = 1800$

$96l = 1800 + 216$

$\frac{96l}{96} = \frac{2016}{96}$

$l = 21$

$s = 21 - 6$

$s = 15$

Number of operators: $s + 2l$

$15 + 2(21) = 57$

So, using all 1,800 square feet, there can be 21 large cubicles and 5 small cubicles. The company would be able to employ 57 operators.

Pool Dimensions (ECR)

Overview

Students will solve quadratic equations to find the dimensions of a pool and a walkway surrounding the pool.

Standards

Create equations that describe numbers or relationships.

HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Solve equations and inequalities in one variable.

HSA-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.1	• 7.EE.B.4 • 8.EE.C.7	1. The length of a rectangular field is 5 feet more than its width. If the area of the field is 2500 square feet, write an equation that could be used to find the dimensions of the field. Define any variables used. a. *w* = width of field in feet; length of field is (*w* + 5) feet. An equation to find the dimensions could be $w(w+5) = 2500$ OR $w^2 + 5w = 2500$. **2.** http://www.illustrativemathematics.org/illustrations/581	• http://www.illustrativemathematics.org/illustrations/986 • http://www.illustrativemathematics.org/illustrations/999 • http://learnzillion.com/lessonsets/120-create-equations-and-inequalities-in-one-variable-and-use-them-to-solve-problems
HSA-REI.B.4	• 8.EE.A.2	1. Solve the equation $3(4x+1)(x) = 9$. a. $x = \frac{3}{4}$ or -1	• http://learnzillion.com/lessonsets/24-

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		2. http://www.illustrativemathematics.org/illustrations/618 3. http://www.illustrativemathematics.org/illustrations/586	solve-quadratic-equations

Real-World Preparation: The following discussions will help prepare students for some of the real-world components of this task:

- Discuss with students that the depth of a pool typically slopes down from the shallow end to the deep end. However, for this task, students will ignore the slope as they calculate the dimensions of the pools described.
- Students will need to remember how to find the volume of a rectangular right prism, which is an expectation of 5th and 6th grade students. For those students who may need the additional scaffolding, the volume formula may be provided.

After the Task

Students may struggle with writing the expressions to represent the dimensions of the pool. Provide students with additional practice in translating verbal expressions into algebraic expressions as needed.

When solving the quadratic equations, students may struggle with multiplying polynomials. Provide additional practice with multiplying polynomials and reinforce the distributive property as needed.

When students have the equation written in factored form, for example $70 = (3d - 1)(d)$, they may set each factor equal to the constant. Discuss with students the Zero Product Property and why setting factors equal to other constants will not always work. Provide additional practice with solving quadratic equations by factoring as needed.

For question 3, students may struggle with writing the expressions for the width of the walkway, as well as the expressions for the total width and length of the pool and walkway. Have students draw a diagram of the scenario and label the information they are given. If necessary, provide a diagram for students who may need the additional scaffolding. Discuss with students which portion of the drawing would represent the area they are given. Then have students identify how someone would likely find the area of that region. Additional practice with finding the area of various regions in given diagrams may be necessary.

Student Extended Constructed Response

A hotel is planning to redesign its pool to accommodate a new diving board and waterslide for guests to use. The existing rectangular pool is 28 feet long. The width of the existing pool is one foot less than three times the measure of the pool's depth.

1. The existing pool can hold up to 1,960 cubic feet of water. What are the dimensions of the existing pool? Be sure to define any variables used. Show all work.

2. In order to accommodate the new diving board and waterslide, the hotel management wants the length of the new rectangular pool to be 30 feet. The deep end of the new pool (where the diving board and waterslide will be placed) will be 10 feet deeper than the shallow end. The new pool's width will equal three feet more than double the depth of the shallow half. The new pool will hold 4,950 cubic feet of water. How deep will each half of the pool be? Show your work and be sure to define any variables used.

3. The walkway to be constructed around the new pool will cover an area of 1,054 square feet, excluding the area of the pool. The width of the walkway will be the same all around the pool. There will be new lounge chairs placed on the walkway for guests to use. The width of the walkway is 1.5 feet more than the length of one lounge chair in order to allow space to walk around the pool. What is the length of a single lounge chair? What is the width of the walkway? Show your work and define any variables used.

Task adapted with permission from Universal Achievement, LLC.

Extended Constructed Response Exemplar Response

A hotel is planning to redesign its pool to accommodate a new diving board and waterslide for guests to use. The existing rectangular pool is 28 feet long. The width of the existing pool is one foot less than three times the measure of the pool's depth.

1. The existing pool can hold up to 1,960 cubic feet of water. What are the dimensions of the existing pool? Be sure to define any variables used. Show all work.

   d = the measure of the pool's depth in feet

   Width of the pool = (3d – 1) feet

   Volume of the pool = 1960 cubic feet

   Volume of the pool = length of pool × width of pool × depth of pool

   $$1960 = 28(3d - 1)(d)$$

   $$1960 = 28(3d^2 - d)$$

   $$\frac{1960}{28} = \frac{28(3d^2 - d)}{28}$$

   $$70 = 3d^2 - d$$

   $$0 = 3d^2 - d - 70$$

   $$0 = (3d + 14)(d - 5)$$

   $$3d + 14 = 0 \; or \; d - 5 = 0$$

   $$d = -\frac{14}{3} \; or \; d = 5$$

   Because d is the depth of the pool in feet, the negative answer is not viable; therefore, the depth of the pool is 5 feet. The width of the pool is 3(5) – 1, which is 14 feet. The length of the pool was given as 28 feet.

   ***Note: This is not the only solution method. Students may also choose to complete the square or use the quadratic formula.*

2. In order to accommodate the new diving board and waterslide, the hotel management wants the length of the new rectangular pool to be 30 feet. The deep end of the new pool (where the diving board and waterslide will be placed) will be 10 feet deeper than the shallow end. The new pool's width will equal three feet more than double the depth of the shallow half. The new pool will hold 4,950 cubic feet of water. How deep will each half of the pool be? Show your work and be sure to define any variables used.

   s = depth of the shallow half of the pool in feet

   s + 10 = depth of the deeper half of the pool in feet

2s + 3 = width of the pool in feet

4950 cubic feet = total volume of the pool

15 feet = the length of each half of the pool (30 feet total ÷ 2)

volume of shallow end + volume of deep end = total volume of the pool

$$15(2s+3)(s)+15(2s+3)(s+10)=4950$$

$$15(2s+3)(s+s+10)=4950$$

$$\frac{15(2s+3)(s+s+10)}{15}=\frac{4950}{15}$$

$$(2s+3)(2s+10)=330$$

$$4s^2+26s+30=330$$

$$4s^2+26s=300$$

$$\frac{4s^2+26s}{4}=\frac{300}{4}$$

$$s^2+\frac{13}{2}s=75$$

$$s^2+\frac{13}{2}s+\frac{169}{16}=75+\frac{169}{16}$$

$$\left(s+\frac{13}{4}\right)^2=\frac{1369}{16}$$

$$s+\frac{13}{4}=\pm\frac{37}{4}$$

$$s=-\frac{13}{4}\pm\frac{37}{4}$$

$$s=\frac{-13+37}{4}\ or\ s=\frac{-13-37}{4}$$

$$s=\frac{24}{4}$$

$$s=6$$

Because s is the depth of the shallow half in feet, the answer cannot be negative. Since the numerator of the second answer (–13 – 37) is negative, that answer is not viable. Therefore, the depth of the shallow half of the pool is 6 feet. The depth of the deeper half of the pool is 16 feet (6 + 10).

***Note: This is not the only solution method. Students may also choose to use the quadratic formula or factor the quadratic equation.*

3. The walkway to be constructed around the new pool will cover an area of 1,054 square feet, excluding the area of the pool. The width of the walkway will be the same all around the pool. There will be new lounge chairs placed on the walkway for guests to use. The width of the walkway is 1.5 feet more than the length of one lounge chair in order to allow space to walk around the pool. What is the length of a single lounge chair? What is the width of the walkway? Show your work and define any variables used.

l = length of one lounge chair in feet

l + 1.5 = the width of the walkway in feet

1054 square feet = total area of the walkway

Total width of pool plus walkway: 15+2(l + 1.5) or (2l + 18) feet (when simplified)

Total length of pool plus walkway: 30+2(l + 1.5) or (2l + 33) feet (when simplified)

Area of the walkway = total area of pool and walkway – area of the pool

$$1054 = (2l + 18)(2l + 33) - (15)(30)$$

$$1054 = 4l^2 + 66l + 36l + 594 - 450$$

$$1054 = 4l^2 + 102l + 144$$

$$0 = 4l^2 + 102l - 910$$

$$l = \frac{-102 \pm \sqrt{102^2 - 4(4)(-910)}}{2(4)}$$

$$l = \frac{-102 \pm \sqrt{24964}}{8}$$

$$l = \frac{-102 \pm 158}{8}$$

$$l = \frac{-102 + 158}{8} \; or \; l = \frac{-102 - 158}{8}$$

$$l = \frac{56}{8}$$

$$l = 7$$

Because l is the length of one lounge chair in feet, the answer cannot be negative. Since the numerator of the second answer (–102 – 158) is negative, that answer is not viable. Therefore, the length of one lounge chair is 7 feet. The width of the walkway is 8.5 feet.

***Note: This is not the only solution method. Students may also choose to complete the square or factor the quadratic equation. Also, students may choose to find the area of four smaller rectangles (around the pool) and four squares (at the corners) and add all of these areas rather than subtracting the pool area from the total area.*

Trophy Boxes (ECR)

Overview

Students will use polynomials to model the volume of boxes used to package trophies.

Standards

Interpret the structure of expressions.

HSA-SSE.A.1 Interpret expressions that represent a quantity in terms of its context. ★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

Perform arithmetic operations on polynomials.

HSA-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Prior to the Task

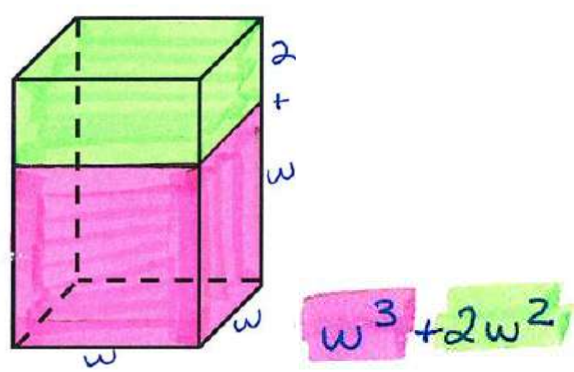
Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-SSE.A.1	• 6.EE.A.2b • 7.EE.A.2	1. The expression $(4x^2 + 6x)$ in.^{2} represents the area of a rectangular region whose width is 2*x* in. Explain the terms of the expression in the context of the area of the region. a. The expression $4x^2$ represents the area of a square region with side lengths of 2*x*. The expression $6x$ represents the area of a rectangular region with a width of 2*x* and a length of 3 in. Together, these regions would be combined to create a rectangular region with an area of $(4x^2 + 6x)$ in.^{2} 2. http://www.illustrativemathematics.org/illustrations/389 3. http://www.illustrativemathematics.org/illustrations/21	• http://www.illustrativemathematics.org/illustrations/1450 • http://learnzillion.com/lessonsets/749-interpret-quadratic-expressions-by-understanding-their-parts • http://learnzillion.com/lessonsets/649-interpret-complicated-expressions-in-context-understanding-the-meaning-of-specific-terms-factors-and-coefficients
HSA-APR.A.1	• 6.EE.A.3 • 6.EE.A.4 • 7.EE.A.1 • 8.EE.A.1	1. Simplify the following: a. $x^3(2x^2 - 3x)$ i. $2x^5 - 3x^4$ b. $(3x+4)\left(\frac{2}{3}x - 1\right)$ i. $2x^2 - \frac{1}{3}x - 4$	• http://www.illustrativemathematics.org/illustrations/542 • http://www.illustrativemathematics.org/illustrations/461 • http://www.illustrativemathematics.org/illustrations/541

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
			• http://www.illustrativemathematics.org/illustrations/395 • http://learnzillion.com/lessonsets/558-understand-that-polynomials-are-closed-under-addition-subtraction-and-multiplication-add-subtract-and-multiply-polynomials • http://learnzillion.com/lessonsets/494-understand-that-polynomials-are-closed-under-addition-subtraction-and-multiplication-perform-these-operations-on-polynomials

After the Task

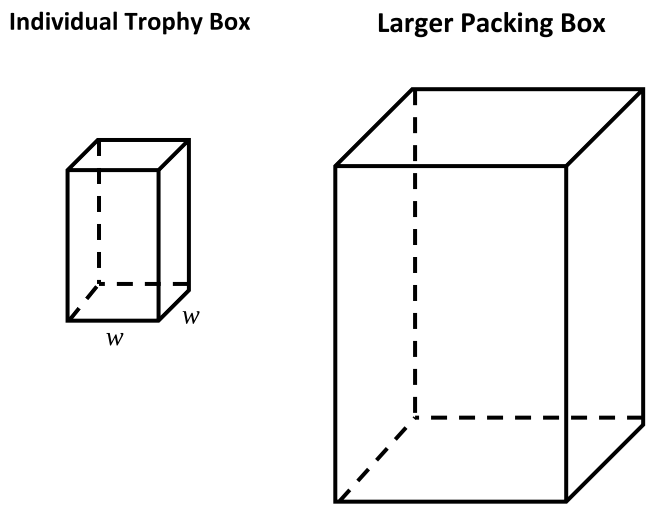
Students will need to remember how to find the volume of a rectangular solid (Grade 6) in order to write the polynomial expressions for the volume of the boxes in the task. When writing a simplified expression for the volume, students might multiply the exponents instead of adding them when multiplying the terms in the polynomials. Provide additional practice as needed with the rules regarding exponents. Students who struggle to interpret the terms of the expression as each relates to the volume of the boxes should be encouraged to draw a diagram and divide the edges of the right rectangular prism into the parts of the expressions they used for length, width, and height. Then they should find the volume of each separate right rectangular prism and relate those expressions to the terms of the expression they found in questions 1 and 2. See the drawing below.



For question 3, students may initially say they can only fit 32 boxes in the packing box since they can only stand two four-inch-tall boxes on top of one another inside of a 10-inch-tall packing box. Have students determine the lengths of the sides of the space that would remain in the box and ask them if they could find a way to fit any additional boxes in the remaining space.

Student Extended Constructed Response

The employees at Trophy Depot are trying to determine how many individual trophy boxes will fit in a larger packing box. Diagrams of the individual trophy box and the larger packing box are shown below. Both the individual trophy box and the larger packing box have square bases and heights that are two inches taller than their widths.



Not drawn to scale

1. Provide a simplified polynomial expression that could be used to model the volume of the individual trophy box. Interpret each term of the simplified expression as it relates to the volume of the individual trophy box.

2. The larger packing box is designed to be as wide as four trophy boxes. Provide a simplified polynomial expression that could be used to model the volume of the larger packing box. Interpret each term of the simplified expression as it relates to the volume of the larger packing box.

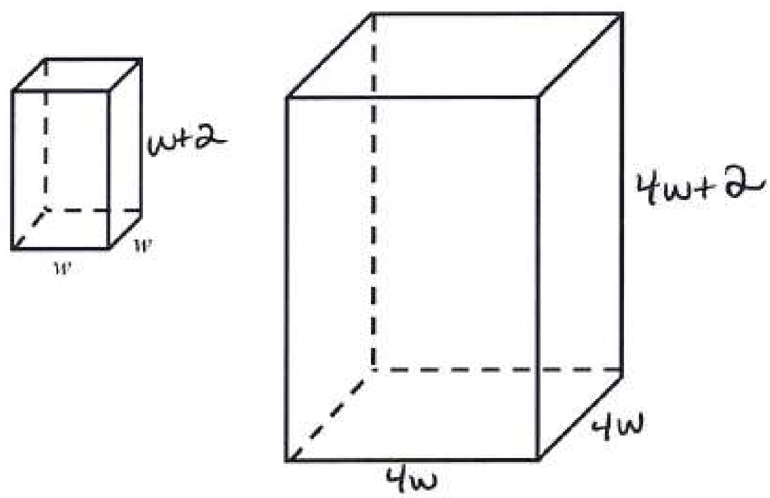
3. If the trophy boxes are three inches wide, what is the greatest number of individual trophy boxes that can fit into one packing box? Explain your reasoning.

Task adapted with permission from Universal Achievement, LLC.

Extended Constructed Response Exemplar Response

The employees at Trophy Depot are trying to determine how many individual trophy boxes will fit in a larger packing box. Diagrams of the individual trophy box and the larger packing box are shown below. Both the individual trophy box and the larger packing box have square bases and heights that are two inches taller than their widths.

Individual Trophy Box **Packing Box**



Not drawn to scale

1. Provide a simplified polynomial expression that could be used to model the volume of the individual trophy box. Interpret each term of the simplified expression as it relates to the volume of the individual trophy box.

Width = w in.; length = w in.; height = (w + 2) in.

Volume = area of the base x height

Volume = $[w^2(w+2)]in^3$

Volume = $(w^3 + 2w^2)\ in^3$

Each term of the expression could represent the volume of two smaller right rectangular prisms, which would be stacked on top of each other inside of the trophy box. The term w^3 *represents the volume of a cube with edge lengths of (w) in. The term* $2w^2$ *represents the volume of a right rectangular prism with a base area of* $(w^2)\ in^2$ *and a height of 2 in.*

2. The larger packing box is designed to be as wide as four trophy boxes. Provide a simplified polynomial expression that could be used to model the volume of the larger packing box. Interpret each term of the simplified expression as it relates to the volume of the larger packing box.

Width = 4w in.; length = 4w in.; height = (4w + 2) in.

Volume = area of the base x height

Volume = $[(4w)^2(4w+2)]in^3$

Volume = $(64w^3 + 32w^2)\ in^3$

Each term of the expression could represent the volume of two smaller right rectangular prisms, which would be stacked on top of each other inside of the packing box. The term $64w^3$ *represents the volume of a cube with edge lengths of (4w) in. The term* $32w^2$ *represents the volume of a right rectangular prism with a base area of* $(16w^2)\ in.^2$ *and a height of 2 in.*

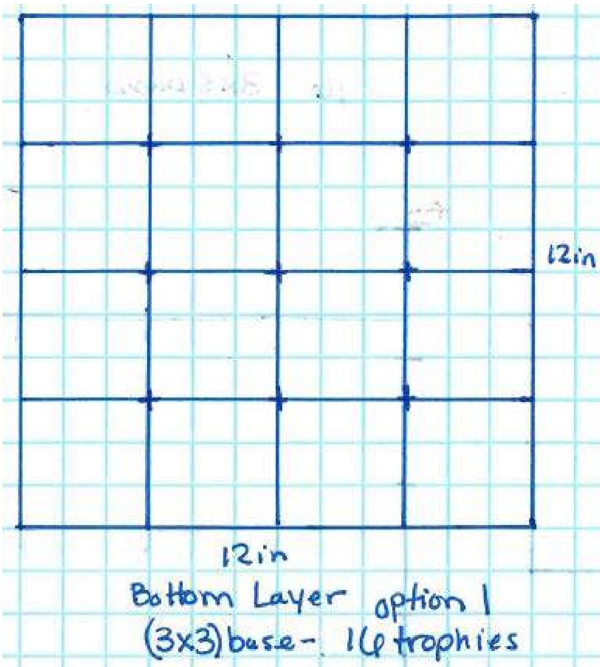
3. If the trophy boxes are three inches wide, what is the greatest number of individual trophy boxes that can fit into one packing box? Explain your reasoning.

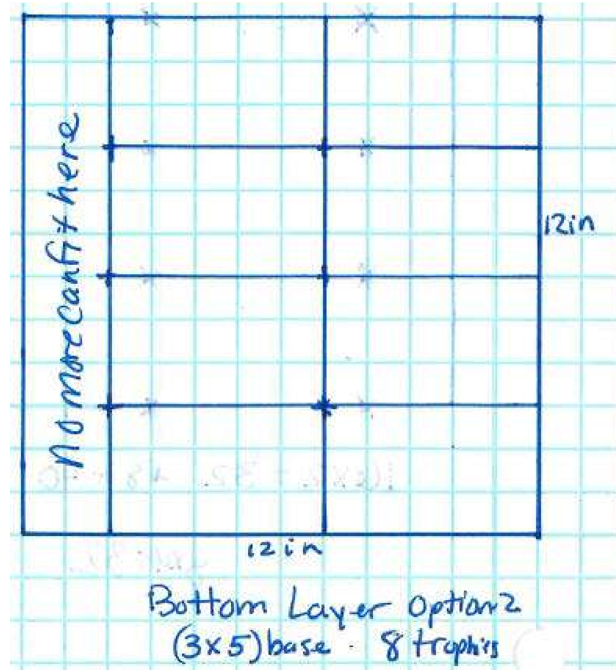
***Note: The ability to find the volume of the boxes in order to answer this question does not rely upon the expressions students wrote in the previous parts. If students do use incorrect expressions from questions 1 and 2, their work will need to be checked for correct reasoning and answers based on the incorrect expressions. This is a sample answer; other approaches and explanations are possible.*

$$\frac{Volume\ of\ Packing\ Box}{Volume\ of\ Ind. Trophy\ Box} = \frac{[64(3)^3 + 32(3)^2]in^3}{[(3)^3 + 2(3)^2]in^3} = \frac{2016}{45} = 44.8$$

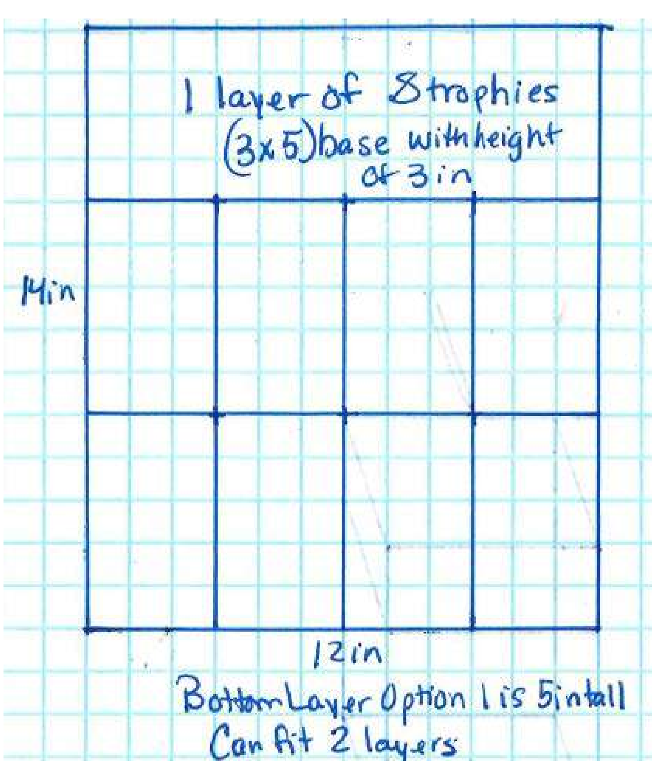
The volume of the packing box is 44.8 times the volume of one individual trophy box. Therefore, up to 44 individual trophy boxes could fit into the packing box. The length and width of the packing box are each 4(3) = 12 in. The height of the packing box is 4(3) + 2 = 14 in. The individual trophy boxes have a length and width of 3 in and a height of 5 in (3 + 2).

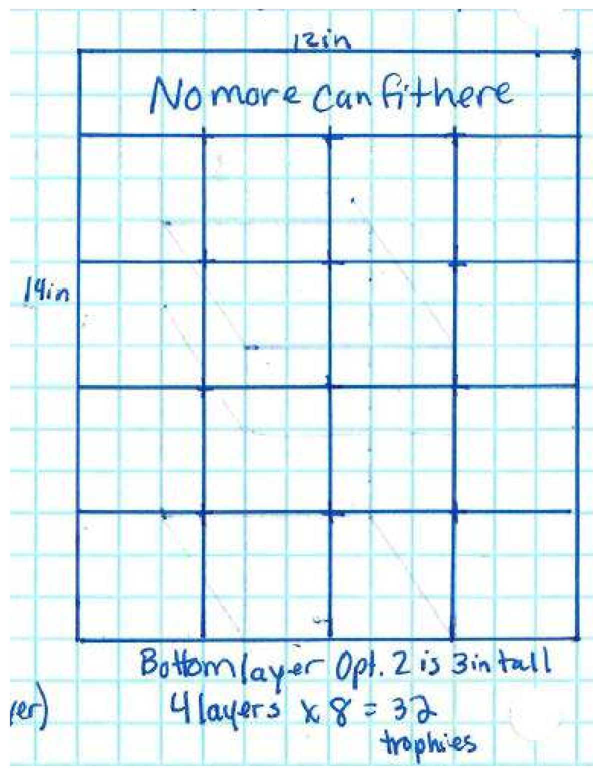
There are two ways I could arrange the trophies in the box—the individual trophy boxes could be laid horizontally to create a base of 3 in x 5 in for each individual trophy box or they could stand up vertically to create a base of 3 in x 3 in for each individual trophy box. Below are two drawings to show the arrangements. Option 1 would create a bottom layer of 16 trophies, while option 2 would create a bottom layer of 8 trophies.





To find how many layers I can fit into one packing box, I used the height of the packing box. Below are the pictures for each option to determine the number of layers.





Option 1 can fit 2 layers to give 32 trophies (16 x 2). In the space at the top, which is 12 in x 12 in x 4 in, I can fit one layer of trophies lying on their side like Option 2. That would be an additional 8 trophies for a total of 40 trophies that could fit in this box. There would be a space of [(2 in x 12 in x 3 in) + (1 in x 12 in x 12 in)] or 216 cubic inches that would not be able to fit any more trophies.

Option 2 can fit 4 layers of 8 trophies, which would be 32 trophies. However, no additional trophies could fit into the space at the top of the box. Also, no additional trophies would fit into the space on the side. So there would be a space of [(2 in x 12 in x 12 in) +(2 in x 12 in x 12 in)] or 576 cubic inches that would not be able to fit any more trophies.

Therefore, the greatest number of individual trophy boxes that are 3 inches wide that could fit into the larger packing box with a width of 12 inches is 40.

Rate of Change (ECR)

Overview

Students will answer questions concerning average rate of change, maximum height, and function transformations based on a given function.

Standards

Solve equations and inequalities in one variable.

HSA-REI.B.4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Interpret functions that arise in applications in terms of the context.

HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Build new functions from existing functions.

HSF-BF.B.3 Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.B.4b	• 7.EE.A.1 • 8.EE.A.2 • HSA-REI.B.4a	1. Solve the equation $x^2 - 26x + 100 = -20$ for *x*. a. $x = 6$ or $x = 20$	• http://www.illustrativemathematics.org/illustrations/541 • http://www.illustrativemathematics.org/illustrations/1827 • http://learnzillion.com/lessonsets/98-

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		2. Solve the equation $5x^2 + 10x - 13 = 0$ for *x*. a. $x = \frac{-5+3\sqrt{10}}{5}$ or $x = \frac{-5-3\sqrt{10}}{5}$ 3. http://www.illustrativemathematics.org/illustrations/375 4. http://www.illustrativemathematics.org/illustrations/586	solve-quadratic-equations-by-inspection-taking-square-roots-completing-the-square-the-quadratic-formula-and-factoring • http://learnzillion.com/lessonsets/26-understand-and-choose-methods-to-solve-quadratic-equations
HSF-IF.B.4	• 8.F.B.5 • HSN-Q.A.1 • HSF-IF.A.1	1. A shot-put throw can be modeled using the equation $y = -0.0241x^2 + x + 5.5$ where *x* is distance traveled (in feet) and *y* is height traveled (in feet). Find the maximum point of this function and explain the meaning of this point. a. The maximum point is approximately (20.75, 15.87). This means that the maximum height of the shot-put throw was 15.87 feet and that maximum height occurred when the shot put had traveled 20.75 feet. 2. http://www.illustrativemathematics.org/illustrations/637 3. http://www.illustrativemathematics.org/illustrations/639	• http://www.illustrativemathematics.org/illustrations/628 • http://www.illustrativemathematics.org/illustrations/632 • http://www.illustrativemathematics.org/illustrations/85 • http://www.illustrativemathematics.org/illustrations/588 • http://www.illustrativemathematics.org/illustrations/630 • http://learnzillion.com/lessonsets/477-graph-quadratic-functions-and-show-intercepts-maxima-and-minima • http://learnzillion.com/lessonsets/470-graph-linear-functions-and-show-intercepts-maxima-and-minima
HSF-IF.B.6	• 8.F.B.4 • HSF-IF.A.2	1. The Just for Fun T-shirt company used the function $P(q) = -100 + 0.5q + 0.01q^2$ to determine the profit $P(q)$, in dollars, of selling *q* T-shirts. Compare the average rate of change between 100 T-shirts and 200 T-shirts to the average rate of change between 200 T-shirts and 300 T-shirts. a. The average rate of change between 100 and 200 shirts is \$3.50 per shirt. The average rate of change between 200 and 300 shirts is \$5.50 per shirt. This shows that the amount of profit per shirt made is increasing as more shirts are sold. 2. http://www.illustrativemathematics.or	• http://www.illustrativemathematics.org/illustrations/120 • http://www.illustrativemathematics.org/illustrations/247 • http://www.illustrativemathematics.org/illustrations/634 • http://www.illustrativemathematics.org/illustrations/626

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		g/illustrations/577 3. http://www.illustrativemathematics.org/illustrations/1500	
HSF-BF.B.3		1. Given the function $f(x) = 2(x-1)^2 + 3$, write the function $h(x) = f(x) + 2$. a. $h(x) = 2(x-1)^2 + 5$. 2. Given the functions $f(x) = 2(x-1)^2 + 3$ and $h(x) = f(x) - 6$, describe how the graphs of $f(x)$and $h(x)$ are different. a. The *y*-intercept of $f(x)$ is (0, 5). The graph of $h(x)$ will shift the graph of $f(x)$ down 6 units, so the *y*-intercept of $h(x)$ will be (0, –1). 3. http://www.illustrativemathematics.org/illustrations/742 4. http://www.illustrativemathematics.org/illustrations/695	• http://learnzillion.com/lessonsets/766-manipulate-quadratic-functions • http://learnzillion.com/lessonsets/764-manipulate-linear-functions

After the Task

Students who struggle with finding the average rate of change for the given intervals may benefit from additional work with finding the slope of lines given two points. Students need to understand that the average rate of change for non-linear functions is the slope of the line connecting two points on the graph of the given function. Have students graph the quadratic function and identify the points on the graph for the specified intervals. Then draw a line through the two identified points. Students should then be able to find the slope of the line they have drawn. For students who need additional scaffolding, use this technique with quadratic functions (and other nonlinear functions) that have simpler numbers.

Students may also struggle with solving the given equation in order to find when the object hits the ground and find the maximum height. Have students state what information they know about the object hitting the ground in terms of time and height. Additionally, relate the solution and the maximum height to the graph of the function and have students answer the questions from the graph. Connect the graphical solution with the algebraic solution.

Student Extended Constructed Response

An object is launched at an initial velocity of 19.6 meters per second from a 58.8-meter-tall platform. This situation can be modeled by the function $h(t) = -4.9t^2 + 19.6t + 58.8$ where t represents the time, in seconds, that the object is in motion, and $h(t)$ is the height of the object.

1. Find the average rate of change from 0 seconds to 1 second. Find the average rate of change from 1 second to 2 seconds. Explain what each average rate of change means based on the problem. Compare the two average rates of change to explain what is happening with the object.

2. Find the average rate of change from 3 seconds to 5 seconds and explain its meaning.

3. What is the maximum height of the object? When does the object reach its maximum height?

4. When does the object hit the ground? How do you know?

5. What is the average rate of change between 1 second and 3 seconds? Why?

6. How would the function and graph of the function change if the object were launched from a platform that is 63.8 meters tall? Explain your reasoning.

Extended Constructed Response Exemplar Response

An object is launched at an initial velocity of 19.6 meters per second from a 58.8-meter-tall platform. This situation can be modeled by the function $h(t) = -4.9t^2 + 19.6t + 58.8$ where *x* represents the time, in seconds, that the object is in motion, and $h(t)$ is the height of the object.

1. Find the average rate of change from 0 seconds to 1 second. Find the average rate of change from 1 second to 2 seconds. Explain what each average rate of change means based on the problem. Compare the two average rates of change to explain what is happening with the object.

   $\frac{h(1)-h(0)}{1-0} = \frac{73.5-58.8}{1} = 14.7$ *meters per second. This means that the object is moving upward (because it's positive) at an average rate of 14.7 meters per second on this time interval.*

   $\frac{h(2)-h(1)}{2-1} = \frac{78.4-73.5}{1} = 4.9$ *meters per second. This means that the object is moving upward at an average rate of 4.9 meters per second on this time interval.*

   The average rate of change during the second interval (1 second to 2 seconds) is slower than the first interval (0 seconds to 1 second) because as the object is traveling upward it will start to slow down as it reaches its maximum height and begins its descent. It is still traveling upward because the average rate of change is positive for both intervals.

2. Find the average rate of change from 3 seconds to 5 seconds and explain its meaning.

   $\frac{h(5)-h(3)}{5-3} = \frac{34.3-73.5}{2} = -19.6$ *meters per second. This means the object is falling at an average rate of 19.6 meters per second. It is falling because the rate is negative.*

3. What is the maximum height of the object? When does the object reach its maximum height?

   The maximum height of the object is reached at 2 seconds and will be located at 78.4 meters.

4. When does the object hit the ground? How do you know?

   The object will hit the ground at 6 seconds. The object is on the ground when h(t) = 0 since h(t) represents the height of the object. So we solve the quadratic equation when set equal to zero, which means to find the values of t that would make the equation true. There are two answers: 6 and –2. Since t represents time in seconds and time cannot be negative, the answer must be 6 seconds.

5. What is the average rate of change between 1 second and 3 seconds? Why?

   The average rate of change between 1 second and 3 seconds is 0 meters per second because the object reached its maximum height at 2 seconds, which means it traveled the same distance from 1 to 2 seconds as it did from 2 to 3 seconds, only in opposite directions. Also, if a line were drawn such that it passes through the two points (1, 73.5) and (3, 73.5), it would be a horizontal line as the h(t) values are the same. Horizontal lines have a slope of zero, and the average rate of change between two points is the slope of the line that passes through the two points.

6. How would the function and graph change if the object were launched from a platform that is 63.8 meters tall? Explain your reasoning.

   The function would become $h(t) = -4.9t^2 + 19.6t + 63.8$ *since the initial height of the object now changed. This shifts the graph of the function up by 5 units because the platform is 5 meters higher than the original. The maximum height would then be 83.4 meters. The time the object hits the ground would be approximately 6.13 seconds.*

Ice-Cream Cones (IT)

Overview

Students will use their knowledge of surface area and area of circles to find a formula to determine the surface area of a cone. Then students will use that formula and the actual measurements of an ice-cream cone to determine the surface area needed to wrap an ice-cream cone. Students will then be able to use their calculations to estimate how many wrappers can be cut from a sheet of paper with specific dimensions.

Standards

Create equations that describe numbers or relationships.

HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.1	• 7.EE.B.4 • 8.EE.C.7	1. A circle with radius *r* is cut out of a square piece of paper whose side lengths are equal to the diameter of the circle. Write an equation, in terms of *r*, to find the area of the square remaining once the circle is removed. a. $A = 4r^2 - \pi r^2$ *OR* $A = r^2(4 - \pi)$ *OR* $A = (2r)^2 - \pi r^2$ 2. http://www.illustrativemathematics.org/illustrations/1124	• http://www.illustrativemathematics.org/illustrations/643 • http://www.illustrativemathematics.org/illustrations/583 • http://www.illustrativemathematics.org/illustrations/999 • http://learnzillion.com/lessonsets/120-create-equations-and-inequalities-in-one-variable-and-use-them-to-solve-problems

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is lateral area?*** Lateral area is calculated for three-dimensional figures. It is the area of the faces of the figure, not including the base.
- ***What is a net?*** In terms of three-dimensional figures, a net is a two-dimensional representation of the surface of a three-dimensional figure such that if you fold the net, it would create the corresponding three-dimensional figure. The net should include all faces, including the base, of the three-dimensional shape.

During the Task

This task is likely best used with groups of 2-3 students so they can work together to write the equations and work through the problem.

Students will be better positioned to complete this task if they are provided with actual ice-cream cones, or other models of cones, which they can manipulate to create the net. Students will have been introduced to the term *net* in 6th grade. While this may be the first time they have created a net for a cone, they should be able to use their previous understanding of the term *net* to complete the task. If some scaffolding is necessary, use the template of the net given at the end of the task.

Students will have to determine the area of the sector of a circle in order to find the lateral area of a cone. While this is a formula reserved for use in geometry, students should be able to apply their understanding of proportional reasoning in order to create the formula for the area of the sector using the variables s and r for slant height and radius, respectively. Students may need assistance to see the lateral area as being a portion of a larger circle. Have students trace multiple copies of the sector with edges touching to help them see how they might determine the area of that one part of the circle. Students should be given some time to struggle with this task and persevere through working the problem in order to arrive at the solution.

Some questions that could be asked to prompt student thinking are:

1. What information do you know about the circular lid of the cone that could help you find some information about the lateral area of the cone?
2. How does the circumference of the circular lid compare to the circumference of the larger circle with the center as the vertex of the cone and the radius length equal to the length of the slant height?

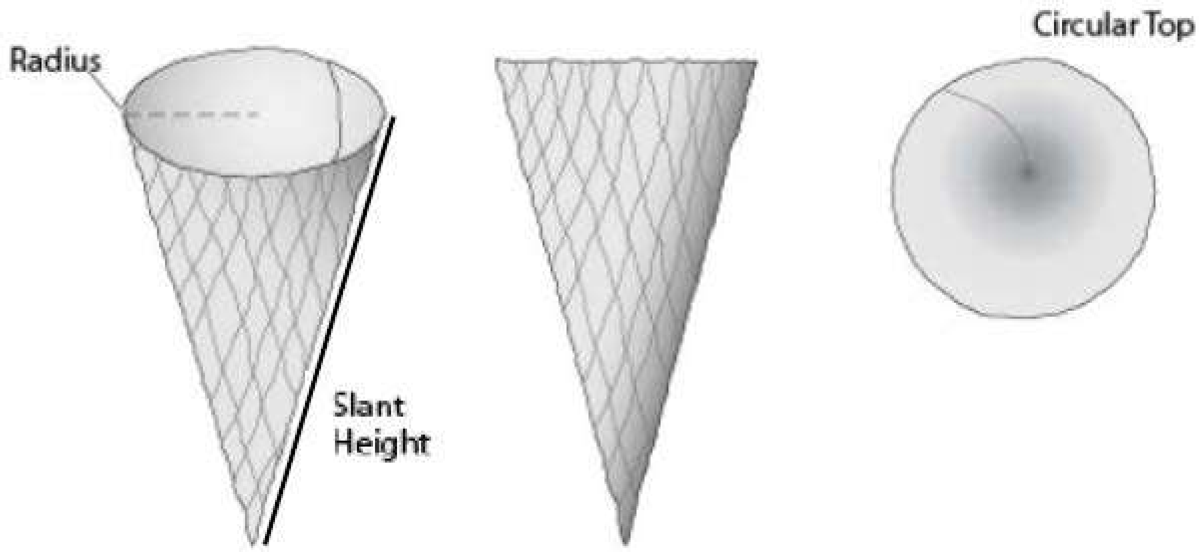
When students are estimating the number of wrappers to be cut in part 5, it may be helpful to give students a copy of the net included in this task or to encourage them to draw a net based on the measurements they found for a cone. It may also be helpful to give students paper they can use to manipulate in order to help make their estimations.

After the Task

Have groups of students explain their estimations for part 5 to the class. Have the class discuss which they think is the best estimation and explain their reasoning. Have students discuss how the answer would change if they had to figure in overlap for the wrapper.

Student Instructional Task

You have been hired by the owner of a local ice-cream parlor to assist in his company's new venture. The company will soon sell its ice-cream cones in the freezer section of area grocery stores. The manufacturing process requires that the ice-cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disk covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones.



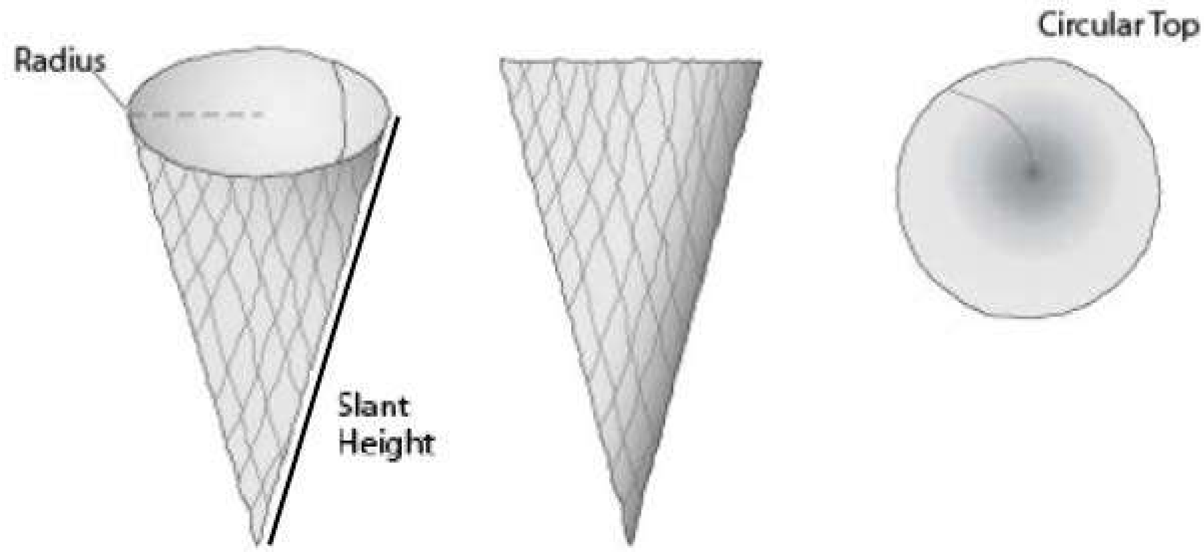
Use a separate sheet of paper for all calculations and explanations.

1. Generate a net of the complete wrapper for the cone and lid, ignoring the overlap required for assembly. The lid should rest on the cone.
2. Use the net of the wrapper without the lid to develop a formula to calculate the lateral area of the cone. The lateral area of a three-dimensional figure is the area of the faces, not including the base. Write or explain the process you used to determine the formula.
3. Provide the ice-cream company owner with a single formula that he can use to find the surface area, *SA*, of a wrapper, including the lid, for any size cone if he knows the radius, *r*, of the base and the slant height, *s*, of the cone.
4. Find the measurements of a real ice-cream cone, either by measuring an actual ice-cream cone, or by researching the dimensions of an ice-cream cone. Use these measurements to calculate the total surface area of the of the ice-cream cone. Show your calculations.
5. The company has large rectangular pieces of paper that measure 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit the cone used in #4 that could be cut from this one piece of paper. Explain your estimate. Would your estimation technique work for wrappers of other sizes or shapes?

Task adapted from http://www.utdanacenter.org/k12mathbenchmarks/tasks/7_coneslaunch.php.

Instructional Task Exemplar Response

You have been hired by the owner of a local ice-cream parlor to assist in his company's new venture. The company will soon sell its ice-cream cones in the freezer section of area grocery stores. The manufacturing process requires that the ice-cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disk covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones.



1. Generate a net of the complete wrapper for the cone and lid, ignoring the overlap required for assembly. The lid should rest on the cone.

   Check that students have an accurate net for the situation. One possible representation is shown below. Note that the placement of the circle may vary.

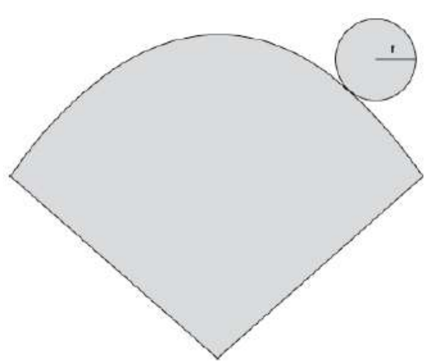
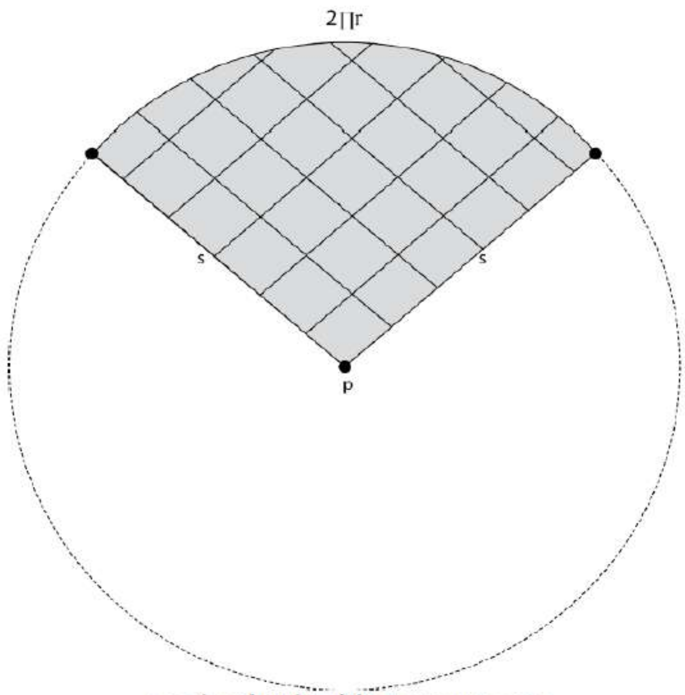


Figure not drawn to scale

2. Use the net of the wrapper without the lid to develop a formula to calculate the lateral area of the cone. The lateral area of a three-dimensional figure is the area of the faces, not including the base. Write or explain the process you used to determine the formula.

