SCARSDALE HIGH SCHOOL
Mathematics Department

Math 424 Final Exam
June 14, 2012
12:45 – 2:45 P.M.

Name: key
Teacher: ______________________

SCIENTIFIC CALCULATORS ARE PERMITTED

Instructions: **Read the directions at the beginning of each part. Show all work for possible partial credit in parts II, III and IV.**

PART I

Directions: **Answer 12 out of 15 questions. Omit 3 questions. Write the letter of the best choice in the space provided. Partial credit is not allowed. (3 points each)**

c 1. Which of the following operational systems is a group?

$(a)(Z_{21},\cdot)$ $(c)(Z_{15},+)$

$(b)\ (Z,\cdot)$ $(d)(Q^*,+)$

d 2. A student wants to prove the following statement indirectly. "If the diagonals of a parallelogram are not congruent, then the parallelogram is not a rectangle" Which of the following statements should be assumed?

(a) The diagonals are congruent.

(b) The quadrilateral is not a parallelogram.

(c) The parallelogram is not a rectangle.

(d) The parallelogram is a rectangle.

b 3. Which of the following is logically equivalent to $[p \vee \sim r] \wedge [r \vee s]$?

$(a) \sim p \rightarrow r$ $(c) \sim r \rightarrow s$

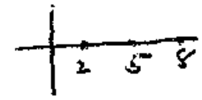
$(b) \sim p \rightarrow s$ $(d) \sim r \rightarrow \sim s$

c 4. Which of the following is the solution set to $x^2 < x$?

$(a) x < 1$ $(c) 0 < x < 1$

$(b) x < 0 \wedge x < 1$ $(d) x < 0 \vee x > 1$

d ______ 5. The locus of points equidistant from the circles whose equations are $x^2+y^2=4$ and $x^2+y^2=64$ is which of the following?



$r=2$ $x^2+y^2=4$

$r=8$ $x^2+y^2=64$

$r=5$

$(a)\ x^2+y^2=39$

$(b)\ x^2+y^2=36$

$(c)\ x^2+y^2=30$

$(d)\ x^2+y^2=25$

b ______ 6. Which of the following is equivalent to $-\frac{11}{9}$ in Z_{13}?

$(a)2$

$(b)6$

$(c)4$

$(d)12$

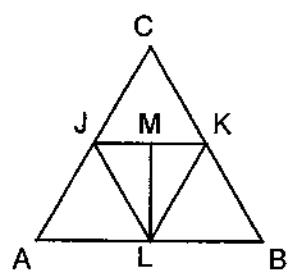
c ______ 7. In the accompanying diagram, $\triangle ABC$ is an equilateral triangle $J, K,$ and L are midpoints as shown. If the perimeter of the $\triangle JKL$ is 12 cm, which of the following could be the length of the altitude $\overline{LM}$?

$(a)\sqrt{3}$

$(b)2$

$(c)2\sqrt{3}$

$(d)4$



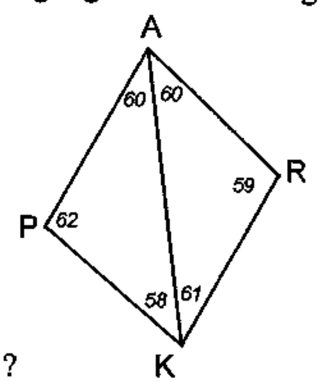
d ______ 8. The accompanying diagram is not drawn to scale, which of the following segments is the longest?

$(a)\ AK$

$(b)\ RK$

$(c)\ PK$

$(d)\ AR$



a ______ 9. Which of the following is the solution set to the equation $\left|\frac{1}{3}x-2\right|=7$?

$(a)\{27,-15\}$

$(b)\{-19,19\}$

$(c)\{15,-27\}$

$(d)\{-8,20\}$

c.

10. Which of the following is the logically equivalent to the statement "If all students pass their finals, then there will be no summer school and the camps will be full"?

(a) If all students don't pass their finals, then there will be summer school or the camps are not full.

(b) If the camp aren't full and there is summer school, then some students did not pass their finals.

(c) If the camp aren't full or there is summer school, then some students did not pass their finals.

