DUSO MATHEMATICS LEAGUE
INDIVIDUAL QUESTIONS - MEET #5
FEBRUARY 11, 2009

1. EL. ALG (MATH A) 5 MINUTES

If 125 is added to 100 or to 164, the result is the square of a positive integer. For what positive integer ***n*** will subtracting ***n*** from 100 or from 164 also result in the square of a positive integer?

2. GEOMETRY (MATH A) 5 MINUTES

An equilateral triangle of perimeter 6 sits atop a square of perimeter 8 with which it shares a side in common. Line segments connect the two vertices of the square that are not also vertices of the triangle to the vertex of the triangle that is not also a vertex of the square. What is the measure of the acute angle between these two line segments?

3. IntALG./TRIG (MATH B) 8 MINUTES

What are **all** (**two**) the pairs of ***positive*** integers **(*x, y*)** which satisfy the following equation:

$$\left(x^2 + y\right) - \left(x + y^2\right) = 38$$

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4. GEOMETRY (MATH B) 7 MINUTES

Quadrilateral Q has an inscribed circle. If the lengths of 3 consecutive sides of Q are 13, 14, and 15, what is the perimeter of Q? Express your answer in simplest form.

5. EL. ALG (MATH A) 6 MINUTES

If $a < b$, what ordered pair of integers (a, b) satisfies the following

$$\sqrt{24 + 2\sqrt{143}} = \sqrt{a} + \sqrt{b}$$

6. IntALG./TRIG (MATH B) 6 MINUTES

What ordered pair of rational numbers (a, b) satisfies the following

$$9^{\log_{81} 16} = 2^a 3^b$$

DUSO MATHEMATICS LEAGUE
RELAY TEAM QUESTION- MEET #5
FEBRUARY 11, 2009

1. Dorack has test scores of 85 and 77. If he takes three more tests, and receives the same grade on each one, what grade must he receive on each one in order to have an average of exactly 87?

2. Governor Lagosh, allegedly, attempted to sell a Senate seat to three different candidates. Candidate #2 offered $30,000 more that candidate #1. Candidate #3 offered $10,000 more than *twice* candidate #1. The three candidates combined, offered a total of $200,000. How many dollars did candidate #1 offer? Now divide your result by 1000 and add it to **TNYWR**.

3. Find the larger root of $3x^2 + x - 14 = 0$, and add it to **TNYWR**.

4. When **(*n* - 1)!** is subtracted from ***n*!** , the result is 322,560. Find the value of *n* and add it to **TNYWR**. (Hint: Guess and Check)

5. Find $\sqrt[3]{5.7}$ to the nearest tenth. (Hint: Guess and Check). Next take your result and multiply it by 10. Add that amount to **TNYWR**.

DUSO MATHEMATICS LEAGUE
SOLUTIONS - MEET #5
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Answers for RELAY TEAM QUESTION

1). 91 **2).** 131 **3).** 133 **4).** 142 **5).** 160

(Solutions for the Relay question)

1). $\dfrac{85+77+x+x+x}{5}=87$ $3x + 162 = 435$ $3x = 273$ $x = 91$
2). $x+x+30000+2x+10000=200000$ $4x+40000=200000$ $4x=160000$ $x=40000$ 40000/1000 = 40   Now 40 + TNYWR = 40 + 91 = 131
3). $3x^2+x-14=0$ $(3x+7)(x-2)=0$ $x = -7/3$ $x = 2$ The larger root is 2   So 2 + TNYWR = 2+131 = 133
4) 7! = 5040   8! = 40320   9! = 362880   So 9! – 8! =362880-40320 = 322560   Thus $n = 9$ Now 9 + TNYWR = 9 + 133 = 142
5) (1.7)(1.7)(1.7) = 4.913   (1.8)(1.8)(1.8) = 5.832   So $\sqrt[3]{5.7} \approx 1.8$ to the nearest tenth   Now 10(1.8) = 18   So 18 + TNYWR = 18 + 142 = 160

Individual Questions written by Steve Conrad www.mathleague.com; edited by Dan Flegler & J.S.
Relay team question written for DUSO by J.A.; DUSO Editor J.S.

DUSO MATHEMATICS LEAGUE
SOLUTIONS - MEET #5 - FEBRUARY 11, 2009

Answers for INDIVIDUAL QUESTIONS

1). 64	**2).** 30 or 30º	**3).** (20, 19) and (11, 9)
4). 56	**5).** (11,13)	**6).** (2, 0)

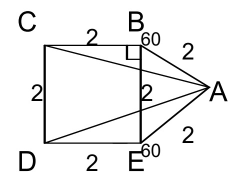
Solutions for the Individual questions)

1). $164 - 64 = 100 = 10^2$

$100 - 64 = 36 = 6^2$

So 64 is the required number

2).



$$m\angle BAC = \frac{180-(90+60)}{2} = \frac{30}{2} = 15$$

$$m\angle CAD = 60-(15+15) = 30$$

3) Rewrite equation: $(x^2 - y^2) - (x-y) = 38$

Factor: $(x-y)(x+y) - (x-y) = 38$

$(x-y)(x+y-1) = 38$

Since $x \ge 1, y \ge 1$, $(x + y - 1)$ is positive, so $(x - y)$ must also be positive, and $(x - y) < (x + y - 1)$.

The equation $(x-y)(x+y-1) = 38$ represents a factorization of 38 into two positive integers.