Teacher notes: To generate a formula for the lateral area of a cone, sketch a circle with the center as the vertex of the cone; the radius has the same measure as the slant height of the cone.

The circumference of the circular top of the cone, where r is the radius of the base of the cone, is 2πr. This is also the length of the arc of the sector.



s = *slant height of the ice cream cone*

To find the lateral area of the cone, set up a proportion of corresponding ratios:

$$\frac{area\ of\ a\ sector}{area\ of\ circle} = \frac{length\ of\ arc\ of\ sector}{total\ circumference\ of\ circle}$$

$$\frac{A}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

$$\pi s^2 \cdot \frac{A}{\pi s^2} = \frac{2\pi r}{2\pi s} \cdot \pi s^2$$

$$A = \pi r s$$

3. Provide the ice-cream company owner with a single formula that he can use to find the surface area, *SA*, of a wrapper, including the lid, for any size cone if he knows the radius, *r*, of the base and the slant height, *s*, of the cone.

   Lateral area (from #2) + Area of circle lid $SA = \pi rs + \pi r^2$

4. Find the measurements of a real ice-cream cone, either by measuring an actual ice-cream cone, or by researching the dimensions of an ice-cream cone. Use these measurements to calculate the total surface area of the of the ice-cream cone. Show your calculations.

   Teacher notes: Measurements for a standard sugar cone are approximately 11.5 cm for slant height and 2.5 cm for radius. (Answers will vary if a different cone size is used and according to the level of measurement precision.)

   Total surface area: $SA = \pi rs + \pi r^2$

   $SA = \pi(2.5)(11.5) + \pi(2.5)^2$

   $SA = 110.0\ cm^2$

5. The company has large rectangular pieces of paper that measure 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit the cone used in #4 that could be cut from this one piece of paper. Explain your estimate. Would your estimation technique work for wrappers of other sizes or shapes?

   Answers will vary. Students may decide to lay out one copy of the net, then another, to explore a pattern for estimating the number of complete wrappers that could be cut from the large paper. The template on the next page could be used to help students in their estimation.

   Sample response:

   If the cone has a slant height (s) of 11.5 cm and a base radius (r) of 2.5 cm, then the lateral area of the cone is

$$A = \pi * 2.5 * 11.5$$

$$A = 28.75\pi\ cm^2$$

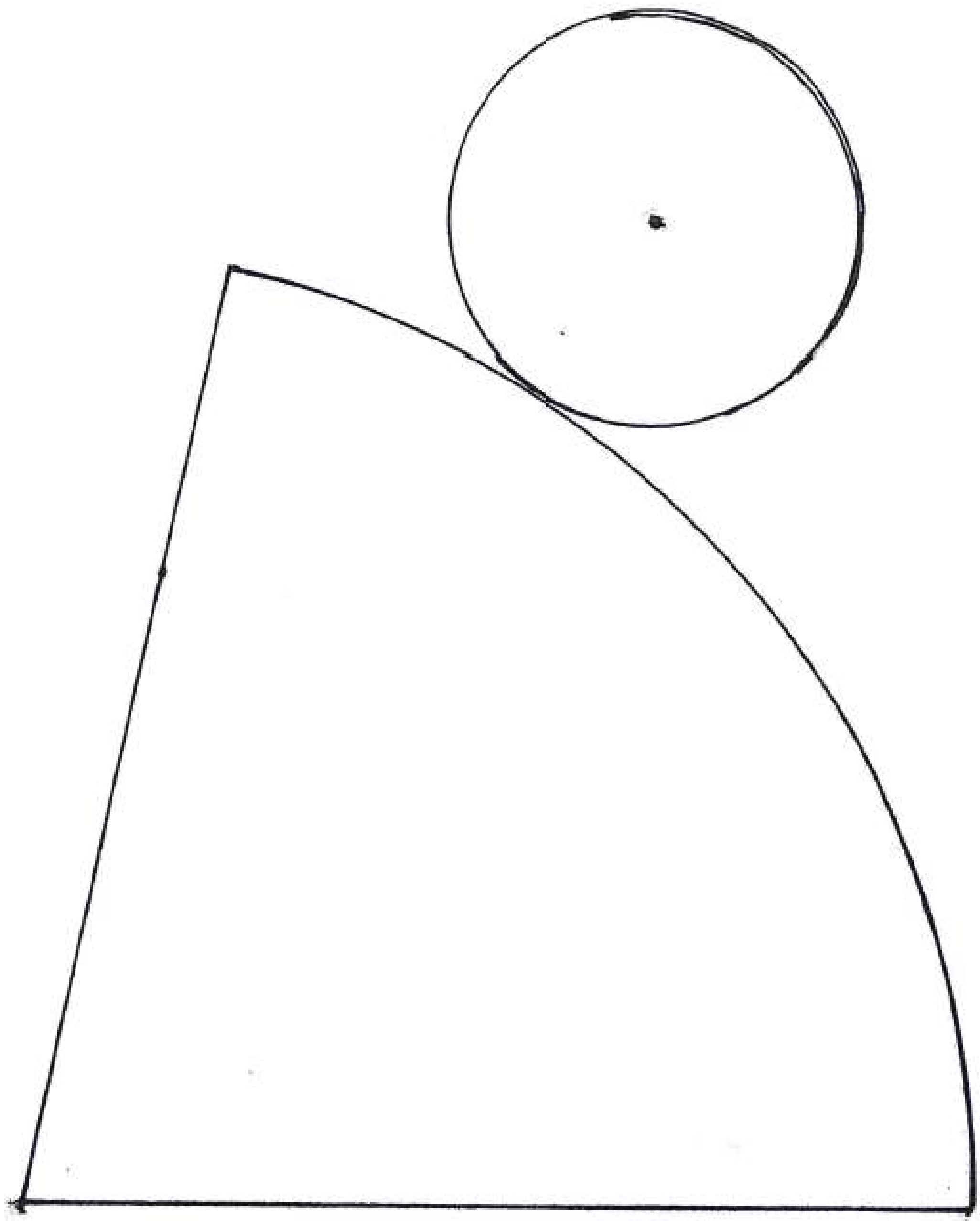
   The area of a circle with a radius of 11.5 cm (the same as the slant height of the cone) is 132.25π cm^2*. Therefore,* $\frac{132.25\pi\ cm^2}{28.75\pi\ cm^2} = 4.6$*, which means I can get 4 sectors with the lateral area of the cone from a circle with a radius of 11.5 cm. If I divide the 100 cm x 150 cm paper into squares with side lengths of 23 cm, I can get 24 circles with a radius of 11.5 cm (or diameter of 23 cm). This would be 6 squares by 4 squares (150/23 ≈ 6.5 and 100/23 ≈ 4). If I can get 4 cone pieces (lateral area only) from one circle, I can get 96 cone pieces (lateral area only) from 24 circles (24 x 4).*

   Now I need to determine if I could get at least 96 cone lids (the circular base) from the remaining paper. From the 150 cm length of the paper, there would be 12 cm remaining (150 cm – 6 x 23 cm = 12 cm). So this rectangle would be 12 cm x 100 cm. The circular base has a diameter of 5 cm (2.5 cm x 2). Using the same reasoning as I did with the large circles, I could divide the 12 cm by 100 cm area into squares of 5 cm on each side—there would be two squares by 20 squares (12/5 = 2.4 and 100/5 = 20). This would mean I could cut 2 x 20 small circles from that area, for a total of 40 circles. The remaining area on the 100 cm side would be 8 cm (100 cm – 4 x 23cm = 8 cm) by 138 cm. I would be able to get one 5 cm square from the 8 cm side and 27 squares from the 138 cm side for a total of 27 additional small *circles. Therefore, I would only be able to get 27 circles from this area. That would be a total of 67 circles (29 fewer than I need). From each of the 24 circles created to produce the lateral sides of the cone, there is a sector remaining for each circle from which one small circle (lid) could be cut, giving an additional 24 small circles. The total is now 91 small circles.*

So my estimate is a total of 91 complete cone wrappers from a sheet of paper 100 cm x 150 cm.

Task adapted from ***http://www.utdanacenter.org/k12mathbenchmarks/tasks/7_coneslaunch.php****.*

Cone template (slant height = 11.5 cm and radius of circle = 2.5 cm; may not be to scale due to document formatting)



Lemonade Stand (IT)

Overview

Students will use systems of inequalities to decide how many pitchers of regular and strawberry lemonade to make and sell.

Standards

Create equations that describe numbers or relationships.

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

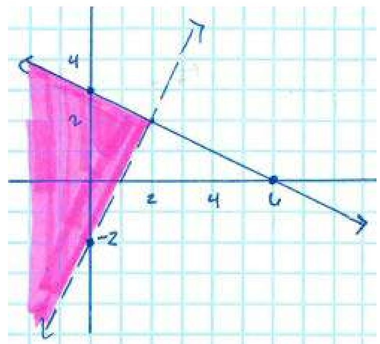
Represent and solve equations and inequalities graphically.

A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.3	• HSA-CED.A.1 • HSA-CED.A.2 • HSA-REI.D.10	1. A local bicycle shop makes $75 on each Model A bike and $90 on each Model B bike. The overhead costs for making the bikes are $1,350. Write an inequality to show how many of each bike model must be sold so the company avoids losing money. a. $75a + 90b \geq 1350$ 2. D'Andre earns some money by helping people with their yards. It takes him an average of 2 hours to weed a flowerbed and an average of 1.5 hours to mow and edge a lawn. He can work no more than 6 hours a day. He charges $60 to weed a flowerbed and $75 to mow and edge a lawn. He needs to cover his expenses of $150 to make a profit. Write a system of inequalities that would help determine how many flowerbeds and lawns he could complete on a given day. a. $2f + 1.5m \leq 6$	• http://www.illustrativemathematics.org/illustrations/582 • http://www.illustrativemathematics.org/illustrations/1351 • http://www.illustrativemathematics.org/illustrations/1066 • http://learnzillion.com/lessonsets/667-represent-constraints-by-equations-inequalities-and-systems • http://learnzillion.com/lessonsets/516-represent-constraints-by-linear-equations-inequalities-and-systems-interpret-solutions-as-viable-and-nonviable

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		$60f + 75m \geq 150$ Where f = # of flowerbeds and m = # of lawns to be mowed and edged 3. http://www.illustrativemathematics.org/illustrations/1351	
HSA-REI.D.12	• HSA-REI.C.5 • HSA-REI.C.6 • HSA-REI.D.11	1. Graph the solution set to the system $\begin{cases} x + 2y \leq 6 \\ y > 2x - 2 \end{cases}$ a.  2. http://www.illustrativemathematics.org/illustrations/644 3. http://www.illustrativemathematics.org/illustrations/1205	• http://www.illustrativemathematics.org/illustrations/1363 • http://www.illustrativemathematics.org/illustrations/1033 • http://www.illustrativemathematics.org/illustrations/618 • http://learnzillion.com/lessonsets/678-graph-the-solutions-to-a-linear-inequality-as-a-halfplane-graph-the-solutions-to-a-system-of-linear-inequalities-as-an-intersection-of-halfplanes • http://learnzillion.com/lessonsets/624-graph-the-solutions-to-a-linear-inequality-as-a-halfplane-and-the-solutions-to-a-system-of-linear-inequalities-as-an-intersection-of-halfplanes

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is a lemonade stand?*** A lemonade stand is a way some young people earn some money. Typically, the person sets up a table in his or her yard or another location that people pass by often. Then the person sells cups of lemonade to those who stop by the stand.

During the Task

- Some students may attempt to write and graph a system of equations rather than a system of inequalities. Ask students to discuss how to represent the term *at most* when constructing the models for this task.
- If students forget to shade in the solution for the inequalities, ask them to identify the solution on their graph. Have the students explain what the solution represents. Then ask students if other ordered pairs in the coordinate plane would satisfy the conditions given in the task. Use probing questions and class discussions to guide students to see that the solutions lie in a region bound by the lines and the axes.

- Students have to make some assumptions about how many servings are in the one-gallon pitchers. Students can choose the serving size they wish. They will need to state this assumption in their explanation to determine the amount of money Lilla and Siriana will earn.
- Encourage students to determine if the number of pitchers of lemonade they chose would result in Lilla and Siriana making the most amount of money. This will require them to try other solutions in the system. Have students explain how they determined the number of pitchers that would produce the most earnings.
- For part 2 of the task, students may be assigned a specific scenario or students may choose the scenario. Some of the given options are more difficult and therefore could be reserved for students who have a strong command of the content, while the other scenarios could be used for those students who are still working to master the content.
- During part 2 of the task, if students are struggling to determine how their original recommendation would change, encourage students to use transparencies or sheet protectors as an overlay to create new graphs so they can see the difference in the solution sets.
- For scenario 4, students may choose to use either price: $0.75 or $1.25. Encourage strong groups to investigate both prices and determine which would be the better option.

After the Task

Once students have completed the task, have them share their answers with the class. Have a class discussion about the various procedures students used to complete the task. Also identify the number of pitchers for each type of lemonade that would produce the largest earnings based on the different serving sizes students used. Have students share their work and reasoning based on the given scenarios for part 2. Allow students to critique the reasoning of other students and facilitate a discussion around which recommendations would be best.

For the first scenario, ask the class to determine if there is a better price to charge for the regular lemonade that would allow the friends to make more money after they have paid for supplies. Also, have the class determine if Lilla and Siriana can pay for the supplies and still make a minimum of $300 based on the number of hours each person wants to work. Have students explain their reasoning using graphs, tables, charts, etc. Also, have students determine if they would recommend that Lilla and Siriana should choose the original plan or if they should choose one of the different scenarios.

Student Instructional Task

Lilla and Siriana plan to open a lemonade stand during the summer. The friends have divided the work as described below. The friends agree to sell regular lemonade for $0.75 per serving and strawberry lemonade for $1.25 per serving.

Lilla decides that she will make two flavors of lemonade: regular and strawberry.

- It takes her 10 minutes to make a one-gallon pitcher of regular lemonade.
- It takes her a 15 minutes to make a one-gallon pitcher of strawberry lemonade.
- She plans to spend no more than four hours making lemonade.

Siriana has decided to work outside and sell the lemonade.

- She plans to spend no more than six hours selling lemonade.
- She estimates she will sell one pitcher of lemonade (of either flavor) every third of an hour.

1. How many pitchers of each type of lemonade would you recommend Lilla and Siriana make and sell? Explain your reasoning. Use equations, inequalities, graphs and/or tables to aid your explanation. In your explanation, also include how much money Lilla and Siriana will earn if they sell every serving in every pitcher of lemonade you recommend they make.

2. Choose **one** of the following scenarios to investigate. Determine how your answer might change based on the scenario you choose. Explain your reasoning. Use equations, inequalities, graphs, and/or tables to aid your explanation.
    - Lilla and Siriana have to pay for the supplies to make the lemonade from their earnings. It costs $10 to make a pitcher of regular lemonade and $12 to make a pitcher of strawberry lemonade.
    - Siriana plans to spend no more than 4 hours selling lemonade.
    - Siriana plans to spend no more than 7 hours selling lemonade.
    - The friends sell both types of lemonade for the same price.
    - Lilla and Siriana want their total earnings to be a minimum of $300.

Task adapted with permission from Universal Achievement, LLC.

Instructional Task Exemplar Response

Lilla and Siriana plan to open a lemonade stand during the summer. The friends have divided the work as described below. The friends agree to sell regular lemonade for $0.75 per serving and strawberry lemonade for $1.25 per serving.

Lilla decides that she will make two flavors of lemonade: regular and strawberry.

- It takes her 10 minutes to make a one-gallon pitcher of regular lemonade.
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- She plans to spend no more than four hours making lemonade.

Siriana has decided to work outside and sell the lemonade.

- She plans to spend no more than six hours selling lemonade.
- She estimates she will sell one pitcher of lemonade (of either flavor) every third of an hour.

1. How many pitchers of each type of lemonade would you recommend Lilla and Siriana make and sell? Explain your reasoning. Use equations, inequalities, graphs and/or tables to aid your explanation. In your explanation, also include how much money Lilla and Siriana will earn if they sell every serving in every pitcher of lemonade you recommended they make.

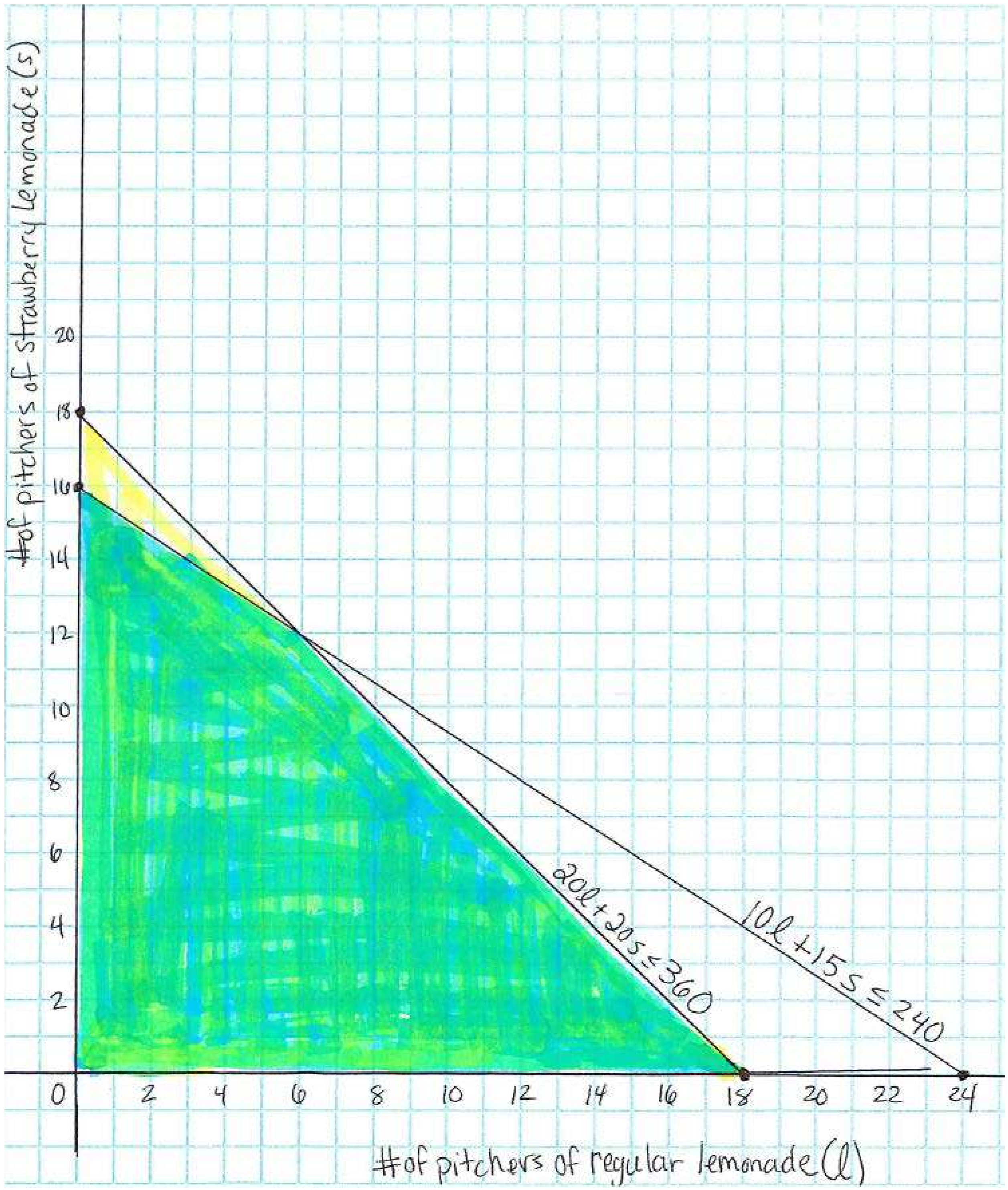
   Sample response: Students have many options to proceed with the work for this task. Below is one way to find an answer.

   Let l = the # of pitchers of regular lemonade to be made and sold; let s = the # of pitchers of strawberry lemonade to be made and sold.

   If Lilla wants to spend no more than four hours making lemonade, then she will spend no more than 4 x 60 minutes or 240 minutes making lemonade. This can be represented by the inequality $10l + 15s \leq 240$, *where 10l represents the amount of time (in minutes) Lilla will take to make l pitchers of regular lemonade, and 15s represents the amount of time (in minutes) Lilla will take to make s pitchers of strawberry lemonade.*

   If Siriana wants to spend no more than six hours selling lemonade, then she will spend no more than 6 x 60 minutes or 360 minutes selling lemonade. This situation can be represented by the inequality $20l + 20s \leq 360$, *where 20l represents the amount of time she would sell l pitchers of regular lemonade and 20s represents the amount of time she would sell s pitchers of strawberry lemonade.*

   Below is a graph of this system of inequalities. The green shaded region represents all possibilities for the number of pitchers of each type of lemonade to be made and sold. I used the axes as additional constraints ($l \geq 0 \text{ and } s \geq 0$*) because Lilla cannot make a negative number of pitchers of lemonade.*



In order to decide the number of pitchers of each type of lemonade to be made and sold, I decided to test different points in the region with the prices they would charge. I think they would want to earn the most money.

First, I assumed that each serving would be 8 oz. of lemonade—in one gallon of lemonade, there would be sixteen 8 oz. servings. The price for one serving of regular lemonade is $0.75; if I multiply that by 16 servings in *one pitcher, the earnings would be $12 per pitcher. The price for one serving of strawberry lemonade is $1.25; if I multiply that by the 16 servings in one pitcher, the earnings would be $20 per pitcher. Therefore, I can use the expression 12l + 20s to find the total earnings for the number of pitchers sold.*

# of pitchers of regular lemonade sold	***# of pitchers of strawberry lemonade sold***	***Total earnings***
6	*12*	*12(6) + 20(12) = $312*
1	*15*	*12(1) + 20(15) = $312*
3	*14*	*12(3) + 20(14) = $316*
4	*13*	*12(4) + 20(13) = $308*
0	*16*	*12(0) + 20(16) = $320*
9	*9*	*12(9) + 20(9) = $288*
18	*0*	*12(18) + 20(0) = $216*

The most amount of money the girls can earn is $320, but that would mean that they would only serve strawberry lemonade. This may not be a good idea if there are people who don't like strawberry lemonade. So, I'm going to choose the next option, which would be 3 pitchers of regular lemonade and 14 pitchers of strawberry lemonade. Lilla would spend exactly 4 hours making the lemonade (because the point lies on boundary line of the graph of the inequality $10l + 15s \leq 240$ *while Siriana would spend 5 hours and 40 minutes selling the lemonade. The girls would make $316 if they serve 8 oz. at a time.*

2. Choose **one** of the following scenarios to investigate. Determine how your answer might change based on the scenario you choose. Explain your reasoning. Use equations, inequalities, graphs, and/or tables to aid your explanation.
    - Lilla and Siriana have to pay for the supplies to make the lemonade from their earnings. It costs $10 to make a pitcher of regular lemonade and $12 to make a pitcher of strawberry lemonade.
    - Siriana plans to spend no more than 4 hours selling lemonade.
    - Siriana plans to spend no more than 7 hours selling lemonade.
    - The friends sell both types of lemonade for the same price.
    - Lilla and Siriana want their total earnings to be a minimum of $300.
    - Lilla and Siriana want their profit to be a minimum of $300. It costs $10 to make a pitcher of regular lemonade and $12 to make a pitcher of strawberry lemonade.

***Note: Below are sample responses for how the students' answers might change. These responses are based on the recommendation from question 1 above.*

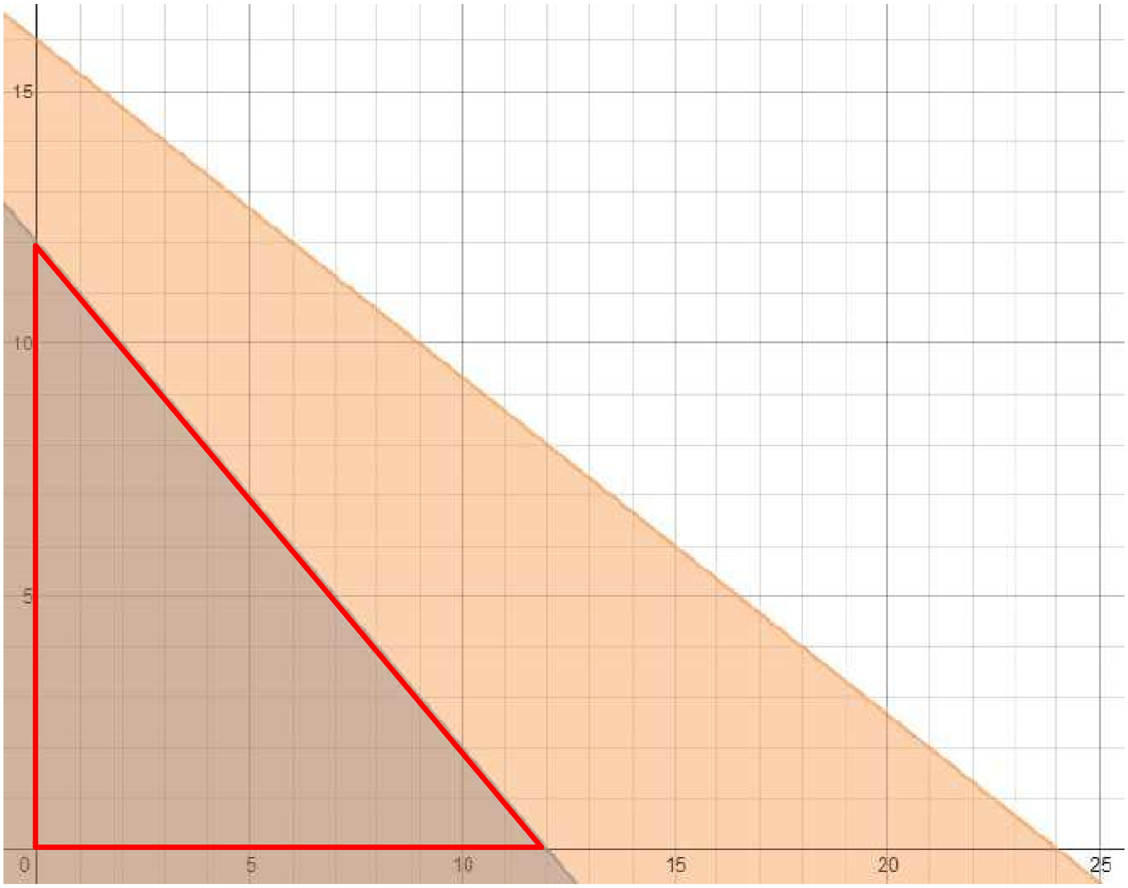
Scenario 1: Lilla and Siriana have to pay for the lemonade with their earnings. It costs $10 to make a pitcher of regular lemonade and $12 to make a pitcher of strawberry lemonade.

The total earnings Lilla and Siriana make can be represented by 12l + 20s, meaning that they will earn $12 per pitcher of regular lemonade and $20 per pitcher of strawberry lemonade. If they have to pay $10 per pitcher of regular lemonade and $12 per pitcher of strawberry lemonade, then they will actually make $2 for every pitcher of regular lemonade (12 – 10) and $8 for every pitcher of strawberry lemonade (20 – 12). Therefore, the amount of money they will have after paying for their supplies can be represented by 2l + 8s. The table below shows their earnings for different combinations of regular and strawberry lemonade.

# of pitchers of regular lemonade sold	***# of pitchers of strawberry lemonade sold***	***Earnings after paying for supplies***
6	*12*	*2(6) + 8(12) = $108*
1	*15*	*2(1) + 8(15) = $122*
3	*14*	*2(3) + 8(14) = $118*
4	*13*	*2(4) + 8(13) = $112*
0	*16*	*2(0) + 8(16) = $128*
9	*9*	*2(9) + 8(9) = $90*
18	*0*	*2(18) + 8(0) = $36*

Based on this table, making 16 pitchers of strawberry lemonade would make the most money again, but I would recommend they make 15 pitchers of strawberry and 1 pitcher of regular in case there are people who do not like strawberry lemonade. If they sell all servings in every pitcher, they would make $122 after paying for supplies.

Scenario 2: Siriana plans to spend no more than 4 hours selling lemonade.



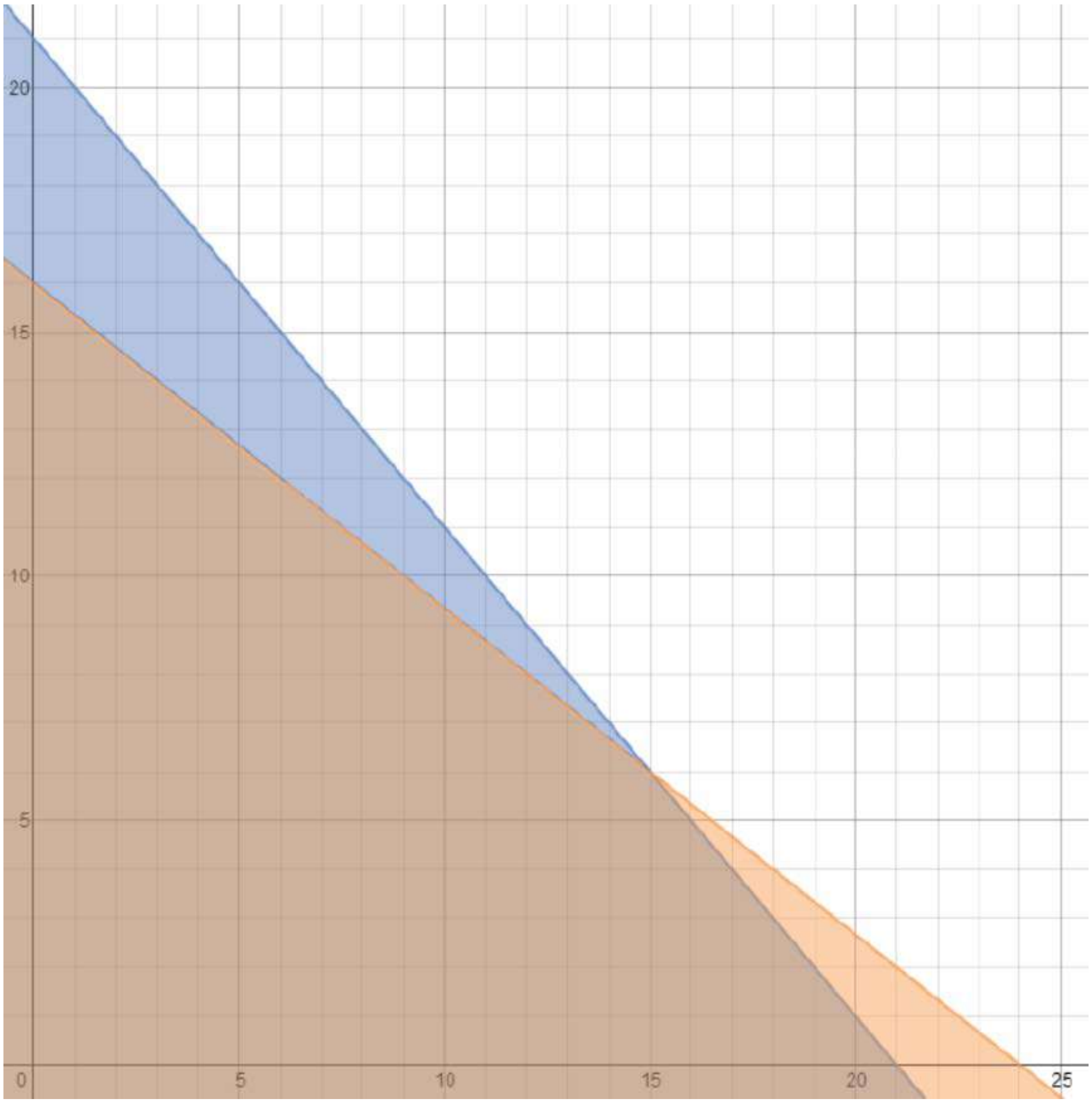
The graph above shows how the solution set changes. There is no intersection, which means that Lilla will always spend less than 4 hours making lemonade. The recommendation must come from the region bounded by the red triangle.

Using the same serving size and cost from my original recommendation, I created the table below to see what combination of pitchers of lemonade would result in the most earnings.

# of pitchers of regular lemonade sold	***# of pitchers of strawberry lemonade sold***	***Total earnings***
0	*12*	*12(0) + 20(12) = $240*
12	*0*	*12(12) + 20(0) = $144*
1	*11*	*12(1) + 20(11) = $232*
1	*10*	*12(1) + 20(10) = $212*
2	*10*	*12(2) + 20(10) = $224*

I know that the maximum number of pitchers to be made is 12 (based on the boundary line for $20l + 20s \leq 420$). I don't think it is a good idea to sell only one type of lemonade, so I recommend making and selling 1 pitcher of regular lemonade and 11 pitchers of strawberry lemonade to earn $232.

Scenario 3: Siriana plans to spend no more than 7 hours selling lemonade.



The graph above shows how the solution set changes. The intersection changes to (15, 6), which means that Lilla will spend exactly 4 hours making lemonade and Siriana will spend exactly 7 hours selling lemonade when they have 15 pitchers of regular lemonade and 6 pitchers of strawberry lemonade. The recommendation must come from the region bounded by the red quadrilateral.

Using the same serving size and cost from my original recommendation, I created the table below to see what combination of pitchers of lemonade would result in the most earnings.

# of pitchers of regular lemonade sold	*# of pitchers of strawberry lemonade sold*	*Total earnings*
15	*6*	*12(15) + 20(6) = $300*
6	*12*	*12(6) + 20(12) = $312*
1	*15*	*12(1) + 20(15) = $312*
3	*14*	*12(3) + 20(14) = $316*
4	*13*	*12(4) + 20(13) = $308*
0	*16*	*12(0) + 20(16) = $320*
9	*9*	*12(9) + 20(9) = $288*

I realize that the change in graph simply allows more combinations to be made. However, the total earnings do not change for the different combinations based on Siriana planning to sell for 7 hours instead of 6. Therefore, I will not change my recommendation to make and sell 3 pitchers of regular lemonade and 14 pitchers of strawberry lemonade to earn $316.

Scenario 4: The friends sell both types of lemonade for the same price.

If Lilla and Siriana spend 4 hours and 6 hours, respectively, on their chosen duties, then the solution set will remain the same. For this scenario, the change is in the total earnings. Based on 8 oz. servings, if the friends sell both types of lemonade for $0.75 per serving, they will earn $12 per pitcher. If they sell both types of lemonade for $1.25 per serving, they will earn $20 per pitcher. To make the most money, they should sell both types of lemonade for $1.25.

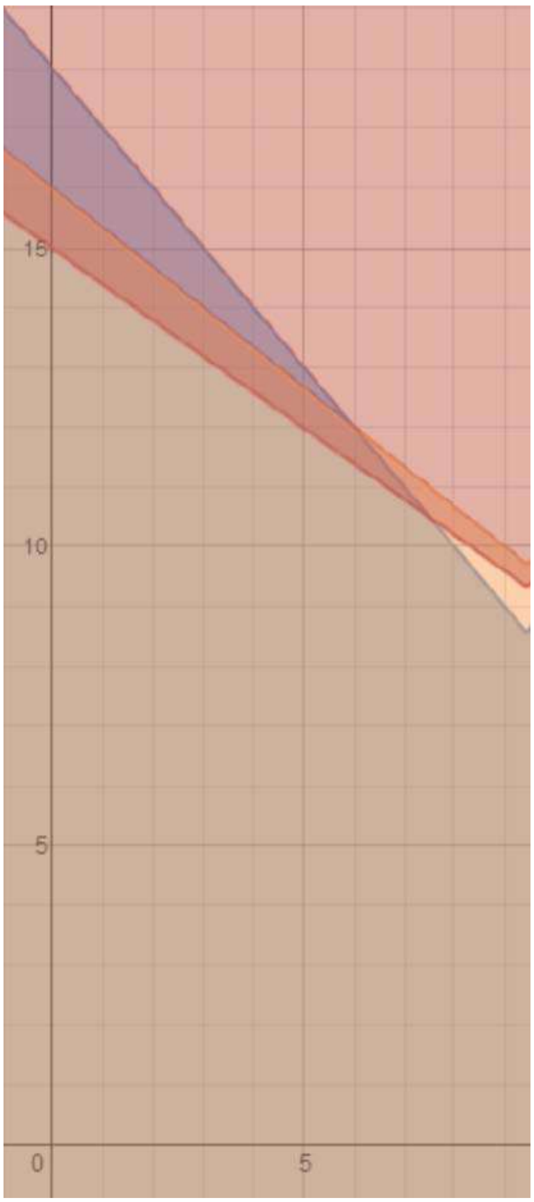
The greatest number of pitchers the friends can make and sell within the time they wish to work is 18. The combination of regular lemonade and strawberry lemonade does not matter here. As long as the friends make and sell a total of 18 pitchers, they will earn $360. Therefore, I recommend that they make and sell 9 pitchers of regular lemonade and 9 pitchers of strawberry lemonade so they might please the most people.

If they decided to sell the lemonade for $0.75, a total of 18 pitchers would still produce the most earnings. However, their earnings will only be $216.

# of pitchers of regular lemonade sold	*# of pitchers of strawberry lemonade sold*	*Earnings at $20 per pitcher*
6	*12*	*20(18) = $360*
1	*15*	*20(16) = $320*

3	*14*	*20(17) = $340*
4	*13*	*20(17) = $340*
9	*9*	*20(18) = $360*
18	*0*	*20(18) = $360*

Scenario 5: Lilla and Siriana want their total earnings to be a minimum of $300.



The graph above shows how the solution set changes with the addition of the constraint $12l + 20s \geq 300$*. There is no intersection of all three boundary lines. The recommendation must come from the region bounded by the red quadrilateral. Because this region is small and the solution must be in whole numbers* *(not allowing for part of a pitcher to be made and sold), the possible solutions are listed in the table below along with the total earnings.*

# of pitchers of regular lemonade sold	***# of pitchers of strawberry lemonade sold***	***Total earnings***
0	*16*	*12(0) + 20(16) = $320*
0	*15*	*12(0) + 20(15) = $300*
1	*15*	*12(1) + 20(15) = $312*
2	*14*	*12(2) + 20(14) = $304*
3	*14*	*12(3) + 20(14) = $316*
4	*13*	*12(4) + 20(13) = $308*
5	*12*	*12(5) + 20(12) = $300*
6	*12*	*12(6) + 20(12) = $312*
7	*11*	*12(7) + 20(11) = $304*

The most money the friends can earn is $320 by selling only strawberry lemonade. However, I believe this would not be a good idea, so my original recommendation to sell 3 pitchers of regular lemonade and 14 pitchers of strawberry lemonade to earn $316 is still my recommendation.

Solving Quadratic Equations (IT)

Overview

Students will reason about the solution methods chosen when solving quadratic equations.

Standards

Understand solving equations as a process of reasoning and explain the reasoning.

HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

HSA-REI.B.4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, using the quadratic formula, and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.A.1	• 7.EE.B.4a • 8.EE.C.7	1. Explain the steps to solving a quadratic equation by completing the square. a. First, if *a* is any value other than 1, divide by *a*. Second, subtract the value of the constant term from both sides. Then divide the coefficient of the linear term by 2 and square the result. Add the squared quotient to both sides of the equation. Next, rewrite the perfect square trinomial as a squared binomial. Then take the square root of both sides. Subtract the constant term in this step from both sides. Finally, simplify the resulting solution(s).	• http://www.illustrativemathematics.org/illustrations/392 • http://www.illustrativemathematics.org/illustrations/550 • http://learnzillion.com/lessonsets/495-justify-solutions-to-equations-in-terms-of-equation-properties • http://learnzillion.com/lessonsets/203-solve-and-explain-simple-algebraic-equations

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.B.4	• 8.EE.A.2	1. Solve $2x^2 - 3x = 7$. a. $x = \frac{3 \pm \sqrt{65}}{4}$ 2. http://www.illustrativemathematics.org/illustrations/375	• http://learnzillion.com/lessonsets/24-solve-quadratic-equations • http://learnzillion.com/lessonsets/98-solve-quadratic-equations-by-inspection-taking-square-roots-completing-the-square-the-quadratic-formula-and-factoring • http://learnzillion.com/lessonsets/26-understand-and-choose-methods-to-solve-quadratic-equations

During the Task

- Be sure students are not resorting to the quadratic formula for every solution. Ask students to think about the initial form of the equation before deciding which method they would choose to use. Discuss with students in which situations it might be better to complete the square rather than use the quadratic formula.
- For question 3, some students may struggle to create an equation with a complex solution. Ask students probing questions to remind them that the radicand in the quadratic formula must be negative in order to get a complex solution. Have them identify values for *a*, *b,* and *c* that will force the radicand to be negative.

After the Task

Allow students to discuss the various solution methods chosen for these problems. Provide students with other examples of quadratic equations that represent real-world situations and have students determine which method would be best to solve those equations.

Student Instructional Task

1. Tamara was asked to solve the equation $x^2 + 7x + 8 = -2$. Her work is shown below.

$$x^2 + 7x + 8 = -2$$
$$x^2 + 7x = -10$$
$$x^2 + 7x + \frac{49}{4} = \frac{9}{4}$$
$$\left(x + \frac{7}{2}\right)^2 = \frac{9}{4}$$
$$x + \frac{7}{2} = \pm\frac{3}{2}$$
$$x = -5, -2$$

   Show a different method of solving $x^2 + 7x + 8 = -2$. Explain why you chose your method and how the method you chose is different from Tamara's method.

2. The equation $200 = 2.2v + \frac{v^2}{20}$ can be used to approximate the speed of a specific model car (in miles per hour) that took 200 feet to brake.
   a. Based on the given equation, which method of solving a quadratic equation would be most appropriate? Explain your reasoning.

   b. Solve the equation, explaining each step in your solution.

3. Create a quadratic equation that has a complex solution. Use the quadratic formula to verify that the equation you created has a complex solution.

Instructional Task Exemplar Response

1. Tamara was asked to solve the equation $x^2 + 7x + 8 = -2$. Her work is shown below.

$$x^2 + 7x + 8 = -2$$
$$x^2 + 7x = -10$$
$$x^2 + 7x + \frac{49}{4} = \frac{9}{4}$$
$$\left(x + \frac{7}{2}\right)^2 = \frac{9}{4}$$
$$x + \frac{7}{2} = \pm\frac{3}{2}$$
$$x = -5, -2$$

Show a different method of solving $x^2 + 7x + 8 = -2$. Explain why you chose your method and how the method you chose is different from Tamara's method.

Teacher note: Students may choose any method that will produce the correct answer. Check students' work for accuracy.

$$x^2 + 7x + 8 = -2$$
$$x^2 + 7x + 10 = 0$$
$$(x + 5)(x + 2) = 0$$
$$x = -5, -2$$

I chose to use the factoring method because when I subtracted 2 from both sides, the constant term on the left side was 10. I know the factors of 10, 5 and 2, will add to seven, so I could write two factors of $(x + 5)$ *and* $(x + 2)$*. This method is different than Tamara's method of completing the square because I did not have to create a squared binomial or take the square root of both sides of the equation.*

2. The equation $200 = 2.2v + \frac{v^2}{20}$ can be used to approximate the speed of a specific model car (in miles per hour) that took 200 feet to brake.
   a. Based on the given equation, which method of solving a quadratic equation would be most appropriate? Explain your reasoning.

      Sample response:

      The most appropriate method to solve the equation would be to use the quadratic formula because the values for a and b ($\frac{1}{20}$ *and 2.2, respectively) do not make the equation easy to factor or to complete the square.*

   b. Solve the equation, explaining each step in your solution.

Solution	Explanation
$200 = 2.2v + \frac{v^2}{20}$	
$0 = \frac{v^2}{20} + 2.2v - 200$	*Subtract 200 from both sides to make the equation equal zero.*
$v = \frac{-2.2 \pm \sqrt{2.2^2 - 4\left(\frac{1}{20}\right)(-200)}}{2\left(\frac{1}{20}\right)}$	*Substitute the values of a, b, and c into the quadratic formula.*
$v = \frac{-2.2 \pm \sqrt{44.84}}{\frac{1}{10}}$	*Simplify the radicand.*
$v = 10\left(-2.2 \pm \sqrt{44.84}\right)$	*Simplify the complex fraction by multiplying the numerator and denominator by 10.*
$v = 10\left(-2.2 + \sqrt{44.84}\right) \text{ or } 10\left(-2.2 - \sqrt{44.84}\right)$	*Separate to find both solutions.*
$v \approx 44.96$	*Simplify the first expression because the second will result in a negative value, which does not make sense in this situation.*

The car was traveling at approximately 45 mph if it took 200 feet to brake.