(d) If some students don't pass their finals, then there will be summer school and the camps are not full.

a

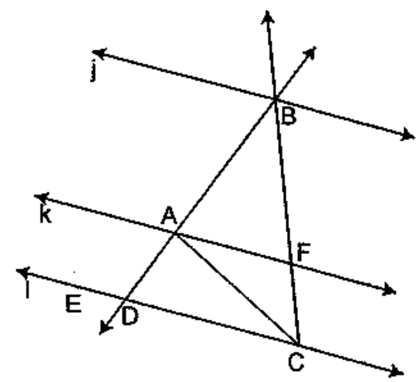
11. In the accompanying figure, lines j, k and l are parallel and $\overline{AB} \cong \overline{AC}$, if $m\angle ABC = x$ and $m\angle ACD = y$, then $m\angle ADE$ is equal to which of the following?

$(a)\ 2x+y$

$(c)\ 3x$

$(b)\ 2y+x$

$(d)\ 180-2x$



b

12. Which of the following is the inverse of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ under the operation $a \circ b$ "a followed by b"?

$(a)\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

$(c)\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$(b)\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

(d) Does not exist.

b

13. A line passes through the points $(-1,5)$ and $(7,|k|)$. Which of the following must be true if the slope of the line is positive?

$(a)\, k>0$ $\quad$ $(c)\, k>5$

$(b)\, k<-5 \vee k>5$ $\quad$ $(d)\, k>-5 \wedge k<5$

$\frac{|k|-5}{7+1} > 0$

$\frac{|k|-5}{8} > 0$

$|k|-5>0$

$|k|>5$

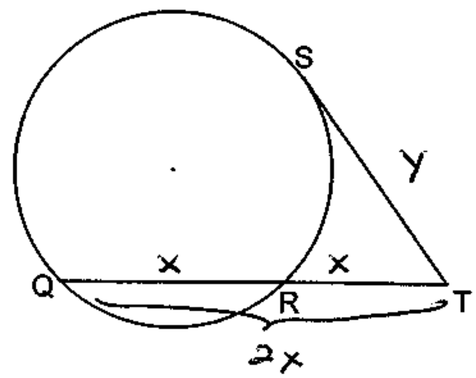
$k<-5 \vee k>5$

c

14. In the figure below R is the midpoint of secant $\overline{TQ}$, $\overline{TS}$ is tangent to the circle at S. The ratio of the tangent to the external segment of the secant is which of the following?

$(a)\, 1:\sqrt{2}$ $\quad$ $(c)\, \sqrt{2}:1$

$(b)\, 1:\sqrt{3}$ $\quad$ $(d)\, \sqrt{3}:1$



$2x(x) = y^2$

$2x^2 = y^2$

$2 = \frac{y^2}{x^2} \longrightarrow \frac{y}{x} = \frac{\sqrt{2}}{1}$

a

15. The intersection of the perpendicular bisectors of the sides of a triangle is called ....?

(a) Circumcenter $\quad$ (c) Incenter

(b) Orthocenter $\quad$ (d) centroid

PART II

Directions: Answer 8 out of 12 questions. **Omit 4 questions**. Show all work in the space provided for possible partial credit. Unless otherwise specified, answers should be given in **simplest radical form**. (4 points each)

Write the question number to be omitted in the space below.

__________, __________, __________, __________

1. Simplify the expression assume no denominator $\dfrac{2x^2+4x}{x^2+4x+4-9y^2}\cdot\dfrac{x+3y+2}{3x^2+6x}$ or variable can equal zero:

$$\frac{2\cancel{x}\cancel{(x+2)}}{(x+2)^2-9y^2}\cdot\frac{x+3y+2}{3\cancel{x}\cancel{(x+2)}} = \frac{2\,\cancel{(x+3y+2)}}{3\,(x+2-3y)\cancel{(x+2+3y)}} = \frac{2}{3(x+2-3y)}$$

+1 +1 +1 +1

2. Find the equation for the locus of points equidistant from $(3,2)$ and $(-9,-2)$. Leave your answer in point-slope form.

