MONTGOMERY HIGH SCHOOL **2014-2015**
CP Pre-Calculus Final Exam Review

The exam will cover the following chapters and concepts:

Chapter 1
1.1 Functions
1.2 Analyzing Graphs of Functions and Relations
1.3 Continuity, End Behavior, and Limits
1.4 Extrema and Average Rates of Change
1.5 Parent Functions and Transformations
1.6 Function Operations and Composition of Functions
1.7 Inverse Relations and Functions

Chapter 2
2.1 Power and Radical Functions
2.2 Polynomial Functions
2.3 The Remainder and Factor Theorems
2.4 Zeros of Polynomial Functions
2.5 Rational Functions
2.6 Nonlinear Inequalities

Chapter 3
3.1 Exponential Functions
3.2 Logarithmic Functions
3.3 Properties of Logarithms
3.4 Exponential and Logarithmic Equations

Chapter 4
4.1 Right Triangle Trigonometry
4.2 Degrees and Radians
4.3 Trigonometry Functions on the Unit Circle
4.4 Graphing Sine and Cosine Functions
4.5 Graphing Other Trigonometric Functions
4.6 Inverse Trigonometric Functions
4.7 The Law of Sines and the Law of Cosines

Chapter 5
5.1 Trigonometric Identities
5.2 Verifying Trigonometric Identities
5.3 Solving Trigonometric Equations
5.4 Sum and Difference Identities
5.5 Multiple-Angle and Product-to-Sum Identities

Chapter 6
6.1 Multivariable Linear Systems and Row Operations
6.2 Matrix Multiplication, Inverses, and Determinants
6.3 Solving Linear Systems Using Inverses and Cramer's Rule

Chapter 7
7.1 Parabolas
7.2 Ellipses and Circles
7.3 Hyperbolas

Chapter 9
9.1 Polar Coordinates
9.2 Graphs of Polar Equations
9.3 Polar and Rectangular Forms of Equations

FORMULAS:

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1 - \cos u}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1 + \cos u}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

CHAPTER 1

Find the domain of the function.

1. $f(x)=\dfrac{3x}{x^2-4}$
2. $f(x)=\sqrt{25-x^2}$
3. $f(x)=\dfrac{\sqrt{x-2}}{x-4}$

Find the inverse of the function.

4. $f(x)=4x^3-3$
5. $f(x)=\sqrt{x+10}$
6. $f(x)=\dfrac{7x+3}{8}$

For each function; A) state the parent function and B) graph using transformations.

7. $f(x)=-2\sqrt{3-x}\ +1$
8. $f(x)=\left(-\dfrac{1}{2}x+1\right)^3-2$

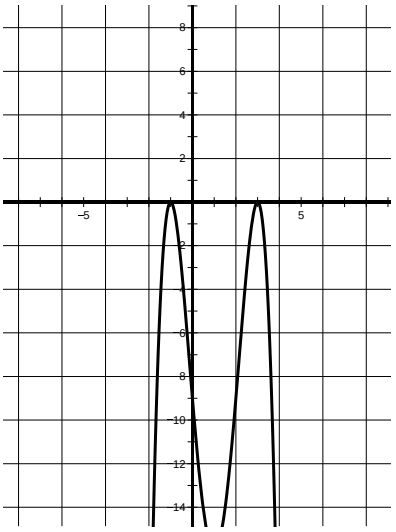
CHAPTER 2

Use the leading coefficient test to describe the end behavior of the following functions.

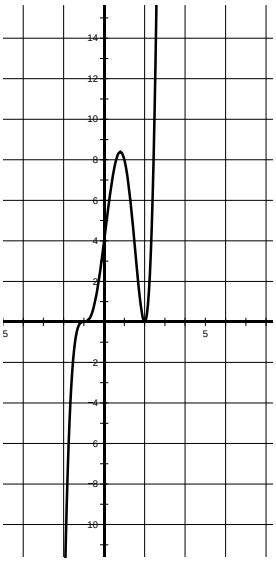
1. $f(x)=-3x^5-8x^4+2x^2+8$
2. $f(x)=x^6+2x^4-2x^3+x^2-6$

Write a function of least degree to describe the graph.

3.



4.



Identify all asymptotes (vertical, horizontal, or slant) and any holes for each rational function. Graph the function.

5. $f(x)=\dfrac{x^2+x-2}{4x^2-16}$
6. $f(x)=\dfrac{2x-10}{x^2-2x-15}$

CHAPTER 3

Evaluate.

1. $\log_3 9$
2. $\log_5 5$
3. $\log_9 3$
4. $\log_7 343$
5. $\ln 0$
6. $\log_{16} \frac{1}{2}$
7. $\log \frac{1}{100}$
8. $\log_2 1$
9. $\log_5 5^3$

State the parent function. Describe the transformations used to graph and then find the y-intercept, the asymptotes, the domain and range.

10. $f(x) = 3^{x+2} - 1$
11. $f(x) = e^{x-3} + 2$
12. $f(x) = 2 + \log_3(x-2)$
13. $f(x) = 1 - \ln(x+2)$

Use the properties of logarithms to expand each expression.