***Note: Students may select other methods to solve this equation. Check their work for accuracy.*

3. Create a quadratic equation that has a complex solution. Use the quadratic formula to verify that the equation you created has a complex solution.

This will have multiple different solutions. Students must use the quadratic formula to show that the equation has a complex solution by having a negative value in the radicand. One possible solution is given below. Check solutions of other students for accuracy.

Sample equation: $3x^2 - 5x = -6$

Sample solution:

$$3x^2 - 5x = -6$$

$$3x^2 + 5x + 6 = 0$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{25 - 72}}{6}$$

$$x = \frac{5 \pm \sqrt{-47}}{6}$$

This results in a complex solution because the radicand is negative.

***Note: Students in Algebra I are NOT required to write the complex solutions to quadratic equations. They are only required to recognize when complex solutions exist.*

Investment Opportunity (IT)
Overview

Students will plot and use data about revenue from wireless customers and the number of wireless connections. Then the students will use the data and graphs to determine various rates of change and to make a decision about investing in a company for the future.

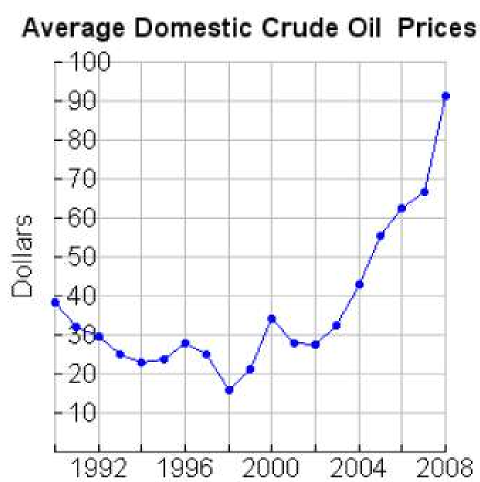
Standards

Interpret functions that arise in applications in terms of the context.

HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness	Sample Remediation Items
HSF-IF.B.6	• 8.F.B.4 • HSF-IF.A.2	1. The graph shows the average cost of a barrel of crude oil from 1998 to 2008. Estimate the average rate of change between 1998 and 2004 based on the graph below. Explain the meaning of average rate of change in terms of the information provided.  a. The average rate of change is approximately \$4.67 per year. This means the average cost of a barrel of crude oil increased by approximately \$4.67 per year from 1998 to 2004. 2. http://www.illustrativemathematics.org/illustrations/577 3. http://www.illustrativemathematics.org/illustrations/1500	• http://www.illustrativemathematics.org/illustrations/1206 • http://www.illustrativemathematics.org/illustrations/383 • http://www.illustrativemathematics.org/illustrations/664

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is revenue?*** Revenue is the amount of money a company makes when providing a service or product for consumers.
- ***What are wireless connections?*** In this task, the term *wireless connections* refers to the number of people with wireless (or cell) service with the company. These connections produce the revenue for the company.

During the Task

- Students may be tempted to find the rate of change by completing the actual math rather than estimating first. Remind them that estimating the rate of change first can help them determine if they have made any major mistakes in the work to find the actual average rate of change.
- For each part of the task, students are finding the average rate of change for two time intervals and then comparing them. The difference is how the information is graphed on the coordinate plane. Students will have to use this information to help them make sense of what the average rate of change means in the context of each part and how that could be useful in determining whether an investment in the company should be made.
- For part 3, students are graphing the revenue with respect to the number of connections. When determining the average rate of change here, students will need to identify the ordered pairs associated with the given time intervals. Also, when calculating the average rate of change, the teacher should monitor groups to be sure that students are not simply dividing the decimal values they obtain from subtracting. Remind them to be precise and pay attention to units for their answers. Guide students to find the average cost per connection rather than cost per million connections.

After the Task

Have groups share their recommendations and their reasoning. Allow students to question each group's recommendations to further their understanding or to critique the reasoning of different groups. This task can be related to finding the average change in the cost of tuition over the past 10-20 years, a factor that may help students decide where to pursue a secondary education.

Student Instructional Task

You have been hired as a financial analyst to help an investor determine whether to invest in various companies. A financial analyst might look at the average rates of change between multiple variables to determine whether the company is in a period of growth.

Below is a table that gives the annual revenue from one company's wireless business and the number of wireless connections that yield this revenue.

Year	Wireless Revenue (billions of dollars)	Wireless Connections (millions)
2006	38.0	59.1
2007	43.9	65.7
2008	49.3	72.1
2009	60.3	96.5
2010	63.4	102.2
2011	70.2	107.8

1. Graph the data for wireless revenue with respect to time. Be sure to label the axes.
    a. Estimate the rate of change between 2006 and 2008. Estimate the average rate of change between 2008 and 2011. Be sure to include units in the answers.
    b. Support each estimate with calculations.
    c. Describe how the average revenue per year changes between the two time intervals.
2. Graph the data for wireless connections with respect to time. Be sure to label the axes.
    a. Estimate the rate of change between 2006 and 2008. Estimate the rate of change between 2008 and 2011. Be sure to include units in the answers.
    b. Support each estimate with calculations.
    c. Describe how the average connections per year change between the two time intervals.
3. Graph the data for wireless revenue with respect to wireless connections. Be sure to label the axes.
    a. Estimate the rate of change between 2006 and 2008. Estimate the rate of change between 2008 and 2011. Be sure to include units in the answers.
    b. Support each estimate with calculations.

c. Describe how the average revenue per connection changes between the two time intervals.

4. Provide a written recommendation to your employer explaining why you believe they should or should not invest in this company. Be sure to use information from the comparisons above to support your recommendation. Also include any other factors you feel may need to be considered in order to make a final decision.

Task adapted from ***http://www.whyseemath.com/pdf212/ave_rate_verizon_wksht.pdf****. More information found at* ***http://whyseemath.com/wp/surveyofcalc/question/how-do-you-calculate-and-interpret-an-average-rate-of-change/***.

Instructional Task Exemplar Response

You have been hired as a financial analyst to help an investor determine whether to invest in various companies. A financial analyst might look at the average rates of change between multiple variables to determine whether the company is in a period of growth.

Below is a table that gives the annual revenue from a company's wireless business and the number of wireless connections that yield this revenue.

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2006	38.0	59.1
2007	43.9	65.7
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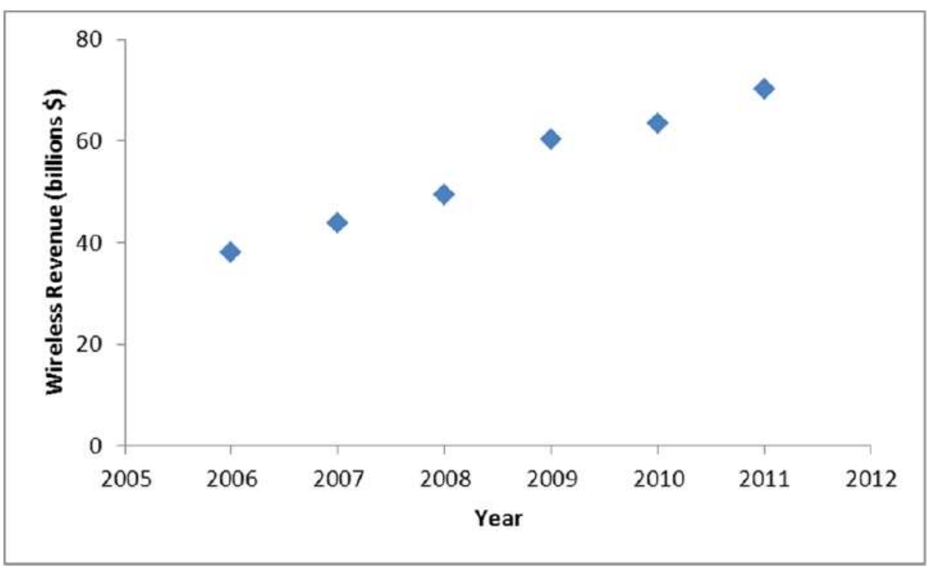
2010	63.4	102.2
2011	70.2	107.8

1. Graph the data for wireless revenue with respect to time. Be sure to label the axes.



Students may choose to use integers to represent the years (i.e., 0 represents 2006, 1 represents 2007, etc.). If so, they must indicate what the values represent.

a. Estimate the average revenue per year between 2006 and 2008. Estimate the average revenue per year between 2008 and 2011.

Estimated revenue per year for 2006-2008: $\frac{50-40}{2} \approx 5$ *billion dollars per year*

Estimated revenue per year for 2008-2011: $\frac{70-50}{3} \approx 6.67$ *billion dollars per year*

***Students may estimate differently but the values should be close to what is here.*

b. Support each estimate with calculations.

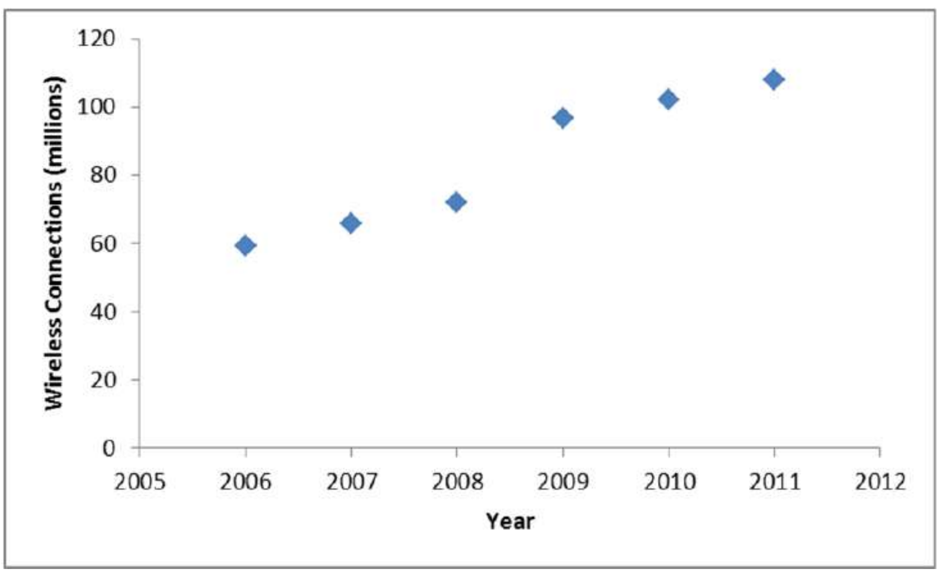
Average revenue per year for 2006-2008: $\frac{49.3-38.0}{2008-2006} = \frac{11.3}{2} \approx 5.65$ *billion dollars per year*

Average revenue per year for 2008-2011: $\frac{70.2-43.9}{2011-2008} = \frac{20.9}{3} \approx 6.97$ *billion dollars per year*

c. Describe how the average revenue per year changes between the two time intervals.

The average amount of revenue per year is higher from 2008 to 2011, indicating that the revenue increased faster from 2008 to 2011 than it did between 2006 and 2008.

2. Graph the data for wireless connections with respect to time. Be sure to label the axes.



Students may choose to use integers to represent the years (i.e., 0 represents 2006, 1 represents 2007, etc.). If so, they must indicate what the values represent.

a. Estimate the average connections per year between 2006 and 2008. Estimate the average connections per year between 2008 and 2011.

Estimated connections per year for 2006-2008: $\frac{70-60}{2} \approx 5$ *million connections per year*

Estimated connections per year for 2008-2011: $\frac{110-70}{3} \approx 13.3$ *million connections per year*

***Students may estimate differently but the values should be close to what is here.*

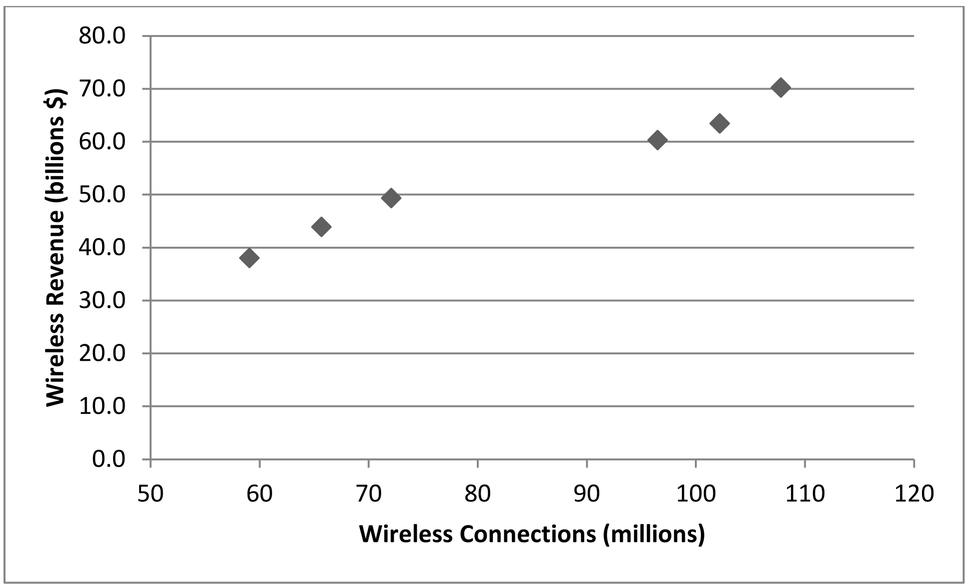
b. Support each estimate with calculations.

Average connections per year for 2006-2008: $\frac{72.1-59.1}{2008-2006}=\frac{13}{2}\approx 6.5$ *million connections per year*

Average connections per year for 2008-2011: $\frac{107.8-72.1}{2011-2008}=\frac{35.7}{3}\approx 11.9$ *million connections per year*

c. Describe how the average connections per year change between the two time intervals.
The average number of connections per year is higher from 2008 to 2011, indicating that each year, more connections are being made.

3. Graph the data for wireless revenue with respect to wireless connections. Be sure to label the axes.



a. Estimate the average revenue per connection between 2006 and 2008. Estimate the average revenue per connection between 2008 and 2011.

Estimated revenue per connection for 2006-2008: $\frac{49-38}{72-60}=\frac{11}{12}\approx 0.917$ *billion dollars per million wireless connections. Since 1 billion divided by 1 million is 1 thousand, this really means $917 per connection from 2006 to 2008.*

Estimated revenue per connection for 2008-2011: $\frac{70-49}{108-73}=\frac{21}{35}\approx 0.6$ *billion dollars per million wireless connections or $600 per connection from 2008 to 2011.*

b. Support each estimate with calculations.

Average revenue per connection for 2006-2008: $\frac{49.3-38.0}{72.1-59.1}=\frac{11.3}{13}\approx 0.869$ *billion dollars per million wireless connections. Since 1 billion divided by 1 million is 1 thousand, this really means $869 per connection from 2006 to 2008.*

Average revenue per connection for 2008-2011: $\frac{70.2-49.3}{107.8-72.1} = \frac{20.9}{35.7} \approx 0.585$ *billion dollars per million wireless connections, or $585 per connection from 2008 to 2011.*

c. Describe how the average revenue per connection changes between the two time intervals.

The comparison of these rates indicates that even though revenue is increasing based on revenue per year, and the number of connections is increasing per year, the amount of money per connection is lower from 2008 to 2011 than it was from 2006 to 2008.

4. Provide a written recommendation to your employer explaining why you believe they should or should not invest in this company. Be sure to use information from the comparisons above to support your recommendation. Also include any other factors you feel may need to be considered in order to make a final decision.

   These recommendations will vary. Students should use the information they determined in the first three situations to support their recommendation.

Task adapted from ***http://www.whyseemath.com/pdf212/ave_rate_verizon_wksht.pdf****. More information on responses can be found at* ***http://whyseemath.com/wp/surveyofcalc/question/how-do-you-calculate-and-interpret-an-average-rate-of-change/****.*

M&M's® Data Analysis (IT)

Overview

Students will analyze given data about bags of M&M's® to determine the relationship between the number of candies in a bag and the net weight of the bag.

Standards

Summarize, represent, and interpret data on two categorical and quantitative variables.

HSS-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

HSS-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

HSS-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

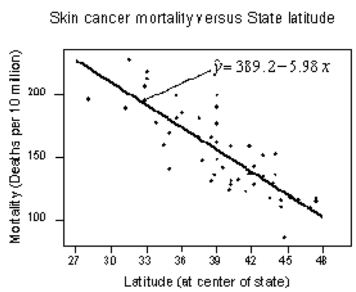
HSS-ID.C.9 Distinguish between correlation and causation.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

<table>
<tr><th>Grade-Level Standard</th><th>The Following Standards Will Prepare Them</th><th>Items to Check for Task Readiness</th><th>Sample Remediation Items</th></tr>
<tr>
<td>HSS-ID.B.6a</td>
<td>• 8.SP.A.2
• 8.SP.A.3
• HSS-ID.B.5</td>
<td>1. Using the data given in the table below, write a linear function to model the relationship between age and glucose level. Use your model to predict the glucose level of a person who is 50 years old.
<table>
<tr><th>Age</th><th>Glucose Level</th></tr>
<tr><td>43</td><td>99</td></tr>
<tr><td>21</td><td>65</td></tr>
<tr><td>25</td><td>79</td></tr>
<tr><td>42</td><td>75</td></tr>
<tr><td>57</td><td>87</td></tr>
<tr><td>59</td><td>81</td></tr>
</table>
a. $y = 0.385x + 65.14$ where *y* is the glucose level and *x* is the age. The predicted glucose level of a person 50 years old is approximately 84.</td>
<td>• http://www.illustrativemathematics.org/illustrations/1558
• http://www.illustrativemathematics.org/illustrations/1370
• http://www.illustrativemathematics.org/illustrations/123
• http://learnzillion.com/lessonsets/553-represent-and-describe-data-on-two-quantitative-variables-on-a-scatter-plot
• http://learnzillion.com/lessonsets/455-represent-data-on-a-scatter-plot-fit-functions-to-the-data-and-assess-fit</td>
</tr>
</table>

<table>
<tr><th>Grade-Level Standard</th><th>The Following Standards Will Prepare Them</th><th>Items to Check for Task Readiness</th><th>Sample Remediation Items</th></tr>
<tr><td></td><td></td><td>2. http://www.illustrativemathematics.org/illustrations/1554
3. http://www.illustrativemathematics.org/illustrations/1307</td><td></td></tr>
<tr><td>HSS-ID.C.7</td><td>• 8.SP.A.3
• HSS-ID.B.6c</td><td>1. The following plot shows the relationship between the state latitude and the mortality rate (per 10 million). A function that can model the relationship is given as $y = 389.2 - 5.98x$. What do the slope and y-intercept represent in this situation?



a. The slope of -5.98 means that for every 1 degree the latitude increases, the mortality rate decreases by 5.98 per 10 million. The y-intercept of 389.2 means that at 0 degrees latitude, the mortality rate is 389.2 deaths per 10 million.
2. http://www.illustrativemathematics.org/illustrations/1028</td><td>• http://learnzillion.com/lessonsets/460-interpret-the-slope-and-intercept-of-a-linear-function-in-context
• http://learnzillion.com/lessonsets/457-interpret-the-slope-and-the-intercept-of-a-linear-model-using-data</td></tr>
<tr><td>HSS-ID.C.8</td><td>• HSS-ID.B.6c</td><td>1. Compute the correlation coefficient of the data in the table below. Explain the meaning of the value.

<table><tr><th>Age</th><th>Glucose Level</th></tr><tr><td>43</td><td>99</td></tr><tr><td>21</td><td>65</td></tr><tr><td>25</td><td>79</td></tr><tr><td>42</td><td>75</td></tr><tr><td>57</td><td>87</td></tr><tr><td>59</td><td>81</td></tr></table></td><td>• http://learnzillion.com/lessonsets/584-find-correlation-coefficient-of-a-linear-fit
• http://learnzillion.com/lessonsets/467-compute-and-interpret-the-correlation-coefficient-of-a-linear-fit</td></tr>
</table>

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		a. The correlation coefficient is approximately 0.530. This suggests that there is a positive linear correlation between age and glucose level. This might suggest that the older a person is, the higher his or her glucose level will be.	
HSS-ID.C.9	• HSS-ID.B.6c	1. Explain the difference between correlation and causation. a. Correlation means that when one variable increases or decreases, so does the other value (even if it is in the opposite direction). Correlation does not imply causation. Causation indicates that one event is the direct result of the other event—causation implies correlation. 2. http://www.illustrativemathematics.org/illustrations/44	• http://learnzillion.com/lessonsets/585-distinguish-between-correlation-and-causation • http://learnzillion.com/lessonsets/468-distinguish-between-correlation-and-causation-and-assess-causation

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is net weight?*** Net weight is the weight of goods in a package and does not include the weight of wrapping material, container, or other packaging. In terms of this task, the net weight would be the weight of the M&M's® in the bag.

During the Task

- Students should be allowed to use technology for all parts of this task. Be sure to review with students the procedure for creating a scatter plot, computing the correlation coefficient, and finding a linear model with the technology available. Graphing calculators or spreadsheet software can be used to complete this task.
- Provide each student with a bag of M&M's® to complete the second part of the task and have students work in groups of 3-4. Students will also need access to a digital scale in order to calculate the net weight of each bag of M&M's®. The science department may be a good source for the digital scales.
- Have students discuss whether the relationship between the number of candies and the weight of the bags is a causal relationship and develop their response to question 8 as a group.

After the Task

It is important to discuss with students the limitations of the linear model they create for question 3. Discuss with students what the intercept means in the context of the task. Ask students to identify what change in the linear model would make the model more accurate.

Have students discuss whether they expected a stronger correlation coefficient. Students should realize that the variance in the weight and the number of candies could be due to the individual weight of the candies, and that this variance might cause the correlation coefficient not to be what one might expect.

Have students use the individual data from the whole class to find the correlation coefficient and a linear model, and then compare it to the data they were given. Have them discuss differences in the function as well as the correlation coefficient.

Student Instructional Task

Part I

The table below gives the color counts and net weight (in grams) for a sample of 30 bags of M&M's®.

Red	Green	Blue	Orange	Yellow	Brown	Weight (g)
15	9	3	3	9	19	49.79
9	17	19	3	3	8	48.98
14	8	6	8	19	4	50.40
15	7	3	8	16	8	49.16
10	3	7	9	22	4	47.61
12	7	6	5	17	11	49.80
6	7	3	6	26	10	50.23
14	11	4	1	14	17	51.68
4	2	10	6	18	18	48.45
9	9	3	9	8	15	46.22
9	11	13	0	7	18	50.43
8	8	6	5	11	20	49.80
12	9	13	2	6	13	46.94
9	7	7	2	18	7	47.98
6	6	6	4	21	13	48.49
4	6	9	4	12	20	48.33
3	5	11	12	11	16	48.72
14	5	6	6	21	6	49.69
5	5	16	12	7	12	48.95
8	9	13	4	15	11	51.71
8	7	7	13	7	18	51.53
9	8	3	8	23	8	50.97
20	2	7	5	13	9	50.01
12	6	1	12	6	19	48.28
8	9	4	6	21	7	48.74
4	6	7	6	14	19	46.72
10	12	11	6	11	7	47.67
5	4	2	9	18	16	47.70
15	11	4	13	7	8	49.40
11	6	7	12	12	13	52.06

Source of data: http://www.math.uah.edu/stat/data/MM.html

1. Create a scatter plot of the data included in the table with the number of candies as the independent variable and the net weight as the dependent variable. Based on the graph you create, describe the relationship between the variables.

2. Find the correlation coefficient of the data. Explain how the correlation coefficient supports your description of the data in the graph.

3. Find a linear function to model the relationship between the variables. Explain what the slope and intercept of the model means in the context of the given data.

4. Based on the linear model, what would be the predicted net weight of a bag of M&M's® that contains 56 candies? Show your work.

5. If the advertised net weight of a bag of M&M's® is 47.9 grams, based on the linear model, how many candies would one expect to be in the bag? Show your work.

Part II

6. Count the number of candies in each bag of M&M's® in your group. Weigh the contents of each bag to the nearest hundredth of a gram. Record the total number of candies and weight for each bag in the table below.

Total Number of Candies	Net Weight

7. Compute the correlation coefficient for the data collected in your group. What does this value tell you about your data?

8. Combine the data from the bags in your group with the original data from part one. Recalculate the correlation coefficient for the new data set (original data plus the group's data). Compare the original correlation coefficient to the new correlation coefficient. Does one correlation coefficient suggest a stronger linear fit than the other? Explain your reasoning.

9. Do you believe a causal relationship exists between the number of candies and the net weight of the bag? Explain your reasoning.

Instructional Task Exemplar Response

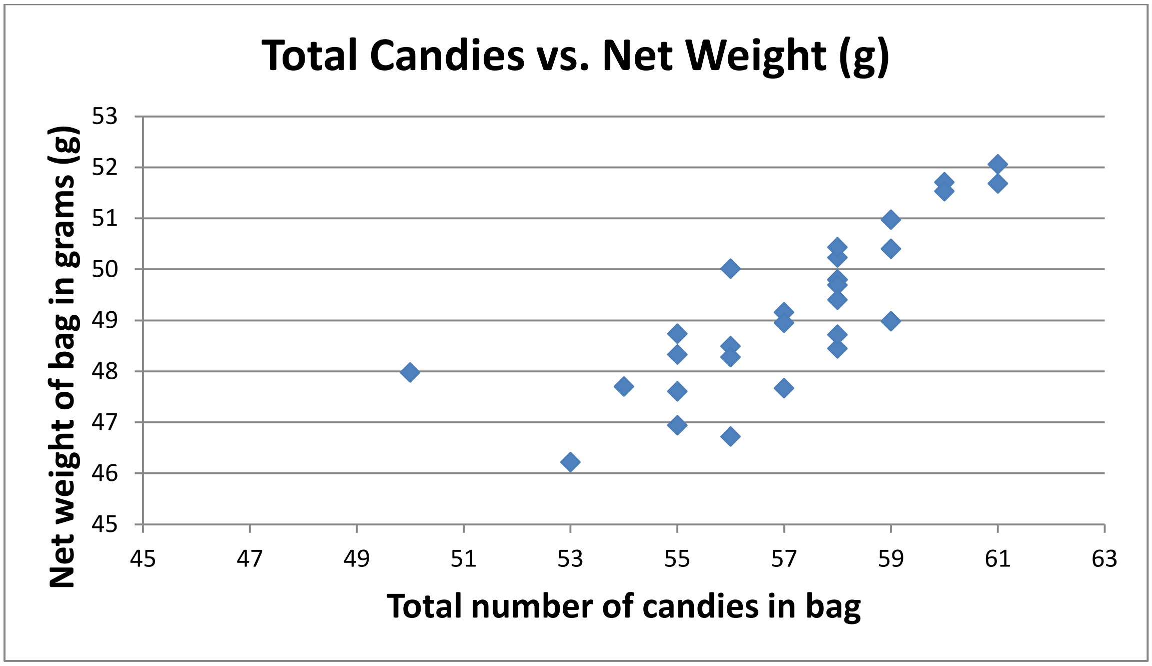
Part I

The table below gives the color counts and net weight (in grams) for a sample of 30 bags of M&M's®.

Red	Green	Blue	Orange	Yellow	Brown	Weight (g)
15	9	3	3	9	19	49.79
9	17	19	3	3	8	48.98
14	8	6	8	19	4	50.40
15	7	3	8	16	8	49.16
10	3	7	9	22	4	47.61
12	7	6	5	17	11	49.80
6	7	3	6	26	10	50.23
14	11	4	1	14	17	51.68
4	2	10	6	18	18	48.45
9	9	3	9	8	15	46.22
9	11	13	0	7	18	50.43
8	8	6	5	11	20	49.80
12	9	13	2	6	13	46.94
9	7	7	2	18	7	47.98
6	6	6	4	21	13	48.49
4	6	9	4	12	20	48.33
3	5	11	12	11	16	48.72
14	5	6	6	21	6	49.69
5	5	16	12	7	12	48.95
8	9	13	4	15	11	51.71
8	7	7	13	7	18	51.53
9	8	3	8	23	8	50.97
20	2	7	5	13	9	50.01
12	6	1	12	6	19	48.28
8	9	4	6	21	7	48.74
4	6	7	6	14	19	46.72
10	12	11	6	11	7	47.67
5	4	2	9	18	16	47.70
15	11	4	13	7	8	49.40
11	6	7	12	12	13	52.06

Source of data: http://www.math.uah.edu/stat/data/MM.html

1. Create a scatter plot of the data included in the table with the number of candies as the independent variable and the net weight as the dependent variable. Based on the graph you created, how would you describe the relationship of the variables?



The graph of the data suggests a positive linear correlation, which would mean that generally, as the number of candies increases, the net weight of the bag of M&M's® increases.

2. Find the correlation coefficient of the data. Explain how the correlation coefficient supports your description of the data in the graph.

   ***Note: The standard for computing the correlation coefficient specifically indicates the use of technology. The correlation coefficient listed below was found using the TI-84 Plus graphing calculator. Other technology devices may produce slightly different results.*

   The correlation coefficient for this data is $r \approx 0.794$. This supports my description that the data has a positive correlation because the value is positive. Correlation coefficients that are close to 1 indicate a strong positive linear relationship, which supports my description as well.

3. Find a linear function to model the relationship between the variables. Explain what the rate of change and constant term of the model represent in the context of the given data.

   ***Note: The linear model for this sample response was found using a TI-84 graphing calculator. Other technology devices may produce slightly different results. Students may also determine a model without the use of technology, in which case students would need to show their work.*

   A linear function to model the relationship could be $y = 0.507x + 20.278$, where x represents the number of candies in the bag and y represents the net weight in grams of the bag of M&M's®. The slope, 0.507, means that for each candy added to the bag, the net weight increases by 0.507 grams. The intercept, 20.278, means that when there are zero candies in the bag, the net weight is 20.278 grams.

4. Based on the linear model, what would be the predicted net weight of a bag of M&Ms® that contains 56 candies? Show your work.

$$y = 0.507(56) + 20.278$$

$$y = 48.67$$

The predicted net weight of a bag of M&M's® that contains 56 candies is 48.67 grams.

5. If the advertised net weight of a bag of *M&M's®* is 47.9 grams, based on the linear model, how many candies would one expect to be in the bag? Show your work.

$$47.9 = 0.507x + 20.278$$

$$47.9 - 20.278 = 0.507x$$

$$27.622 = 0.507x$$

$$\frac{27.622}{0.507} = \frac{0.507x}{0.507}$$

$$54.48 \approx x$$

For the advertised net weight of 47.9 grams, one would expect there to be about 54 candies in the bag.

Part II

6. Count the number of candies in each bag of *M&M's®* in your group. Weigh the contents of each bag to the nearest hundredth of a gram. Record the total number of candies and weight for each bag in the table below.

Total Number of Candies	Net Weight (g)
61	*49.14*
59	*51.62*
52	*48.30*
51	*47.72*

***Note: The data in the table above was randomly generated in order to be able to provide sample responses.*

7. Compute the correlation coefficient for the data collected in your group. What does this value tell you about your data?

***Note: The standard for computing the correlation coefficient specifically indicates the use of technology. The correlation coefficient listed below was found using the TI-84 Plus graphing calculator. Other technology devices may produce slightly different results.*

The correlation coefficient for this data is $r \approx 0.698$*. This value indicates there is a strong positive relationship between the number of candies in a bag and the net weight for the group's data.*

8. Combine the data from the bags in your group with the original data from part one. Recalculate the correlation coefficient for the new data set (original data plus the group's data). Compare the original correlation coefficient (from question 2) to the new correlation coefficient. Does one correlation coefficient suggest a stronger linear fit than the other? Explain your reasoning.

   The correlation coefficient for the new data set is $r \approx 0.742$*. The correlation coefficient for the original data set was* $r \approx 0.794$*. The original data set has a stronger linear fit because the correlation coefficient for the original data set is closer to 1 than the correlation coefficient for the new data set.*

9. Do you believe a causal relationship exists between the number of candies and the net weight of the bag? Explain your reasoning.

   The relationship is causal because the net weight of each bag will only increase if candies are added to the bag (likewise, the net weight would decrease if candies were removed from the bag). The correlation coefficients suggest a strong linear relationship, which supports my belief that this is a causal relationship.

GEOMETRY TOOLS

GEOMETRY TOOLS

Geometry Remediation Guide

As noted in "Remediation" on page 11 isolated remediation helps target the skills students need to more quickly access and practice on-grade level content. This chart is a reference guide for teachers to help them more quickly identify the specific remedial standards necessary for every geometry math standard[6].

Geometry Standards	Previous Grade or Course Standards	Geometry Standards to be Taught before (Scaffolded)	Geometry Standards to be Taught Concurrently
HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	• 4.MD.C.5 • 4.G.A.1 • 4.G.A.2		
HSG-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	• 8.G.A.1 • 8.G.A.2 • 8.G.A.3 • 8.G.A.4 • HSF-BF.B.3		• HSG-CO.A.3 • HSG-CO.A.5
HSG-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	• 8.G.A.2 • 8.G.A.3		• HSG-CO.A.2 • HSG-CO.A.4 • HSG-CO.A.5
HSG-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	• 8.G.A.1 • 8.G.A.3	• HSG-CO.A.1	• HSG-CO.A.3
HSG-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	• 8.G.A.2 • 8.G.A.3		• HSG-CO.A.2 • HSG-CO.A.3
HSG-CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	• 8.G.A.2	• HSG-CO.A.5	
HSG-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	• 8.G.A.2	• HSG-CO.B.6	

[6] This content comes from research found here: http://www.edutron.com/0/Math/ccssmgraph.htm

Geometry Standards	Previous Grade or Course Standards	Geometry Standards to be Taught before (Scaffolded)	Geometry Standards to be Taught Concurrently
HSG-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	• 8.G.A.2	• HSG-CO.B.7 • HSG-SRT.A.2	
HSG-CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	• 4.MD.C.7 • 7.G.B.5 • 8.G.A.5	• HSG-CO.A.1	
HSG-CO.C.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	• 7.G.A.2 • 8.G.A.5	• HSG-CO.B.8 • HSG-CO.C.9 • •	
HSG-CO.C.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	• 5.G.B.3	• HSG-CO.B.8 • HSG-CO.C.9	
HSG-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	• 4.MD.C.6 • 7.G.A.2	• HSG-CO.A.1	
HSG-CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	• 7.G.A.2	• HSG-CO.C.9 • HSG-CO.D.12	
HSG-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	• 8.G.A.4	• HSG-CO.A.2	

Geometry Standards	Previous Grade or Course Standards	Geometry Standards to be Taught before (Scaffolded)	Geometry Standards to be Taught Concurrently
HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	• 8.G.A.4	• HSG-SRT.A.1	
HSG-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	• 8.G.A.4	• HSG-SRT.A.2	
HSG-SRT.B.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	• 8.G.B.6	• HSG-SRT.A.3	
HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.		• HSG-SRT.A.3 • HSG-CO.B.8	
HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		• HSG-SRT.A.2	
HSG-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.	• 7.G.B.5	• HSG-SRT.C.6	
HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	• 8.G.B.7 • HSN-RN.B.3	• HSG-SRT.B.4	
HSG-C.A.1 Prove that all circles are similar.		• HSG-SRT.A.2	
HSG-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*		• HSG-CO.C.10	
HSG-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.		• HSG-C.A.2	
HSG-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.		• HSG-C.A.2 • HSG-CO.B.8 • HSG-GMD.A.1	

Geometry Standards	Previous Grade or Course Standards	Geometry Standards to be Taught before (Scaffolded)	Geometry Standards to be Taught Concurrently
HSG-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	• 8.G.B.8 • HSA-REI.B.4	• HSG-SRT.B.4 • HSG-SRT.C.8	
HSG-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*	• 8.G.B.8	• HSG-GPE.B.5	
HSG-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	• 8.EE.B.6 • 8.F.A.3		
HSG-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		• HSG-SRT.A.2 • HSG-CO.C.9	
HSG-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	• 8.G.B.8	• HSG-SRT.B.4	
HSG-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	• 7.G.B.4		
HSG-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	• 8.G.C.9	• HSG-GMD.A.1	
HSG-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	• 7.G.A.3		
HSG-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★	• 6.G.A.4 • 7.G.B.6		
HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★	• 7.G.B.6 • 8.G.C.9	• HSG-GMD.A.1 • HSG-GMD.A.3	
HSG-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★	• 7.G.A.1 • 7.G.B.6 • 8.G.C.9		

Geometry Tasks at a Glance

There are 10 sample tasks included in this guidebook that can be used to supplement any curriculum.

The tasks for geometry include:

- **5 Extended Constructed Response (ECR):** These short tasks, aligned to the standards, mirror the extended constructed response items students will see on their end of year state assessments.
- **5 Instructional Tasks (IT):** These complex tasks are meant to be used for instruction and assessment. They will likely take multiple days for students to complete. They can be used to help students explore and master the full level of rigor demanded by the standards. Teachers can use the table below to find standards associated with current instruction and add in these practice items to supplement any curriculum. These tasks should be used after students have some initial understanding of the standard. They will help students solidify and deepen their understanding of the associated content.

This is an overview of the geometry tasks included on the following pages.

Title	Type	Task Standards	Task Remedial Standards
Constructions and Proof Page 111	ECR	• HSG-CO.C.9 • HSG-CO.D.12	• 4.MD.C.6 • 4.MD.C.7 • 7.G.A.2 • 7.G.B.5 • 8.G.A.5
Equations of Circles Page 117	ECR	• HSG-C.A.1 • HSG-GPE.A.1 • HSG-GPE.B.4	• 7.G.B.4 • 8.G.B.8 • HSG-SRT.A.2 • HSA-REI.B.4 • HSG-SRT.B.4 • HSG-SRT.C.8 • HSG-GPE.B.5
Trigonometric Ratios Page 122	ECR	• HSG-SRT.A.2 • HSG-SRT.C.6 • HSG-SRT.C.7	• 7.G.B.5 • 8.G.A.4 • 8.G.B.8 • HSG-SRT.A.1 • HSG-SRT.A.2 • HSG-SRT.C.6
Similar Triangles Page 128	ECR	• HSG-SRT.A.2	• 8.G.A.4 • HSG-SRT.A.1
Solving Right Triangles Page 133	ECR	• HSG-SRT.C.8	• 8.G.B.7 • HSG-SRT.B.4 • HSG-SRT.C.6 • HSG-SRT.C.7
Understanding Congruence in Terms of Rigid Motions Page 140	IT	• HSG.CO.A.5 • HSG.CO.B.7	• 8.G.A.2 • 8.G.A.3 • HSG-CO.B.6

Title	Type	Task Standards	Task Remedial Standards
City Map Page 147	IT	• HSG-CO.A.1 • HSG-CO.C.9 • HSG-CO.D.12	• 4.G.A.1 • 4.G.A.2 • 4.MD.C.5 • 4.MD.C.6 • 4.MD.C.7 • 7.G.A.2 • 7.G.B.5 • 8.G.A.5 • HSG-CO.A.1
Parallelogram Congruence Page 155	IT	• HSG-CO.C.11 • HSG-SRT.B.5	• 5.G.B.3 • HSG-CO.B.8 • HSG-CO.C.9 • HSG-SRT.A.3
Modeling with Three Dimensional Figures Page 160	IT	• HSG-MG.A.2 • HSG-GMD.A.3	• 7.G.B.6 • 8.G.C.9 • HSG-GMD.A.1 • HSG-GMD.A.3 • 8.G.C.9 • HSG-GMD.A.1
Exploring Similar Triangles Page 166	IT	• HSG-SRT.A.3	• 8.G.A.4 • HSG-SRT.A.2

Constructions and Proof (ECR)

Overview

Students will perform basic constructions and prove two lines are parallel.

Standards

Prove geometric theorems.

HSG-CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly equidistant from the segment's endpoints.*

Make geometric constructions.

HSG-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-CO.C.9	• 4.MD.C.7 • 7.G.B.5 • 8.G.A.5	1. Explain how you would show that two lines cut by a transversal are parallel. a. To show that two lines, which are cut by a transversal, are parallel, show that (a) corresponding angles are congruent; or (b) alternate interior angles are congruent; or (c) alternate exterior angles are congruent; or (d) consecutive interior angles are supplementary. **2.** http://www.illustrativemathematics.org/illustrations/967	• http://www.illustrativemathematics.org/illustrations/59 • http://www.illustrativemathematics.org/illustrations/56 • http://www.illustrativemathematics.org/illustrations/1501 • http://www.illustrativemathematics.org/illustrations/1503

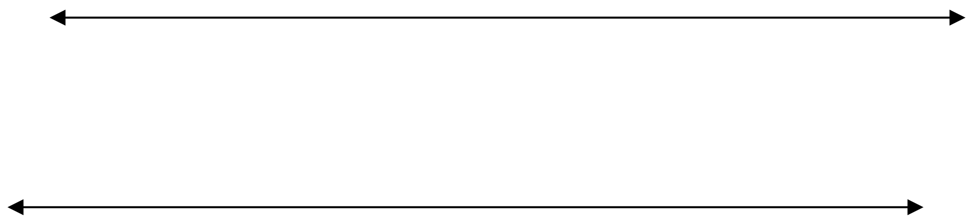
Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-CO.D.12	• 4.MD.C.6 • 7.G.A.2	1. http://www.illustrativemathematics.org/illustrations/1320 2. http://www.illustrativemathematics.org/illustrations/966	• http://www.illustrativemathematics.org/illustrations/909

After the Task

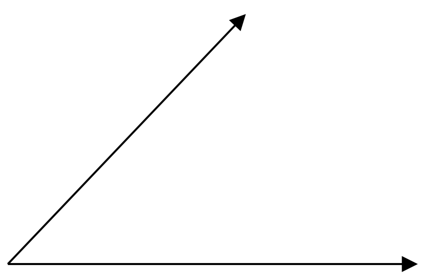
- ***For problem 1***, students are able to draw any transversal they choose and should use a protractor to measure the angles. Students may need assistance with using a protractor or remembering the different ways to show that two lines are parallel.
- ***For problems 2-4***, students should use a compass and a straightedge to complete the constructions. Students who are struggling can visit the websites listed in the exemplar response to see how the constructions are performed and practice them with new segments and angles. Students should be discouraged from using a ruler or protractor to measure segments or angles in order to make the constructions.
- ***For problem 4***, students may fail to see the 90° triangle that is formed. Students should be encouraged to label the figure throughout the construction to mark congruent segments, angles, or the measures of angles they know. Provide students with additional practice with constructions throughout the year.

Student Extended Constructed Response

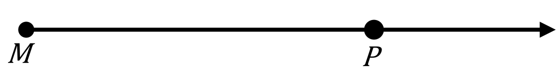
1. Using the following diagram, draw a transversal. Use the transversal and a protractor to explain why these lines are parallel.



2. Copy the angle below. Then, bisect the copied angle.



3. Construct $\angle M$ given $\overrightarrow{MP}$ such that $\angle M$ is a right angle.

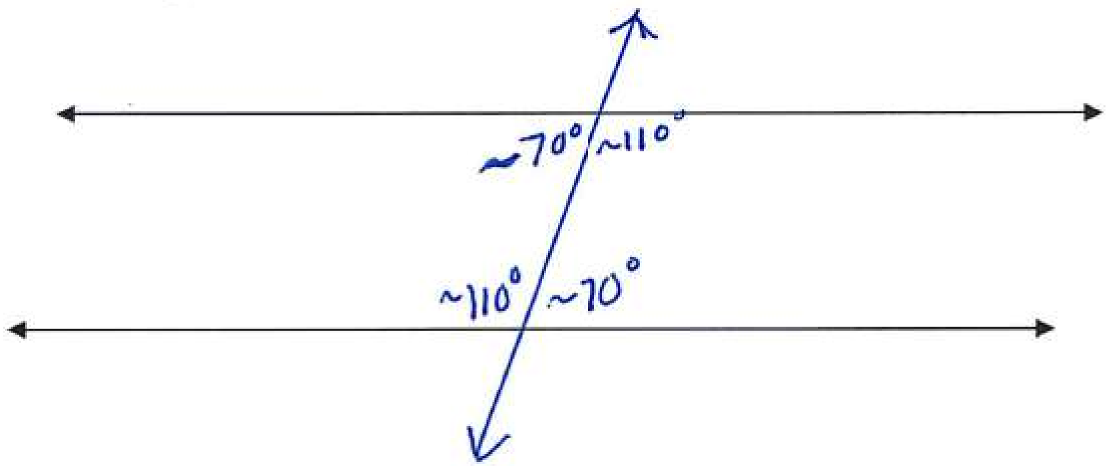


4. Given $\overline{BC}$, perform the following constructions and answer the question.
   a. Construct the perpendicular bisector of $\overline{BC}$. Label the midpoint of $\overline{BC}$ as *M*.
   b. Construct $\overline{MP}$ so that the length of $\overline{MP}$ is equal to the length of $\overline{BM}$ and so that $\overline{MP} \perp \overline{BC}$.
   c. Draw a line connecting *B* and *P*.