$$m=\frac{-2-2}{-9-3}$$

Midpoint: $(-3,\ 0)$ (+1)

$$m=\frac{-4}{-12}$$

$$m=\frac{1}{3}\ (+1)$$

(+1)

$$y-0=-3(x+3)\quad (+1)$$

OR

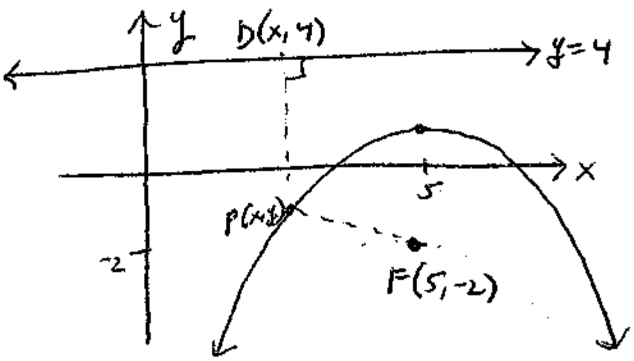
$$y=-3(x+3)$$

3. Simplify the expression: $\dfrac{\dfrac{2a+4}{a}}{3-12a^{-2}}$

$$\frac{\dfrac{2a+4}{a}}{3-\dfrac{12}{a^2}}\cdot\frac{\dfrac{a^2}{1}}{\dfrac{a^2}{1}} = \frac{a(2a+4)}{3a^2-12} = \frac{2a\cancel{(a+2)}}{3\cancel{(a+2)}(a-2)} = \boxed{\frac{2a}{3(a-2)}}\ (+1)$$

(+1) (+1) (+1)

4. Find the standard form equation for the locus of points equidistant from $(5,-2)$ and the line $y=4$.



$PF = PD$ +1

$$\sqrt{(x-5)^2+(y+2)^2}^{\,2} = \sqrt{(x-x)^2+(y-4)^2}^{\,2}$$

$$(x-5)^2+(y+2)^2 = (y-4)^2$$

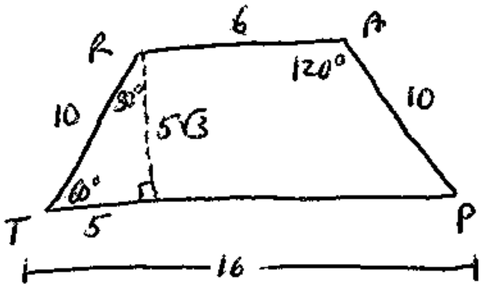
$$x^2-10x+25+y^2+4y+4 = y^2-8y+16$$ +2

$$x^2-10x+4y+29 = -8y+16$$

$$x^2-10x+13 = -12y$$

$$-\tfrac{1}{12}(x^2-10x+13) = y \text{ or } y = -\frac{x^2}{12}+\frac{5}{6}x-\frac{13}{12}$$ +1

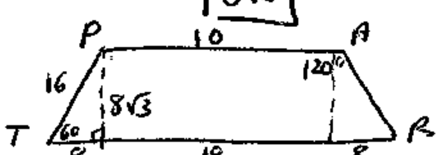
5. Find the area of the isosceles trapezoid *TRAP* (not shown), if $AP = 10$, $m\angle A = 120$, and $PT = 16$. (leave your answer in simplest radical form)



$$A = \frac{5\sqrt{3}}{2}(6+16)$$ (+2) (+1)

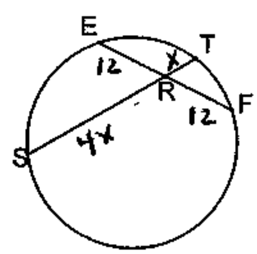
$$A = 55\sqrt{3}$$ (+1)

$\boxed{\text{OR}}$



$$A = \frac{8\sqrt{3}}{2}(26+10) = 144\sqrt{3}$$

6. In the accompanying diagram, chord $\overline{ST}$ bisects chord $\overline{EF}$ at *R*. If $EF = 24$, find *SR*, given $SR : RT = 4:1$.



$$(4x)\cdot x = 12 \cdot 12 \quad (+2)$$

$$4x^2 = 144$$

$$x^2 = 36$$

$$x = 6 \quad (+1)$$

$$SR = 24 \quad (+1)$$

7. Prove or disprove the following: Multiplication distributes over $*$, where $a*b=a^2-b$, $\forall a,b \in \Re$? (Substitution of numbers for variables will not be accepted.)