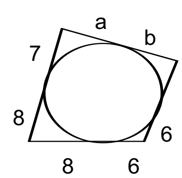
Factors of 38 are: 38 and 1 or 19 and 2

So

$x - y = 1$	or	$x - y = 2$
$x + y - 1 = 38$		$x + y - 1 = 19$
$2x = 40$		$2x = 22$
$x = 20$, $y = 19$		$x = 11$, $y = 9$

Thus the solutions are (20, 19) and (11, 9)

4).



A possible quadrilateral is shown

Sides of quadrilateral Q are tangent segments to the inscribed circle.

So a = 7 and b = 7

Perimeter is 13 + 14 + 15 + 14 = 56

Theorem: For any quadrilateral with an inscribed circle, the sum of the lengths of each pair of opposite sides of the quadrilateral is the same.

5). $\sqrt{24 + 2\sqrt{143}} = \sqrt{(11+13) + 2\sqrt{(11)(13)}}$

$= \sqrt{\left(\sqrt{11} + \sqrt{13}\right)^2}$

$= \sqrt{11} + \sqrt{13}$

So $(a, b) = (11,13)$

6) Squaring both sides of $9^{\log_{81} 16} = 2^a 3^b$ we get $81^{\log_{81} 16} = 2^{2a} 3^{2b}$ but $81^{\log_{81} 16} = 16$

so $2^{2a} 3^{2b} = 16$

Since $16 = 2^4$, $2a = 4$ and $2b = 0$

Thus $a = 2$ and $b = 0$ and our ordered pair is (2,0)

Or

since

$$\log_{81} 16 = \frac{\log 16}{\log 81} = \frac{\log 4^2}{\log 9^2} = \frac{2\log 4}{2\log 9} = \frac{\log 4}{\log 9} = \log_9 4$$

we have $9^{\log_{81} 16} = 9^{\log_9 4} = 2^a 3^b$

but since $9^{\log_9 4} = 4$ we get $4 = 2^a 3^b$

Thus it follows that $a = 2$ and $b = 0$ and our ordered pair is (2,0)

February 2009

Dear Section Secretary:

Attached is a tiebreaker question, to be used at our last regular meet in order to break potential ties for the section champion and/ or potential ties among individuals attempting to qualify for States.

All Mathletes will do this question. It will be contested ***after*** the regular Meet 5 questions are completed and scored. Should any teams be tied with the highest cumulative total in their section for all 5 regular meet questions, the tied teams will designate 5 members to compete as a team. (Only tied teams will have members competing as teams and everyone else will compete as individuals.) Prior to the beginning the tiebreaker, announce that the solutions of **competing teams** will be collected similar to relays. Only competing teams solutions will be collected and numbered in order of submission, until the end of the given time period. At the end of the time period, the remaining solutions will be collected from ***everyone***. The competing team with the most correct solutions will be the Section Champion. Should there still be a tie for most correct solutions, the team that submitted the first correct answer will be the Section Champion. When you have determined a winner, place a star (*) next to the name and score of the Section Champion on your Meet 5 results. Do not add in tiebreaker points to the Meet 5 results. Record tiebreaker results separately on the grid below and submit that along with the Meet 5 results.

All Mathlete responses to the tiebreaker will be collected. Tiebreaker results will not be added to the individual totals. Instead the papers of any and all Mathletes with an individual cumulative score of 7 or higher prior to meet 5 should be graded and put in a separate envelope. These should be brought to Sectionals in order to assist, if necessary, in determining contested positions on the DUSO State teams. Be sure that all the heading information (name, team, grade level,T-shirt size and sex) is filled in. Please include a cover sheet of names and scores for these results with the envelope, and send a copy of the cover sheet with your results.

If you have any questions regarding this, please contact Anchala Sobrin, Maureen Black or myself.

Thanks,
June Schreck -

TIEBREAKER RESULTS

DATE_______________ SECTION_____CHAMPION TEAM________________

TIEBREAKER QUESTION :

COMPETING TEAMS	NUMBER CORRECT

TIE BREAKER # 1 **GEOMETRY** **(MATH A)** **5 MINUTES**

ANSWER

Name ______________________________

Team ______________________________

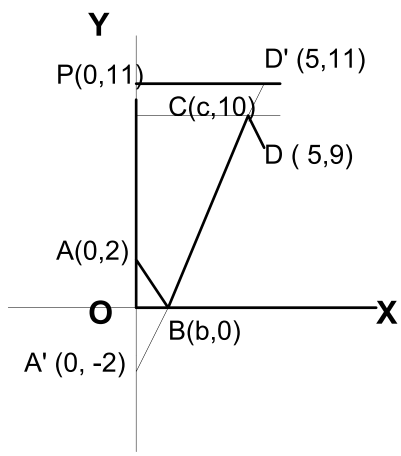
Grade Level_______________

T shirt size____________M/F _____

What is the numerical value of ***b*** for which the length of the path from A(0, 2) to B(***b***, 0) to C(c, 10) to D(5, 9) will be a **minimum**? Express your answer as a fraction in simplest form. [HINT, since the shortest distance between *two* points is a straight line segment, draw $\overline{A'BCD'}$, such that its length is equal to the path $\overline{AB}$ to $\overline{BC}$ to $\overline{CD}$.]

DUSO MATHEMATICS LEAGUE
MEET#5 - TIE-BREAKER SOLUTIONS – 2009

SOLUTION FOR TIE BREAKER # 1 **b = 10/13**



By reflections, the length of $\overline{A'BCD'}$ is equal to the length of the path $\overline{AB}$ to $\overline{BC}$ to $\overline{CD}$.

$$\Delta A'OB \sim \Delta A'PD'$$

So $\frac{2}{b} = \frac{13}{5}$ thus $b = \frac{10}{13}$