What is the measure of $\angle PBM$? Explain your reasoning.

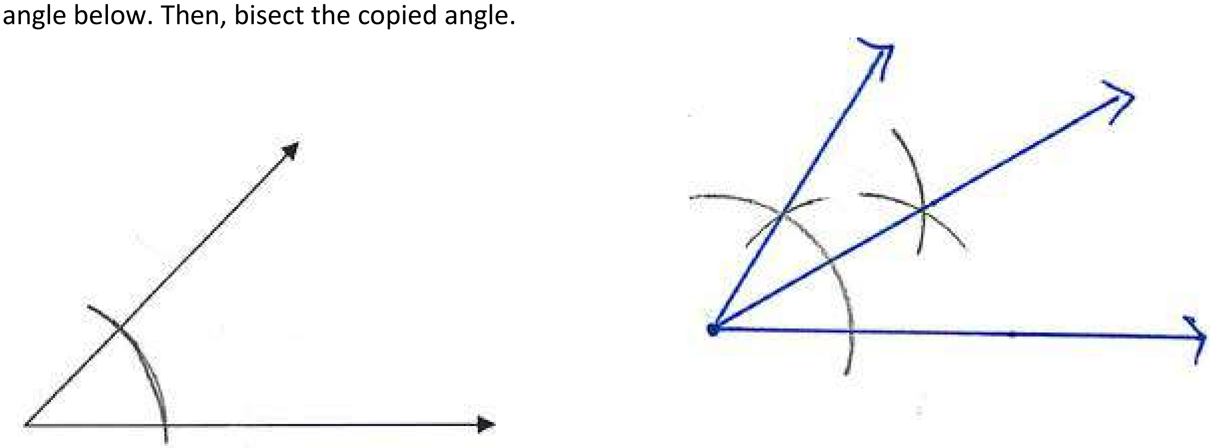
Extended Constructed Response Exemplar Response

1. Using the following diagram, draw a transversal. Use the transversal and a protractor to explain why these lines are parallel.



After drawing in the transversal, I measured the angles and found the measures of the angles shown. Since the alternate interior angles have the same measure, they are congruent. If two lines are cut by a transversal such that the alternate interior angles are congruent, then the lines are parallel. Thus, the lines above are parallel.

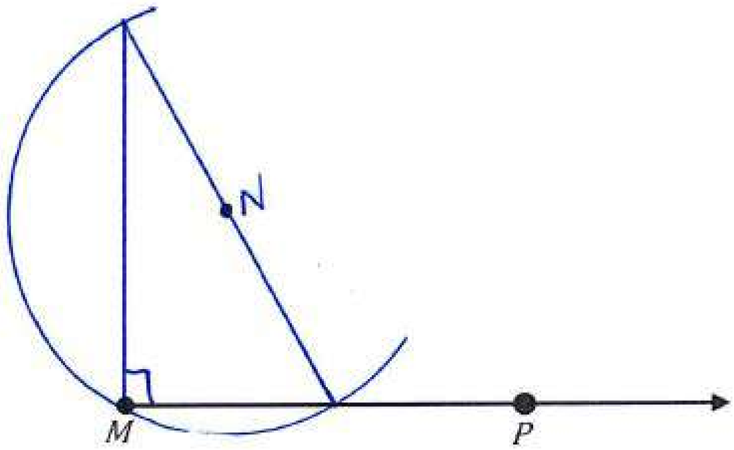
2. Copy the angle below. Then, bisect the copied angle.



Teacher Note: For the steps to copy an angle, visit http://www.mathopenref.com/printcopyangle.html.

For the steps to bisect an angle, visit http://www.mathopenref.com/printbisectangle.html.

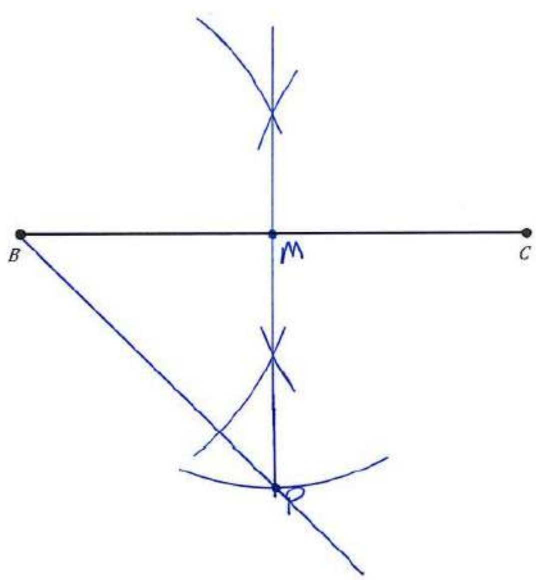
3. Construct $\angle M$ given $\overrightarrow{MP}$ such that $\angle M$ is a right angle.



Teacher Note: For the steps to construct a perpendicular at the endpoint of a ray, visit http://www.mathopenref.com/constperpendray.html.

4. Given $\overline{BC}$, perform the following constructions and answer the question.
    a. Construct the perpendicular bisector of $\overline{BC}$. Label the midpoint of $\overline{BC}$ as *M*.
    b. Construct $\overline{MP}$ so that the length of $\overline{MP}$ is equal to the length of $\overline{BM}$ and so that $\overline{MP} \perp \overline{BC}$.
    c. Draw a line connecting *B* and *P*.

What is the measure of $\angle PBM$? Explain your reasoning.



The measure of $\angle PBM$ is 45°. Triangle BMP is an isosceles right triangle because the length of $\overline{MP}$ is the same as the length of $\overline{BM}$ and the two segments are perpendicular. In an isosceles right triangle, the acute angles are both 45°.

Teacher Note: For the steps to construct a 45-degree angle, visit http://www.mathopenref.com/constangle45.html.

Equations of Circles (ECR)

Overview

Using information about two circles, students will derive the equations of the circles. These equations will be used to prove that the two circles are similar and to prove or disprove that a point lies on a circle.

Standards

Understand and apply theorems about circles.

HSG-C.A.1 Prove that all circles are similar.

Translate between the geometric description and the equation for a conic section.

HSG-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically.

HSG-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-C.A.1	• 7.G.B.4 • HSG-SRT.A.2	1. Draw two circles on your page. Are they similar? a. Yes; all circles are similar. 2. http://www.illustrativemathematics.org/illustrations/1368	• http://www.illustrativemathematics.org/illustrations/34 • http://www.illustrativemathematics.org/illustrations/603
HSG-GPE.A.1	• 8.G.B.8 • HSA-REI.B.4 • HSG-SRT.B.4 • HSG-SRT.C.8	1. What is the equation of a circle with center (-2, 7) and a radius of 4? a. $(x+2)^2 + (y-7)^2 = 16$ 2. http://www.illustrativemathematics.org/illustrations/1425	• http://www.illustrativemathematics.org/illustrations/375 • http://www.illustrativemathematics.org/illustrations/1568 • http://www.illustrativemathematics.org/illustrations/1095 • http://www.illustrativemathematics.org/illustrations/962

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
			• http://learnzillion.com/lessonsets/763-derive-the-equation-of-a-circle-using-the-pythagorean-theorem • http://learnzillion.com/lessonsets/724-derive-the-equation-of-a-circle-and-complete-the-square-to-find-the-center-and-radius • http://learnzillion.com/lessonsets/28-understand-and-use-equations-for-circles
HSG-GPE.B.4	• 8.G.B.8 • HSG-GPE.B.5	1. Does the point (2, 22) lie on the parabola with equation $y = x^2 + 5x + 8$? a. Yes 2. http://www.illustrativemathematics.org/illustrations/605	• http://www.illustrativemathematics.org/illustrations/1556 • http://www.illustrativemathematics.org/illustrations/1347 • http://www.illustrativemathematics.org/illustrations/1348 • http://learnzillion.com/lessons/286-prove-whether-a-point-is-on-a-circle

After the Task

The following list identifies possible student difficulties and suggestions for helping students by problem number:

- Students may struggle with completing the square. They may need to be reminded to move 56 to the other side of the equation. They may also stop when they encounter fractions. Remind them that equations of circles can have fractions in them.
- Encourage students to draw a picture before attempting to find the equation.
- Students may solve this problem using their equations from part 2 or using the Pythagorean Theorem.
- Students may only say that all circles are similar. Encourage them to explain how they know that all circles are similar.

Student Extended Constructed Response Task

James has two circles, Circle A and Circle B. Use the information given below to answer the questions that follow.

Circle A:

Equation: $100x^2 + 100y^2 - 100x + 240y - 56 = 0$

Circle B:

$center$ $(0, -4)$ with a radius of 3

1. Complete the square in order to find the center and radius of Circle A. Show your work and state the center and radius.

2. Derive the equation of Circle B using the Pythagorean Theorem. Explain how you found your equation.

3. Does the point $(\sqrt{8}, -3)$ lie on Circle B? Support your answer with evidence.

4. Are Circle A and Circle B similar? Prove your answer.

Extended Constructed Response Exemplar Response

James has two circles, Circle A and Circle B. Use the information given below to answer the questions that follow.

Circle A:

Equation: $100x^2 + 100y^2 - 100x + 240y - 56 = 0$

Circle B:

$center$ $(0, -4)$ with a radius of 3

1. Complete the square in order to find the center and radius of Circle A. Show your work and state the center and radius.

$$100x^2 + 100y^2 - 100x + 240y - 56 = 0$$
$$100x^2 - 100x + 100y^2 + 240y = 56$$
$$x^2 - x + y^2 + \frac{12}{5}y = \frac{14}{25}$$
$$\left(x^2 - x + \frac{1}{4}\right) + \left(y^2 + \frac{12}{5}y + \frac{36}{25}\right) = \frac{14}{25} + \frac{1}{4} + \frac{36}{25}$$
$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{6}{5}\right)^2 = \frac{9}{4}$$
$$\left(x - \frac{1}{2}\right)^2 + \left(y - \left(-\frac{6}{5}\right)\right)^2 = \left(\frac{3}{2}\right)^2$$

The center of Circle A is $\left(\frac{1}{2}, -\frac{6}{5}\right)$, and the radius of Circle A is $\frac{3}{2}$ units.

2. Derive the equation of Circle B using the Pythagorean Theorem. Explain how you found your equation.

$center$: $(0, -4)$ with a radius of 3

The center of the circle is at (0, -4). Label a point (x, y) on the circle.

From the center, you would move over to x. The length of this leg of the triangle would be $|x - 0|$ or x. You would move from -4 to y, so the length of the second leg would be $|y - -4|$ or $y + 4$.

Connecting this point, (x, y), to the center on the circle, (0, -4), would form the hypotenuse of a right triangle.

Using the Pythagorean Theorem,

$$(x)^2 + (y + 4)^2 = 3^2$$
$$(x)^2 + (y + 4)^2 = 9$$

3. Does the point $(\sqrt{8}, -3)$ lie on Circle B? Support your answer with evidence.

Yes, the point $(\sqrt{8}, -3)$ *does lie on Circle B.*

$$(x)^2 + (y+4)^2 = 9$$
$$\left(\sqrt{8}\right)^2 + (-3+4)^2 = 9$$
$$\left(\sqrt{8}\right)^2 + (1)^2 = 9$$
$$8 + 1 = 9$$
$$9 = 9$$

Alternate solution:

$center$: $(0, -4)$ *with a radius of 3*

The center of the circle is at (0, -4). Label a point $(\sqrt{8}, -3)$ *on the circle.*

From the center, you would move over to $\sqrt{8}$*. The length of this leg of the triangle would be* $\left|\sqrt{8} - 0\right|$ *or* $\sqrt{8}$*. You would move from -4 to* -3*, so the length of the second leg would be* $|-3 - -4|$ *or* 1*.*

Connecting this point, $(\sqrt{8}, -3)$*, to the center on the circle, (0, -4), would form the hypotenuse of a right triangle.*

Using the Pythagorean Theorem,

$$\left(\sqrt{8}\right)^2 + (1)^2 = 3^2$$
$$8 + 1 = 9$$
$$9 = 9$$

Yes, the point $(\sqrt{8}, -3)$ *does lie on Circle B.*

Note: If a student's answer is correct based on an incorrect equation in part 2, then this part should be marked correct.

4. Are Circle A and Circle B similar? Prove your answer.

Yes, Circle A and Circle B are similar (all circles are similar). A translation and a dilation transform Circle A to Circle B. A translation of $\left(x - \frac{1}{2}, y - \frac{14}{5}\right)$ *and a dilation by a scale factor of 2 maps Circle A onto Circle B.*

Note: If a student's answer is correct based on an incorrect equation in part 1, then this part should be marked correct.

Trigonometric Ratios (ECR)

Overview

This task requires students to decide if right triangles are similar and then develop a conceptual understanding of trigonometric ratios for acute angles of right triangles using properties of similarity.

Standards

Understand similarity in terms of similarity transformations.

HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Define trigonometric ratios and solve problems involving right triangles.

HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

HSG-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.A.2	• 8.G.A.4 • 8.G.B.8 • HSG-SRT.A.1	1. Triangle ABC has coordinates $A(-3,-1), B(-5,-4)$, and $C(-$. Triangle XYZ has coordinates $X(4,5),\ Y(2,2)$ and $Z(5,1)$. Are Triangle ABC and Triangle XYZ similar? Explain your reasoning. a. Yes. Triangle ABC is translated seven units to the right and six units up. A translation is a rigid transformation, so it preserves both congruence and similarity. **2.** http://www.illustrativemathematics.org/illustrations/603	• http://www.illustrativemathematics.org/illustrations/602 • http://www.illustrativemathematics.org/illustrations/1556 • http://learnzillion.com/lessonsets/774-decide-if-two-figures-are-similar-and-explain-the-meaning-of-triangle-similarity

Grade Level Standards	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.C.6	• HSG-SRT.A.2	1. When two triangles are similar triangles, what do you know about the ratios of their corresponding side lengths? a. You know that they are equivalent. **2.** http://www.illustrativemathematics.org/illustrations/1635	• http://www.illustrativemathematics.org/illustrations/603 • http://learnzillion.com/lessonsets/767-understand-side-ratios-in-right-triangles-are-properties-of-the-angles-define-trigonometric-ratios-for-acute-angles-of-a-right-triangle • http://learnzillion.com/lessonsets/524-relate-side-ratios-to-angles-and-derive-trigonometric-ratios
HSG-SRT.C.7	• 7.G.B.5 • HSG-SRT.C.6	1. In a figure, angle A measures 30°. What would be the measure of an angle complementary to angle A? Explain your reasoning. a. 60° because complementary angels are angles whose sum is 90°. Therefore, 90 - 30 = 60. **2.** http://www.illustrativemathematics.org/illustrations/1443	• http://learnzillion.com/lessonsets/768-relate-the-sine-and-cosine-of-complementary-angles • http://learnzillion.com/lessonsets/768-relate-the-sine-and-cosine-of-complementary-angles

After the Task

To complete various portions of this task, students will need to find the length of the sides of the given triangles. Students may struggle with finding the length of each side of the triangles. They might count the length of the legs of the right triangles (horizontal and vertical sides) and use the Pythagorean Theorem or the distance formula to find the hypotenuse of each right triangle, or they may use the Pythagorean Theorem or the distance formula to find the length of all sides.

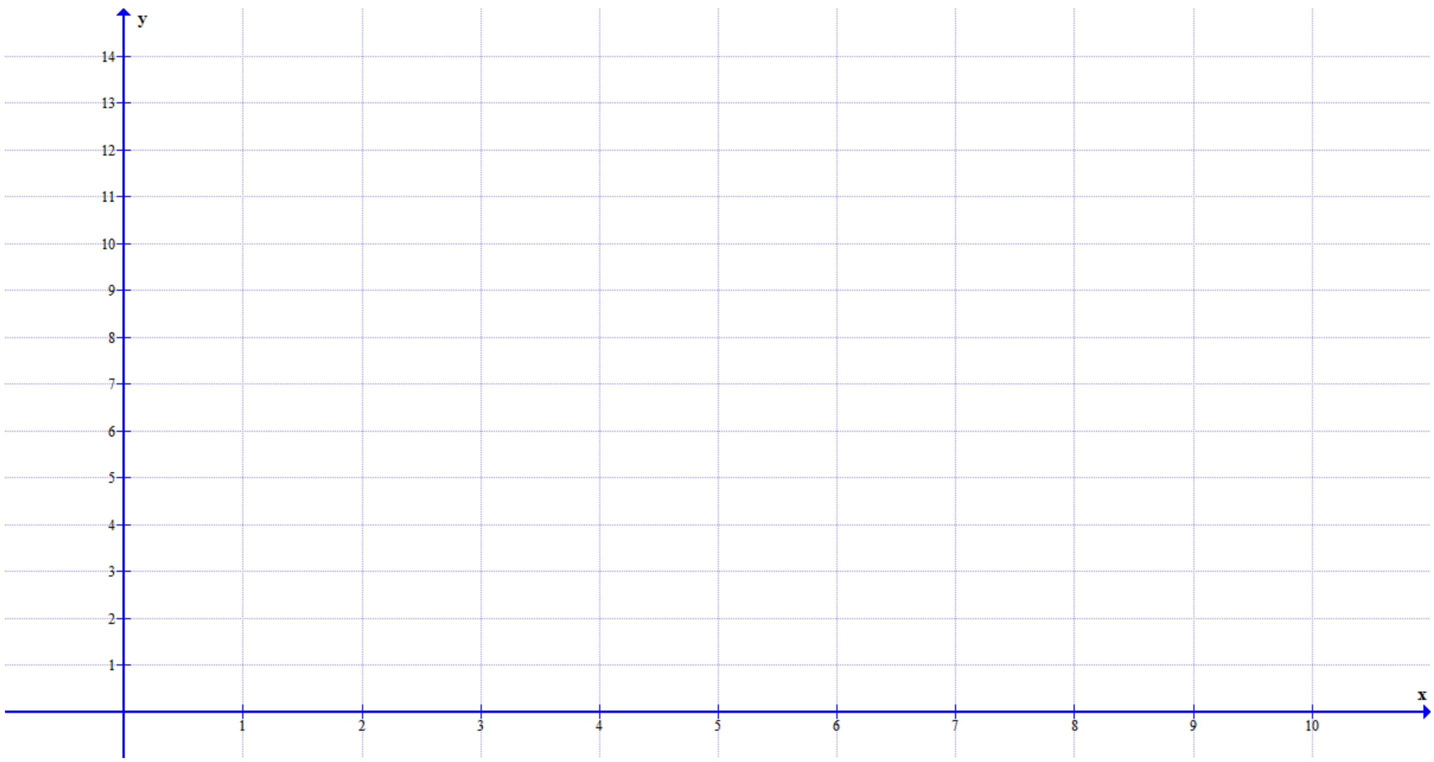
Student Extended Constructed Response

Plot the following triangles on the graph paper below. Label all vertices.

$Triangle\ ABC$: $A(2, 2)$, $B(5, 2)$, and $C(2, 7)$

$Triangle\ XYZ$: $X(4, 4)$, $Y(10, 4)$, and $Z(4, 14)$

$Triangle\ PQR$: $P(1, 1)$, $Q(2.5, 1)$, and $R(1, 3.5)$



Use your graph as needed to answer the following questions.

1. Are Triangle ABC and Triangle XYZ similar? Support your statement with evidence.

2. Are Triangle XYZ and Triangle PQR similar? Support your statement with evidence.

3. Find the sine of $\angle B$ and the sine of $\angle Y$. Write your answers as fractions. What do you notice about the sine values of these two angles? Explain your observation using properties of similarity.

4. Find the cosine of $\angle C$. Write your answer as a fraction. Compare the cosine of $\angle C$ to the sine of $\angle B$. Explain the comparison using properties of similarity or definitions/properties of trigonometric ratios.

5. Using your work from parts 2 and 3, find the cosine of $\angle Z$. Explain your reasoning using properties of similarity or definitions/properties of trigonometric ratios.

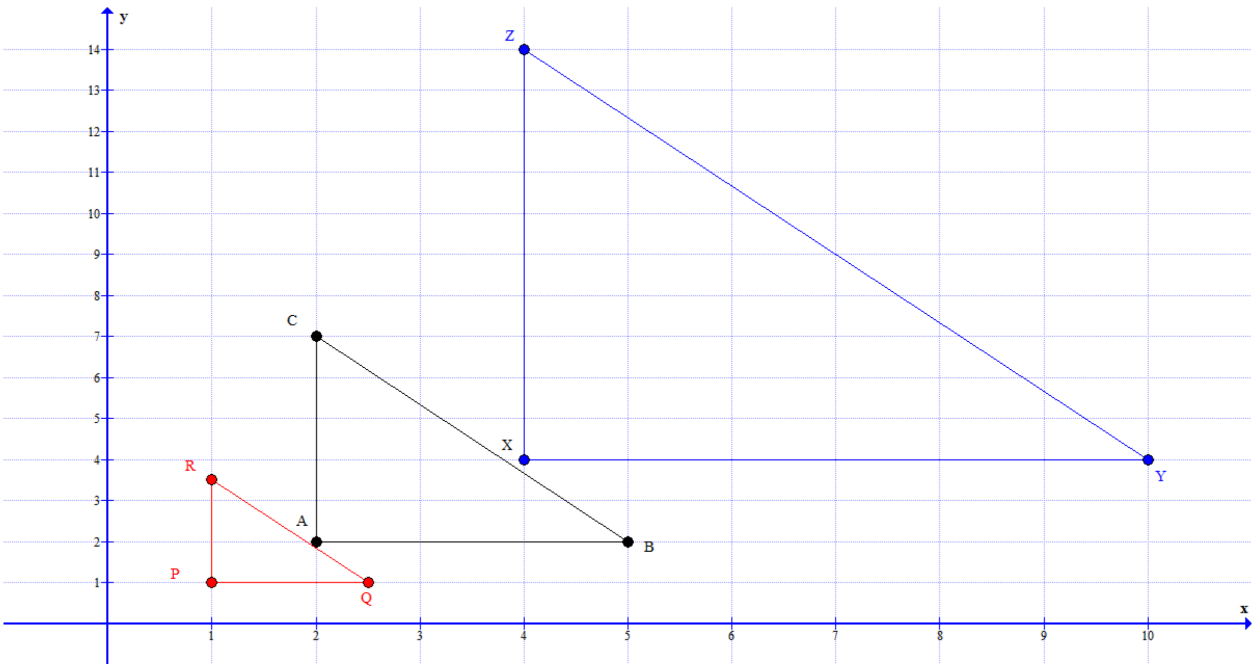
Extended Constructed Response Exemplar Response

Plot the following triangles on the graph paper below. Label all vertices.

$Triangle\ ABC$: $A(2,2)$, $B(5,2)$, and $C(2,7)$

$Triangle\ XYZ$: $X(4,4)$, $Y(10,4)$, and $Z(4,14)$

$Triangle\ PQR$: $P(1,1)$, $Q(2.5,1)$, and $R(1,3.5)$



Use your graph as needed to answer the following questions.

1. Are Triangle ABC and Triangle XYZ similar? Support your statement with evidence.

   Yes, the triangles are similar. One way that I can tell that the triangles are similar is by looking at the ordered pairs. If I make Triangle ABC the pre-image, then Triangle XYZ is a dilation with the origin as center with a scale factor of 2. I know that dilations are a similarity transformation, so Triangles ABC and XYZ are similar.

   Alternate explanation:

   I can tell that the three triangles are similar using side lengths and angle measures.

$$\frac{AB}{XY} = \frac{AC}{XZ}$$

$$\frac{3}{6} = \frac{5}{10}$$

$$\frac{1}{2} = \frac{1}{2}$$

Angle A and Angle X are both 90°, so they are congruent. Triangle ABC is similar to Triangle XYZ by SAS Similarity.

2. Are Triangle XYZ and Triangle PQR similar? Support your statement with evidence.

   Yes, the triangles are similar. One way that I can tell that the triangles are similar is by looking at the ordered pairs. If I make Triangle XYZ the pre-image, Triangle PQR is a dilation with the origin as center with a scale factor of $\frac{1}{4}$. I know that dilations are a similarity transformation, so Triangles XYZ and PQR are similar.

   Alternate explanation:

$$\frac{XY}{PQ} = \frac{XZ}{PR}$$

$$\frac{6}{1.5} = \frac{10}{2.5}$$

$$4 = 4$$

   Angle X and Angle P are both 90°, so they are congruent. Triangle XYZ is similar to Triangle PQR by SAS Similarity.

3. Find the sine of $\angle B$ and the sine of $\angle Y$. Write your answers as fractions. What do you notice about the sine values of these two angles? Explain your observation using properties of similarity.

$$\sin B = \frac{5}{\sqrt{34}}$$

$$\sin Y = \frac{10}{\sqrt{136}}$$

$$= \frac{10}{2\sqrt{34}}$$

$$= \frac{5}{\sqrt{34}}$$

   The sine values of these two angles are the same. This occurs because $\angle B$ and $\angle Y$ are corresponding angles in similar triangles. Therefore, the ratios $\frac{AC}{BC}$ and $\frac{XZ}{YZ}$ are equal. This is verified because when these expressions are evaluated using the side lengths, they both equal $\frac{5}{\sqrt{34}}$.

4. Find the cosine of $\angle C$. Write your answer as a fraction. Compare the cosine of $\angle C$ to the sine of $\angle B$. Explain the comparison using properties of similarity or definitions/properties of trigonometric ratios.

   $\cos C = \frac{5}{\sqrt{34}}$

   The cosine of $\angle C$ is the same as the sine of $\angle B$. The sine and cosine of complementary angles are the same because in a right triangle, the opposite side of one complementary angle is the adjacent side of the other complementary angle.

   $\sin \theta = \frac{opposite}{hypotenuse}$ *and* $\cos \theta = \frac{adjacent}{hypotenuse}$

Since the opposite and adjacent sides are the same sides for complementary angles, then $\sin\theta = \cos(90-\theta)$.

5. Using your work from parts 2 and 3, find the cosine of $\angle Z$. Explain your reasoning using properties of similarity or definitions/properties of trigonometric ratios.

   The cosine of $\angle Z$ *will be* $\frac{5}{\sqrt{34}}$ *because the cosine of* $\angle C$ *is* $\frac{5}{\sqrt{34}}$. *These two angles are corresponding angles in similar triangles, so they will have the same cosine values.*

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

   Since these two angles are corresponding angles of similar triangles, then we know that the corresponding sides of the two triangles are proportional, so $\frac{XZ}{YZ} = \frac{AC}{BC}$, so *cosine of* $\angle Z$= *cosine of* $\angle C$.

Similar Triangles (ECR)

Overview

This task requires that students determine the similarity of figures using given information.

Standards

Understand similarity in terms of similarity transformations.

HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.A.2	• 8.G.A.4 • HSG-SRT.A.1	1. Explain what you know about the corresponding pairs of angles and sides in similar triangles. a. In similar triangles, all pairs of corresponding angles are congruent, and all pairs of corresponding sides are proportional.	• http://www.illustrativemathematics.org/illustrations/602 • http://learnzillion.com/lessonsets/774-decide-if-two-figures-are-similar-and-explain-the-meaning-of-triangle-similarity

During the Task

In this problem, students are given a figure in which two triangles appear to be similar, but their similarity cannot be proven without further information. Asking students to provide a sequence of similarity transformations that maps one triangle to the other focuses them on the work of standard HSG-SRT.A.2, using the definition of similarity in terms of similarity transformations. Teachers will need to remind students to show that their sequences of similarity transformations have the intended effect; that is, that all parts of one triangle get mapped to the corresponding parts of the other triangle.

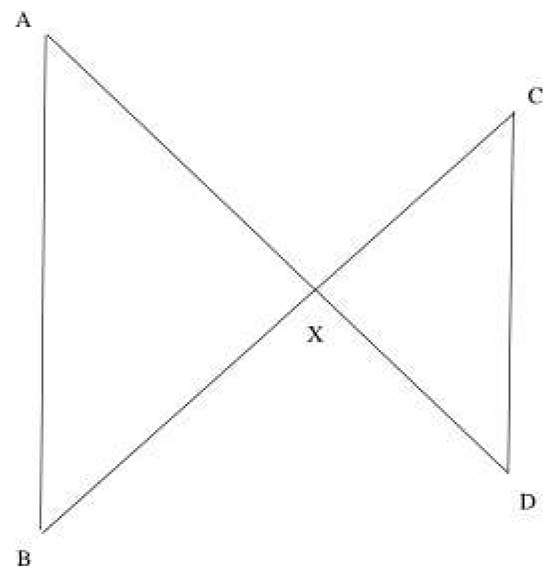
As noted in the solution, almost any attempt to draw a counterexample works. But on this part, a teacher may have students observe that they do not seem to have enough information to prove similarity, rather than showing that it is impossible to prove similarity.

After the Task

After establishing similarity through the use of transformations, ask students to prove or disprove that the triangles are similar for each problem using properties of parallel lines and the definition of similarity. This shifts the focus of this task from G-SRT.2 to G-SRT.5.

Student Extended Constructed Response

In the picture given below, $\overline{AD}$ and $\overline{BC}$ intersect at *X*. $\overline{AB}$ and $\overline{CD}$ are drawn, forming two triangles, *AXB* and *CXD*.



In each part (a-d) below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar; and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one triangle to the other. If not, explain why not.

a. The lengths *AX* and *XD* satisfy the equation 2*AX* = 3*XD*.

b. The lengths *AX, BX, CX*, and *DX* satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$.

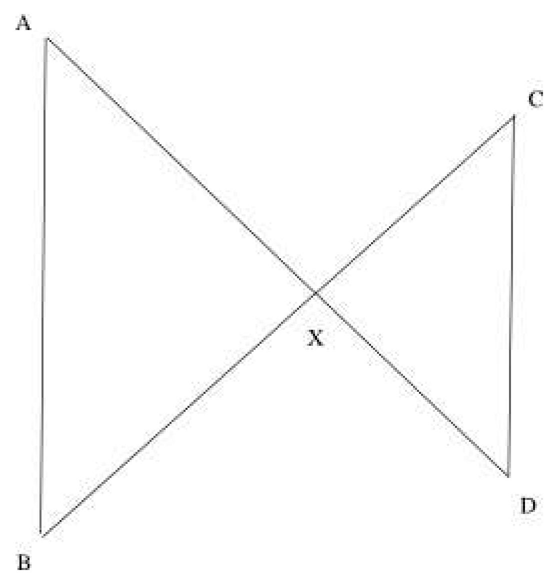
c. $\overline{AB}$ and $\overline{CD}$ are parallel.

d. $\angle XAB \cong \angle XCD$.

Task adapted from ***http://www.illustrativemathematics.org/illustrations/603.***

Extended Constructed Response Exemplar Response

In the picture given below, $\overline{AD}$ and $\overline{BC}$ intersect at *X*. $\overline{AB}$ and $\overline{CD}$ are drawn, forming two triangles, *AXB* and *CXD*.



In each part (a-d) below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar; and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one triangle to the other. If not, explain why not.

a. The lengths *AX* and *XD* satisfy the equation 2*AX* = 3*XD*.

I am given that 2AX = 3XD. This is not enough information to prove similarity. To see that in a simple way, I can draw an arbitrary triangle △AXB. Extend AX and choose a point D on the extended line so that 2AX = 3XD. Extend BX and choose a point C on the extended line so that 2BX = XC. Now triangles AXB and CXD satisfy the given conditions but are not similar.

b. The lengths *AX, BX, CX*, and *DX* satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$.

I am given that $\frac{AX}{BX} = \frac{DX}{CX}$*. Rearranging this proportion gives* $\frac{AX}{DX} = \frac{BX}{CX}$*. Let* $k = \frac{AX}{DX}$*. Suppose I rotate the triangle DXC 180 degrees about point X: Since AD is a straight line, DX and AX align upon rotation of 180 degrees, as do CX and BX, and so* $\angle DXC$ *and* $\angle AXB$ *coincide after this rotation. Then dilate the Triangle DXC by a factor of k about the center X. This dilation moves the point D to A, since k(DX) = AX, and moves C to B, since k(CX) = BX. Then, since the dilation fixes X, and dilations take line segments to line segments, I see that the Triangle DXC is mapped to Triangle AXB. So the original triangle DXC is similar to Triangle AXB.*

c. $\overline{AB}$ and $\overline{CD}$ are parallel.

Again, rotate triangle DXC so that $\angle DXC$ *coincides with* $\angle AXB$*. Then the image of the side CD under this rotation is parallel to the original side CD, so the new side CD is still parallel to side AB. Now, apply a dilation about point X that moves the vertex C to point B. This dilation moves the line CD to a line through B parallel to the previous line CD. I already know that line AB is parallel to line CD, so the dilation must move the line CD onto the line AB. Since the dilation moves D to a point on the ray XA and on the line AB, D must move to A. Therefore, the rotation and dilation map the Triangle DXC to the Triangle AXB. Thus, triangle DXC is similar to Triangle AXB.*

d. $\angle XAB \cong \angle XCD$.

Suppose I draw the bisector of $\angle AXC$*, and reflect the triangle CXD across this angle bisector. This maps the segment XC onto the segment XA; and since reflections preserve angles, it also maps segment XD onto segment XB. Since* $\angle XAB \cong \angle XCD$*, I also know that the image of side CD is parallel to side AB. Therefore, if I apply a dilation about the point X that takes the new point C to A, then the new line CD will be mapped onto the line AB, by the same reasoning used in (c). Therefore, the new point D is mapped to B, and thus, the Triangle XCD is mapped to Triangle XAB. So Triangle XCD is similar to Triangle XAB.*

Solving Right Triangles (ECR)

Overview

This task requires students to solve real-world problems using trigonometric ratios.

Standards

HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.C.8	• 8.G.B.7 • HSG-SRT.B.4 • HSG-SRT.C.6 • HSG-SRT.C.7	1. The angle of elevation from a point on the ground to the top of a building is 50^{0}. The point on the ground is 6 feet from the bottom of the building. How tall is the building? a. Approximately 7.15 feet	• http://www.illustrativemathematics.org/illustrations/655 • http://www.illustrativemathematics.org/illustrations/1635 • http://www.illustrativemathematics.org/illustrations/1443 • http://learnzillion.com/lessonsets/769-solve-problems-using-trigonometric-ratios-and-the-pythagorean-theorem • http://learnzillion.com/lessonsets/663-solve-right-triangles-using-trigonometric-ratios-and-the-pythagorean-theorem

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- **What is a clinometer?** A clinometer is an instrument used to find the measure of the angle of elevation in order to find the height of objects such as trees or buildings. (http://www.merriam-webster.com/dictionary/clinometer)

- **Why would someone buy a house and have trees cut down?** Trees may be removed for many reasons. Sometimes trees have a disease or get struck by lightning, and branches fall to the ground, causing damage to people or property. Another reason trees may be removed is because they are too close to a house, other structure, or power lines. If a tree is too close to a house, the house may become un-level.

- **What is stump grinding?** Stump grinding is the use of a machine to chip up the remainder of a tree stump. The machine chips the base of the tree into small pieces of wood until the tree stump is below ground level.

After the Task

Common errors/misunderstandings by problem number:

- **Problem 1**: Some students may forget to add in 6 feet for the height of the clinometer from the ground. Also, students may forget to put their calculator in degree mode.
- **Problem 2**: Students may give the incorrect answer of $1,854 because they do not notice that the total to remove the trees is over $1,200.
- **Problem 4**: Remind students to draw a picture. Also, students may forget to put their calculator in degree mode.

Student Extended Constructed Response

Bob just bought a new house, and he has three trees that he needs to have cut down. He has saved $1,000 to get the trees removed, but he is not sure if he has enough money. He contacts Jason's Tree Service, a local tree removal service, to get some preliminary information. The information below was provided by the company.

Jason's Tree Service Pricing

	Small Trees **25 feet or shorter**	**Medium Trees** **25-50 feet**	**Large Trees** **50-90 feet**
Tree Removal	$8 per foot	$9 per foot	$10 per foot
Stump Grinding	$75	$150	$300

When calculating the height of the trees, round up to the next whole number of feet.

If the cost to remove all trees is more than $1,200, we will throw in the stump grinding for free.

1. Bob needs to estimate the height of each tree to determine if he has enough money to have them removed. Bob uses a clinometer to measure the angle of elevation to the top of the tree. **He holds the clinometer 6 feet from the ground**. Complete the chart below to determine the height of each tree. Show all work.

	Angle of Elevation	**Bob's Distance from the** **Base of the Tree (feet)**	**Height of the Tree (feet)**
Tree 1	74°	4	
Tree 2	80°	13	
Tree 3	45°	35	

2. Based on the information from Jason's Tree Service, how much should Bob be charged for the removal and stump grinding of the three trees? Show all work and explain your reasoning.

3. Does Bob have enough money to get the 3 trees removed from his property? Show all work and explain your reasoning.

4. Bob has a tree in his backyard that he is going to chop down himself. In order to keep the tree from falling toward him, he decides to tie a rope to the top of the tree and stake it to the ground. If there is 25 feet of rope between the stake and the top of the tree, and the tree is 15 feet tall, what angle does the rope make with the ground? Show all work.

Extended Constructed Response Exemplar Response

Bob just bought a new house, and he has three trees that he needs to have cut down. He has saved $1,000 to get the trees removed, but he is not sure if he has enough money. He contacts Jason's Tree Service, a local tree removal service, to get some preliminary information. The information below was provided by the company.

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	Small Trees **25 feet or shorter**	**Medium Trees** **25-50 feet**	**Large Trees** **50-90 feet**
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If the cost to remove all trees is more than $1,200, we will throw in the stump grinding for free.

1. Bob needs to estimate the height of each tree to determine if he has enough money to have them removed. Bob uses a clinometer to measure the angle of elevation to the top of the tree. **He holds the clinometer 6 feet from the ground**. Complete the chart below to determine the height of each tree. Show all work.

	Angle of Elevation	Bob's Distance from the Base of the Tree (feet)	Height of the Tree (rounded to the nearest foot)
Tree 1	74°	4	***20***
Tree 2	80°	13	***80***
Tree 3	45°	35	***41***

Height of Tree 1:

$$\tan 74° = \frac{x}{4}$$
$$4 \times tan74° = x$$
$$13.95 \approx x$$
$$\approx 14\ feet$$
$$14\ ft + 6\ ft = 20\ feet$$

Height of Tree 2:

$$\tan 80° = \frac{x}{13}$$
$$13 \times tan\,80° = x$$
$$73.73 \approx x$$
$$\approx 74\,feet$$
$$74\,ft + 6\,ft = 80\,feet$$

Height of Tree 3:

$$\tan 45° = \frac{x}{35}$$
$$35 \times tan\,45° = x$$
$$35 = x$$
$$\approx 35\,feet$$
$$35ft + 6\,ft = 41\,feet$$

Alternate Solution for Height of Tree 3:

This is an isosceles triangle ($45°, 45°, 90°$*), so the second leg would be congruent to the first. As a result, that side is 35 feet, and the total height of the tree is 35 + 6 = 41 feet.*

2. Based on the information from Jason's Tree Service, how much should Bob be charged for the removal and stump grinding of the three trees? Show all work and explain your reasoning.

Tree 1:

$$20\,feet\ \times \$8 = \$160$$
$$\$160 + 75 = \$235$$

Tree 2:

$$80\,feet\ \times \$10 = \$800$$
$$\$800 + 300 = \$1100$$

Tree 3:

$$41\,feet\ \times \$9 = \$369$$
$$\$369 + 150 = \$519$$

If Bob had to pay for all three trees to be removed and the stumps ground, it would be as follows:
$\$235 + \$1100 + \$519 = \$1854$*.*

However, the cost for the three trees to be removed is as follows:
$\$160 + \$800 + \$369 = \$1329$*. This total is more than $1,200, so the stump grinding will be free.*

The total estimate is $1,329 to have all 3 trees removed from Bob's property and the stumps ground.

3. Does Bob have enough money to get the 3 trees removed from his property? Show all work and explain your reasoning.

Bob does not have enough money to have the tree removed. $\$1329 - \$1000 = \$329$ *Bob needs to save an additional $329 to be able to afford to have the three trees removed from his property.*

Note: If a student's answer is correct based on an incorrect equation in part 2, then this part should be marked correct.

4. Bob has a tree in his backyard that he is going to chop down himself. In order to keep the tree from falling toward him, he decides to tie a rope to the top of the tree and stake it to the ground. If there is 25 feet of rope between the stake and the top of the tree, and the tree is 15 feet tall, what angle does the rope make with the ground?

$$sin\,\alpha = \frac{15}{25}$$
$$\mathrm{m}\angle\alpha = \sin^{-1}\frac{15}{25}$$
$$m\angle\alpha \approx 36.87°$$

Understanding Congruence in Terms of Rigid Motions (IT)

Overview

This task allows students to explore triangle congruence in terms of rigid motions.

Standards

HSG.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

HSG.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG.CO.A.5	• 8.G.A.2 • 8.G.A.3	1. http://www.illustrativemathematics.org/illustrations/31 2. http://www.illustrativemathematics.org/illustrations/1549 3. http://www.illustrativemathematics.org/illustrations/1547	• http://www.illustrativemathematics.org/illustrations/646 • http://www.illustrativemathematics.org/illustrations/1231 • http://www.illustrativemathematics.org/illustrations/1232
HSG.CO.B.7	• 8.G.A.2 • HSG-CO.B.6	1. http://www.illustrativemathematics.org/illustrations/1637	• http://www.illustrativemathematics.org/illustrations/1228 • http://www.illustrativemathematics.org/illustrations/1230

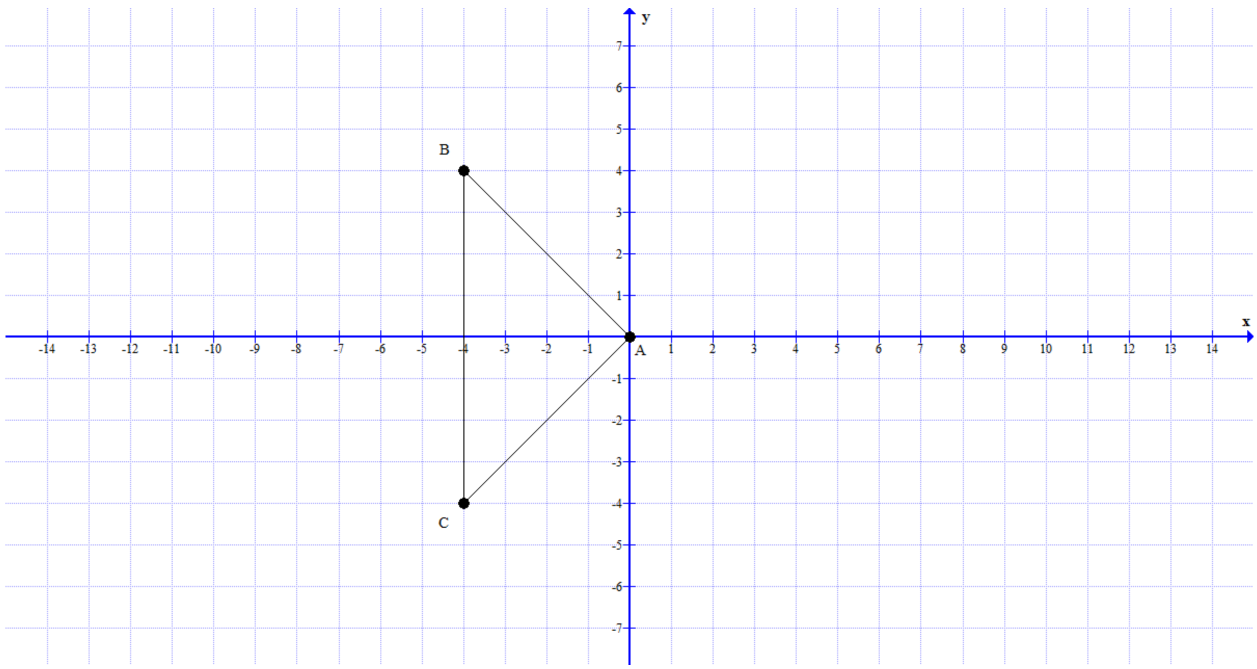
During the Task

While working on this task, students may need help finding the measures of the triangles' angles and sides. Students may choose to use appropriate tools to measure the angles or, as in the Instructional Task Exemplar Response, they may draw an additional segment to form right angles.

After the Task

Have students share the triangles that they created in step 5 with the class. Be sure that they explain how they know that the two triangles are congruent.

Student Instructional Task



1) Reflect Triangle ABC about the y-axis and label the new figure Triangle A'B'C'.