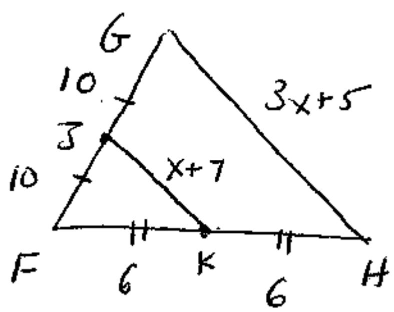
$c(a*b) \stackrel{?}{=} (ca*cb)$ +2

$c(a^2-b) \stackrel{?}{=} (ca)^2-cb)$

$a^2c-bc \neq a^2a2-cb$ +1

$\therefore$ mult. does not distribute over +1

8. In $\triangle FGH$, J and K are the midpoints of $\overline{FG}$ and $\overline{FH}$ respectively. If $FG=20, FH=12$, $GH=3x+5$, and $JK=x+7$, find the perimeter of $JKHG$.



$x+7=\frac{1}{2}(3x+5)$ +2

$2x+14=3x+5$

$-x=-9$

$x=9$ +1

$JK=16$

$GH=32$

Perimeter of JKHG = 64 +1

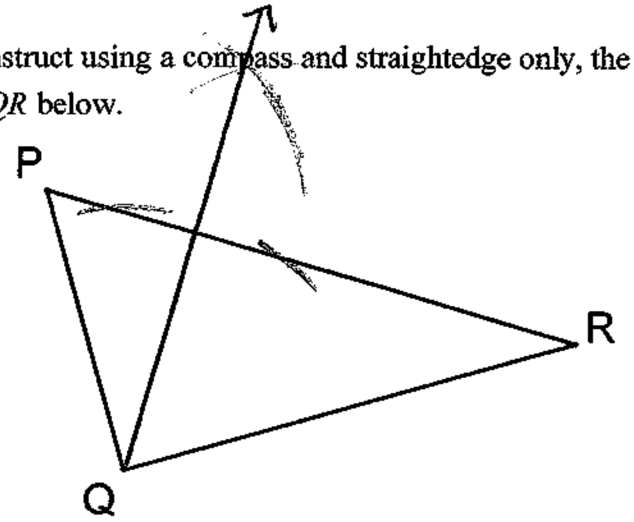
9. Find the length of the median $\overline{CM}$ in $\triangle ABC$, where $A(-5,-5), B(7,1)$, and $C(-2,2)$.

Midpoint of $\overline{AB} = M(1,-2)$ +1

$CM=\sqrt{(-2-1)^2+(2+2)^2}$ +2

$CM=\sqrt{9+16} = \boxed{5}$ +1

10. Construct using a compass and straightedge only, the altitude from vertex Q to $\overline{PR}$ in $\triangle PQR$ below.



11a. Find the number of permutations that can be formed from the letters in the word "CONTESTANT"

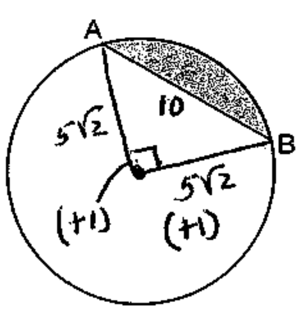
$$\frac{10!}{2!\,3!} = \boxed{302,400}$$

11*b*. A book collection contains 6 different music books, 4 different art books, and 3 different literature books. A sample is selected from the collection, how many 5-book samples are there that have 3 music books and 2 literature books?

$${}_6C_3 \cdot {}_3C_2 = 20 \cdot 3 = \boxed{60}$$

(+1) (+1)

12. In the figure below, find the area of the shaded region bounded by the chord $\overline{AB}$ and $\overset{\frown}{AB}$, if $AB = 10$ and area of the circle is 50π. Leave your answer in terms of π.



