

Trig

Angle Addition/Subtraction

Lets try an experiment?

What is $\cos(30^\circ + 60^\circ)$? $\cos 90 = 0$

Is it equal to $\cos 30^\circ + \cos 60^\circ$? $\frac{\sqrt{3}}{2} + \frac{1}{2}$

This demonstrates that trigonometric functions are **not** distributive.

Imagine $\sqrt{(4 + 9)} \neq 2 + 3$

The Formulas

Based on Geometry and distance Formula

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

The Formulas

Based on Geometry and distance Formula

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Examples

Find the following

1. $\cos 75^\circ \rightarrow \cos(\overset{\text{A}}{45} + \overset{\text{B}}{30})$

$$\cos A \cos B - \sin A \sin B$$

$$\cos 45 \cos 30 - \sin 45 \sin 30$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Prove $\sin 90^\circ = 1$ by angle addition formulas.

$$\sin(45 + 45)$$

$$\sin A \cos B + \sin B \cos A$$

$$\sin 45 \cos 45 + \sin 45 \sin 45$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = 1$$

$$\cos 202^{\circ} \cos 22^{\circ} + \sin 202^{\circ} \sin 22^{\circ}$$

$$\cos A \cos B + \sin A \sin B$$

$$\cos(A - B)$$

$$\cos(202 - 22)$$

$$\cos(180)$$

$$-1$$

$$\frac{\tan \frac{\pi}{16} + \tan \frac{3\pi}{16}}{1 - \tan \frac{\pi}{16} \tan \frac{3\pi}{16}}$$

$$\tan(A + B)$$

Group Problems

Find the following

$$\sin 165^\circ$$

$$\cos 255^\circ$$

$$\tan 105^\circ$$