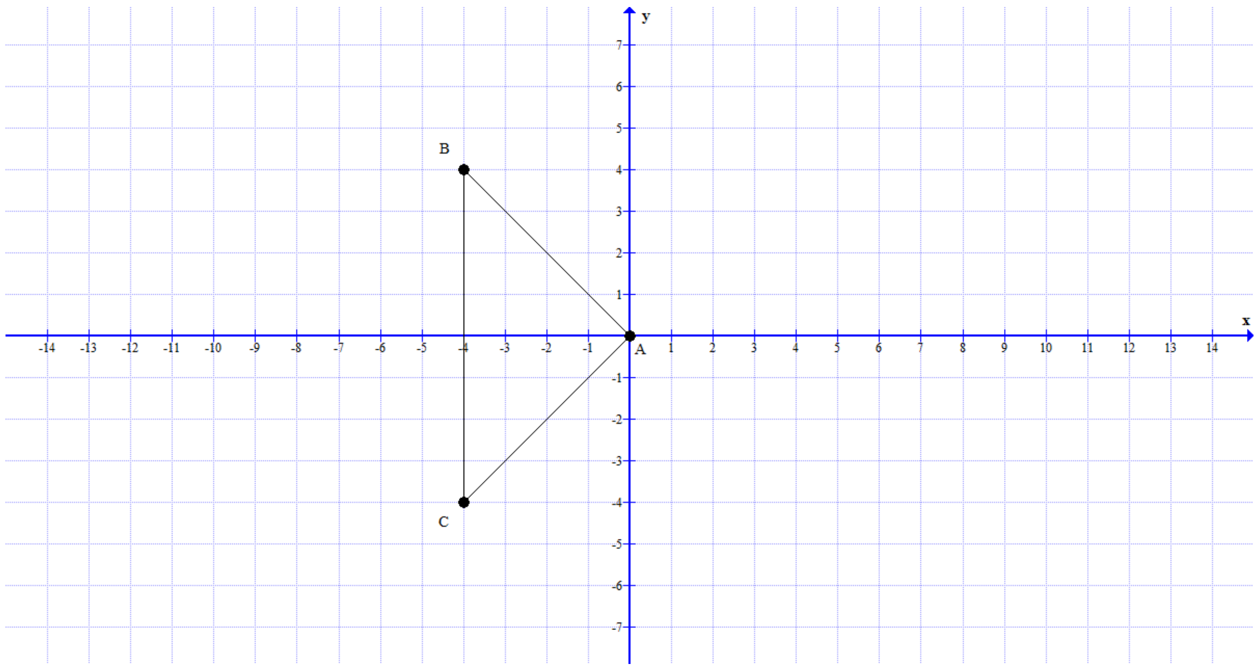
2) Give a sequence of 2 rigid transformations that would also carry Triangle ABC onto Triangle A'B'C'.

3) Find the side lengths and angle measures of both triangles. Show your work, and explain your reasoning.

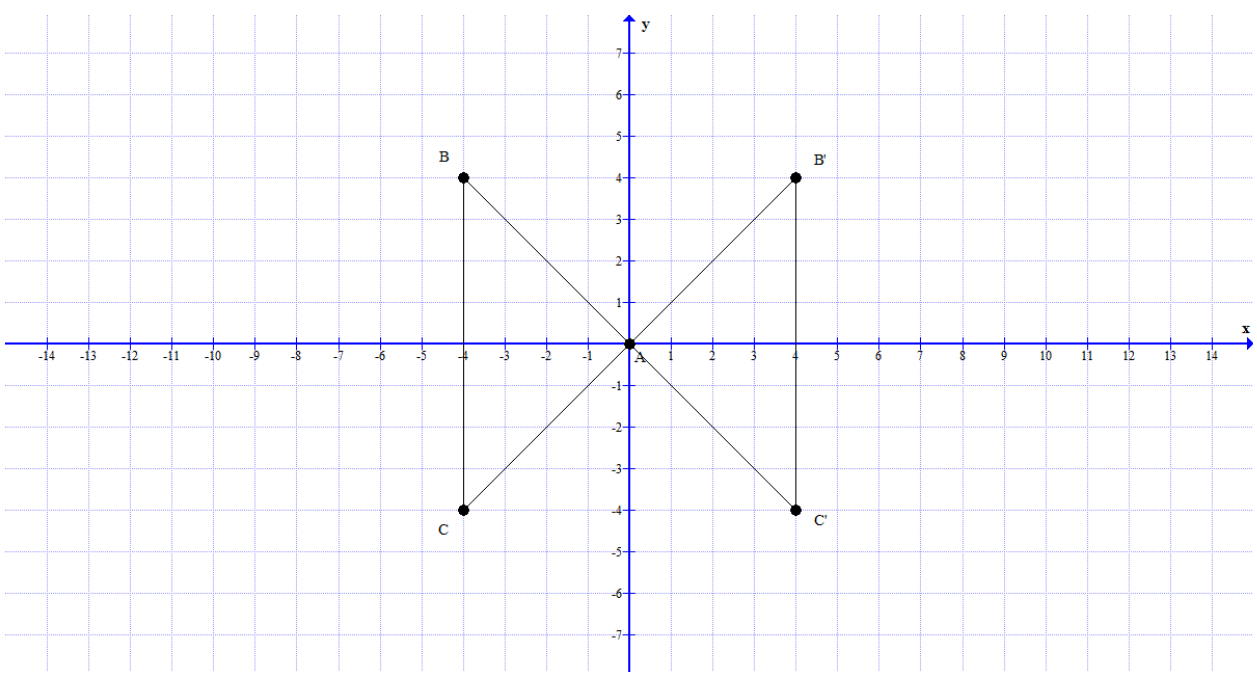
4) What do you know about the two triangles? Explain your reasoning.

5) Draw a new Triangle PQR that is congruent to Triangle ABC, and in a different location than either Triangle ABC or Triangle A'B'C'. Explain how you know Triangle PQR is congruent to Triangle ABC using at least 2 transformations.

Instructional Task Exemplar Response



1) Reflect Triangle ABC about the y-axis and label the new figure Triangle A'B'C'.



2) Give a sequence of 2 rigid transformations that would also carry Triangle ABC onto Triangle A’B’C’.
A rotation of 180° with the origin as the center and a reflection about the x-axis would carry Triangle ABC onto Triangle A’B’C’.

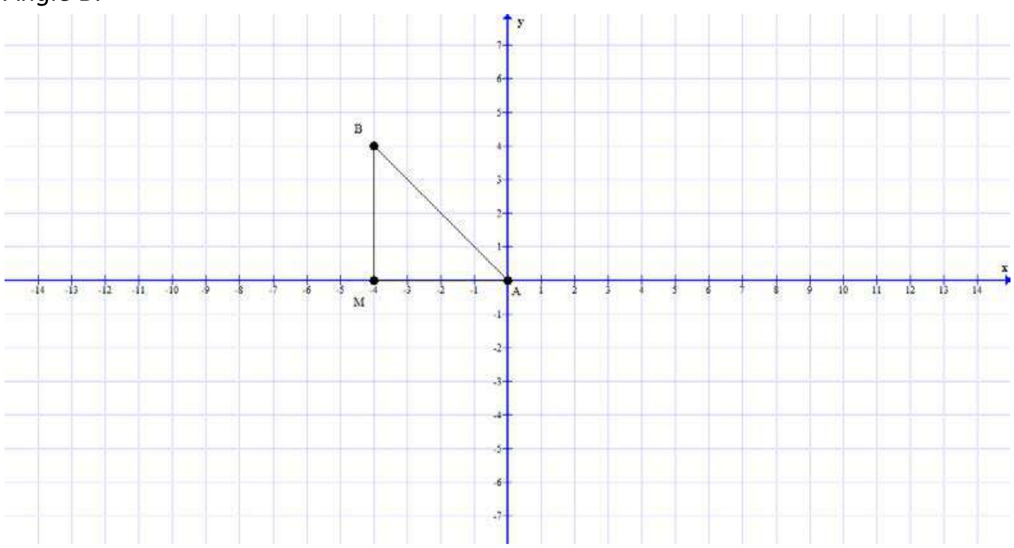
Alternate responses might include:

- *A reflection across the line y = x followed by a rotation of 90° counterclockwise*
- *A reflection across the line y = –x followed by a rotation of 90° clockwise*
- *A translation of 8 units to the right followed by a reflection across the line x = 4*

3) Find the side lengths and angle measures of both triangles. Show your work, and explain your reasoning.

Segment BC: $|4 - -4| = 8$ *units*
Alternate Response: Students might state that since this is a vertical segment, they counted 8 units.

Angle B:



In the above picture, I drew in Point M. This forms an isosceles right triangle. The base angles in an isosceles triangle are congruent, so Angle B measures 45°.

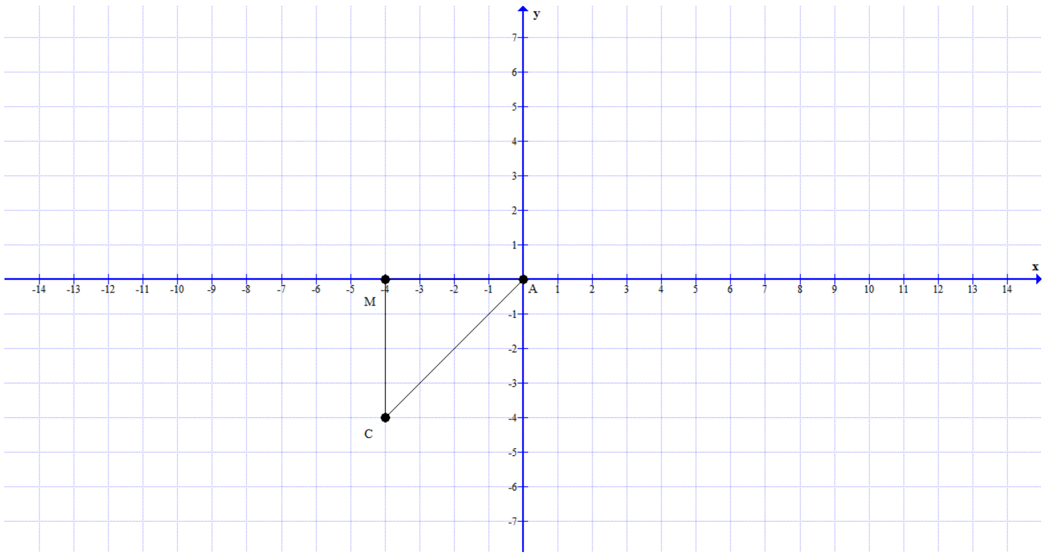
Alternate Response: Students may use a protractor to measure Angle B.

Segment AB:
I can use the Pythagorean Theorem.

$$4^2 + 4^2 = c^2$$
$$16 + 16 = c^2$$
$$32 = c^2$$
$$c = \sqrt{32}$$

Note: Students may find the length of Segment AB using the Distance Formula.

Angle C:



In the above picture, I drew in Point M. This forms an isosceles right triangle. The base angles in an isosceles triangle are congruent, so Angle C measures 45°.

Alternate Response: Students may use a protractor to measure Angle C.

Segment AC:
I can use the Pythagorean Theorem.

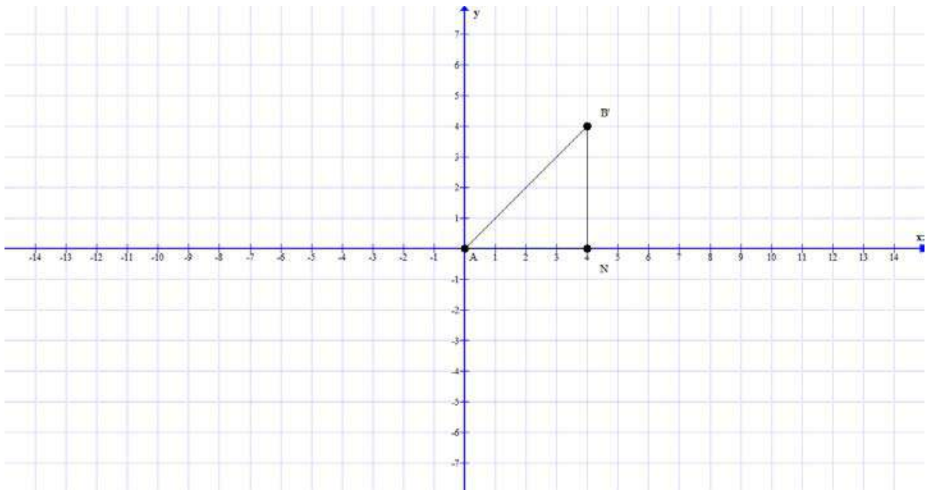
$$4^2 + 4^2 = c^2$$
$$16 + 16 = c^2$$
$$32 = c^2$$
$$c = \sqrt{32}$$

Note: Students may find the length of Segment AC using the Distance Formula.

Angle A: Angle C and Angle B both measure 45°. Therefore, the measure of Angle A is $80° - 45° - 45°$ *or* $90°$.

Segment B'C': $|4 - -4| = 8$
Alternate Response: Students might state that since this is a vertical segment, they counted 8 units.

Angle B':



In the above picture, I drew in Point N. This forms an isosceles right triangle. The base angles in an isosceles triangle are congruent, so Angle B' measures 45°.

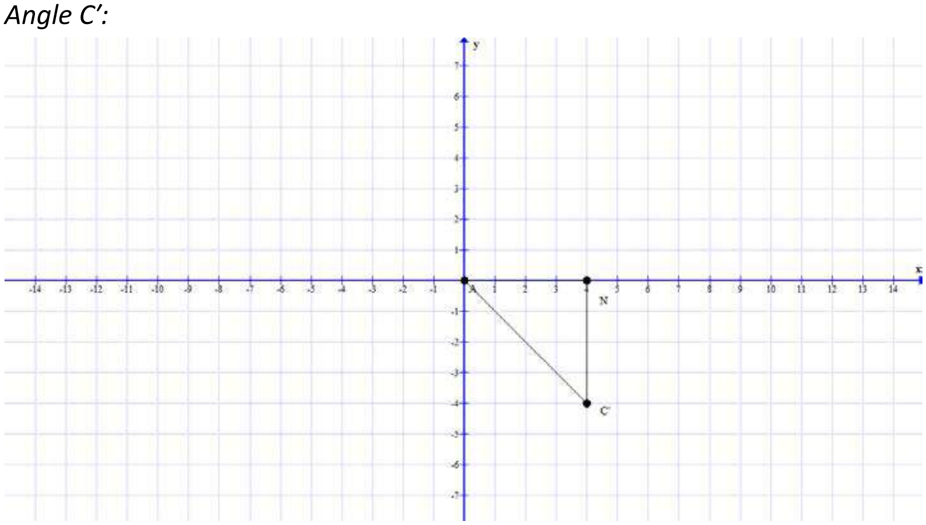
Alternate Response: Students may use a protractor to measure Angle B'.

Segment AB':
I can use the Pythagorean Theorem.

$$4^2 + 4^2 = c^2$$
$$16 + 16 = c^2$$
$$32 = c^2$$
$$c = \sqrt{32}$$

Note: Students may find the length of Segment AB' using the Distance Formula.

Angle C':



In the above picture, I drew in Point N. This forms an isosceles right triangle. The base angles in an isosceles triangle are congruent, so Angle C' measures 45°.

Alternate Response: Students may use a protractor to measure Angle C'.
Segment A'C':
I can use the Pythagorean Theorem.

$$4^2 + 4^2 = c^2$$
$$16 + 16 = c^2$$
$$32 = c^2$$
$$c = \sqrt{32}$$

Note: Students may find the length of Segment A'C' using the Distance Formula.

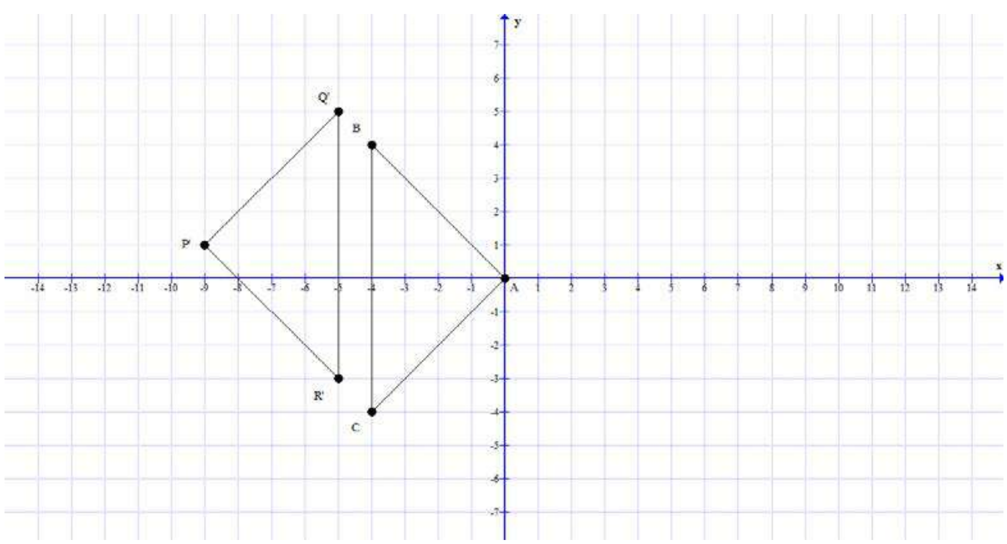
Angle A': Angle C' and Angle B' both measure 45°. Therefore, the measure of Angle A' is $180° - 45° - 45°$ *or* $90°$.

4) What do you know about the two triangles? Explain your reasoning.

The two triangles are congruent. From part 3, I can see that the corresponding pairs of sides and angles are congruent, so the two triangles are congruent. Also, since Triangle ABC can be transformed into Triangle A'B'C' through a series of rigid motions, then the two triangles are congruent.

5) Draw a new Triangle P'Q'R' that is congruent to Triangle ABC, and in a different location than either Triangle ABC or Triangle A'B'C'. Explain how you know Triangle PQR is congruent to Triangle ABC using at least 2 transformations.

There are many triangles that would answer this question. Here is a sample:



These triangles are congruent because a series of rigid motions transforms Triangle ABC into Triangle P'Q'R'. Triangle ABC can be reflected across the line, $x = -4.5$, *and translated up 1 unit.*

City Map (IT)

Overview

Students will use their knowledge of geometric terms and their skills with constructions to draw a city map with specific guidelines. After the map has been constructed, students will write proofs and explanations to answer questions about the lines and angles in the map.

Standards

Experiment with transformations in the plane

HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Prove geometric theorems

HSG-CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent, and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

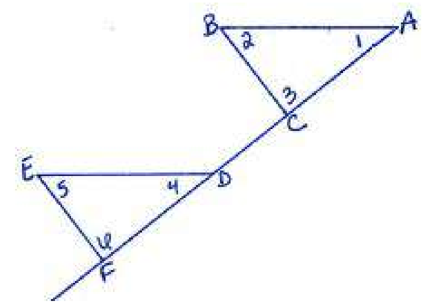
Make geometric constructions

HSG-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-CO.A.1	• 4.MD.C.5 • 4.G.A.1 • 4.G.A.2	1. Define parallel lines. a. Parallel lines are two coplanar lines that do not intersect. **2.** http://www.illustrativemathematics.org/illustrations/1543 3. http://www.illustrativemathematics.org/illustrations/1544	• http://www.illustrativemathematics.org/illustrations/1272 • http://www.illustrativemathematics.org/illustrations/1274 • http://learnzillion.com/lessonsets/808-define-geometric-terms-precisely

HSG-CO.C.9	• 4.MD.C.7 • 7.G.B.5 • 8.G.A.5 • HSG-CO.A.1	1. Given the diagram below, if $\overline{EF} \perp \overline{AF}$, $\overline{BC} \perp \overline{AF}$, and $\angle 5 \cong \angle 2$, prove that $\overline{AB} \parallel \overline{DE}$.  a. By the definition of perpendicular lines, $\angle 6$ and $\angle 3$ are right angles. Since all right angles are congruent, $\angle 6 \cong \angle 3$. By the definition of congruence, $m\angle 5 = m\angle 2$ and $m\angle 6 = m\angle 3$. The sum of the interior angles of a triangle is 180 degrees, so $m\angle 4 + m\angle 5 + m\angle 6 = 180$ and $m\angle 1 + m\angle 2 + m\angle 3 = 180$. By the substitution property of equality $m\angle 4 + m\angle 5 + m\angle 6 = 180$ and $m\angle 1 + m\angle 5 + m\angle 6 = 180$. Then, $m\angle 4 = 180 - (m\angle 5 + m\angle 6)$ and $m\angle 1 = 180 - (m\angle 5 + m\angle 6)$ Therefore, $m\angle 4 = m\angle 1$ and $\angle 4 \cong \angle 1$. $\angle 4$ and $\angle 1$ are formed by two lines and a transversal, which make them corresponding angles. When corresponding angles are congruent, the lines cut by the transversal are parallel. Therefore $\overline{AB} \parallel \overline{DE}$.	• http://www.illustrativemathematics.org/illustrations/1168 • http://www.illustrativemathematics.org/illustrations/59 • http://www.illustrativemathematics.org/illustrations/56 • http://www.illustrativemathematics.org/illustrations/1501 • http://www.illustrativemathematics.org/illustrations/1503
HSG-CO.D.12	• 4.MD.C.6 • 7.G.A.2 • HSG-CO.A.1	**1.** http://www.illustrativemathematics.org/illustrations/966	• http://www.illustrativemathematics.org/illustrations/909

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- **What is a compass?** A compass is used to tell the direction one is traveling. In the case of a map, the compass provides a means of orienting someone reading the map to the directions north, south, east, and west so they can better plan routes for trips.

- **What is a subway?** A subway is a mode of public transportation in some cities. Most subways run on rails like train tracks and are located underground so the subway does not create additional congestion on the highways and surface streets.

During the Task

- The directions in the task indicate students should use a poster board; however, this task can be completed on a regular letter size (8½" x 11") piece of paper as well. The size of the triangle may need to be adjusted if the task is changed so that students submit the product using a different medium than the poster.

- Students should use a compass, straightedge (ruler), and a protractor for the constructions described in this task.

- Students may attempt to draw parallel lines using their straightedge only. Ask probing questions to get students to think about the properties of parallel lines that will ensure the lines are parallel (if two lines are intersected by a transversal such that corresponding angles are congruent, then the lines must be parallel). Have students use constructions that would ensure congruent angles to create the parallel lines.

- Encourage students to use precise mathematical language as they write their explanations and proofs.

- Look for different explanations from students and have students share their explanations. Allow students to respectfully critique the reasoning of their classmates.

After the Task

Have students construct other polygons like squares, regular hexagons, regular octagons, etc., by copying angles and segments and creating perpendicular lines.

Student Instructional Task

Directions for Map Construction:

Note: All streets (lines) constructed should be extended to “run off” the poster board.

1. Begin by sketching a compass to indicate the directions north, south, east, and west. Draw this in the upper left-hand corner of the poster board.
2. Construct an equilateral triangle in the center of your poster board. The sides of the triangle should each measure 4 inches. Use a straightedge to extend the lines, including the sides of each triangle, so the lines “run off” the poster board. Label the vertices A, B, and C. This will give you three streets: $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
3. Construct a street parallel to street $\overline{BC}$. Name this street $\overline{AD}$.
4. Construct a street perpendicular to street $\overline{AD}$ so that it lies to the east of the triangle but does not pass through any point on $\Delta\, ABC$. Label the intersection of the perpendicular street and street $\overline{AD}$ as point E. Label the intersection of the perpendicular street and street $\overline{BC}$ as point F.
5. Label the intersection of streets $\overline{AB}$ and $\overline{EF}$ as point G.
6. Construct a street perpendicular to street $\overline{BC}$ so that it lies to the east of street $\overline{EF}$. Label the intersection of this perpendicular street and street $\overline{AD}$ as point H. Label the intersection of the perpendicular street and street $\overline{BC}$ as point J.
7. Label the intersection of streets $\overline{AB}$ and $\overline{HJ}$ as point K.

After completing the constructions, answer the questions on the following page.

On a separate sheet of paper, answer the following questions based on the map you created. Use complete sentences in your explanations and justifications. For proofs, you may choose the type of proof you create. Be sure to include logical reasoning to avoid leaving relevant information out of the proof.

1. State the measures of the following angles. Explain how you know the measures of the angles without using a protractor. Verify the angle measures.

    a. measure of Angle BAC

    b. measure of Angle DAB

    c. measure of Angle ABF

    d. measure of Angle BGF

2. What does it mean for two lines to be parallel? Prove that streets $\overline{AD}$ and $\overline{BC}$ are parallel.

3. Are there other pairs of streets that are parallel? Explain your reasoning using the properties of angles and parallel lines.

4. Imagine the section of the city depicted by your map has a subway that runs entirely underground and directly beneath street $\overline{EF}$.

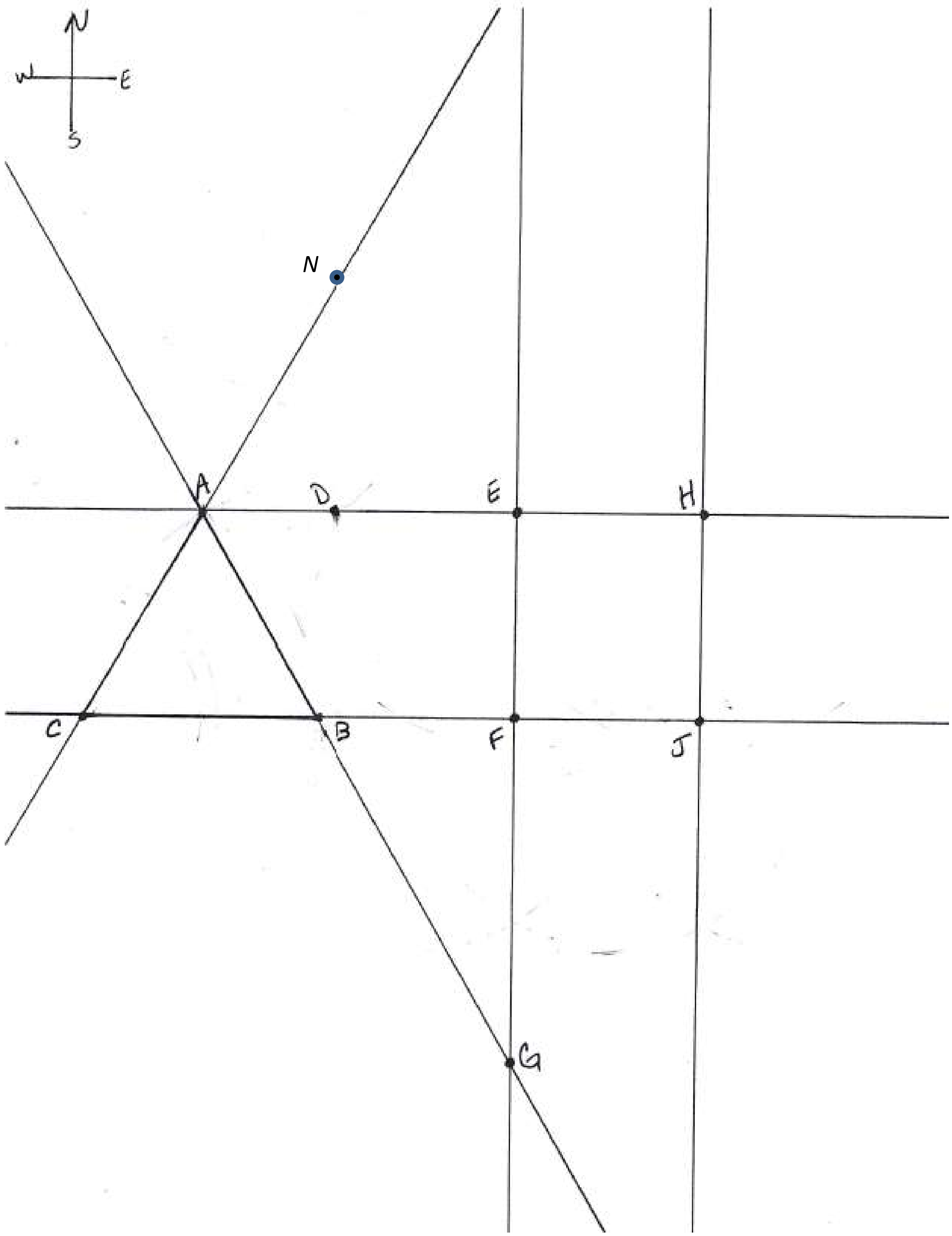
    a. Is the subway parallel to $\overline{EF}$? Explain.

    b. Is the subway parallel to another street? Explain.

    c. Since the subway runs underneath the streets on this map, it will never intersect with any of the streets shown on the map. Does this mean the subway is parallel to some or all of these streets? Explain.

Task adapted from: http://www.radford.edu/rumath-smpdc/Resources/src/Walstrum_CivilEng.pdf

Instructional Task Exemplar Response



Teacher Note: This picture is missing the intersection labeled K. Also, this is not drawn to the scale identified in the task. The construction marks were also removed. Point N was placed to aid in explanations in part two of the task.

Answer the following questions based on the map you created. Use complete sentences in your explanations and justifications. For proofs, you may choose the type of proof you create. Be sure to include logical reasoning to avoid leaving relevant information out of the proof.

1. State the measures of the following angles. Explain how you know the measures of the angles without using a protractor. Verify the angle measures.

    a. measure of Angle BAC
    The measure of Angle BAC is 60 degrees. Angle BAC is an angle in the equilateral triangle. All angles in an equilateral triangle are 60 degrees.

    b. measure of Angle DAB
    The measure of Angle DAB is also 60 degrees. Angle DAN is a copy of Angle ACB so it has the same measure as Angle ACB, which is 60 degrees. Angle DAN, Angle CAB, and Angle DAB are adjacent, and their non-common rays form a line, which means the sum of the angle measures is 180 degrees. Since the sum of Angle CAB and Angle DAN is 120 degrees, the measure of Angle DAB is also 60 degrees.

    c. measure of Angle ABF
    The measure of Angle ABF is 120 degrees. Angle ABF and Angle ABC are adjacent, and their non-common rays form a line, so the sum of the measures of the two angles is 180 degrees. Angle ABC is an angle in the equilateral triangle; therefore, its measure is 60 degrees. 180 degrees – 60 degrees = 120 degrees.

    d. measure of Angle BGF
    The measure of Angle BGF is 30 degrees. Angle FBG is 60 degrees because Angle ABC and Angle FBG are vertical angles and vertical angles are congruent, which means they have the same measure. Angle BFG is 90 degrees because street $\overline{EF}$ is perpendicular to street $\overline{BC}$ at point F, which means the angles formed at the intersection (labeled point F) are right angles. Right angles are 90-degree angles. Together, Angle FBG, Angle BFG, and Angle BGF are three angles in right triangle BFG. The sum of the measures of the three angles of a triangle is 180 degrees. 180 degrees – (60 degrees + 90 degrees) = 30 degrees.

2. What does it mean for two lines to be parallel? Prove that streets $\overline{AD}$ and $\overline{BC}$ are parallel.

    If two lines are parallel, they lie in the same plane and they do not intersect. I constructed $\overline{AD}$ by copying Angle ACB to construct Angle NAD. Copying an angle creates two congruent angles so Angle ACB is congruent to Angle NAD. Street $\overline{AC}$ is a transversal of streets $\overline{AD}$ and $\overline{BC}$, which makes Angle ACB and Angle NAD corresponding angles by the definition of corresponding angles. If two lines are cut by a transversal so that two corresponding angles are congruent, then the lines are parallel. Therefore streets $\overline{AD}$ and $\overline{BC}$ *are parallel.*

3. Are there any other pairs of streets that are parallel? Explain your reasoning using the properties of angles and parallel lines.

    Yes, streets $\overline{EF}$ and $\overline{HJ}$ are parallel. Street $\overline{EF}$ was constructed to be perpendicular to street $\overline{AD}$. All four angles formed at the intersection labeled E are right angles, and all right angles are congruent. Because $\overline{EF}$ is a transversal intersecting streets $\overline{AD}$ and *$\overline{BC}$, and they are parallel, street $\overline{EF}$ must be perpendicular to street $\overline{BC}$. Street $\overline{HJ}$ was constructed to be perpendicular to street $\overline{BC}$. By the same reasoning used earlier, street $\overline{HJ}$ must be perpendicular to street $\overline{AD}$. If two lines are perpendicular to the same line then the lines are parallel. Therefore streets $\overline{EF}$ and $\overline{HJ}$ are parallel.*

4. Imagine the section of the city depicted by your map has a subway that runs entirely underground and directly beneath street $\overline{EF}$.

   a. Is the subway parallel to $\overline{EF}$? Explain.

      Yes, the subway is parallel to $\overline{EF}$. Even though the subway is not on the street level, a plane can be formed between the street and the subway. According to the map, both street $\overline{EF}$ and the subway would be running north and south. Since they will never intersect, the street and the subway are parallel.

   b. Is the subway parallel to another street? Explain.

      Yes, the subway is also parallel to $\overline{HJ}$. Even though the subway is not on the street level, a plane can be formed between the street and the subway. According to the map, both street $\overline{HJ}$ and the subway would be running north and south. Since they will never intersect, the street and the subway are parallel.

   c. Since the subway runs underneath the streets on this map, it will never intersect with any of the streets shown on the map. Does this mean the subway is parallel to some or all of these streets? Explain.

      The subway is only parallel to those streets that run north and south. Streets that run any other direction (on the map they would intersect streets $\overline{EF}$ and $\overline{HJ}$) would not be in any same plane as the subway. Two lines that do not intersect and are not in the same plane are considered skew lines, not parallel. Therefore, the subway would be considered skew to all streets that do not run north and south.

Parallelogram Congruence (IT)

Overview

This task allows students to explore congruence and similarity using parallelograms.

Standards

Prove geometric theorems.

HSG-CO.C.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and, conversely, rectangles are parallelograms with congruent diagonals*.

Prove theorems involving similarity.

HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-CO.C.11	• 5.G.B.3 • HSG-CO.B.8 • HSG-CO.C.9	1. http://www.illustrativemathematics.org/illustrations/1321 2. http://www.illustrativemathematics.org/illustrations/1511 3. http://www.illustrativemathematics.org/illustrations/35	• http://www.illustrativemathematics.org/illustrations/109 • http://www.illustrativemathematics.org/illustrations/339 • http://www.illustrativemathematics.org/illustrations/110 • http://www.illustrativemathematics.org/illustrations/967 • http://learnzillion.com/lessonsets/810-prove-theorems-concerning-triangles-and-parallelograms
HSG-SRT.B.5	• HSG-SRT.A.3 • HSG-CO.B.8	1. http://www.illustrativemathematics.org/illustrations/651	• http://www.illustrativemathematics.org/illustrations/1422 • http://www.illustrativemathematics.org/illustrations/340 • http://learnzillion.com/lessonsets/668-solve-problems-using-congruence-and-similarity-criteria-for-triangles

During the Task

This task addresses similarity and congruence for a specific class of quadrilaterals, namely parallelograms. This task is ideal for hands-on work. For example, only one triangular shape is possible when using three toothpicks, namely an equilateral triangle. For quadrilaterals, on the other hand, four toothpicks can be put together to make any number of rhombuses with that side length.

After the Task

Have students make and verify conjectures about how much information is needed to determine if two quadrilaterals are congruent. For example, for squares one side is enough; for rectangles two adjacent sides are sufficient. Ask students "What information would be needed to show that any two arbitrary quadrilaterals are congruent?"

Student Instructional Task

Rhianna has learned the SSS and SAS congruence tests for triangles, and she wonders if these tests might work for parallelograms.

a. Suppose *ABCD* and *EFGH* are two parallelograms in which all corresponding sides are congruent; that is $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FG}$, $\overline{CD} \cong \overline{GH}$, $\overline{DA} \cong \overline{HE}$. Is it always true that *ABCD* is congruent to *EFGH*? Explain and show your reasoning.

b. Suppose *ABCD* and *EFGH* are two parallelograms with a pair of congruent corresponding sides, $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{FG}$. Suppose also that the included angles $\angle ABC \cong \angle EFG$ are congruent. Are *ABCD* and *EFGH* congruent? Explain and show your reasoning.

Task adapted from ***http://www.illustrativemathematics.org/illustrations/1517***

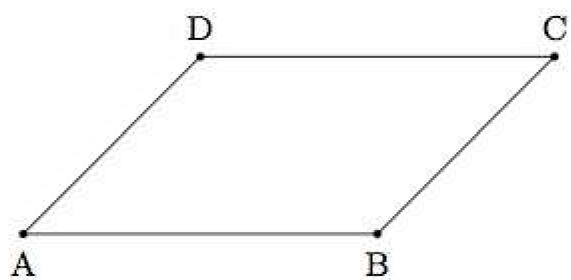
Instructional Task Exemplar Response

Rhianna has learned the SSS and SAS congruence tests for triangles, and she wonders if these tests might work for parallelograms.

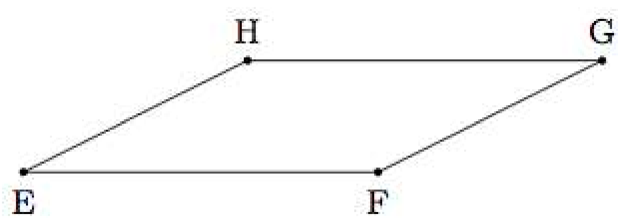
a. Suppose *ABCD* and *EFGH* are two parallelograms in which all corresponding sides are congruent; that is $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FG}$, $\overline{CD} \cong \overline{GH}$, $\overline{DA} \cong \overline{HE}$. Is it always true that *ABCD* is congruent to *EFGH*? Explain and show your reasoning.

   Sample response:

   I assumed that the statement "ABCD is always congruent to EFGH if all pairs of corresponding sides are congruent" was true. I attempted to find a counterexample to prove this assumption false. I began by drawing or building a parallelogram. The opposite sides of a parallelogram are congruent, so I will need two pairs of congruent segments:



   Leave AD fixed, but push it to the right as if it were hinged to AB at point A. This movement does not change the side lengths, but it does change the angle measures. Therefore, I get a new parallelogram as shown below:

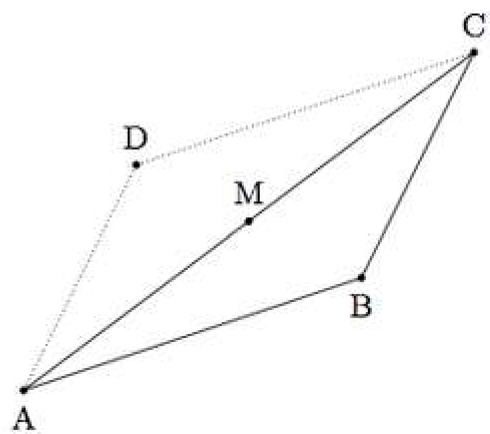


   Parallelograms ABCD and EFGH have four pairs of corresponding congruent sides, but the parallelograms are not congruent since they have different angle measures. So it is not always true that ABCD will be congruent to EFGH because I have at least one counterexample to the assumption that they are always congruent.

b. Suppose *ABCD* and *EFGH* are two parallelograms with a pair of congruent corresponding sides, $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{FG}$. Suppose also that the included angles are congruent. Are *ABCD* and *EFGH* congruent? Explain and show your reasoning.

Sample Response:

I know from the SAS triangle congruence test that △ABC is congruent to △EFG. In order to see what happens with the parallelograms ABCD and EFGH, I focus first on ABCD. Note that the vertex D is obtained by rotating vertex B 180 degrees about the midpoint M of $\overline{AC}$. *This is pictured below with the image of B labeled D:*



In other words, the parallelogram ABCD is obtained by connecting to △ABC a second triangle, △CDA, which is congruent to △ABC. The same is true of parallelogram EFGH (which is obtained by connecting △GHE to △EFG), and since △ABC is congruent to △EFG (which implies △CDA is congruent to △GHE), I can conclude that parallelogram ABCD is congruent to parallelogram EFGH.

Task adapted from http://www.illustrativemathematics.org/illustrations/1517

Modeling with Three-Dimensional Figures (IT)

Overview

Students will make decisions about the purchase of an air conditioner based on concepts of density based on volume.

Standards

Apply geometric concepts in modeling situations.

HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

Explain volume formulas and use them to solve problems.

HSG-GMD-A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-MG.A.2	• 7.G.B.6 • 8.G.C.9 • HSG-GMD.A.1 • HSG-GMD.A.3	1. The volume of a room is 3,300 cubic feet. If 2.5 BTUs per cubic foot are required to cool this room, how many total BTUs are needed? a. 8,250 BTUs **2.** http://www.illustrativemathematics.org/illustrations/1439	• http://www.illustrativemathematics.org/illustrations/266 • http://www.illustrativemathematics.org/illustrations/521 • http://www.illustrativemathematics.org/illustrations/1567 • http://www.illustrativemathematics.org/illustrations/1565 • https://www.khanacademy.org/math/geometry/basic-geometry/volume_tutorial/e/surface-and-volume-density-word-problems
HSG-GMD.A.3	• 8.G.C.9 • HSG-GMD.A.1	1. What is the volume of a rectangular closet with width 5 feet, length 6 feet, and height 8 feet? a. 240 cubic feet **2.** http://www.illustrativemathematics.org/illustrations/527 3. http://www.illustrativemathematics.org/illustrations/514	• http://learnzillion.com/lessonsets/535-use-volume-formulas-for-cylinders-pyramids-cones-and-spheres-to-solve-problems

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is an air conditioner?*** An air conditioner is a machine used to lower the temperature in a room or building.
- ***What is the difference between a window unit and a central air conditioner?*** A window unit is usually installed in the window of a room and usually cools just one room. A central air conditioner is usually installed outside of a house and usually cools the entire house or building.

During the Task

Students may struggle with finding the prices of air conditioners. You may want to provide a list of possible websites to students. For example, the reference for the Instructional Task Exemplar Response is ***http://www.homedepot.com***.

After the Task

This task shows students how volume is useful in the real world. They are able to see how purchasing decisions can rely on volume.

As an extension activity, students could measure the classroom, calculate the BTUs needed, and decide on an air conditioner.

Student Instructional Task

Sara is in a rock band. Her band practices several nights a week in a building at the back of her parents' property. The building is 22 feet by 24 feet and has 8-foot ceilings.

The central air conditioner in the building broke last week. Sara's parents told her that since she is the only one who uses the building, Sara has to pay to put in a new air conditioner if she wants the building to be cool when her band practices. Sara has to decide if they should purchase a replacement central air conditioning unit or a window unit.

1) A BTU, or British Thermal Unit, is a measurement of heat used to determine the cooling capacity of an air conditioner. One BTU is approximately the amount of heat put off by one kitchen match, so running a 1,000 BTU air conditioner is like extinguishing 1,000 matches in one hour.

   A good estimate for the cooling requirement of a building with 8-foot ceilings is 2.5 BTUs per cubic foot. Based on this estimate, how many BTUs does Sara's air conditioner need to have? Explain your reasoning and show your work. Source: **http://www.ehow.com/info_10044382_recommended-btu-air-conditioner-per-cubic-foot.html**

2) For rooms that regularly contain more than two people, an air conditioner needs an extra 600 BTUs per person. Sara's band has a total of seven people. How many additional BTUs does she need to add to her required BTUs? Show your work.

3) Air conditioners are available with BTUs in increments of 1,000. Based on your calculations, what is the minimum number of BTUs her air conditioner needs to have? Justify your answer.

4) Central air conditioners are often measured using tons in increments of half a ton. One ton of refrigeration can remove 12,000 BTUs of heat in one hour. How many tons would Sara's unit need to be? Justify your answer.

5) Research the prices and operating costs of central air conditioners and window units online. Select a central air conditioner and a window unit for Sara. Create equations for the central unit and window unit you chose that would tell the total cost, C, for operating the unit after x years.

6) Which type of air conditioner, a replacement central air conditioner or a window unit, should Sara buy? Explain your reasoning using the information you gathered about the cost of the air conditioners.

Instructional Task Exemplar Response

Sara is in a rock band. Her band practices several nights a week in a building at the back of her parents' property. The building is 22 feet by 24 feet and has 8-foot ceilings.

The central air conditioner in the building broke last week. Sara's parents told her that since she is the only one who uses the building, Sara has to pay to put in a new air conditioner if she wants the building to be cool when her band practices. Sara has to decide if they should purchase a replacement central air conditioning unit or a window unit.

1) A BTU, or British Thermal Unit, is a measurement of heat used to determine the cooling capacity of an air conditioner. One BTU is approximately the amount of heat put off by one kitchen match, so running a 1,000 BTU air conditioner is like extinguishing 1,000 matches in one hour.

   A good estimate for the cooling requirement of a building with 8-foot ceilings is 2.5 BTUs per cubic foot. Based on this estimate, how many BTUs does Sara's air conditioner need to have? Explain your reasoning and show your work. Source: http://www.ehow.com/info_10044382_recommended-btu-air-conditioner-per-cubic-foot.html
   First, find the volume of the room, and then multiply by the number of BTUs per cubic foot.