$A = 50\pi$

$\pi r^2 = 50\pi$

$r^2 = 50$

$r = 5\sqrt{2}$

(+1) (+1)

$A_{\text{shaded}} = \boxed{\frac{50\pi}{4} - 25}$ OR $\boxed{\frac{25\pi}{2} - 25}$

PART III

Directions: **Answer two (2) questions from this part. Omit 1 problem. Show all work.**
(8 pts. each)

Write the question number to be omitted in the space below.

____

1. Given: $r \rightarrow \sim j$
$\sim(\sim r \wedge k)$
$\sim(\sim p \rightarrow \sim k)$

Prove: $\sim j \vee t$

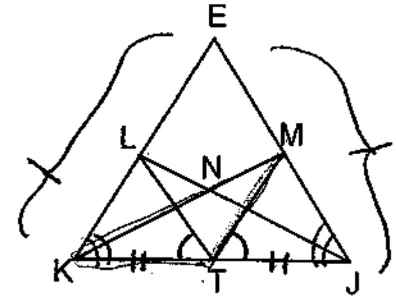
Statement	Reason
1. $\sim(\sim p \rightarrow \sim k)$	1. Given
2. $\sim p \wedge \sim(\sim k)$	2. Def^n of negation of a conditional (1)
3. $\sim(\sim k)$	3. Def^n of true conjunction (2)
4. $\sim(\sim r \wedge k)$	4. Given
5. $r \vee \sim k$	5. De Morgan's Laws (4)
6. r	6. Law of Disjunctive Inference (3,5)
7. $r \rightarrow \sim j$	7. Given
8. $\sim j$	8. Law of Detachment (6, 7)
9. $\sim j \vee t$	9. Law of Disjunctive Addition (8)

-1 if no reference #'s

2. Given: $\overline{KE} \cong \overline{JE}, \angle LTK \cong \angle MTJ$,

T is the midpoint of $\overline{KJ}$

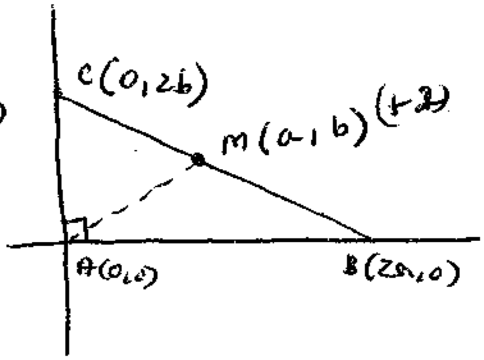
Prove: $\overline{KM} \cong \overline{JL}$



Statement	Reason
1. $\overline{KE} \cong \overline{JE}$	1. Given
2. $\angle EKT \cong \angle EJT$	2. If 2 sides of a Δ are $\cong$ then the $\angle$'s opp. those sides are $\cong$.
3. T is the midpt. of $\overline{KJ}$	3. Given
4. $\overline{KT} \cong \overline{JT}$	4. Def^n of midpoint.
5. $\angle LTK \cong \angle MTJ$	5. Given
6. $\Delta LKT \cong \Delta MJT$	6. ASA Axiom (+3)
7. $\overline{LT} \cong \overline{MT}$	7. Corr. parts of $\cong \Delta$'s are $\cong$.
8. $\angle LTm \cong \angle LTm$	8. Reflexive Property
9. $m\angle LTK = m\angle MTJ$ $m\angle LTm = m\angle LTm$	9. Def^n of $\cong \angle$'s.
10. $m\angle LTK + m\angle LTm = m\angle mTJ + m\angle LTm$	10. Add^n prop. of equality
11. $m\angle KTm = m\angle LTK + m\angle LTm$ $m\angle LTJ = m\angle MTJ + m\angle LTm$	11. $\angle$ Add^n Axiom
12. $m\angle KTM = m\angle LTJ$	12. Substitution Prop. (10, 11)
13. $\angle KTm \cong \angle LTJ$	13. Def^n of $\cong \angle$'s
14. $\Delta MTK \cong \Delta LTJ$	14. SAS Axiom (+4)
15. $\overline{Km} \cong \overline{JL}$	15. Corr. parts of $\cong \Delta$'s are $\cong$. (+1)

3. Prove using coordinate geometry: The midpoint of the hypotenuse of a right triangle is equidistant from the vertices.

(+2)



Given: Right △ ABC, w/ rt. ∡ A.
M is the midpt. of $\overline{BC}$

Prove: M is equidistant from A, B and C.

