

**AP<sup>®</sup> CALCULUS AB**  
**2014 SCORING GUIDELINES**

**Question 5**

$x$	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions  $f$  and  $g$  are defined for all real numbers  $x$ . Values of  $f$ ,  $f'$ ,  $g$ , and  $g'$  for various values of  $x$  are given in the table above.

- (a) Find the  $x$ -coordinate of each relative minimum of  $f$  on the interval  $[-2, 3]$ . Justify your answers.
- (b) Explain why there must be a value  $c$ , for  $-1 < c < 1$ , such that  $f''(c) = 0$ .
- (c) The function  $h$  is defined by  $h(x) = \ln(f(x))$ . Find  $h'(3)$ . Show the computations that lead to your answer.
- (d) Evaluate  $\int_{-2}^3 f'(g(x))g'(x) dx$ .

- (a)  $x = 1$  is the only critical point at which  $f'$  changes sign from negative to positive. Therefore,  $f$  has a relative minimum at  $x = 1$ .

1 : answer with justification

- (b)  $f'$  is differentiable  $\Rightarrow f'$  is continuous on the interval  $-1 \leq x \leq 1$

$$\frac{f'(1) - f'(-1)}{1 - (-1)} = \frac{0 - 0}{2} = 0$$

Therefore, by the Mean Value Theorem, there is at least one value  $c$ ,  $-1 < c < 1$ , such that  $f''(c) = 0$ .

$$2 : \begin{cases} 1 : f'(1) - f'(-1) = 0 \\ 1 : \text{explanation, using Mean Value Theorem} \end{cases}$$

- (c)  $h'(x) = \frac{1}{f(x)} \cdot f'(x)$

$$h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14}$$

$$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$$

- (d)  $\int_{-2}^3 f'(g(x))g'(x) dx = [f(g(x))]_{x=-2}^{x=3}$   
 $= f(g(3)) - f(g(-2))$   
 $= f(1) - f(-1)$   
 $= 2 - 8 = -6$

$$3 : \begin{cases} 2 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$$