   Volume = length X width X height
$$V = 22 \times 24 \times 8$$
$$V = 4224\,ft^3$$
$$4224\,ft^3 \times 2.5\,BTUs/ft^3$$
$$= 10{,}560\,BTUs$$

2) For rooms that regularly contain more than two people, an air conditioner needs an extra 600 BTUs per person. Sara's band has a total of seven people. How many additional BTUs does she need to add to her required BTUs? Show your work.

$$7\,people - 2\,people = 5\,people$$
$$= 5 \times 600 = 3000\,BTUs$$

3) Air conditioners are available with BTUs in increments of 1,000. Based on your calculations, what is the minimum number of BTUs her air conditioner needs to have? Justify your answer.

$$10{,}560\,BTUs + 3000\,BTUs$$
$$= 13{,}560\,BTUs$$
   Sara will need an air conditioner for $14{,}000\,BTUs$ *in order to meet the minimum of 13,560 BTUs.*

4) Central air conditioners are often measured using tons in increments of half a ton. One ton of refrigeration can remove 12,000 BTUs of heat in one hour. How many tons would Sara's unit need to be? Justify your answer.
$$14{,}000\,BTUs \div 12{,}000\,BTUs$$
$$\approx 1.167\,tons$$
   Sara will need $1.5\,tons$ *of refrigeration because 1 ton will not be enough.*

5) Research the prices and operating costs of central air conditioners and window units online. Select a central air conditioner and a window unit for Sara. Create equations for the central unit and window unit you chose that would tell the total cost, *C*, for operating the unit after *x* years.

Sample response:
Sara found a replacement central air conditioner for $1,799. It has an estimated yearly operating cost of $122.

Central air conditioner:

$$C = 1799 + 122x, x = \#years\ used\ and\ C = Cost$$

Sara found a window unit for $399. It has an estimated yearly operating cost of $206:

$$C = 399 + 206x, x = \#years\ used\ and\ C = Cost$$

Source for air conditioner prices: ***http://www.homedepot.com***

6) Which type of air conditioner, a replacement central air conditioner or a window unit, should Sara buy? Explain your reasoning using the information you gathered about the cost of the air conditioners.

This answer will vary based on the prices and equations in part 5. Sample response:

I decided to find out how long it would take for the cost to run the different types of air conditioners to be the same.

$$C = 1799 + 122x$$
$$C = 399 + 206x$$
$$1799 + 122x = 399 + 206x$$
$$1400 = 84x$$
$$16.67 = x$$

At 16.67 years the cost of the two units would be the same.

$$C = 399 + 206(16.67)$$
$$C = 399 + 3{,}434.02$$
$$C = 3{,}833.02$$

The cost would be $3,833.02.

Note: If the cost is calculated using $\frac{1400}{84}$ *instead of 16.67, the answer is $3,832.33.*

If Sara is planning on using the practice space less than 16.67 years, she should purchase the window unit. If Sara is planning on using the practice space more than 16.67 years, she should replace the central air conditioner.

Exploring Similar Triangles (IT)

Overview

Students will be given two triangles and use a sequence a transformations to prove that they are similar.

Standards

Understand similarity in terms of similarity transformations.

HSG-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.A.3	• 8.G.A.4 • HSG-SRT.A.2	1. Explain using words how to prove that a two-dimensional figure is similar to another. a. In order to prove that a two-dimensional figure is similar to another two-dimensional figure, you must be able to draw the second figure from the first using a sequence of rotations, reflections, translations, and/or dilations.	• http://www.illustrativemathematics.org/illustrations/603 • http://learnzillion.com/lessonsets/775-establish-the-aa-criterion-for-triangle-similarity • http://learnzillion.com/lessonsets/540-use-the-properties-of-similarity-transformations-to-establish-the-aa-criterion

During the Task

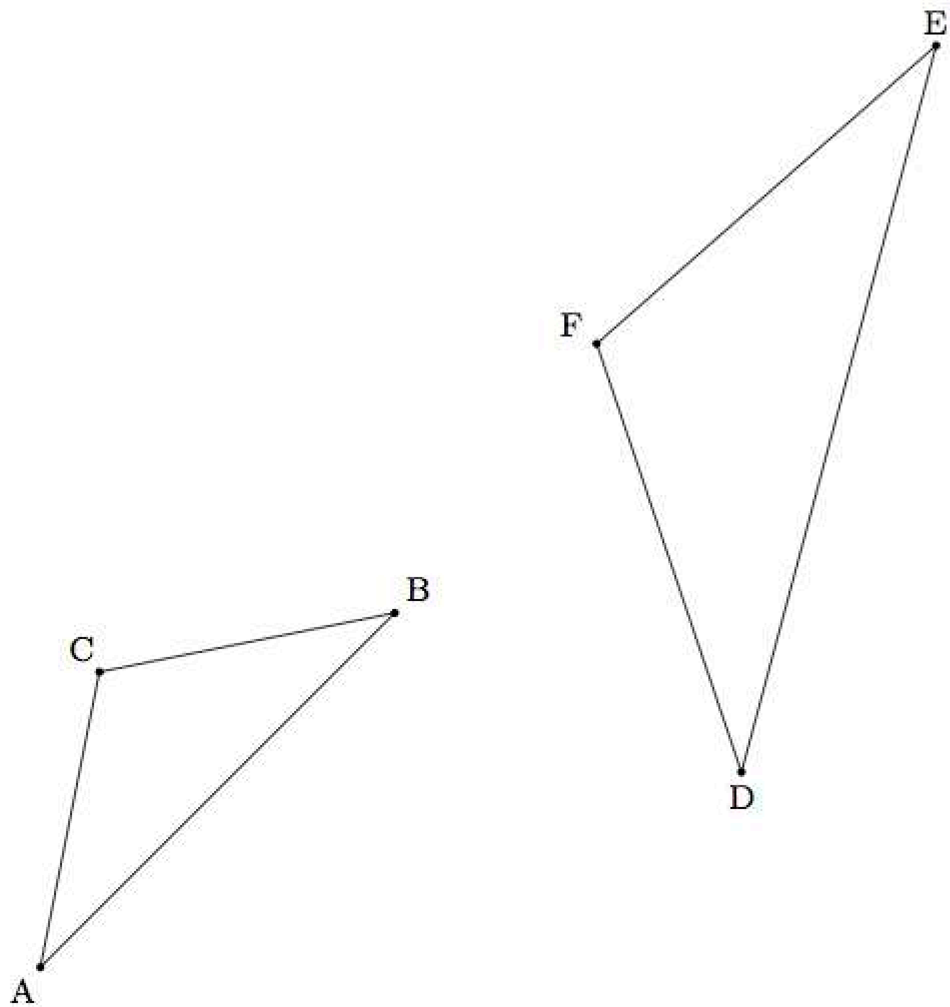
- As with all problems where a sequence of similarity transformations is requested, there are many possibilities. Keeping to a translation followed by a rotation and dilation, as in the solution provided, we could instead translate vertex *B* to vertex *E* (or vertex *C* to vertex *F*). A rotation will then align angles *ABC* and *DEF* (or *BCA* and *EFD*) and then a dilation will finish showing the similarity.
- For students who are just beginning to experiment with these transformations, dynamic geometry software or application would be invaluable to build an intuition for the different steps in the construction. It would also provide students an opportunity to exhibit the similarity in different ways, for example using a sequence of reflections and then a dilation.

After the Task

The teacher can have students compare their solutions as there are many possible series of transformations that demonstrate the similarity.

Student Instructional Task

In the two triangles pictured below, $m\angle A = m\angle D$ and $m\angle B = m\angle E$.

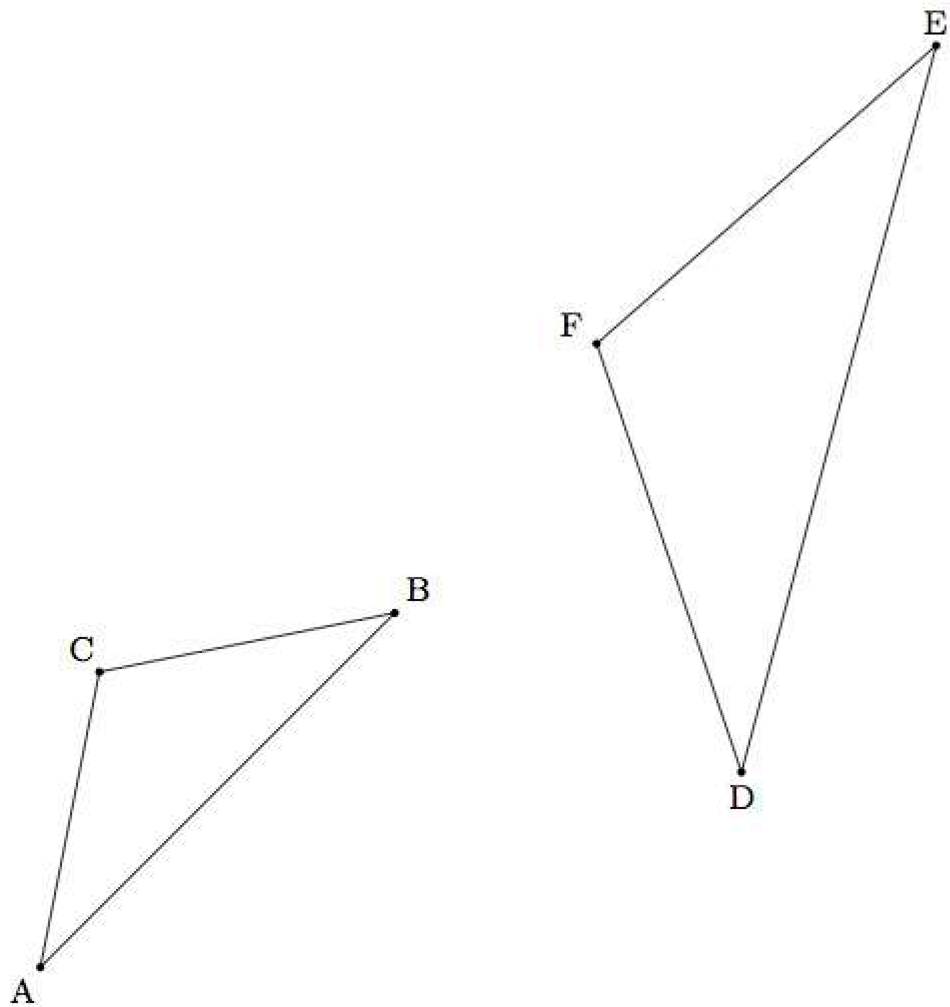


Using a sequence of translations, rotations, reflections, and/or dilations, show that ΔABC is similar to ΔDEF.

Task adapted from http://www.illustrativemathematics.org/illustrations/1422.

Instructional Task Exemplar Response

In the two triangles pictured below, $m\angle A = m\angle D$ and $m\angle B = m\angle E$.



Using a sequence of translations, rotations, reflections, and/or dilations, show that ΔABC is similar to ΔDEF.

Task adapted from ***http://www.illustrativemathematics.org/illustrations/1422****.*

Sample Response:

There are many ways to complete this Instructional Task. This sample will use the following:

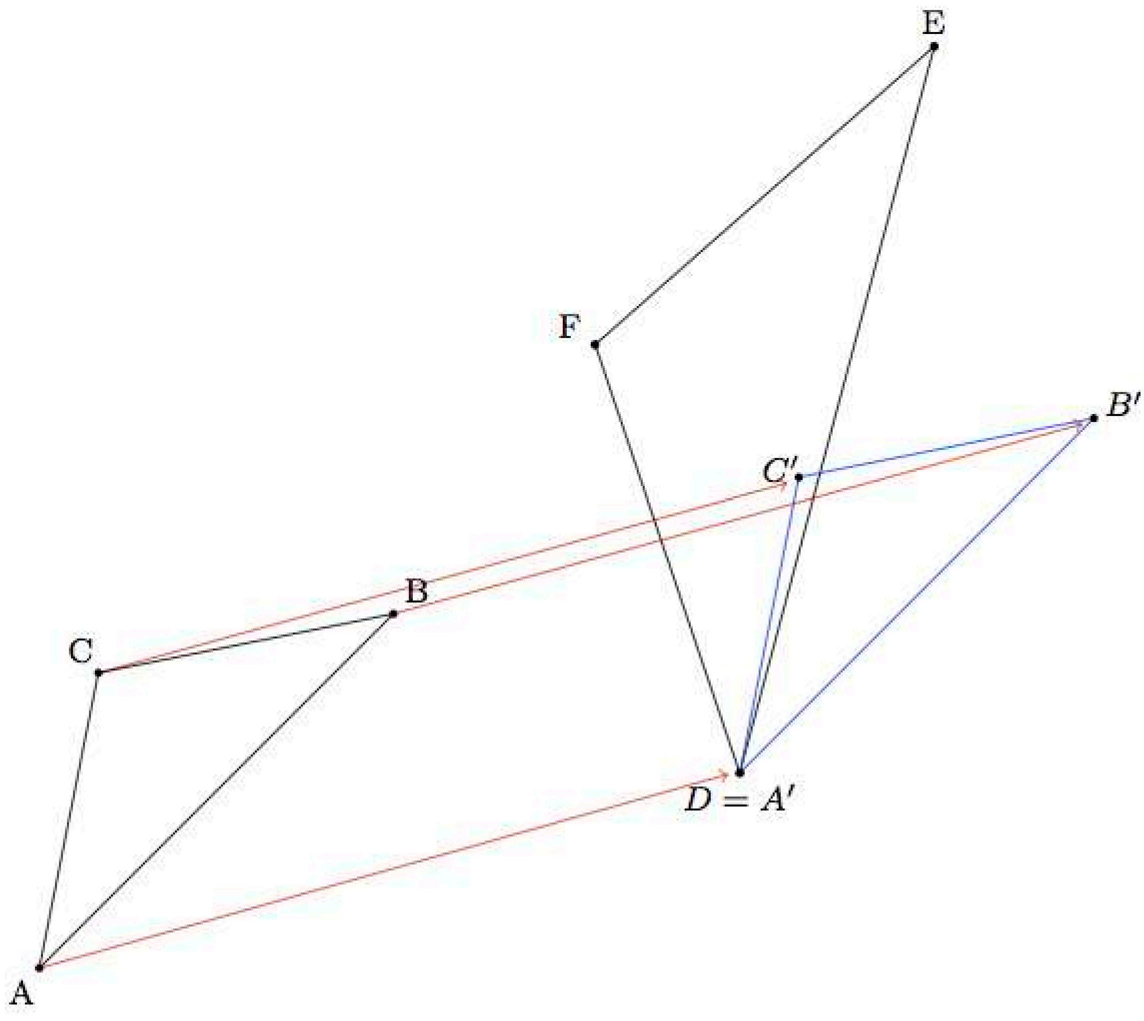
Step 1: a translation to map A to D

Step 2: a rotation to align two of the sides of the two triangles, and

Step 3: a dilation that completes the similarity transformation.

Step 1: Translation

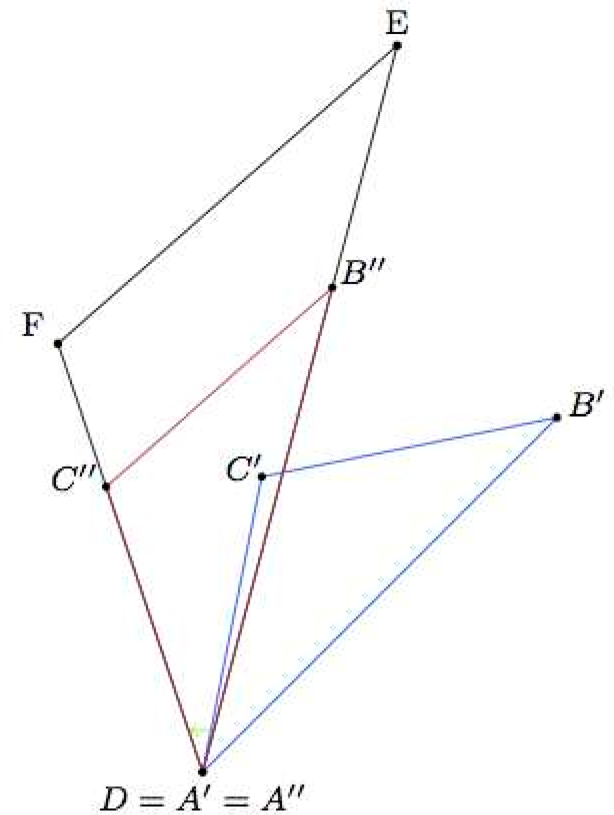
The translation taking *A* to *D* is pictured below, with the image of $\triangle ABC$ being denoted by $A'B'C'$.



Note that *A* = *D* because the translation is chosen precisely so as to map *A* to *D*.

Step 2: Rotation

Rotate $\triangle A'B'C'$ by angle $C'A'F'$ as pictured below where the angle of rotation is marked in green:



Also pictured is the image $A''B''C''$ of $\triangle A'B'C'$ under this rotation. Note that $\overrightarrow{A''C''} = \overrightarrow{DF}$. This is true by the choice of my angle of rotation. Note too that $\overrightarrow{A''B''} = \overrightarrow{DE}$. This is true because $m(\angle B''A''C'')=m(\angle BAC)$, since rigid motions preserve angle measures, and $m(\angle BAC) = m(\angle EDF)$ by hypothesis.

Step 3: Dilation

I have already moved A'' to D and so I chose D as the center of our dilation. I would like to move B'' to E and the dilation factor that will accomplish this is $\frac{|DE|}{|A''B''|}$. To check that C'' maps to F note that $m(\angle DEF) = m(\angle A''B''C'')$. Angles are preserved by dilations and so this means that $\overline{B''C''}$ must map to $\overline{EF}$ (if C'' mapped to a point different than F, then there would be two rays from E to points on $\overleftrightarrow{DF}$ making the same angle with $\overline{ED}$, which is not possible).

ALGEBRA II TOOLS

ALGEBRA II TOOLS

Algebra II Remediation Guide

As noted in the "Remediation" on page 11 isolated remediation helps target the skills students need to more quickly access and practice on-grade level content. This chart is a reference guide for teachers to help them more quickly identify the specific remedial standards necessary for every Algebra II math standard[7].

Algebra II Standards	Previous Grade or Course Standards	Algebra II Standards to be Taught before (Scaffolded)	Algebra II Standards to be Taught Concurrently
HSN-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	• 8.EE.A.1 • 8.EE.A.2		
HSN-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.		• HSN-RN.A.1	
HSN-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.	• HSN-Q.A.1		
HSN-CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.	• 8.EE.A.2		• HSN-CN.A.2
HSN-CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	• 7.EE.A.1		• HSN-CN.A.1
HSN-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.		• HSN-CN.A.1 • HSN-CN.A.2	
HSA-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see x^4-y^4 as $(x^2)^2-(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$.*	• HSA-SSE.A.1		
HSA-SSE.B.3c Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*		• HSA-SSE.A.2	
HSA-SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★		• HSA-SSE.B.3c	

[7] This content comes from research found here: http://www.edutron.com/0/Math/ccssmgraph.htm

Algebra II Standards	Previous Grade or Course Standards	Algebra II Standards to be Taught before (Scaffolded)	Algebra II Standards to be Taught Concurrently
HSA-APR.B.2 Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by $x - a$ is *p(a)*, so *p(a)* = 0 if and only if $(x - a)$ is a factor of *p(x)*.	• HSA-SSE.B.3a	• HSA-APR.B.3	• HSA-APR.D.6
HSA-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	• HSA-SSE.B.3a	• HSA-SSE.A.2	
HSA-APR.C.4 Prove polynomial identities and use them to describe numerical relationships. F*or example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*		• HSA-SSE.A.2	
HSA-APR.D.6 Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x)* + *r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.	• 7.NS.A.2b	• HSA-SSE.A.2	• HSA-APR.B.2
HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	• HSA-REI.B.4a		• HSA-REI.A.1
HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.			• HSA-CED.A.1
HSA-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.		• HSA-REI.A.1	
HSA-REI.B.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.	• HSA-REI.B.4a • HSA-SSE.B.3a		
HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	• HSA-CED.A.3 • HSA-REI.C.5		• HSA-REI.D.11
HSA-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.		• HSA-REI.C.6	• HSA-REI.D.11

Algebra II Standards	Previous Grade or Course Standards	Algebra II Standards to be Taught before (Scaffolded)	Algebra II Standards to be Taught Concurrently
HSA-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★			• HSA-REI.C.6 • HSA-REI.C.7
HSF-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	• HSF-IF.A.1 • HSF-IF.A.2		• HSF-IF.C.7c
HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	• HSF-IF.A.1 • HSN-Q.A.1		• HSF-BF.A.1a • HSF-F.IF.C.7c • HSF-F.IF.C.7e
HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	• HSF-IF.A.2		
HSF-IF.C.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	• HSF-IF.C.8a • HSA-APR.B.3		• HSF-IF.B.4
HSF-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	• HSF-IF.A.1 • HSF-LE.A.1c		• HSF-IF.B.4 • HSF-IF.C.8b • HSF-BF.B.3 • HSF-TF.B.5
HSF-IF.C.8b Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*		• HSN-RN.A.1	• HSF-F.IF.C.7e • HSF-BF.B.3
HSF-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	• HSF-IF.C.8a	• HSF-IF.B.4 • HSF-IF.C.8b	

Algebra II Standards	Previous Grade or Course Standards	Algebra II Standards to be Taught before (Scaffolded)	Algebra II Standards to be Taught Concurrently
HSF-BF.A.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context.			• HSF-IF.B.4 • HSF-LE.A.2
HSF-BF.A.1 Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	• HSA-APR.B.3	• HSF-BF.A.1a	
HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★		• HSF-BF.A.1a • HSF-IF.A.3	
HSF-BF.B.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	• HSF-IF.C.7a • HSF-IF.C.7b		• HSF-F.IF.C.7e • HSF-IF.C.8b
HSF-BF.B.4a Solve an equation of the form *f(x) = c* for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	• HSA-CED.A.4	• HSA-REI.A.2 • HSA-REI.C.6	
HSF-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	• HSF-LE.A.1b • HSF-LE.A.1c		• HSF-BF.A.1a • HSF-BF.A.2
HSF-LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.		• HSA-SSE.B.3c • HSF-IF.C.8b	
HSF-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.		• HSF-BF.B.3 • HSF-LE.A.2 • HSF-IF.C.7e	
HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	• HSG-C.B.5		

Algebra II Standards	Previous Grade or Course Standards	Algebra II Standards to be Taught before (Scaffolded)	Algebra II Standards to be Taught Concurrently
HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	• HSG-SRT.C.8 • HSG-GPE.A.1	• HSF-TF.A.1	
HSF-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	• HSF-LE.A.1b	• HSF-BF.B.3 • HSF-IF.B.4	• HSF-IF.C.7e
HSF-TF.C.8 Prove the Pythagorean identity $\sin^2(\emptyset) + \cos^2(\emptyset) = 1$ and use it to find $\sin(\emptyset)$, $\cos(\emptyset)$, or $\tan(\emptyset)$ given $\sin(\emptyset)$, $\cos(\emptyset)$, or $\tan(\emptyset)$ and the quadrant of the angle.		• HSF-TF.A.2	
HSG-GPE.A.2 Derive the equation of a parabola given a focus and directrix.	• HSG-GPE.A.1		
HSS-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	• 6.SP.B.5 • HSS-ID.A.1 • HSS-ID.A.2		
HSS-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	• 8.SP.A.3 • HSF-LE.A.1 • HSA-CED.A.2	• HSF-LE.A.2 • HSF-BF.A.1	
HSS-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	• 7.SP.A.2		
HSS-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	• 7.SP.C.7		
HSS-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.		• HSS-IC.A.1	
HSS-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.		• HSS-IC.A.2 • HSS-IC.B.3	

Algebra II Standards	Previous Grade or Course Standards	Algebra II Standards to be Taught before (Scaffolded)	Algebra II Standards to be Taught Concurrently
HSS-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.		• HSS-IC.A.2 • HSS-IC.B.3	
HSS-IC.B.6 Evaluate reports based on data.		• HSS-IC.B.4 • HSS-IC.B.5	
HSS-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	• 7.SP.C.8		
HSS-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.		• HSS-CP.A.1	
HSS-CP.A.3 Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.		• HSS-CP.A.1 • HSS-CP.A.2	
HSS-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	• HSS-ID.B.5	• HSS-CP.A.2 • HSS-CP.A.3	
HSS-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*		• HSS-CP.A.3	
HSS-CP.B.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.		• HSS-CP.A.3	
HSS-CP.B.7 Apply the Addition Rule, *P(A* or *B)* = *P(A)* + *P(B)* – *P(A* and *B)*, and interpret the answer in terms of the model.		• HSS-CP.A.1 • HSS-CP.A.3	

Algebra II Tasks at a Glance

There are 10 sample tasks included in this guidebook that can be used to supplement any curriculum.

The tasks for Algebra II include:

- **5 Extended Constructed Response (ECR):** These short tasks, aligned to the standards, mirror the extended constructed response items students will see on their end of year state assessments.
- **5 Instructional Tasks (IT):** These complex tasks are meant to be used for instruction and assessment. They will likely take multiple days for students to complete. They can be used to help students explore and master the full level of rigor demanded by the standards. Teachers can use the table below to find standards associated with current instruction and add in these practice items to supplement any curriculum. These tasks should be used after students have some initial understanding of the standards. They will help students solidify and deepen their understanding of the associated content.

This is an overview of the Algebra II tasks included on the following pages.

Title	Type	Task Standards	Task Remedial Standards
Zeros of Polynomials Page 181	ECR	• HSA-APR.B.2 • HSA-APR.B.3	• HSA-SSE.B.3a • HSA-SSE.A.2 • HSA-SSE.B.3a
Solving Exponential Equations Page 186	ECR	• HSA-REI.D.11	• HSA-REI.C.5 • HSA-REI.C.6 • HSA-REI.C.7
Arithmetic and Geometric Sequences Page 191	ECR	• HSF-BF.A.2	• HSF-BF.A.1a • HSF-IF.A.3
Student Well-being Page 198	ECR	• HSS-IC.B.4	• HSS-IC.A.2 • HSS-IC.A.3
Radical Table Page 204	ECR	• HSN-RN.A.2	• HSN-RN.A.1
Preferred Chocolate Page 207	IT	• HSS-IC.B.3	• HSS-IC.A.1
Growing Radishes Page 224	IT	• HSS-IC.B.5	• HSS-IC.A.2 • HSS-IC.B.3
Interpreting Functions Page 232	IT	• HSF-IF.B.4 • HSF-IF.B.6	• HSF-IF.A.1 • HSN-Q.A.1 • HSF-IF.A.2
Lifetime Savings Page 237	IT	• HSA-SSE.B.4	• HSA-SSE.B.3c
Radical Equations Page 245	IT	• HSA-REI.A.1 • HSA-REI.A.2	• HSA-REI.B.4 • HSA-REI.A.1

Zeros of Polynomials (ECR)

Overview

Apply the Remainder Theorem to identify zeros of a function and determine a missing coefficient. Students will also sketch the graph of a polynomial function using the zeros of the function.

Standards

Understand the relationship between zeros and factors of polynomials.

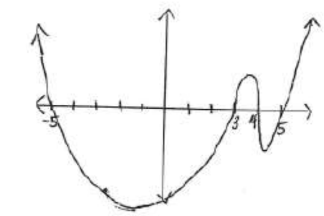
HSA-APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

HSA-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-APR.B.2	• HSA-SSE.B.3a	1. Use the Remainder Theorem to determine whether $(x-5)$ is a factor of $x^3+3x^2-25x-75$. a. $(x-5)$ is a factor of the polynomial because $p(5)=0$. 2. http://www.illustrativemathematics.org/illustrations/592	• http://www.illustrativemathematics.org/illustrations/388 • http://learnzillion.com/lessonsets/605-know-and-apply-the-remainder-theorem • http://learnzillion.com/lessonsets/504-know-and-apply-the-remainder-theorem
HSA-APR.B.3	• HSA-SSE.A.2 • HSA-SSE.B.3a	1. Find all zeros of $p(x)=(x^2-25)(x^2-7x+12)$. a. Zeros: –5, 3, 4, 5 2. Construct a rough graph of $p(x)=(x^2-25)(x^2-7x+12)$ showing all zeros of the function.	• http://www.illustrativemathematics.org/illustrations/87 • http://www.illustrativemathematics.org/illustrations/21 • http://learnzillion.com/lessonsets/606-identify-zeros-of-polynomials-and-construct-rough-graphs-of-polynomial-functions • http://learnzillion.com/lessonsets/559-identify-zeros-and-construct-graphs-of-

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		a. 	2degree-polynomials

After the Task

Common errors/misconceptions by problem number:

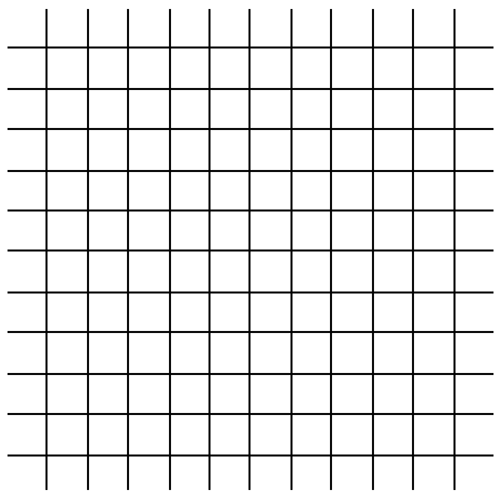
- ***Problem 1:*** Students may use 3 for the value of *a* instead of –3 when evaluating the polynomial. Remind students to rewrite the binomial $(x + 3)$ as $(x - (-3))$ to find the correct value of *a*.
- ***Problem 2:*** Students may struggle to find the remaining zeros. Remind students that one factor was given in problem 1. To find the remaining zeros, students will need to find the other factors. Ask students to think about methods used to find an unknown factor when given a product and one of the factors. Then guide students to recall the ways they know to divide polynomials. Students will also need to be reminded of how to factor a quadratic trinomial from Algebra I. Extra practice with division of polynomials and factoring may be necessary for struggling students.
- ***Problem 3:*** Students' responses should demonstrate that they are able to apply the Remainder Theorem. If students use long division or synthetic division, ask them if there is a more efficient method to find the remainder of the given polynomial.
- ***Problem 4:*** Students who are struggling with applying the Remainder Theorem will attempt to solve this by using other methods (e.g., long division or synthetic division). Ask students to identify what the remainder would be if the given polynomial were divided evenly by the binomial. Students should understand that the remainder would be zero and that *p*(*a*) should be zero.

Student Extended Constructed Response

Use the polynomial function $p(x) = 7x^3 + 29x^2 + 25x + 3$ to answer questions 1 and 2 below.

1. Show that $(x + 3)$ is a factor of $p(x)$ using the Remainder Theorem. Explain your reasoning.

2. Sketch a graph of $p(x)$ showing all zeros of the function. Show all work to find the zeros.



3. Consider the polynomial $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$. Determine the remainder of $f(x) \div k(x)$ if $k(x) = (x - 3)$. Show your work.

4. The polynomial function $q(x) = x^4 + ax^3 - 19x^2 - 46x + 120$ has a root of –4. What is the value of the missing coefficient, *a*? Show your work and explain your reasoning.

Extended Constructed Response Exemplar Response

Use the polynomial function $p(x) = 7x^3 + 29x^2 + 25x + 3$ to answer questions 1 and 2 below.

1. Show that $(x + 3)$ is a factor of $p(x)$ using the Remainder Theorem. Explain your reasoning.

   If $(x + 3)$ *is a factor of* $p(x)$*, then* $p(-3) = 0$*. Substitute –3 for x and evaluate the expression.*

$$p(-3) = 7(-3)^3 + 29(-3)^2 + 25(-3) + 3$$

$$p(-3) = -189 + 261 - 75 + 3$$

$$p(-3) = 0$$

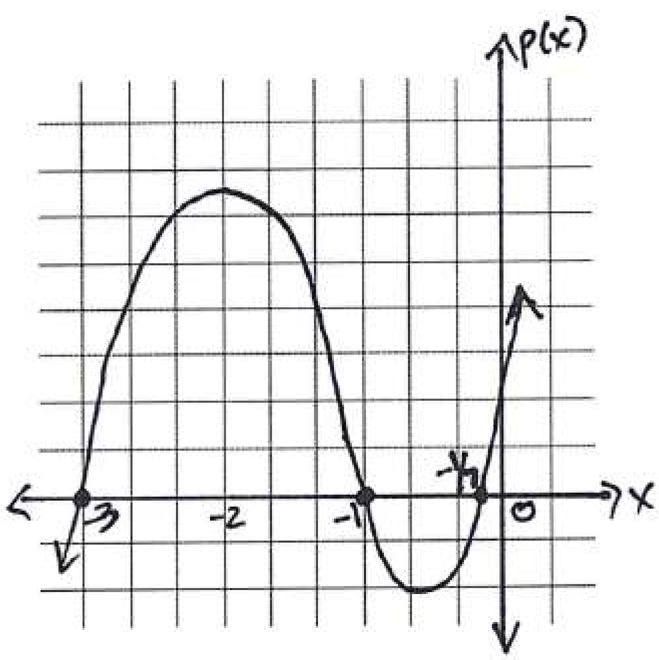
2. Sketch a graph of $p(x)$ showing all zeros of the function. Show all work to find the zeros.

   Find the remaining factors first.

-3	7	29	25	3
		-21	-24	-3
	7	8	1	0

$7x^2 + 8x + 1 = (7x + 1)(x + 1)$; *set factors equal to zero and solve for x.*

Zeros of the function: $-3,\ -1, -\frac{1}{7}$



3. Consider the polynomial $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$. Determine the remainder of $f(x) \div k(x)$ if $k(x) = (x - 3)$. Show your work.

$$f(3) = (3)^4 + 2(3)^3 - 7(3)^2 - 8(3) + 12$$

$$f(3) = 60$$

The remainder of $f(x) \div k(x)$ if $k(x) = (x - 3)$ is 60.

4. The polynomial function $q(x) = x^4 + ax^3 - 19x^2 - 46x + 120$ has a root of –4. What is the value of the missing coefficient, *a*? Show your work and explain your reasoning.

If –4 is a zero of $q(x)$, then $q(-4) = 0$. So replace all x variables with –4, set the function equal to 0, and solve for a.

$$(-4)^4 + a(-4)^3 - 19(-4)^2 - 46(-4) + 120 = 0$$

$$256 - 64a - 304 + 184 + 120 = 0$$

$$-64a + 256 = 0$$

$$-64a = -256$$

$$a = 4$$

Solving Exponential Equations (ECR)

Overview

Using technology, students will solve exponential equations by finding the x-coordinate(s) of the point(s) of intersection.

Standard

Represent and solve equations and inequalities graphically.

HSA-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are prerequisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.D.11	• HSA-REI.C.5 • HSA-REI.C.6 • HSA-REI.C.7	1. Using technology, approximate the solution to the equation $f(x) = g(x)$ if $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = 3$. a. $x \approx -1.585$ 2. http://www.illustrativemathematics.org/illustrations/645	• http://www.illustrativemathematics.org/illustrations/1363 • http://www.illustrativemathematics.org/illustrations/1833 • http://www.illustrativemathematics.org/illustrations/223 • http://www.illustrativemathematics.org/illustrations/576 • http://learnzillion.com/lessonsets/692-identify-the-points-of-intersection-between-y-fx-and-y-gx-as-the-solutions-to-fx-gx • http://learnzillion.com/lessonsets/662-understand-relationships-between-functions-and-why-the-xcoordinates-of-where-the-graphs-of-fx-and-gx-intersect-are-the-solutions-of-fxgx

After the Task

In all three problems, students will be required to explain how they used technology to find their solution. If students do not include an explanation, guide them to write the steps they perform on the calculator to help with their explanation. Students may also be allowed to sketch what the screen of their calculator looks like to support the explanation. For problems 1 and 3, if students have learned how to solve exponential equations using logarithms, the task could be modified to ask students to solve the problem using pencil and paper in place of technology.

Student Extended Constructed Response

1. Consider the functions $f(x) = 5e^x$ and $g(x) = 25$. Using technology, approximate the *x*-coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and $y = g(x)$. Explain how you found your answer.

2. Approximate the solution of the equation $b(x) = c(x)$, given $b(x) = \log(2x)$ and $c(x) = 5 - e^x$. Explain how you found your answer.

3. A company estimates that the monthly cost of gradually implementing a new process in its factory can be modeled by the function $f(n) = \frac{1}{4}(2)^{0.25n}$ where *n* is the number of months since implementation began. This cost continues to change up to a maximum monthly cost of $m(n) = 200$ dollars. Once the monthly cost reaches the maximum of $200, the process is fully implemented.

    a. Write an equation to determine the number of months until full implementation.

    b. Determine the approximate number of months until the process is fully implemented. Explain how you found your answer.

Extended Constructed Response Exemplar Response

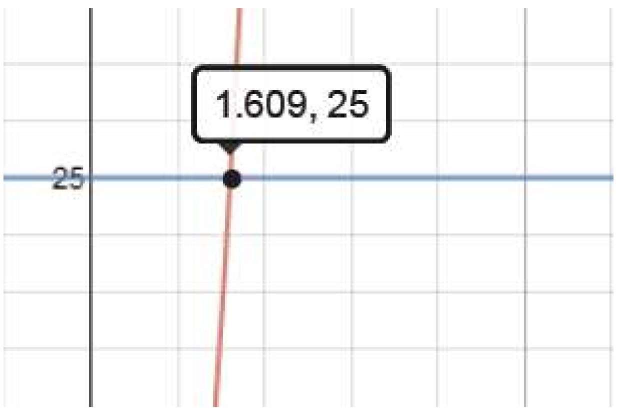
1. Consider the functions $f(x) = 5e^x$ and $g(x) = 25$. Using technology, approximate the *x*-coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and $y = g(x)$. Explain how you found your answer.

   Note: As students find the approximation, they may use technology to create a table of values or create a graph and find the point of intersection. The approximation may vary based on the chosen methods.

   Sample response:

   I graphed each function and used the calculate feature on the graphing calculator to find the point of intersection. The x-coordinate is approximately 1.609.

   The window used to create the graph below is:
   Xmin: -1; Xmax: 6; X scale: 1; Ymin: 21; Ymax: 28; Yscale: 1

   The online calculator at https://www.desmos.com/calculator/ was used to create this graph.

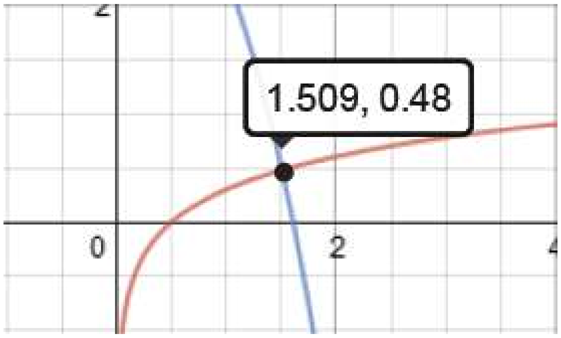
2. Approximate the solution of the equation $b(x) = c(x)$, given $b(x) = \log(2x)$ and $c(x) = 5 - e^x$. Explain how you found your answer.

   Note: As students find the approximation, they may use technology to create a table of values or create a graph and find the point of intersection. The approximation may vary based on the chosen methods.

   Sample response:

   I graphed each function and used the calculate feature on the graphing calculator to find the point of intersection. The x-coordinate of the point of intersection is the solution to the equation $b(x) = c(x)$. The solution is approximately 1.509.

The window used to create the graph below is:
Xmin: -1; Xmax: 4; X scale: 0.5; Ymin: -1; Ymax: 2; Yscale: 0.5



The online calculator at https://www.desmos.com/calculator/ was used to create this graph.

3. A company estimates that the monthly cost of gradually implementing a new process in their factory can be modeled by the function $f(n) = \frac{1}{4}(2)^{0.25n}$ where *n* is the number of months since implementation began. This cost continues to change up to a maximum monthly cost of $m(n) = 200$ dollars. Once the monthly cost reaches the maximum of $200, the process is fully implemented.

   a. Write an equation to determine the number of months until full implementation.

$$\frac{1}{4}(2)^{0.25n} = 200$$

   b. Determine the approximate number of months until the process is fully implemented. Explain how you found your answer.

   Note: As students find the approximation, they may use technology to create a table of values or create a graph and find the point of intersection. The approximation may vary based on the chosen methods.

   Sample response:

   The process will be fully implemented at approximately 38.6 months. I created a table of values with my calculator. I entered the function into Y_1 = and started the table at 1 with a change in x-value of 1. Next, I found that the function value was 181.02 at x = 38 and 215.27 at x = 39. Then I changed the table to start at 38 with a change in x-value of 0.1. At x = 38.5, the value of the function is 197.4. At x = 38.6, the value of the function is 200.85.

   Note: If students get a correct response based on an incorrect equation from part a, they should be awarded credit for this part.

Arithmetic and Geometric Sequences (ECR)

Overview

Students will write recursive and explicit formulas for arithmetic and geometric sequences and use the formulas to solve a mathematical problem.

Standard

Build a function that models a relationship between two quantities.

HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are prerequisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSF-BF.A.2	• HSF-BF.A.1a • HSF-IF.A.3	1. Are the following sequences geometric or arithmetic? a. $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ i. geometric b. 7, 16, 25, 34, 43, . . . i. arithmetic c. $\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, \ldots$ i. arithmetic d. 2, 6, 18, 54, 162, . . . i. geometric e. 1, 2, 4, 8, 16, . . . i. geometric 2. Write a formula to represent the sequence: 7, 16, 25, 34, 43, . . . a. $a_n = 7 + 9(n-1)$ 3. Write a formula to represent the sequence: 2, 6, 18, 54, 162, . . a. $a_n = (2)(3)^{n-1}$	• http://www.illustrativemathematics.org/illustrations/572 • http://www.illustrativemathematics.org/illustrations/573 • http://www.illustrativemathematics.org/illustrations/218

Real-World Preparation: The following question will prepare students for some of the real-world components of this task:

- ***What is a book of stamps?*** Stamps are available individually or in packets called *books*. Books of stamps commonly hold either 10 or 20 stamps.

After the Task

Students may struggle with writing the explicit and recursive formulas. Provide students with additional practice with writing both the explicit and recursive forms of arithmetic and geometric sequences.

Students may choose to use the recursive formula to answer part c on problem 1. Discuss with them how they could have found the answer more quickly using the explicit formula. Discuss different situations when both the recursive formula and the explicit formula would be useful.

Have students identify a real-world situation that can be modeled with a geometric and arithmetic sequence. Have students write the formulas for each situation and then use those formulas to answer questions.

Student Extended Constructed Response

Jerome works at the United States Post Office.

1. Jerome is selling books of 10 stamps. As he is selling them to customers, he notices that 2 books cost $9.80, 3 books cost $14.70, 4 books cost $19.60, and 5 books cost $24.50.

    a. Is this sequence arithmetic, geometric, or neither? Show or explain your thinking.

    b. Write the recursive and explicit formulas for this sequence.

    c. Use either of the formulas that you created in the step above to show how much 17 books of stamps would cost. Show your work.

2. A customer wants to hang a poster on Jerome's bulletin board. Jerome notices that the poster is too tall to fit in the space that he has available, so he decides to use his copier to reduce the height of the poster. Jerome knows that he will have to reduce the poster more than one time to make it fit in the space that he has available. The original height of the poster is 20 inches. For each reduction, the copier will reduce the poster's height to 64% of the previous height.

    a. Is this sequence arithmetic, geometric, or neither? Show or explain your thinking.

    b. Write the recursive and explicit formulas for this sequence.

    c. Use either of the formulas that you created in the step above to find the height of the photograph after three reductions. Show your work.

3. Last year, Jerome's town had a population of approximately 5,000 people. The population this year is 1.01 times the population last year.

   a. Assuming that the population continues to grow at this rate, write a formula for this sequence.

   b. Find the population on the third year of the sequence. Show your work.

   c. If each person uses approximately 20 stamps per year, how many stamps should Jerome order for the third year in the sequence? Show your work.

Extended Constructed Response Exemplar Response

Jerome works at the United States Post Office.

1. Jerome is selling books of 10 stamps. As he is selling them to customers, he notices that 2 books cost \$9.80, 3 books cost \$14.70, 4 books cost \$19.60, and 5 books cost \$24.50.

   a. Is this sequence arithmetic, geometric, or neither? Show or explain your thinking.

   Sample response:

   This sequence is arithmetic.

   - $14.70 - 9.80 = 4.90$
   - $19.60 - 14.70 = 4.90$
   - $24.50 - 19.60 = 4.90$

   The sequence is arithmetic because there is a common difference. The common difference is \$4.90.

   b. Write the recursive and explicit formulas for this sequence.

   Sample response:

   Explicit:

$$a_n = a_1 + d(n-1)$$

$$a_n = 4.90 + 4.90(n-1)$$

   Alternate explicit formula: $a_n = 4.90n$

   Recursive:

$$\begin{cases} a_1 = start \\ a_n = a_{n-1} + d \end{cases}$$

$$\begin{cases} a_1 = 4.90 \\ a_n = a_{n-1} + 4.90 \end{cases}$$

   c. Use either of the formulas that you created in the step above to show how much 17 books of stamps would cost. Show your work.

$$a_n = 4.90 + 4.90(n-1)$$

$$a_n = 4.90 + 4.90(17-1)$$
$$a_n = 4.90 + 4.90(16)$$
$$a_n = 4.90 + 78.40$$
$$a_n = 83.30$$

   ***Note: Students may also choose to use the recursive formula here, which would require them to find all of the values for 6 – 16 books of stamps.*

2. A customer wants to hang a poster on Jerome's bulletin board. Jerome notices that the poster is too tall to fit in the space that he has available, so he decides to use his copier to reduce the height of the poster. Jerome knows that he will have to reduce the poster more than one time to make it fit in the space that he has available. The original height of the poster is 20 inches. For each reduction, the copier will reduce the poster's height to 64% of the previous height.

   a. Is this sequence arithmetic, geometric, or neither? Show or explain your thinking.