(+3) D.F.

(+1) Conclusion

$CM = \sqrt{(a-0)^2 + (b-2b)^2}$

$CM = \sqrt{a^2 + b^2}$

$BM = \sqrt{(2a-a)^2 + (0-b)^2}$

$Bm = \sqrt{a^2+b^2}$

$Am = \sqrt{(a-0)^2 + (b-0)^2}$

$Am = \sqrt{a^2+b^2}$

$Am = Bm = CM$

∴ M is equidistant from A, B and C.

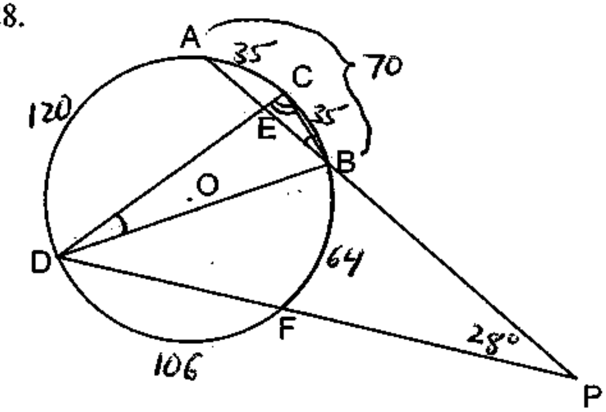
PART IV

Directions: **Answer any two (2) questions. Omit 2 problems. Show all work. Final answers must be in simplest radical form. (8 points each)**

Write the question number to be omitted in the space below.

__________, __________

1. Given: Circle O, C is the midpoint of $\widehat{AB}$, chords $\overline{BC}, \overline{CD}$, and $\overline{DB}$ are drawn. Secants $\overline{PBA}$ and $\overline{PFD}$ are drawn from external point P. $\overline{PBA}$ intersects chord $\overline{CD}$ at E. $m\widehat{AB} = 70, m\widehat{BF} = 64$, and $m\angle P = 28$.



+2 (a) Find $m\widehat{AD}$

$$28 = \frac{1}{2}(m\widehat{AD} - 64)$$

$$\boxed{m\widehat{AD} = 120}$$

+2 (b) Find $m\angle BED$

$$m\angle BED = \frac{1}{2}(m\widehat{BD} + m\widehat{AC}) = \frac{1}{2}(170 + 35) = \boxed{102.5}$$

+2 (c) Find $m\angle DBP$

$$\boxed{120°}$$

+2 (d) In the figure name the triangle similar but not congruent to $\triangle BEC$.

$\triangle DBC$

2*a*. Show that the equation expressing the condition that the point (x, y) is twice as far from the point $(3,-1)$ as it is from $(4,6)$ is $3x^2+3y^2-16x+20y-12=0$.

this problem is wrong

$$\left[2\sqrt{(x-3)^2+(y+1)^2}\right]^2 = \sqrt{(x-4)^2+(y-6)^2}. \qquad (+1)$$

$$4\left[(x-3)^2+(y+1)^2\right] = (x-4)^2+(y-6)^2$$

$$4(x^2-6x+9+y^2+2y+1) = x^2-8x+16+y^2-12y+36$$

$$4x^2-24x+36+4y^2+8y+4 = x^2-8x+y^2-12y+52 \qquad (+2)$$

$$4x^2-24x+4y^2+8y+40 = x^2-8x+y^2-12y+52$$

$$\boxed{3x^2+3y^2-16x-20y-12=0}$$

2*b*. A cyclist travels 200 km, part at 30 km/h and the rest at 20 km/h. How far does the cyclist travel at each speed if the trip takes 7.5 hours?