      Sample response:

      This sequence is geometric. Each time, the height is going to be multiplied by .64 to obtain the next height. Since each term is multiplied by a common factor to obtain the next term, this is a geometric sequence.

   b. Write the recursive and explicit formulas for this sequence.

      Sample response:

      Explicit:

$$a_n = (a_1)(r)^{n-1}$$

$$a_n = (12.8)(.64)^{n-1}$$

      Recursive:

$$\begin{cases} a_1 = start \\ a_n = (a_{n-1})(r) \end{cases}$$

$$\begin{cases} a_1 = 12.8 \\ a_n = (a_{n-1})(.64) \end{cases}$$

   c. Use either of the formulas that you created in the step above to find the height of the photograph after three reductions. Show your work.

$$a_n = (12.8)(.64)^{n-1}$$

$$a_n = (12.8)(.64)^{3-1}$$

$$a_n = (12.8)(.64)^{2}$$

$$a_n = (12.8)(.4096)$$

$$a_n = 5.24288$$

      The height of the poster after three reductions is about 5.3 inches.

      ***Note: Students may also choose to use the recursive formula here, which would require them to find all of the values for all three reductions.*

3. Last year, Jerome's town has a population of approximately 5,000 people. The population this year is 1.01 times the population last year.

   a. Assuming that the population continues to grow at this rate, write a formula for this sequence.

   $$a_n = (5050)(1.01)^{n-1}$$

   b. Find the population on the third year of the sequence. Show your work.

   $$a_n = (5050)(1.01)^{n-1}$$
   $$a_n = (5050)(1.01)^{3-1}$$
   $$a_n = (5050)(1.01)^2$$
   $$a_n = (5050)(1.0201)$$
   $$a_n = 5151.505$$

   The population on the third year will be approximately 5,151 people.

   c. If each person uses approximately 20 stamps per year, how many stamps should Jerome order for the third year in the sequence? Show your work.

   $$5151 \times 20 = 103020$$

   Jerome needs to order 102,030 stamps.

Student Well-Being (ECR)

Overview

Students will use sample survey data to calculate population mean, margin of error, and sample size.

Standard

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

HSS-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are prerequisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSS-IC.B.4	• HSS-IC.A.2 • HSS-IC.A.3	1. Suppose that the Gallup organization's latest poll sampled 1,000 people from the United States, and the results show that 520 people (52%) think the president is doing a good job, compared to 48% who don't think so. Find the margin of error that corresponds to a 95% confidence level for this poll. a. .0310 or 3.1% **2.** http://www.illustrativemathematics.org/illustrations/1411	• http://www.illustrativemathematics.org/illustrations/244 • http://www.illustrativemathematics.org/illustrations/125 • http://www.illustrativemathematics.org/illustrations/1099 • http://www.illustrativemathematics.org/illustrations/1029 • http://learnzillion.com/lessonsets/531-use-survey-data-to-estimate-means-and-proportions-develop-a-margin-of-error-through-simulation-models-and-evaluate-reports-based-on-data • http://learnzillion.com/lessonsets/401-use-data-from-a-sample-survey-and-evaluate-reports-based-on-data

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What are some key factors that drive student success?*** According to the Gallup Poll, the following were relevant to student success:
    - Ideas and energy students have for the future (hopeful)
    - Involvement in and enthusiasm for school (engagement)
    - How students think about and experience their lives (well-being)
- ***How was the general well-being of students in grades 5-12 measured?***
    - The Gallup Student Poll is a 20-question survey that measures the hope, engagement, and well-being of students in grades 5-12.
    - The primary application was to measure noncognitive metrics that predict student success in academic and other youth development settings.
- ***What is meant by* noncognitive*?***
    - *Noncognitive* means related to personality or preferences rather than intelligence.

Source: **http://www.gallupstudentpoll.com/121688/Online-Learning-Webinars.aspx**

After the Task

Students may struggle with trying to remember how to find the margin of error. Provide students with additional practice finding the margin of error for various confidence intervals. For problem 2, students need to select only one of the 6 items. Be sure to check the accuracy of their work and statements. When working problem 3, students may overlook the fact that they are to assume there is no information about the population mean. They must use a constant value of 0.25 when finding the sample size. Students who overlook the given information for problem 3 will likely struggle with problem 4.

Student Extended Constructed Response

In 2013, a Gallup Poll was administered to 589,997 students in grades 5-12. The purpose of this study was to determine the overall well-being of students. It is important to measure such noncognitive areas because they serve as a predictor of student success in academic and other youth development settings. The following chart includes the results of 6 items included on the survey.

Well-Being Survey Items	**% of Students Responding "Yes"**
Were you treated with respect all day yesterday?	**69%**
Did you smile or laugh a lot yesterday?	**85%**
Did you learn or do something interesting yesterday?	**76%**
Did you have enough energy to get things done yesterday?	**75%**
Do you have health problems that keep you from doing any of the things other people your age normally do?	**16%**
If you are in trouble, do you have family or friends you can count on to help whenever you need them?	**93%**

Source Data: http://www.gallupstudentpoll.com/166037/2013-gallup-student-poll-overall-report.aspx

1. Assuming that the sample is a simple random sample, find the margin of error that corresponds to a 95% confidence level for each item.

2. Select 1 of the 6 items and use the calculated margin of error to define the confidence interval and write a statement that could be used to report this data.

3. A member of your community would like to complete a similar study. How many students in grades 5-12 must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points? Assume that we have no prior information of the population mean.

4. The mean “yes” response rate across all 6 items is 69%. Based on this new piece of data, how many students in grades 5-12 must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points?

Extended Constructed Response Exemplar Response

In 2013, a Gallup Poll was administered to 589,997 students in grades 5-12. The purpose of this study was to determine the overall well-being of students. It is important to measure such noncognitive areas because they serve as a predictor of student success in academic and other youth development settings. The following chart includes the results of 6 items included on the survey.

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Do you have health problems that keep you from doing any of the things other people your age normally do?	**16%**
If you are in trouble, do you have family or friends you can count on to help whenever you need them?	**93%**

Source Data: http://www.gallupstudentpoll.com/166037/2013-gallup-student-poll-overall-report.aspx

1. Assuming that the sample is a simple random sample, find the margin of error that corresponds to a 95% confidence level for each item.

Formula for calculating the margin of error: $E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

Item 1. $E = 1.96\sqrt{\frac{(.69)(.31)}{589{,}997}} = .0012$ Margin of error: 0.12%

Item 2. $E = 1.96\sqrt{\frac{(.85)(.15)}{589{,}997}} = .00091$ Margin of error: 0.09%

Item 3. $E = 1.96\sqrt{\frac{(.76)(.24)}{589{,}997}} = .0011$ Margin of error: 0.11%

Item 4. $E = 1.96\sqrt{\frac{(.75)(.25)}{589{,}997}} = .0011$ Margin of error: 0.11%

Item 5. $E = 1.96\sqrt{\frac{(.16)(.84)}{589{,}997}} = .00094$ Margin of error: 0.10%

Item 6. $E = 1.96\sqrt{\frac{(.93)(.07)}{589{,}997}} = .00065$ Margin of error: 0.07%

2. Select 1 of the 6 items and use the calculated margin of error to define the confidence interval and write a statement that could be used to report this data.

(Note: Exemplar response is provided for Item 3. Responses for all items will be similar.)

Item 3. Exemplar:

$$\hat{p} - E < p < \hat{p} + E$$

$$.76 - .0011 < p < .76 + .0011$$

$$.75891 < p < .76109$$

$$75.9\% < p < 76.1\%$$

It is estimated that 76% of students in grades 5-12 indicated they learned or did something interesting yesterday, with a margin of error of plus or minus 0.109%.

3. A member of your community would like to complete a similar study. How many students in grades 5-12 must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points? Assume that we have no prior information of the population mean.

$$n = \frac{\left[z_{\alpha/2}\right]^2 \cdot 0.25}{E^2}$$

$$n = \frac{(1.96)^2 \cdot 0.25}{(.04)^2}$$

$$n = 600.25$$

In order to be in error by no more than 4% and 95% confident in our results, 601 students in grades 5-12 must be surveyed.

4. The mean “yes” response rate across all 6 items is 69%. Based on this new piece of data, how many students in grades 5-12 must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points?

$$n = \frac{\left[z_{\alpha/2}\right]^2 \hat{p}\hat{q}}{E^2}$$

$$n = \frac{(1.96)^2(.69)(.31)}{(.04)^2}$$

$$n = 513.574$$

Using the known population mean of 69%, we will need a sample size of 514 students in grades 5-12 in order to be 95% confident with an error of no more than 4%.

Radical Table (ECR)

Overview

Students will use the properties of exponents to simplify expressions while demonstrating an understanding of the notations for radicals in terms of rational exponents.

Standard

Extend the properties of exponents to rational exponents.

HSN-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are prerequisites for student success with this task's standards.

Grade Level Standard	The Following Standard Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSN-RN.A.2	• HSN-RN.A.1	1. Rewrite the following in rational notation: $\sqrt[5]{x^2y^3}$ a. $x^{\frac{2}{5}}y^{\frac{3}{5}}$ 2. Simplify: $\sqrt{\frac{16x^5}{9y^3}}$ a. $\frac{4x^2\sqrt{x}}{3y\sqrt{y}} = \frac{4x^2\sqrt{xy}}{3y^2}$ 3. http://www.illustrativemathematics.org/illustrations/1220 4. http://www.illustrativemathematics.org/illustrations/1842	• http://www.illustrativemathematics.org/illustrations/1823 • http://learnzillion.com/lessonsets/646-rewrite-expressions-involving-radicals-and-rational-exponents

After the Task

Students who struggle with simplifying the expressions with rational exponents in this task may need additional practice applying the properties of integer exponents for scaffolding purposes.

For item 4, students may leave the radical in the denominator, which is acceptable depending on the situation. If students do leave the radical expression in the denominator, discuss with them the process of rationalizing the denominator and in which situations that process would be helpful.

It may be helpful to have students discuss as a whole class the different strategies they used to simplify some of the expressions in the task in order to help struggling students understand the different procedures they can use.

Student Extended Constructed Response

Part A. Complete the following table to so that each item is represented both in radical and rational forms. Simplify when possible while assuming all variables represent positive real numbers.

Item #	Radical Notation	Rational (Fraction) Notation	Simplified Radical Form
1.	$\sqrt[4]{x^3}$		
2.		$z^{-\frac{1}{3}}$	
3.	$\sqrt[4]{128x^5}$		
4.		$\left(\frac{27}{z^2}\right)^{\frac{1}{3}}$	
5.		$\frac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$	
6.	$\frac{\sqrt[3]{\frac{1}{z}}}{\sqrt[3]{z^2}}$		
7.		$\left(y \cdot y^{\frac{1}{4}}\right)^{\frac{4}{3}}$	
8.	$\sqrt{\frac{243a^{10} \cdot b^3}{3b}}$		

Extended Constructed Response Exemplar Response

Part A. Complete the following table to so that each item is represented both in radical and rational forms. Include the simplest form while assuming all variables represent positive real numbers.

(Note: Red text indicates item filled in by students.)

Item #	Radical Notation	Rational (Fraction) Notation	Simplified Radical Form
1.	$\sqrt[4]{x^3}$	$x^{\frac{3}{4}}$	$\sqrt[4]{x^3}$
2.	$\sqrt[3]{\frac{1}{z}}$	$z^{-\frac{1}{3}}$	$\sqrt[3]{\frac{1}{z}}$ Or equivalent form
3.	$\sqrt[4]{128x^5}$	$(128x^5)^{\frac{1}{4}}$	$2x\sqrt[4]{8x}$
4.	$\sqrt[3]{\frac{27}{z^2}}$	$\left(\frac{27}{z^2}\right)^{\frac{1}{3}}$	$\frac{3\sqrt[3]{z}}{z}$
5.	$\sqrt[4]{\frac{x^3}{x}}$ or $\frac{\sqrt[4]{x^3}}{\sqrt[4]{x}}$	$\frac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$	$\sqrt{x}$ or $x^{\frac{1}{2}}$
6.	$\frac{\sqrt[3]{\frac{1}{z}}}{\sqrt[3]{z^2}}$	$\frac{z^{-\frac{1}{3}}}{z^{\frac{2}{3}}}$	$\frac{1}{z}$ or z^{-1}
7.	$(\sqrt[3]{y^4}) \cdot (\sqrt[3]{y})$	$\left(y \cdot y^{\frac{1}{4}}\right)^{\frac{4}{3}}$	$y\sqrt[3]{y^2}$ or $y^{\frac{5}{3}}$
8.	$\sqrt{\frac{243a^{10} \cdot b^3}{3b}}$	$\left(\frac{243a^{10}b^3}{3b}\right)^{\frac{1}{2}}$	$9a^5b$

Preferred Chocolate (IT)

Overview

Students will design and conduct a study to investigate a simple question about which brand of chocolate is preferred. Using results gathered from the study conducted, students will make a recommendation about which chocolate brand should continue to be sold at athletic events for the school.

Standards

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

HSS-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSS-IC.B.3	• HSS-IC.A.1	1. http://www.illustrativemathematics.org/illustrations/122 2. http://www.illustrativemathematics.org/illustrations/1029	• http://www.illustrativemathematics.org/illustrations/186 • http://www.illustrativemathematics.org/illustrations/191 • http://learnzillion.com/lessonsets/475-distinguish-between-surveys-experiments-and-observational-studies-relate-randomization-to-each • http://learnzillion.com/lessonsets/400-recognize-the-purposes-of-and-differences-among-research-methods-including-how-they-relate-to-randomization

Setup/Organization:

- Divide the class into groups of 4-5 students and assign each group the title of Survey, Experiment, or Observational Study. The number of groups will vary but there should be a minimum of three groups so that each type of study is conducted.
- Determine which two brands of chocolate will be used. One possible pairing of brands is Hershey's and Nestlé because of the option to use chocolate baking chips for the group(s) assigned to conduct an experiment. Both brands of baking chips will look the same and limit the opportunity for inaccurate results.

During the Task

- During the planning and discussion phase of the task, the teacher should circulate to not only respond to student questions but to ensure students are considering necessary aspects. Guiding questions will vary by group based on whether they are designing a survey, an experiment, or an observational study.

ALL GROUPS	
Look-Fors (observables)	***Guiding Questions***
Discussions are related to the purpose of the task (to determine which brand of chocolate should continue to be sold).	• What was the original question posed in the task? • Why was this question posed to you? • Do you understand what you are being asked to do? • How can you formulate a research question that captures the problem posed by the task?
Discussions include consideration of design elements (population, brands, location), which will lead to the collection of relevant data.	• What elements are you considering? • Who will make up the population? • Does the task include reference to a required level of confidence? • Is it important to consider where your research will take place? If so, what do you need to consider? • Which elements do you think are essential in answering the research question and providing a meaningful recommendation?
Discussions are centered around data collection.	• What data will be collected? • Is demographic data important? If so, how will you collect this information? • Is the data qualitative or quantitative?
Discussions focus on randomization.	• What is the size of your target population? • Is it necessary to sample the entire population? If no, how will you select your subjects? If yes, what is your rationale? • What sample size is adequate to arrive at a conclusion? • How do you determine an adequate sample size?
Discussions are centered around the feasibility study.	• Is it possible to carry out your study in the required amount of time? • Have you considered the availability of/access to the population? • What tools/resources will you need in order to complete the study? • Do you have access to everything required?
Group members are assigned/take on individual roles.	• What role will each member play in each phase of the process? • Who will lead the efforts? • How will you remain accountable for your tasks?

SURVEY GROUP(S)	
Look-Fors (observables)	***Guiding Questions***
Discussions should include how the population will be surveyed.	• How will you collect the data? • What supplies/tools will be needed in order to collect the data? • Is a paper survey the only option? • What is the most efficient way to capture the needed information?
Discussions are related to how the questions will be asked (regardless of the choice of written or verbal questions).	• How will you ensure the questions are unbiased? • Is it important to consider the order of the questions?
Discussions about the process of conducting the survey should extend beyond the questions included in the survey.	• Who will approach people about completing the survey? • How will you respond when people refuse? • Will this attempt be recorded? • What to partially completed surveys?

EXPERIMENT GROUP(S)	
Look-Fors (observables)	***Guiding Questions***
Discussions should include the logistics of the study.	• How will you conduct the experiment? • What will be needed in order to collect the data?
Discussions focus on whether it is necessary to conduct a blind or double-blind study.	• Does a placebo effect apply to your design? • Will the subjects know what they are sampling? Why or why not? • Will **YOU** know the brand they are sampling?
Discussions about the process of choosing subjects should extend to include the specific data to be collected and the potential for errors or alternate responses.	• Who will approach people about participating in the experiment? • What are the possible responses/outcomes for this trial? • How will you respond when people refuse? • Will this attempt be recorded?

OBSERVATIONAL STUDY GROUP(S)	
Look-Fors (observables)	***Guiding Questions***
Discussions should include details about how students will conduct the experiment relative to the nature of an observational study.	• How will you conduct the observational study? • Will you approach the subjects or will you wait for them to approach you? • What will be needed in order to collect the data?
Based on the research question, students should be discussing the element of choice.	• What role does choice play in your study? • What is the significance of this role?
Discussions about the process of choosing subjects should extend to include the specific data to be collected and the potential for errors or alternate responses.	• How will you respond when people refuse? • Will this attempt be recorded? • How will you record data for a subject who does not follow your instructions—for example, he/she takes both brands of chocolate?

- After the groups have decided on a plan, collect copies of the designs, give feedback with time to make adjustments, and plan to attend the event where each study will take place. The role of the teacher during this time will be that of a supporter and guide if students hit roadblocks that would significantly limit the completion of their study.

After the Task

Have students make connections between what they learned through conducting their own studies and the results of studies they see and/or hear in the news or on the Internet.

Student Instructional Task

Because of new business practices, your school must limit the number of brands to be sold in concession stands during school athletic events. The decision about which brand of chocolate to be sold has been narrowed to two brands: Hershey's and Nestlé. Design and conduct a study that will assist you in making a recommendation of which brand should continue to be sold at athletic events.

Design: All designs must include the following components.

1. Research Question
2. Population
3. Method
    a. Study type
    b. Selection of sample population (sample description, time, place, event)
    c. Data collection
    d. Data analysis

Study Results:

1. Data Summary and Analysis
2. Recommendation

Presentation: Each group will present their design, results, and recommendations to the class. This should include a one-page summary to be distributed to your classmates. As each group presents their results and recommendations, reflect on the questions below.

Class Recommendation: As a group, discuss the following questions and record your responses. This information will be used to formulate a class response.

1. Does the data presented across all groups imply that there may be differences between what people believe they prefer, what people truly prefer, and what people would be likely to buy? If so, explain. Further, how should this be included as a factor when making a recommendation?

2. What role did randomization play in each study?

3. Looking at the data gathered across all of the studies, identify the groups that can be used to make generalizations about the school as a whole. Use data to support and explain these generalizations.

Instructional Task Exemplar Response

Because of new business practices, your school must limit the number of brands to be sold in concession stands during school athletic events. The decision about which brand of chocolate to be sold has been narrowed to two brands: Hershey's or Nestlé. Design and conduct a study that will assist you in making a recommendation of which brand should continue to be sold at athletic events.

SURVEY GROUP EXEMPLAR *(Note: Responses must include all elements as outlined below—because the data will vary by school, the responses will look different but the overall content should be similar. Students should utilize data collection and analysis tools as available.)*

Design:

1. **Research Question**
   Should the school choose Hershey's or Nestlé brand chocolate items to be sold in concession stands at school athletic events?
2. **Population**
   Our survey will be sent out electronically through the East High School communication program to all 1,000 registered users (parents, teachers/staff members, students) since they are the people who are likely to attend school athletic events.
3. **Method**
   a. **Study type** *Survey*
   b. **Selection of sample population** (sample description, time, place, event)
   The survey will be sent to all 1,000 registered users—current East High School parents, teachers/staff members, and students. Responses will be captured through an electronic survey, and to be confident in our recommendation, our response rate needs to be 50% (500 responses). If the response rate is not approaching 50% after two email reminders, we will attend a variety of athletic events to obtain more responses. We will ask all people who enter if they received and responded to the email regarding their preference of Hershey's or Nestlé chocolate. If they have not responded to the electronic survey, we will ask the survey questions and record their responses. If they refuse to complete the survey, we will thank them for their time and will not record this response.
   c. **Data collection** *(Note: Surveys may vary but should include opportunities to collect all data relevant to the research question and described situation. As the teacher reviews group designs, it may be helpful for him or her to ask, "Will these components collectively provide data that translates to information we can use to make a recommendation?")*

   Preferred Brand of Chocolate Survey

   The following items will be included on the survey to be distributed as described above.

   1. *Gender: ___Male ___Female*
   2. *Select the category that best describes you (select ONE):*
      ____student _____teacher/staff member _____ relative of a current student ____other
   3. *On average, how many athletic events do you attend: ____0-3 ____4-5____6 or more*

4. *When purchasing chocolate from school athletic event concession stands, which brand are you more likely to select?____Hershey's ____Nestlé _____no preference*

d. **Data analysis** *(Note: At the design step of the task, students will include the chart they will use to analyze the responses. A sample set of data is included for this exemplar. All student responses should align to the items included in the survey.)*

The following charts will be used to compile and analyze the responses.

TABLE 1							
Sample Population Summary Data		**Total**	**% of Total**	**# Male**	**% Male**	**# Female**	**% Female**
	Total Surveyed	510	**100**	245	**48.0**	265	**52.0**
Completion Type	**Completed Online**	248	**48.6**	100	**40.3**	148	**59.7**
	Completed in Person	262	**51.3**	145	**55.3**	117	**44.7**
Respondent Type	**Student**	215	**42.2**	133	**61.9**	82	**38.1**
	Teacher/Staff Member	96	**18.8**	29	**30.2**	67	**69.8**
	Relative	199	**39.0**	105	**52.8**	94	**47.2**
Average Annual Attendance	**0**	45	**8.8**	4	**8.9**	41	**91.1**
	1-3	123	**24.1**	70	**56.9**	53	**43.1**
	4-5	301	**59.0**	202	**67.1**	99	**32.9**
	6 or >	41	**8.0**	33	**98.4**	8	**1.6**

The highest response percentage for each subgroup is noted with a blue shaded cell. The gender with the highest percentage within each category is identified with a yellow shaded cell.

TABLE 2									
Chocolate Preference Data Summary		Total	# Hershey's	% Hershey's	# Nestlé	% Nestlé	# No Pref.	% No Pref.
	Total Surveyed	510	300	58.8	183	35.9	27	5.3
Gender	Male	245	194	79.2	44	17.9	7	2.9
	Female	265	106	40.0	139	52.5	20	7.5
Completion Type	Completed Online	248	104	41.9	129	52.0	15	6.1
	Completed in Person	262	196	74.8	54	20.6	12	4.6
Respondent Type	Student	215	166	77.2	35	16.3	14	6.5
	Teacher/Staff Member	96	41	44.8	46	47.9	9	7.3
	Relative	199	93	46.7	102	51.3	4	2.0
Average Annual Attendance	0	45	12	26.7	27	60.0	6	13.3
	1-3	123	102	82.9	14	11.4	7	5.7
	4-5	301	171	56.8	122	40.5	8	0
	6 or >	41	15	36.6	20	48.8	6	14.6

The highest preferred percentage in each category is noted with a blue shaded cell.

Study Results: *(Note: The exemplar response below is relative to Tables 1 and 2. Actual student summaries will vary but should accurately reflect the data collected, include an overall summary of the survey respondents, identify key pieces of data related to the research question, and indicate any potential limitations of the study. While most data will be pulled from the summary charts, students may revisit certain aspects of the survey as they work to make a recommendation. The key is that all data presented should remain focused on answering the research question.)*

Population Summary:

1. *Approximately 1,000 people were given an opportunity to complete the survey. Responses were collected from 510 people (51%). The majority of the responses (51.3%) were collected in person during 2 athletic events. Requests for the survey to be completed electronically yielded 248 (48.6%) responses.*
2. *All people who entered the chosen athletic events were asked to complete the survey (if they had not already done so). All potential in-person respondents participated in the survey.*

3. *Females contributed to 52% of responses.*
4. *Based on respondent type, the highest percentages were students (42.2%) and relatives (39%). The category of "other" yielded no data.*
5. *The majority of responses in the area of average annual attendance (67%) came from respondents who attend an average of 4 or more events per year. 8.8% of respondents indicated they attend 0 athletic events per year.*

Data Summary and Analysis:

Because the majority of the response came from in-person respondents, females, students, relatives, and those who attend an average of 4 or more athletic events per year, we chose to focus on the preferences of these subgroups.

Subgroups	% of Total Responses	% Preferring Hershey's	% Preferring Nestlé	% No Preference
Total Population	**100**	**58.8**	**35.9**	**5.3**
Average Attendance: 4 or more	**67.0**	**54.4**	**41.5**	**4.1**
Females	**52.0**	**40.0**	**52.5**	**7.5**
Completed In Person	**51.3**	**74.8**	**20.6**	**4.6**
Students	**42.2**	**77.2**	**16.3**	**6.5**
Relatives	**39.0**	**46.7**	**51.3**	**2.0**

1. *Of the five subgroups selected for further analysis, three groups (average attendance > 4, completed in person, students) prefer Hershey's, and two groups (females and relatives) prefer Nestlé.*
2. *As a group, students have the greatest range in their preference level based on the survey, which indicates 77.2% prefer Hershey's compared to 16.3% preferring Nestlé.*
3. *While females contributed to a large portion of the respondents (52%), the data indicates they are not the group that attends the greatest number of athletic events on average each year. Only 40% (107/265, Table 1) of females indicate they attend 4 or more events on average each year.*
4. *67% (342/510 from Table 1) of respondents indicate they attend 4 or more athletic events each year. When surveyed, these respondents prefer Hershey's over Nestlé chocolate.*

Recommendation: *(Note: Exemplar responses for this portion of the task should include the data used to make a recommendation, limitations of the study, and a final recommendation.)*

Survey Group Recommendation: It is our recommendation that the school choose Hershey's as the chocolate to be sold in athletic event concession stands. The student group was identified with the highest percentage preferred when looking at both brands of chocolate. 77.2% of students surveyed prefer Hershey's compared to 16.3% preferring Nestlé. 67% of the population surveyed indicates they attend 4 or more athletic events per year, and since this group is engaging in activities that give them access to the concession stands more frequently, it is important to look at which chocolate they *prefer. 54.4% prefer Hershey's over Nestlé. Additionally, 58.8% of the 510 survey respondents indicated they prefer Hershey's chocolate. Therefore, the school should select Hershey's chocolate to be sold in athletic event concession stands.*

Limitations: While we made efforts to ensure participants completed the survey one time, duplicate responses may have been recorded. If this occurred, we do not feel it would impact our recommendation, considering those responses given face to face were disaggregated from online responses.

EXPERIMENTAL GROUP EXEMPLAR *(Note: Student responses must include all elements as outlined below—because the data will vary by school, the responses will look different but the overall content should be similar. Students should utilize data collection and analysis tools as available.)*

Design:

1. **Research Question**
   Should the school choose Hershey's or Nestlé brand chocolate items to be sold in concession stands at school athletic events?
2. **Population**
   The experimental study will obtain data by sampling the population of people attending athletic events that have concession stands available. Further selection will be enhanced by placing the testing area near the concession stands at each event.
3. **Method**
   a. **Study type** *Double-blind experiment*
   b. **Selection of sample population** (sample description, time, place, event): *The subjects will come from the population of those people attending athletic events that have concession stands available. We will attend a variety of athletic events in order to obtain as many responses as possible. All ages will be considered, and for young participants, adults may help them complete the data collection form. We will not work to eliminate duplicate participants from the study, but if they say they have already completed the taste test, we will not proceed. Our taste test area will be located near the concession stand, and as people approach the concession stand, we will ask them to participate. If they decline, we will thank them for their time and record this as a "declined" attempt.*
   c. **Data collection** *(Note: For an experimental study, the data collection process may vary but should include opportunities to collect all data relevant to the research question and described situation. As the teacher reviews group designs, it may be helpful for him or her to ask, "Will these components collectively provide data that translates to information we can use to make a recommendation?")*

      Subjects will be given two brands of chocolate (Hershey's and Nestlé) and asked to rank their choice with most or least preferred. A response of "no preference" will not be an option. Each subject will be given identical cups from which to taste. Each cup will be filled with one brand of chocolate chips. The subjects will not know which brand is in each cup. Preferences will not be stated verbally in order to minimize the potential for bias of those waiting to participate in the experiment. For double-blinding of the experiment, the chocolate chips will be scooped from two containers labeled only as "X" and "Y." The group member given the role of study manager will be the only one who know which brand is labeled with "X" and which is labeled with "Y." Also, *the order each subject samples the chocolate will be randomized because both cups will be placed in front of the subject and he or she will choose which to sample first.*

After each sampling, the subject will complete the following form:

Chocolate Taste Test Experiment
*Check the **one** category that best describes you:*

1. *Gender: ______ Male ______Female*
2. *Age in Years:_____ 0-8 ____ 9-13 ____14-18 ____19-23 ____24-30 ____31-40 ____ >40*
3. *Average Athletic Events Attended per Year: _____1-3 _____4-6 _____7 or more*
4. *Which brand did you prefer? ____Brand X _____Brand Y*

d. **Data Analysis** *(Note: At the design step of the task, students should include the chart they will use to analyze the responses. For a sample set of data (completed tables), see the Survey Group Exemplar. All elements should align to the data being collected.)*

The following tables will be used to compile and analyze the results of the experiment.

TABLE A

Sample Population Summary Data		Total	% of Total	# Male	% Male	# Female	% Female
	Total Asked to Participate			NA	NA	NA	NA
	Declined to Participate			NA	NA	NA	NA
	Completed Experiment						
Age in Years	0-8						
	9-13						
	14-18						
	19-23						
	24-30						
	31-40						
	>40						
Average Annual Attendance	1-3						
	4-6						
	7 or >						

TABLE B						
Chocolate Preference Data Summary		**Total**	**# Hershey's**	**% Hershey's**	**# Nestlé**	**% Nestlé**
	Total Subjects					
Gender	**Male**					
	Female					
Age in Years	**0-8**					
	9-13					
	14-18					
	19-23					
	24-30					
	31-40					
	>40					
Average Annual Attendance	**1-3**					
	4-6					
	7or >					

Study Results: *(Note: The exemplar response below provides a shell of what the results of an experiment of this type might look like. Actual student summaries will vary but should accurately reflect the data collected, include an overall summary of the subjects, identify key pieces of data related to the research question, and indicate any potential limitations of the study. While most data will be pulled from the summary charts, students may revisit certain aspects of the experiment as they work to make a recommendation. The key is that all data presented should remain focused on answering the research question.)*

Population Summary:

1. *Taste-testing took place at __(#)_ events over the course of __(#)_ days.*
2. *__(#)__ athletic event attendees were asked to participate in the taste test. __(#)__ agreed to participate, and __(#)__ declined.*
3. *Females contributed to ____% of subjects.*
4. *The majority of the subjects (___%) were _(#)___ years of age or older.*
5. *The majority of subjects (_____%) indicated they attend an average of __(#)___ or more events per year.*

Data Summary and Analysis:

Because we had an equal number of male and female subjects, the majority of the subjects were in the age category of _(#)__ years of age or older and those who attend _(#)__ or more athletic events per year, we chose to focus on the preferences of these subgroups.

Subgroups (will vary depending on data collected)	**% of Total Responses**	**% Preferring Hershey's**	**% Preferring Nestlé**
Total Population			
Average Attendance: __#__ or more			
Males			
Females			
Age: __#__ or older			

1. *Of the (#) subgroups selected for further analysis, _(#)_ groups (list groups) prefer Hershey's and (#) groups (list groups) prefer Nestlé.*
2. *(Note: Exemplars should highlight a variety of data from the experiment. See Survey Group Exemplar for more examples.)*

Recommendation: *(Note**:** Exemplar responses for this portion of the task should include the data used to make a recommendation, limitations of the study, and a final recommendation. See Survey Group Exemplar for an example of a recommendation. Potential limitations for this experimental design may be associated with access to a large enough population because of a limited number of athletic events, etc. Students may also point out that while respondents chose one brand over the other based on taste, this may not match their actual purchasing behavior.)*

OBSERVATIONAL STUDY GROUP EXEMPLAR *(Note: Responses must include all elements as outlined below—because the data will vary by school, the responses will look different but the overall content should be similar. Students should utilize data collection and analysis tools as available.)*

Design:

1. **Research Question**
   Should the school choose Hershey's or Nestlé brand chocolate items to be sold in concession stands at school athletic events?
2. **Population**
   The observational study will obtain data by sampling the population of people attending athletic events who purchase chocolate from school athletic events.
3. **Method**
   a. **Study type** *Observational*
   b. **Selection of sample population** (sample description, time, place, event)

The subjects will come from the population of people attending athletic events AND those who purchase chocolate from the school concession stand. We will attend a variety of athletic events in order to maximize the data collected. All people who purchase an item from the concession stand during the event will be considered. When people approach the concession as a group, and one person pays, this will be recorded as one subject. If a person comes to the concession stand more than once, each order will be treated as a new subject.

c. **Data collection** *(Note: For an observational study, the data collection process may vary but should include opportunities to collect all data relevant to the research question and described situation. As the teacher reviews group designs, it may be helpful for him or her to ask, "Will these components collectively provide data that translates to information we can use to make a recommendation?")*

As people approach the concession stand, data will be collected using a chart similar to the one below. We will have a list (including pictures) of Hershey's and Nestlé chocolate items to make brand identification easier.

Subject #	***# of Items Purchased***	***# of Chocolate Items Purchased***	***# of Hershey's Chocolate Items***	***# of Nestlé Chocolate Items***	***# of Other Chocolate Items***
1.					
2.					
3.					
4.					
5.					
6.					
.....					

d. **Data analysis** *(Note: At the design step of the task, students should include the chart they will use to analyze the responses. For a sample set of data (completed tables), see the Survey Group Exemplar. All elements should align to the data being collected.)*

The following table will be used to compile and analyze the results of the observational study.

Observational Study Data Summary	
Total Events Attended	
Total Subjects Observed	
Mean # of Subjects per Event	
Total Number of Items Purchased	
Mean # of Items per Event	
Total # of Chocolate Items Purchased	
Mean # of Chocolate Items per Subject	

Event Summary Data					
Event	**Total Items Purchased**	**# of Hershey's Products**	**% of Hershey's (#H/total items)**	**# of Nestlé Products**	**% of Nestlé (#N/total items)**
Event 1					
Event 2					
Event 3					
Event 4					
Event 5					
Event 6					
Total All					

Events					

Study Results: *(Note: The exemplar response below provides a shell of what the data of an observational study might produce. Actual student summaries will vary but should accurately reflect the data collected, include an overall summary of the subjects, identify key pieces of data related to the research question, and indicate any potential limitations of the study. While most data will be pulled from the summary charts, students may revisit certain aspects of the experiment as they work to make a recommendation. The key is that all data presented should remain focused on answering the research question.)*

Population Summary:

1. *Observations took place at __(#)_events over the course of __(#)_days.*
2. *__(#)__athletic event attendees who purchased items at the concession stand were observed.*
3. *A mean of __(#)_chocolate items per event were purchased compared to a mean of _(#)__total items purchased per event.*
4. *The percentage of Hershey's to total items purchased is ___%.*
5. *The percentage of Nestlé to total items purchased is ___%.*

Data Summary and Analysis:

(Note: Exemplar responses for this section of the observational study should highlight a variety of data from the experiment. See Survey Group Exemplar for more examples.)

Recommendation: *(Note: Exemplar responses for this portion of the task should include the data used to make a recommendation, limitations of the study, and a final recommendation. See Survey Group Exemplar for an example of a recommendation. Potential limitations for this observational design may be associated with the potential for errors on the part of the observers or the concession stand not having an adequate supply of either or both brands of chocolate.)*

ALL GROUPS—EXEMPLAR *(Note: The guidance for the presentation and class recommendation sections is similar regardless of group type.)*

Presentation: *(Note: Each group will present their design, results, and recommendations to the class. This should include a one-page summary to be distributed to classmates. The one-page summary will be a selection of the information from the above sections [data analysis, study results, and recommendation]. As students listen to the results and recommendations, they should be reflecting on the questions under the Class Recommendation section.)*

Class Recommendation: As a group, discuss the following questions and record your responses. This information will be submitted to your teacher, who will review the recommendations and formulate a class response.

1. Does the data presented across all groups imply there may be differences between what people believe they prefer, what people truly prefer, and what people would be likely to buy? If so, explain. Further, how should this be included as a factor when making a recommendation?

(Note: Responses will vary depending on the data collected by all three groups. For example, if all three groups present data summaries that clearly indicate a preference of one chocolate over another, the answer to this question would be: "Based on the data from all three groups, it appears there is no difference between what people believe they prefer, what they truly prefer, and what they would likely buy.")
If there is a difference between the experimental group and the survey or observational study groups, the response should identify the specific data while pointing out the aspect of choice when selecting an item to buy at a concession stand. It should be clear that students understand the link between choice and the two types of studies (survey and observational study).

2. What role did randomization play in each study? *(Note: Responses should note the preplanning and inclusion of participant selection in the method section for each study.)*

   In order to ensure randomization, groups defined the method for identifying their participants prior to the start of the study. The methods presented by each group clearly defined the population and who would participate in the study. They did not change their method during or after the study.

   All users registered for the school communication system were given the same opportunity to reply to the survey, and additional data was collected by giving all people who entered an athletic event a chance to respond. Both approaches satisfy elements of randomization.

   The experimental study was conducted at athletic events, which relates to the research question. Their method was a double-blind taste test in which those administering the samples and the subjects were unaware of which brand was being tasted.

   Groups completing the observational study recorded choices made by the participants attending athletic events when purchasing concessions. The group recorded the choices of all subjects, including those who did not purchase items of either brand of chocolate. No one in the group discussed the purchases with the participants either before or after the purchases were recorded.

3. Looking at the data gathered across all of the studies, identify the groups that can be used to make generalizations about the school as a whole. Use data to support and explain these generalizations. *(Note: Responses will vary depending on data presented but should not include groups that do not relate to the research question and/or groups that were not included in the data collection. For example, generalizations about 14- to 15-year-old students could only be made if this was a piece of data collected. Generalization is related to the population of interest, and in this task, that population is people attending athletic events.)*

Growing Radishes (IT)

Overview

Students use given data from an experiment involving different fertilizers and a simulation based on re-randomizing the given data to describe differences between the treatment groups in the experiment. Students also determine whether the observed differences are significant.

Standards

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

HSS-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide whether differences between parameters are significant.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSS-IC.B.5	• HSS-IC.A.2 • HSS-IC.B.3	1. Describe the concept of statistical significance. a. A difference is statistically significant if we are reasonably sure that observed differences are not due to sampling variation. If the observed difference would commonly occur due to natural sampling variation, the difference is not significant. 2. Explain how a simulation can determine statistical significance between the means of two groups. a. To determine whether the difference between two groups is statistically significant, all data from both groups can be combined and re-randomized. Then, the combined data is repeatedly, randomly assigned to two groups, and the differences in means is recalculated. The observed difference in means is then compared to the simulated differences in means. If the observed difference, or a value greater than the observed difference, is commonly found in the simulated means, then the observed difference is not significant.	• http://www.illustrativemathematics.org/illustrations/244 • http://www.illustrativemathematics.org/illustrations/125 • http://www.illustrativemathematics.org/illustrations/122 • http://www.illustrativemathematics.org/illustrations/1029 • http://learnzillion.com/lessonsets/402-use-data-from-a-randomized-experiment-to-compare-treatments-evaluate-reports-based-on-data

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is fertilizer, and what is it used for?*** Fertilizer refers to any of a large number of natural and synthetic materials, including manure and compounds containing nitrogen, phosphorus, and potassium, spread on or worked into soil to increase its capacity to support plant growth. Fertilizer is used to increase plant size, healthiness, and/or growth rate.

During the Task

- Students need to understand the difference between an observed difference in two means and a statistically significant difference between two means. If they are unsure of how to answer #1 and/or how to approach #2, discussing this difference may be useful.
- Students may also struggle to understand why re-randomization of data, which have been combined from two groups, gives insight into the statistical significance of the observed differences. To help students see the reasoning behind re-randomizing combined data, have them list all of the data from two groups together. The question students are trying to answer is: “Is there really any difference in these two groups that have been combined?” If one assumes that there is NOT any difference in the two groups, then combining the two groups together and randomly picking half of the data for one group and the remaining data for the other group should produce differences in the means similar to the observed difference. The students could do a few trials of this to see what averages (and what differences) they get to help them understand the data in the re-randomization table.

After the Task

This task helps to establish the distinction between a difference and a statistically significant difference. Differences between groups or subgroups are reported commonly in newspapers (or online news articles). Using a current article making a claim about a difference between groups may be a good way to follow this task. Students could discuss the reported difference and the statistical significance of the difference.

Student Instructional Task

A local farmer has been growing radishes to sell at a farmers market for years. This year, he decided to test two different types of fertilizers. He planted 30 radish plants in an area with consistent lighting conditions. He randomly assigned each plant to one of the following treatments: (1) no fertilizer [control group], (2) Fertilizer A, and (3) Fertilizer B. This resulted in 10 plants per treatment. After the radishes had been growing for four weeks, he measured the height of all of the plants. The heights are shown in the table below.

Plant Heights after 4 Weeks (cm)

No Fertilizer	Fertilizer A	Fertilizer B
4.9	5.0	4.7
3.7	4.5	4.6
4.1	5.9	4.1
4.0	5.7	5.5
3.5	5.2	3.0
4.7	6.2	5.2
5.1	5.2	5.3
3.3	5.1	4.1
4.7	6.0	4.5
4.0	5.4	4.9

1. Compare the center and spread of the three groups. Assume that the three data distributions are approximately unimodal and symmetric.

All of the data from the control group and from the Fertilizer A group were combined and randomly divided into two groups of 10 samples. The means of these new groups (shown as “Sample Group 1 Mean” and “Sample Group 2 Mean”) were calculated, and the differences between the mean heights were calculated. This process was repeated for the control group with Fertilizer B. The re-randomization of the data was performed 25 times for each treatment. The results of this re-randomization of the data are shown in the tables below.

Re-randomization of Data

Control and Fertilizer A Group			
Trial #	Sample Group 1 Mean	Sample Group 2 Mean	Difference
1	4.98	4.64	-0.34
2	5.09	4.53	-0.56
3	4.79	4.83	0.04
4	4.68	4.94	0.26
5	4.82	4.8	-0.02
6	4.62	5	0.38
7	5.06	4.56	-0.5
8	4.83	4.79	-0.04
9	4.95	4.67	-0.28
10	4.68	4.94	0.26
11	4.9	4.72	-0.18
12	4.68	4.94	0.26
13	5.16	4.46	-0.7
14	4.81	4.81	0
15	4.75	4.87	0.12
16	4.43	5.19	0.76
17	4.73	4.89	0.16
18	4.77	4.85	0.08
19	5	4.62	-0.38
20	5.01	4.61	-0.4
21	4.57	5.05	0.48
22	4.91	4.71	-0.2
23	4.87	4.75	-0.12
24	4.9	4.72	-0.18
25	4.74	4.88	0.14

Control and Fertilizer B Group			
Trial #	Sample Group 1 Mean	Sample Group 2 Mean	Difference
1	4.7	4.09	-0.61
2	4.59	4.2	-0.39
3	4.32	4.47	0.15
4	4.37	4.42	0.05
5	4.3	4.49	0.19
6	4.18	4.61	0.43
7	4.63	4.16	-0.47
8	4.37	4.42	0.05
9	4.4	4.39	-0.01
10	4.32	4.47	0.15
11	4.36	4.43	0.07
12	4.23	4.56	0.33
13	4.45	4.34	-0.11
14	4.34	4.45	0.11
15	4.36	4.43	0.07
16	4.14	4.65	0.51
17	4.27	4.52	0.25
18	4.4	4.39	-0.01
19	4.63	4.16	-0.47
20	4.37	4.42	0.05
21	4.22	4.57	0.35
22	4.35	4.44	0.09
23	4.18	4.61	0.43
24	4.44	4.35	-0.09
25	4.29	4.5	0.21

2. Based on these data, is the difference between the control group and the group with Fertilizer A significant? Explain how you arrived at your answer.

3. Based on these data, is the difference between the control group and the group with Fertilizer B significant? Explain how you arrived at your answer.

4. Next year, the farmer plans to grow all of the radishes the same way (either with no fertilizer, Fertilizer A, or Fertilizer B). If the cost of Fertilizer A is three times the cost of Fertilizer B, which of the three options would you recommend the farmer choose to use next year? Explain your reasoning, using statistics to support your answer.