	r	t	D
PART A	30	$\frac{d}{30}$	d
PART B	20	$\frac{200-d}{20}$	200-d

+2

$$\frac{60}{1}\cdot\left(\frac{d}{30}+\frac{200-d}{20}\right) = \frac{15}{2}\cdot\frac{60}{1} \qquad +1$$

$$2d+3(200-d)=450$$

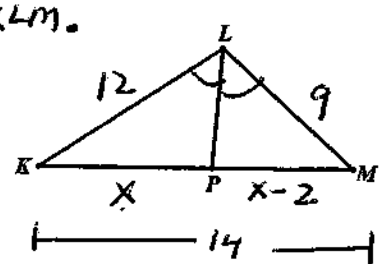
$$2d+600-3d=450$$

$$-d=-150$$

$$d=150$$

$$\boxed{\begin{array}{l} 150\text{ km} @ 30\ \frac{\text{km}}{\text{h}} \quad +1 \\ 50\text{ km} @ 20\ \frac{\text{km}}{\text{h}} \quad +1 \end{array}}$$

3*a*. In $\triangle KLM, \overline{LP}$ is drawn to $\overline{MK}$ such that $\angle KLP \cong \angle MLP$. MP is two less than KP, $LM = 9$ and $KL = 12$. Find the perimeter of the triangle. KLM.



$$\frac{x}{x-2} = \frac{12}{9} \qquad (+2)$$

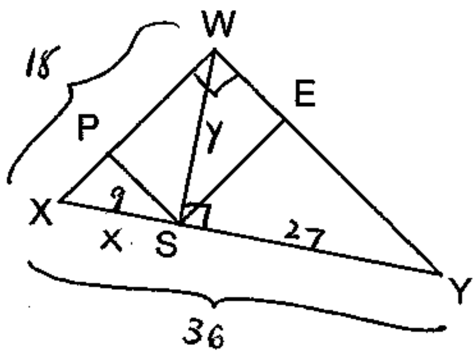
$$12x - 24 = 9x$$

$$-24 = -3x \qquad (+1)$$

$$8 = x$$

$$P_{\triangle KLM} = 35. \qquad (+1)$$

3*b*. In the figure below, $PWES$ is a rectangle and $\overline{WS} \perp \overline{XY}$. If $WX = 18$ and $XY = 36$, find WS.



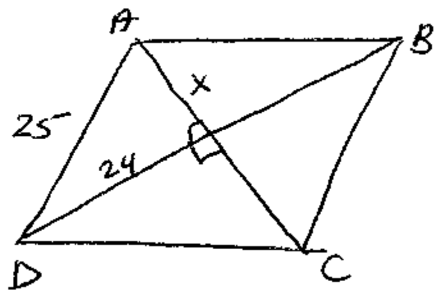
$$(+1) \quad \frac{36}{18} = \frac{18}{x} \qquad\qquad \frac{9}{y} = \frac{y}{27} \quad (+1)$$

$$324 = 36x \qquad\qquad y^2 = 243$$

$$(+1) \quad \boxed{9 = x} \qquad\qquad y = \sqrt{246}$$

$$\boxed{y = 9\sqrt{3}} \quad (+1)$$

4*a*. Find the length of the shorter diagonal of a rhombus whose perimeter is 100, and whose longer diagonal is 48 *cm*.



$x^2 + 24^2 = 25^2$ +1

$x^2 + 576 = 625$

$x^2 = 49$

$x = 7$ +1

$\boxed{AC = 14}$ +1

4*b*. Solve: $\dfrac{x}{x^2-1} + \dfrac{2}{x+1} = 1 + \dfrac{1}{2x-2}$

$\dfrac{2(x+1)(x-1)}{1}\left[\dfrac{x}{(x+1)(x-1)} + \dfrac{2}{x+1}\right] = \left(1 + \dfrac{1}{2(x-1)}\right)\dfrac{2(x+1)(x-1)}{1}$ +1

$2x + 4(x-1) = 2(x+1)(x-1) + 1(x+1)$ +1

$2x + 4x - 4 = 2(x^2-1) + x + 1$

$6x - 4 = 2x^2 - 2 + x + 1$

$6x - 4 = 2x^2 + x - 1$

$0 = 2x^2 - 5x + 3$

+1

$0 = (2x - 3)(x - 1)$ +1

$2x - 3 = 0 \vee x - 1 = 0$

$x = \frac{3}{2} \vee \cancel{x = 1}$

+1 reject

-1 if not rejected.