Instructional Task Exemplar Response

A local farmer has been growing radishes to sell at a farmers market for years. This year, he decided to test two different types of fertilizers. He planted 30 radish plants in an area with consistent lighting conditions. He randomly assigned each plant one of the following treatments: (1) no fertilizer [control group], (2) Fertilizer A, and (3) Fertilizer B. This resulted in 10 plants per treatment. After the radishes had been growing for four weeks, he measured the height of all of the plants. The heights are shown in the table below.

Plant Heights after 4 Weeks (cm)

No Fertilizer	Fertilizer A	Fertilizer B
4.9	5.0	4.7
3.7	4.5	4.6
4.1	5.9	4.1
4.0	5.7	5.5
3.5	5.2	3.0
4.7	6.2	5.2
5.1	5.2	5.3
3.3	5.1	4.1
4.7	6.0	4.5
4.0	5.4	4.9

1. Compare the center and spread of the three groups. Assume that the three data distributions are approximately unimodal and symmetric.

   If the distributions are unimodal and symmetric, then the most appropriate measures of center and spread are mean and standard deviation. The mean height with no fertilizer is 4.20 cm, the mean height with Fertilizer A is 5.42 cm, and the mean height with Fertilizer B is 4.59 cm.

   The standard deviation of the data from the group with no fertilizer is 0.59 cm. The standard deviation of the data with Fertilizer A is 0.50 cm and with Fertilizer B is 0.70 cm.

   Both types of fertilizer seemed to produce taller plants (on average, based on the calculated means) than using no fertilizer at all. Also, compared to the group with no fertilizer, the amount of variation in the plant heights was slightly greater with Fertilizer A, and even greater with Fertilizer B.

   ***Note: The standard deviation of the data for all three groups was calculated using a TI-84 Plus calculator. Other technology may produce slightly different results. Students may also find the standard deviation by hand.*

All of the data from the control group and from the Fertilizer A group were combined and randomly divided into two groups of 10 samples. The means of these new groups (shown as “Sample Group 1 Mean” and “Sample Group 2 Mean”) were calculated, and the differences between the mean heights were calculated. This process was repeated for the control group with Fertilizer B. The re-randomization of the data was performed 25 times for each treatment. The results of this re-randomization of the data are shown in the tables below.

Re-randomization of Data

Control and Fertilizer A Group			
Trial #	**Sample Group 1 Mean**	**Sample Group 2 Mean**	**Difference**
1	4.98	4.64	-0.34
2	5.09	4.53	-0.56
3	4.79	4.83	0.04
4	4.68	4.94	0.26
5	4.82	4.8	-0.02
6	4.62	5	0.38
7	5.06	4.56	-0.5
8	4.83	4.79	-0.04
9	4.95	4.67	-0.28
10	4.68	4.94	0.26
11	4.9	4.72	-0.18
12	4.68	4.94	0.26
13	5.16	4.46	-0.7
14	4.81	4.81	0
15	4.75	4.87	0.12
16	4.43	5.19	0.76
17	4.73	4.89	0.16
18	4.77	4.85	0.08
19	5	4.62	-0.38
20	5.01	4.61	-0.4
21	4.57	5.05	0.48
22	4.91	4.71	-0.2
23	4.87	4.75	-0.12
24	4.9	4.72	-0.18
25	4.74	4.88	0.14

Control and Fertilizer B Group			
Trial #	**Sample Group 1 Mean**	**Sample Group 2 Mean**	**Difference**
1	4.7	4.09	-0.61
2	4.59	4.2	-0.39
3	4.32	4.47	0.15
4	4.37	4.42	0.05
5	4.3	4.49	0.19
6	4.18	4.61	0.43
7	4.63	4.16	-0.47
8	4.37	4.42	0.05
9	4.4	4.39	-0.01
10	4.32	4.47	0.15
11	4.36	4.43	0.07
12	4.23	4.56	0.33
13	4.45	4.34	-0.11
14	4.34	4.45	0.11
15	4.36	4.43	0.07
16	4.14	4.65	0.51
17	4.27	4.52	0.25
18	4.4	4.39	-0.01
19	4.63	4.16	-0.47
20	4.37	4.42	0.05
21	4.22	4.57	0.35
22	4.35	4.44	0.09
23	4.18	4.61	0.43
24	4.44	4.35	-0.09
25	4.29	4.5	0.21

2. Based on these data, is the difference between the control group and the group with Fertilizer A significant? Explain how you arrived at your answer.

   The difference in the average heights between the control group and the group with Fertilizer A was 1.22 cm. This difference seems very large relative to the total heights of the plants. Once the data were re-randomized (using only the data from the control group and the group with Fertilizer A), the difference in average heights ranged from -0.7 cm to 0.76 cm. Out of 25 trials of re-randomizing the data, an average height difference of 1.22 cm was never found. The greatest difference in averages was 0.76 cm—well below the observed difference of

1.22 cm. Therefore, it appears that the difference in heights was significant and that using Fertilizer A made a significant difference in the heights of radish plants.

3. Based on these data, is the difference between the control group and the group with Fertilizer B significant? Explain how you arrived at your answer.

   The difference in the average heights between the control group and the group with Fertilizer B was 0.39 cm. This difference is notable. However, once the data were re-randomized (using only the data from the control group and the group with Fertilizer B), the difference in average heights ranged from -0.61 cm to 0.51 cm. A difference of 0.39 cm is well within this range. Looking at only the absolute value of the differences in average heights, a difference of 0.39 cm or greater was found 7 out of 25 times. Therefore, according to the simulation, approximately 28% of the time, a difference of 0.39 cm might be observed due to sampling variation. Therefore, I am not very confident that the difference in observed heights is a direct result of using Fertilizer B as opposed to normal sampling variation. The observed difference is not significant.

4. Next year, the farmer plans to grow all of the radishes the same way (either with no fertilizer, Fertilizer A, or Fertilizer B). If the cost of Fertilizer A is three times the cost of Fertilizer B, which of the three options would you recommend the farmer choose to use next year? Explain your reasoning, using statistics to support your answer.

   Sample response:

   Because I am not very confident in the effectiveness of Fertilizer B, I would not choose to purchase that fertilizer at all. The decision for me would be between no fertilizer and Fertilizer A. Because I found a statistically significant difference when using Fertilizer A, I would use this fertilizer for next year's crop, even at the higher cost.

Interpreting Functions (IT)

Overview

Students will interpret functions in a real-world context.

Standards

Interpret functions that arise in applications in terms of the context.

HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSF-IF.B.4	• HSF-IF.A.1 • HSN-Q.A.1	1. http://www.illustrativemathematics.org/illustrations/637 2. http://www.illustrativemathematics.org/illustrations/639 3. http://www.illustrativemathematics.org/illustrations/649	• http://www.illustrativemathematics.org/illustrations/588 • http://www.illustrativemathematics.org/illustrations/589 • http://www.illustrativemathematics.org/illustrations/598 • http://www.illustrativemathematics.org/illustrations/85 • http://www.illustrativemathematics.org/illustrations/473 • http://learnzillion.com/lessonsets/477-graph-quadratic-functions-and-show-intercepts-maxima-and-minima • http://learnzillion.com/lessonsets/470-graph-linear-functions-and-show-intercepts-maxima-and-minima

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSF-IF.B.6	• HSF-IF.A.2	1. What is the rate of change between the points (3, 20) and (-2, 8)? a. 2.4 2. http://www.illustrativemathematics.org/illustrations/577 3. http://www.illustrativemathematics.org/illustrations/686 4. http://www.illustrativemathematics.org/illustrations/1500	• http://www.illustrativemathematics.org/illustrations/599 • http://www.illustrativemathematics.org/illustrations/634 • http://www.illustrativemathematics.org/illustrations/625

Real-World Preparation:

- ***What sport does the New Orleans Pelicans team play?*** The New Orleans Pelicans are a NBA basketball team.
- ***What is a team's record?*** Teams often track their records. A record is a comparison of wins to losses. The fraction formed by the number of wins and the total number of games played is the winning percentage.

During the Task

Remind students that the equation and the graph model the same data. They can use either representation to answer questions. They should decide which is most appropriate for each question.

After the Task

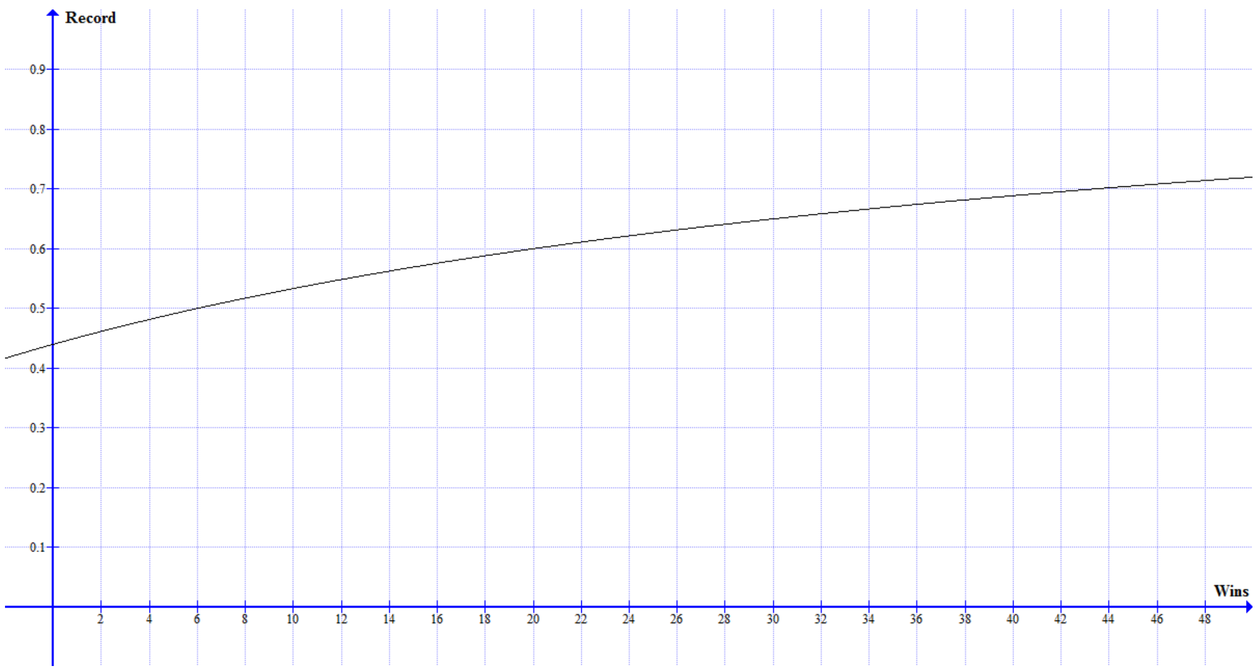
The teacher can have students compare the graph and the team's actual record at various points in the season.

Student Instructional Task

Jeremy is a huge New Orleans Pelicans fan. During the 2013-2014 season, at week 15 the team has 22 wins and 28 losses. Jeremy is worried that they might end the season with a losing record. He decides to see what will happen to the team's record if they win every game for the rest of the season. He writes the equation below. In his equation, *x* represents the number of consecutive wins.

$$W(x) = \frac{22 + x}{50 + x}$$

Jeremy creates the graph below to model his equation.



1. What is the y-intercept of this graph? Support your answer using either the equation or graph. What does it represent?

2. Is the function increasing or decreasing? Support your answer using either the equation or graph. Explain your answer in the context of the New Orleans Pelicans' winning percentage.

3. Find the average rate of change from 2 wins to 6 wins and from 14 wins to 18 wins. Compare your answers and explain your answer in the context of the rate of change of the New Orleans Pelicans' winning percentage. Show all of your work.

4. The New Orleans Pelicans will play 82 games during the 2013-2014 season. Is it possible that the team will finish with a winning percentage of 60%? Explain and show your reasoning.

5. Does this function have a horizontal asymptote? Explain your answer in the context of the team's winning percentage.

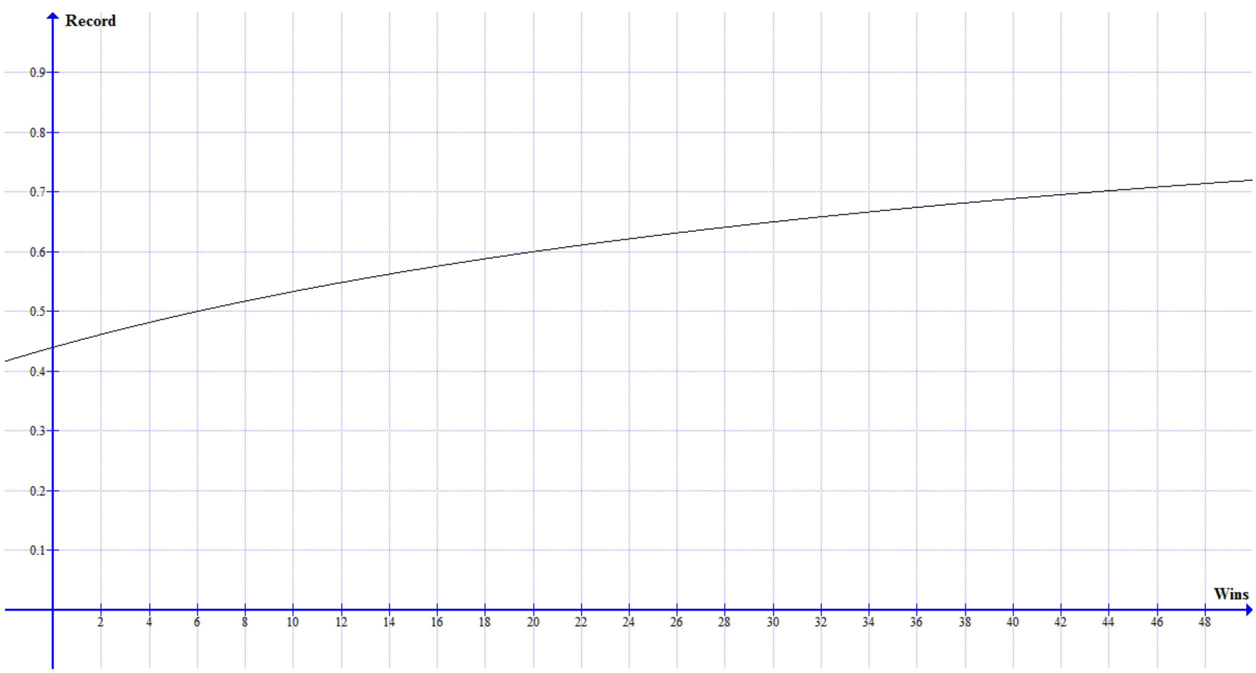
Information found at http://espn.go.com/nba/team/rankings/_/name/no/new-orleans-pelicans.

Instructional Task Exemplar Response

Jeremy is a huge New Orleans Pelicans fan. During the 2013-2014 season, at week 15 the team has 22 wins and 28 losses. Jeremy is worried that they might end the season with a losing record. He decides to see what will happen to the team's record if they win every game for the rest of the season. He writes the equation below. In his equation, *x* represents the number of consecutive wins.

$$W(x) = \frac{22 + x}{50 + x}$$

Jeremy creates the graph below to model his equation.



1. What is the y-intercept of this graph? Support your answer using either the equation or graph. What does it represent?

$$W(x) = \frac{22 + 0}{50 + 0}$$

$$W(x) = \frac{22}{50}$$

The y-intercept is $\frac{22}{50}$ *or 0.44. This y-intercept represents the team's winning percentage at week 15.*

2. Is the function increasing or decreasing? Support your answer using either the equation or graph. Explain your answer in the context of the New Orleans Pelicans' winning percentage.

The function is increasing. The graph is only increasing. This means that the equation is representing the team's winning percentage as they win every game.

3. Find the average rate of change from 2 wins to 6 wins and from 14 wins to 18 wins. Compare your answers and explain your answer in the context of the rate of change of the New Orleans Pelicans' winning percentage. Show all of your work.

   From 2 to 6:

$$\frac{0.5 - 0.44}{6 - 0}$$
$$= \frac{0.06}{6}$$
$$= 0.01$$

   From 14 to 18:

$$\frac{0.58824 - 0.5625}{18 - 14}$$
$$= \frac{0.02574}{4}$$
$$= 0.006435$$

$$0.01 > 0.006435$$

   This means that the New Orleans Pelicans' winning percentage was increasing at a faster rate between wins 2 and 6 than it was between wins 14 and 18.

4. The New Orleans Pelicans will play 82 games during the 2013-2014 season. Is it possible that the team will finish with a winning percentage of 60%? Explain and show your reasoning.

   Yes, it is possible that the team finishes with a winning percentage of 60%. I see the point (20, 0.6) on the graph. This means that it would take 20 consecutive wins for the team to have a winning percentage of 60%. They have already played 50 games. If they play an additional 20 games, they will only have played 70 games, less than the total 82 games that they will play during the season.

5. Does this function have a horizontal asymptote? Explain your answer in the context of the team's winning percentage.

   This function does have a horizontal asymptote. From the graph, I can see that if I extend the graph, the function will not cross 1. In terms of the team's winning percentage, the team cannot have a record of over 100% wins.

   Information found at **http://espn.go.com/nba/team/rankings/_/name/no/new-orleans-pelicans**

Lifetime Savings (IT)

Overview

Students will derive the formula for the sum of a finite geometric series using a real-world context.

Standards

Write expressions in equivalent forms to solve problems.

HSA-SSE.B.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-SSE.B4	• HSA-SSE.B.3c	1. http://www.illustrativemathematics.org/illustrations/442 2. http://www.illustrativemathematics.org/illustrations/805 3. http://www.illustrativemathematics.org/illustrations/929 4. http://www.illustrativemathematics.org/illustrations/1797	• http://www.illustrativemathematics.org/illustrations/1305

Real-World Preparation:

- ***What does the phrase "compounded annually" mean?*** Interest that is compounded annually is computed at the end of each year and added into the account, thereby increasing the amount of money in the account.
- ***What is an annuity?*** When you contribute the same amount each year to an account, it is called an *annuity*.

During the Task

The task assumes that students have already developed the formula for a geometric series themselves; having students recognize the need for this formula (and look it up, if necessary) allows them to apply their understanding of a geometric series.

When students begin to work on problem 2, be sure that they understand they are only finding the value of the $100 deposited at the end of the second year. Natural inclination might be to find the total value of the account, including the money from the first year. Draw students' attention to the wording in order for them to make sense of the problem.

For problem 5, when students solve the equation, they may round the answer and not check to see if the rounded amount will produce the $150,000. Encourage students to test their answer and try to determine the smallest amount Ms. McCarty would need to save to accumulate a minimum of $150,000 after 70 years.

After the Task

Have students research different savings plans. Using the plans they find, have students determine how much they could accumulate after 70 years if they started saving while in school. Have students determine how much they would have to save each year if they want to reach a particular savings goal. Discuss how developing a savings plan like the one in this task could help students in their future.

Student Instructional Task

For 70 years, Oseola McCarty earned a living washing and ironing other people's clothing in Hattiesburg, Mississippi. Although she did not earn much money, she budgeted her money wisely, lived within her means, and began saving at a very young age. Before she died, she drew worldwide attention by donating $150,000 to the University of Southern Mississippi for a scholarship fund in her name. The fact that Ms. McCarty was able to save so much money and generously gave it away is an inspiration to many others. She was honored with the Presidential Citizens Medal for her generosity. How did she do it?

1. Suppose Ms. McCarty saved $100 and then deposited it at the end of the year in an account that earns 5% interest, compounded annually.

    a. How much will it be worth at the end of the second year? At the end of the third year? At the end of the 70th year? Show your work.

    b. Write an expression that represents the value of an investment of C dollars after 70 years. Assume that C is deposited at the end of the first year in an account that earns 5% interest, compounded annually.

2. Now suppose Ms. McCarty saved another $100 in the **second** year and then deposited it in her account at the end of that year.

    a. How much will this $100 deposit be worth at the end of the third year? At the end of the fourth year? At the end of the 70th year? Show your work.

    b. Write an expression that represents the value of an investment of C dollars after 69 years. Assume that C is deposited at the end of the second year in an account that earns 5% interest, compounded annually.

3. Suppose Ms. McCarty saved $100 **each and every year** for 70 years. Each time, she deposited it in her account at the end of the year.

    a. How much money would Ms. McCarty have saved without interest? What would be the total amount in the account after 70 years? Show your work.

    b. Write an expression that represents the value of an investment of *C* dollars deposited each year for 70 years. Assume as above that it is always deposited at the end of the year in an account that earns 5% interest, compounded annually.

4. Had Ms. McCarty saved $1,000 each year, how much would she have had after 70 years under the same conditions? Justify your answer.

5. How much would she have to save each year in order to accumulate $150,000 after 70 years? Are you surprised by the answer? Show your work and explain your reasoning.

6. The *future value* (*FV*) of an annuity is the total value of the annuity after a certain number of years. The formula for the future value of an annuity is shown below.

$$FV = C \cdot \frac{(1+r)^t - 1}{r}$$

Based on the work you did above, what is the meaning of *C* in this context? What is the meaning of *r* in this context? What is the meaning of *t* in this context?

Task adapted from http://www.illustrativemathematics.org/illustrations/1283.

Instructional Task Exemplar Response

For 70 years, Oseola McCarty earned a living washing and ironing other people's clothing in Hattiesburg, Mississippi. Although she did not earn much money, she budgeted her money wisely, lived within her means, and began saving at a very young age. Before she died, she drew worldwide attention by donating $150,000 to the University of Southern Mississippi for a scholarship fund in her name. The fact that Ms. McCarty was able to save so much money and generously gave it away is an inspiration to many others. She was honored with the Presidential Citizens Medal for her generosity. How did she do it?

1. Suppose Ms. McCarty saved $100 and then deposited it at the end of the year in an account that earns 5% interest, compounded annually.

   a. How much will it be worth at the end of the second year? At the end of the third year? At the end of the 70th year? Show your work.

   Sample answer:

   Note that computations are rounded to the nearest cent.

Year	***Value (in dollars) at the end of the year***
1	*100*
2	$100 \cdot (1.05) = 105$
3	$[100 \cdot (1.05)] \cdot 105 = 100 \cdot (1.05)^2 = 110.25$
4	$[100 \cdot (1.05)^2] \cdot 105 = 100 \cdot (1.05)^3 = 115.76$
70	$100 \cdot (1.05)^{69} = 2897.76$

   The investment will be worth $105 at the end of the second year, $110.25 at the end of the third year, and $2,897.76 at the end of the 70th year.

   b. Write an expression that represents the value of an investment of *C* dollars after 70 years. Assume that *C* is deposited at the end of the first year in an account that earns 5% interest, compounded annually.

   Replacing 100 by C, an expression that represents the value of an investment of C dollars after 70 years is $C \cdot (1.05)^{69}$.

2. Now suppose Ms. McCarty saved another \$100 in the **second** year and then deposited it in her account at the end of that year.

   a. How much will this \$100 deposit be worth at the end of the third year? At the end of the fourth year? At the end of the 70th year? Show your work.

Year	***Value (in dollars) at the end of the year***
2	100
3	$100 \cdot (1.05) = 105$
4	$[100 \cdot (1.05)] \cdot 105 = 100 \cdot (1.05)^2 = 110.25$
5	$[100 \cdot (1.05)^2] \cdot 105 = 100 \cdot (1.05)^3 = 115.76$
70	$100 \cdot (1.05)^{68} = 2759.77$

   \$100 deposited at the end of the second year will be worth \$105 at the end of the third year, \$110.25 at the end of the fourth year, and \$2,759.77 at the end of the 70th year.

   b. Write an expression that represents the value of an investment of *C* dollars after 69 years. Assume that *C* is deposited at the end of the second year in an account that earns 5% interest, compounded annually.

   An expression that represents the value of an investment of C dollars deposited at the end of the second year after 69 years is $C \cdot (1.05)^{68}$.

3. Suppose Ms. McCarty saved \$100 **each and every year** for 70 years. Each time, she deposited the money in her account at the end of the year.

   a. How much money would Ms. McCarty have saved without interest? What would be the total amount in the account after 70 years? Show your thinking.

   Ms. McCarty saved $100 \cdot 70$ *or \$7,000 over the 70 years. The statements below show how to find the ending value of each of these deposits:*

- *The \$100 invested at the end of year 1 is worth* $100 \cdot (1.05)^{69}$ *at the end of the 70 years.*
- *The \$100 invested at the end of year 2 is worth* $100 \cdot (1.05)^{68}$ *at the end of the 70 years.*
- *The \$100 invested at the end of year 3 is worth* $100 \cdot (1.05)^{67}$ *at the end of the 70 years.*
- *...and so on, until...*
- *The \$100 invested at the end of year 69 is worth* $100 \cdot (1.05)^{1}$ *at the end of the 70 years.*
- *The \$100 invested at the end of year 70 is worth* $100 \cdot (1.05)^{0}$ *at the end of the 70 years.*

Now add these up.

$$100 \cdot (1.05)^{69} + 100 \cdot (1.05)^{68} + 100 \cdot (1.05)^{67} + \ldots + 100 \cdot (1.05)^{1} + 100 \cdot (1.05)^{0}$$

The sum is a finite geometric series, and can be evaluated as

$$100 \cdot \frac{(1.05)^{70} - 1}{1.05 - 1} = 58852.85$$

I conclude that she would have accumulated \$58,852.85 altogether by setting aside \$100 each year for 70 years.

***Note: the calculation could also be written as*

$$100 \cdot \frac{1 - (1.05)^{70}}{1 - 1.05} = 58852.85$$

b. Write an expression that represents the value of an investment of *C* dollars deposited each year for 70 years. Assume as above that it is always deposited at the end of the year in an account that earns 5% interest, compounded annually.

An expression that represents the value of an investment of C dollars deposited at the end of each year for 70 years is

$$C \cdot \frac{(1.05)^{70} - 1}{.05}$$

***Note: the expression could also be written as*

$$C \cdot \frac{1 - (1.05)^{70}}{1 - 1.05}$$

4. Had Ms. McCarty saved \$1,000 each year, how much would she have had after 70 years under the same conditions? Justify your answer.

I can use the last part of the problem above to find this amount:

$$1000 \cdot \frac{(1.05)^{70} - 1}{.05} = 588528.51$$

She would have saved \$588,528.51 altogether.

Alternate response:

Ms. McCarty is saving 10 times as much each year, so the total she saves will be 10 times as great. She would have saved \$588,528.51 altogether.

5. How much would she have to save each year in order to accumulate \$150,000 after 70 years? Are you surprised by the answer? Show your work and explain your reasoning.

   Using the expression I wrote in part 3:

$$C \cdot \frac{(1.05)^{70} - 1}{.05} = 150000$$

$$C \cdot 588.53 = 150000$$

$$C = \frac{150000}{588.53}$$

$$C = 254.87$$

   Sample student response:

   If Ms. McCarty saves \$254.87 each year, she would actually be short by \$1.74. If she saves \$255.88 each year, she would have \$150,004.15, \$4.15 more than she needs. Therefore, Ms. McCarty would need to save approximately \$254.88 each year to accumulate at least \$150,000 in 70 years.

   This is very surprising. I would have thought that she needed to save a lot more each month to accumulate that much money.

   ***Note: If student work is correct based on incorrect expressions from earlier problems, the answer should be counted as correct. Also, students may provide various responses about whether the amount of money Ms. McCarty would need to save is surprising—students should be able to justify their reasoning.*

6. The *future value* (*FV*) of an annuity is the total value of the annuity after a certain number of years. The formula for the future value of an annuity is shown below.

$$FV = C \cdot \frac{(1 + r)^{t} - 1}{r}$$

   Based on the work you did above, what is the meaning of *C* in this context? What is the meaning of *r* in this context? What is the meaning of *t* in this context?

   Based on the work above, C is the amount of money added to the annuity every year, r is the annual interest rate (expressed as a decimal), and t is the number of years the same amount of money is deposited in the annuity.

Radical Equations (IT)

Overview

Students will apply their knowledge of solving radical equations to identify errors and determine whether given equations are always true, sometimes true, or never true using the set of real numbers as the domain.

Standards

Understand solving equations as a process of reasoning and explain the reasoning.

HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

HSA-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.A.1	• HSA-REI.B.4	1. Explain the steps you would take to solve $\sqrt{4x-3}=7$. a. Assuming there is a value for x that would make this equation true, I would begin by squaring both sides of the equation to get $4x-3=49$. Then I would solve this linear equation by adding 3 to both sides, then dividing by 4. This gives the result of x = 13. Finally, I would check my solution by substituting 13 for x in order to be sure it is not an extraneous solution. $\sqrt{4(13)-3}=\sqrt{49}=7$, so the solution is 13.	• http://www.illustrativemathematics.org/illustrations/618 • http://www.illustrativemathematics.org/illustrations/1690 • http://learnzillion.com/lessonsets/495-justify-solutions-to-equations-in-terms-of-equation-properties • http://learnzillion.com/lessonsets/203-solve-and-explain-simple-algebraic-equations

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.A.2	• HSA-REI.A.1	1. Solve $\sqrt{x+3}=2$. State any extraneous solutions. a. $x=1$; there are no extraneous solutions. 2. Solve $5-x=\sqrt{2x-10}$. State any extraneous solutions. a. $x=5$; there is one extraneous solution: $x=7$. 3. Solve $\sqrt{x^2+16}=x+4$. State any extraneous solutions. a. $x=0$; there are no extraneous solutions. 4. http://www.illustrativemathematics.org/illustrations/391	• http://learnzillion.com/lessons/1458-solve-radical-algebraic-equations • http://learnzillion.com/lessons/1459-fina-extraneous-solutions-in-algebraic-radical-expressions

During the Task

- Students may struggle to explain their reasoning for each step in problem 1. Ask students to think about why they are choosing certain operations as they go through each step. Encourage them to use precise vocabulary when explaining their reasoning for each step.
- Students who have struggled with multiplication of polynomials may not realize the mistake in problem 2. Have students try to work the same problem on their own before looking at the steps for Ashley's work. Students should always check their own answers for extraneous solutions. If students find a solution that works from the values they found, students can then compare their work with the steps given for Ashley.
- Allow students to work together for problem 3 in determining whether the statements are always true, sometimes true, or never true. Students will spend time trying to determine how to show that their choice is correct. In most cases, solving the equation in the traditional manner will work. However, for some of these items, students will need to use their number sense and reasoning to determine what is happening with the solutions to the given equations. Encourage students to explain to each other why they believe a given equation is always true, sometimes true, or never true, and have students critique each other's arguments.
- At the end of problem 3, it is a good idea to have all of the groups share their responses to see how the class labeled each item. Have a class discussion in which students defend their choices and others critique their reasoning.

After the Task

Lead a discussion with the class to have students make connections between solving radical equations and the work with rational exponents and simplifying radical expressions. After discussing the solutions to problem 3, have students use graphing technology to verify their claims based on the truth of the given statements.

Student Instructional Task

1. Find all real solutions for $4 = \sqrt{3x + 7}$. Show your work and explain your reasoning for each step. Identify any extraneous solutions.

2. Ashley solved the equation $5x - 3 = \sqrt{9 - 15x}$. Her work is shown below.

$$5x - 3 = \sqrt{9 - 15x}$$

Step 1: $(5x - 3)^2 = \left(\sqrt{9 - 15x}\right)^2$

Step 2: $25x^2 + 9 = 9 - 15x$

Step 3: $25x^2 + 9 - 9 + 15x = 0$

Step 4: $25x^2 + 15x = 0$

Step 5: $(5x)(5x + 3) = 0$

Step 6: $5x = 0$ or $5x + 3 = 0$

Step 7: $x = 0$ or $x = -\frac{3}{5}$

Ashley made an error in her work. Identify Ashley's error and find the correct solution set. Be sure to state any extraneous solutions. Show all of your work.

3. Decide whether each of the statements below is always true, sometimes true, or never true for the set of real numbers. If you decide a statement is sometimes true, state the values of *x* for which the statement is true. Explain why the statement is true for only those values. If you decide the statement is always true or never true, explain your reasoning. Remember, substituting a few values for *x* is not enough to explain your reasoning.

   a. $\sqrt{2x-3} = \sqrt{2x} - \sqrt{3}$

   b. $\sqrt{-x} = -\sqrt{x}$

   c. $\sqrt[3]{-x} = -\sqrt[3]{x}$

   d. $-\sqrt{x^2} = -x$

   e. $\sqrt{8x+3} = -1$

   f. $\sqrt{16x^2} \cdot \sqrt{4} = 8x$

Instructional Task Exemplar Response

1. Find all real solutions for $4 = \sqrt{3x+7}$. Show your work and explain your reasoning for each step. Identify any extraneous solutions.

$4 = \sqrt{3x+7}$	*Assume there is a value for x that will make this equation true.*
$4^2 = \left(\sqrt{3x+7}\right)^2$	*The inverse operation for the square root is to square the expression, so I squared both sides of the equation.*
$16 = 3x + 7$	*Each side of the equation is simplified.*
$16 - 7 = 3x + 7 - 7$	*Subtract 7 from both sides in order to isolate the variable expression, then simplify.*
$9 = 3x$	
$\frac{9}{3} = \frac{3x}{3}$	*Divide both sides of the equation by 3 to isolate x, then simplify.*
$x = 3$	

The solution is $x = 3$. There are no extraneous solutions. I know this because substituting 3 for x in the original equation results in 4 = $\sqrt{16}$, which is a true statement.

2. Ashley solved the equation $5x - 3 = \sqrt{9 - 15x}$. Her work is shown below.

$$5x - 3 = \sqrt{9 - 15x}$$

Step 1: $(5x-3)^2 = \left(\sqrt{9-15x}\right)^2$

Step 2: $25x^2 + 9 = 9 - 15x$

Step 3: $25x^2 + 9 - 9 + 15x = 0$

Step 4: $25x^2 + 15x = 0$

Step 5: $(5x)(5x+3) = 0$

Step 6: $5x = 0$ or $5x + 3 = 0$

Step 7: $x = 0$ or $x = -\frac{3}{5}$

Ashley made an error in her work. Identify Ashley's error and find the correct solution(s). Be sure to state any extraneous solutions. Show all of your work.

Ashley's error occurred in Step 2. Ashley did not square the binomial correctly. She should have gotten $25x^2 - 30x + 9 = 9 - 15x$ for Step 2. The correct solution is $x = \frac{3}{5}$. There is one extraneous solution, $x = 0$. I know this because substituting 0 for x in the original equation results in the equation – 3 = 3, which is not true.

$25x^2 - 30x + 9 - 9 + 15x = 0$
$25x^2 - 15x = 0$
$(5x)(5x-3) = 0$
$5x = 0$ or $5x - 3 = 0$
$x = 0$ or $x = \frac{3}{5}$

3. Decide whether each of the statements below is always true, sometimes true, or never true for the set of real numbers. If you decide a statement is sometimes true, state the values of *x* for which the statement is true. Explain why the statement is true for only those values. If you decide the statement is always true or never true, explain your reasoning. Remember, substituting a few values for *x* is not enough to explain your reasoning.

   a. $\sqrt{2x-3} = \sqrt{2x} - \sqrt{3}$

   Sometimes true—the equation is only true when $x = \frac{3}{2}$. When I solved the equation for x, I only obtained one value for the solution. When I checked by substitution, $\frac{3}{2}$ made the equation true.

   $\left(\sqrt{2x-3}\right)^2 = \left(\sqrt{2x} - \sqrt{3}\right)^2$
   $2x - 3 = 2x - 2\sqrt{6x} + 3$
   $-6 = -2\sqrt{6x}$
   $3 = \sqrt{6x}$
   $(3)^2 = \left(\sqrt{6x}\right)^2$
   $9 = 6x$
   $\frac{3}{2} = x$

   b. $\sqrt{-x} = -\sqrt{x}$

   Sometimes true—the equation is only true when x = 0. If x is positive, then the left side of the equation becomes the square root of a negative number, which does not exist in the real number system. If x is negative, then the right side contains the square root of a negative number, which does not exist in the real number system. Since 0 does not have an opposite, when x = 0, the equation will be true.

   c. $\sqrt[3]{-x} = -\sqrt[3]{x}$

   Always true—the cube root of a negative number is negative, and the cube root of a positive number is positive.

   $\left(\sqrt[3]{-x}\right)^3 = \left(-\sqrt[3]{x}\right)^3$
   $-x = -x$

   Since the resulting equation is a true statement, this is always true.

   d. $\sqrt{8x+3} = -1$

   Never true—the left side of the equation asks for the positive square root of the quantity. The positive root will never be equal to a negative number.

   e. $\sqrt{16x^2} \cdot \sqrt{4} = 8x$

   Sometimes true—this is only true for $x \geq 0$. When the left side is simplified, the result is 8|x|. This is to ensure that we obtain a positive result for the left side. If x is negative, then the equation would not be true when evaluated.

APPENDIX

Understanding Mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as (a+b)(x+y) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding (a+b+c)(x+y). Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

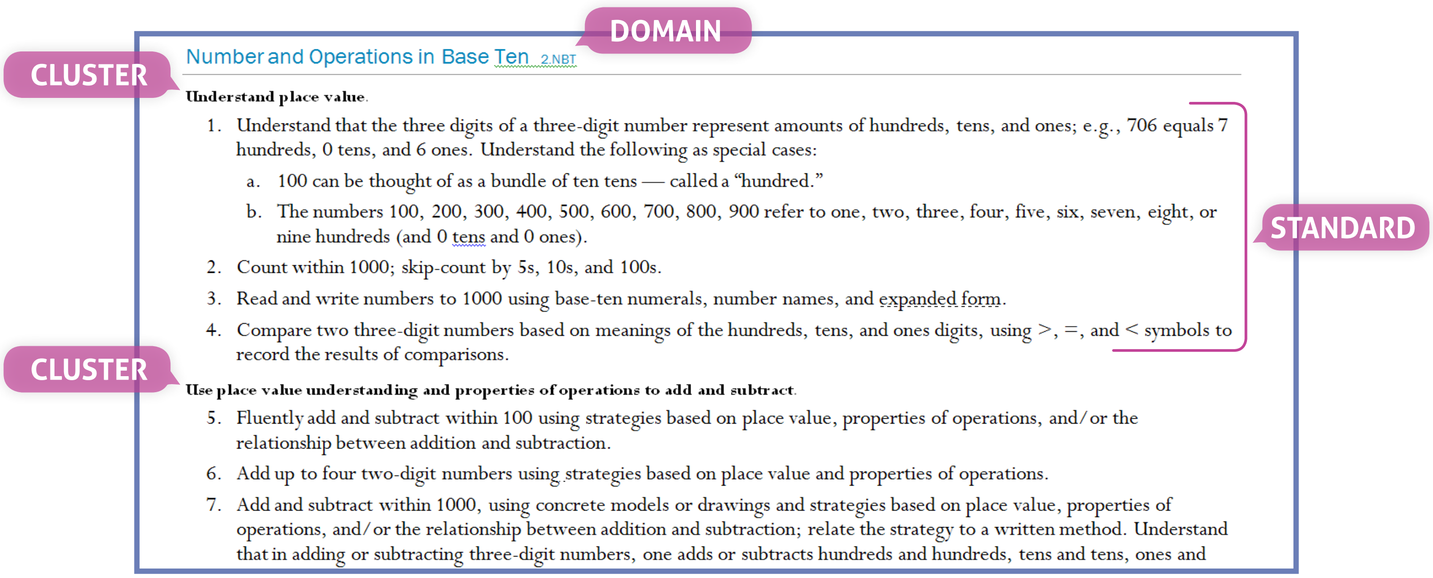
The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.



Number and Operations in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y–2)/(x–1)=3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Mathematics | High School—Number and Quantity

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3.... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 51 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Overview

Math Standards		Math Practives
The Real Number System	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Quantities	• Reason quantitatively and use units to solve problems.	
The Complex Number System	• Perform arithmetic operations with complex numbers. • Represent complex numbers and their operations on the complex plane. • Use complex numbers in polynomial identities and equations.	
Vector and Matrix Quantities	• Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications.	

The Real Number System (N-RN)

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers are rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities★ (N-Q)

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)^3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus 2 and argument 120°.*
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (N-VM)

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $||v||$, v).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Mathematics | High School—Algebra

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p+0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p+0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p+0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Overview

<table>
<tr><th colspan="2">Math Standards</th><th>Math Practives</th></tr>
<tr><td>Seeing Structure in Expressions</td><td>• Interpret the structure of expressions.
• Write expressions in equivalent forms to solve problems.</td><td rowspan="4">1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Arithmetic with Polynomials and Rational Expressions</td><td>• Perform arithmetic operations on polynomials.
• Understand the relationship between zeros and factors of polynomials.
• Use polynomial identities to solve problems.
• Rewrite rational expressions.</td></tr>
<tr><td>Creating Equations</td><td>• Create equations that describe numbers or relationships.</td></tr>
<tr><td>Reasoning with Equations and Inequalities</td><td>• Understand solving equations as a process of reasoning and explain the reasoning.
• Solve equations and inequalities in one variable.
• Solve systems of equations.
• Represent and solve equations and inequalities graphically.</td></tr>
</table>

Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.★
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

Arithmetic with Polynomials and Rational Expressions (A-APR)

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[8]

[8] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations★ (A-CED)

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Reasoning with Equations and Inequalities (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Overview

Math Standards		Math Practives
Interpreting Functions	• Understand the concept of a function and use function notation. • Interpret functions that arise in applications in terms of the context. • Analyze functions using different representations.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Building Functions	• Build a function that models a relationship between two quantities. • Build new functions from existing functions.	
Linear, Quadratic, and Exponential Models	• Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	
Trigonometric Functions	• Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions. • Prove and apply trigonometric identities.	

Interpreting Functions (F-IF)

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f *is the graph of the equation* $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions (F-BF)

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Build new functions from existing functions.

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
   a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models★ (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $a b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Trigonometric Functions (F-TF)

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\emptyset) + \cos^2(\emptyset) = 1$ and use it find $\sin(\emptyset)$, $\cos(\emptyset)$, or $\tan(\emptyset)$ given $\sin(\emptyset)$, $\cos(\emptyset)$, or $\tan(\emptyset)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

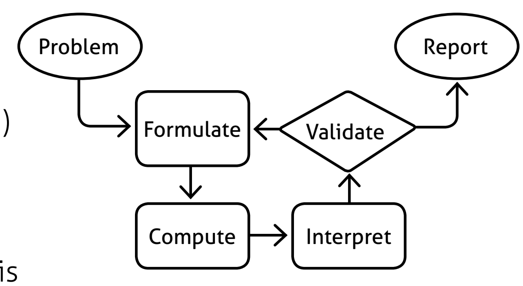
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.

- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry Overview

<table>
<tr><th colspan="2">Math Standards</th><th>Math Practives</th></tr>
<tr><td>Congruence</td><td>• Experiment with transformations in the plane.
• Understand congruence in terms of rigid motions.
• Prove geometric theorems.
• Make geometric constructions.</td><td rowspan="6">1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Similarity, Right Triangles, and Trigonometry</td><td>• Understand similarity in terms of similarity transformations.
• Prove theorems involving similarity.
• Define trigonometric ratios and solve problems involving right triangles.
• Apply trigonometry to general triangles.</td></tr>
<tr><td>Circles</td><td>• Understand and apply theorems about circles.
• Find arc lengths and areas of sectors of circles.</td></tr>
<tr><td>Expressing Geometric Properties with Equations</td><td>• Translate between the geometric description and the equation for a conic section.
• Use coordinates to prove simple geometric theorems algebraically.</td></tr>
<tr><td>Geometric Measurement and Dimension</td><td>• Explain volume formulas and use them to solve problems.
• Visualize relationships between two-dimensional and three-dimensional objects.</td></tr>
<tr><td>Modeling with Geometry</td><td>• Apply geometric concepts in modeling situations.</td></tr>
</table>

Congruence (G-CO)

Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions.

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry (G-SRT)

Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity.

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Apply trigonometry to general triangles.

9. (+) Derive the formula $A = ½\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles (G-C)

Understand and apply theorems about circles.

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations (G-GPE)

Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically.

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Geometric Measurement and Dimension (G-GMD)

Explain volume formulas and use them to solve problems.

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry (G-MG)

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Mathematics | High School—Statistics and Probability ★

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Overview

<table>
<tr><th colspan="2">Math Standards</th><th>Math Practives</th></tr>
<tr><td>Interpreting Categorical and Quantitative Data</td><td>• Summarize, represent, and interpret data on a single count or measurement variable.
• Summarize, represent, and interpret data on two categorical and quantitative variables.
• Interpret linear models.</td><td rowspan="4">1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Making Inferences and Justifying Conclusions</td><td>• Understand and evaluate random processes underlying statistical experiments.
• Make inferences and justify conclusions from sample surveys, experiments and observational studies.</td></tr>
<tr><td>Conditional Probability and the Rules of Probability</td><td>• Understand independence and conditional probability and use them to interpret data.
• Use the rules of probability to compute probabilities of compound events in a uniform probability model.</td></tr>
<tr><td>Using Probability to Make Decisions</td><td>• Calculate expected values and use them to solve problems.
• Use probability to evaluate outcomes of decisions.</td></tr>
</table>

Interpreting Categorical and Quantitative Data (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions (S-IC)

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability (S-CP)

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions (S-MD)

Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Note on courses and transitions

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics—in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3)—will be made available shortly after the release of the final Common Core State Standards. It is expected that additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.