14. $\log_a \frac{y^4\sqrt{x}}{wz^4}$
15. $\ln\sqrt{x^2(x+2)}$
16. $\ln\left(\frac{x}{\sqrt{x^2+1}}\right)^3$

Use the properties of logarithms to condense each expression.

17. $2\log_b z - \log_b y$
18. $\ln x - 3\ln(x+1) - \ln y$
19. $\frac{1}{2}\left[\ln(x+1) + 2\ln(x-1)\right] + 3\ln x$
20. $2\left[\ln x - \ln(x+1) - \ln(x-1)\right]$

Solve the equation.

21. $8^a = 2^{-a}$
22. $625^{-2x} = 125^{2x-3}$
23. $4e^{2x-3} = 2$
24. $4^{x-2} = 5^{3x+2}$
25. $e^{2x} - 3e^x + 2 = 0$
26. $\log_{17}(n+6) = \log_{17}(-5n-6)$
27. $\log_3(x+9) + \log_3 7 = 1$
28. $\log_3(x^2+2) - \log_3 6 = 1$
29. $\ln(x-2) + \ln(2x-3) = \ln x$

CHAPTER 4

In which quadrant is the terminal side of each angle?

1. $\theta = -\frac{9\pi}{10}$

2. $455°$

Name the complement and supplement if possible.

3. $\theta = \frac{4\pi}{15}$

4. $57°$

5. A bicycle wheel with a radius of 13 inches makes 2.1 revolutions per second. What is the speed of the bicycle?

6. A point on the rim of a wheel has a linear speed of 14 cm/sec. If the radius of the wheel is 20 cm, what is the angular speed of the wheel in radians per second?

7. The needle of the scale in a bulk food section of a supermarket is 28 cm long. Find the distance the tip of the needle travels if it rotates 174°

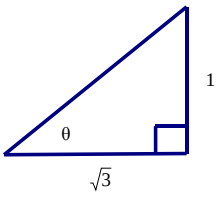
Evaluate.

8. $\cos\frac{9\pi}{4}$

9. $\tan\left(-\frac{4\pi}{3}\right)$

10. $\csc\left(-\frac{7\pi}{6}\right)$

11. Find the exact values of the sine and cosine for the angle θ.



If θ is an acute angle, find the indicted trigonometric function:

12. if $\sin\theta = \frac{15}{17}$, find $\sec\theta$

13. if $\csc\theta = \sqrt{26}$, find $\cot\theta$

14. $\sin t = -\frac{4}{19}$, find $\sin\left(\frac{\pi}{2} - t\right)$ for $\pi < t < \frac{3\pi}{2}$

15. Given $\tan\theta = -\frac{12}{35}$ and $\sin\theta > 0$, find the other five trigonometric functions.

Find the reference angle.

16. $\theta = 3.5$ 17. $\theta = \frac{5\pi}{3}$ 18. $\theta = -159°$

Sketch a graph of the following functions through 1 full period. Use $6 \text{ units} = \pi$ **as the scale for the x-axis.**

19. $y = -4\sin\left(x + \frac{\pi}{2}\right)$ 20. $\frac{1}{2}\cos(2x - \pi) - 1$

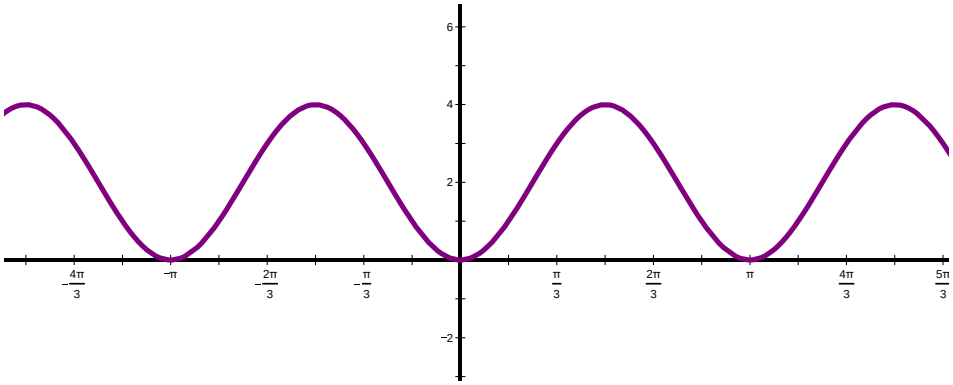
Sketch a graph of the following functions through 1 full period. Use $6 \text{ units} = \pi$ **as the scale for the x-axis. Determine the equation for the asymptotes. State the range and give the maximum point and minimum point.**

21. $y = \sec(3x + \pi) - 1$ 22. $y = -\csc\left(\frac{x}{2} + \frac{\pi}{4}\right)$

Sketch a graph of the following functions through 1 full period. Use $6 \text{ units} = \pi$ **as the scale for the x-axis. Determine the equation for the asymptotes and state the three key points.**

23. $y = 1 + \cot\left(\frac{3x}{4}\right)$ 24. $y = 3\tan\left(\frac{x}{3}\right)$

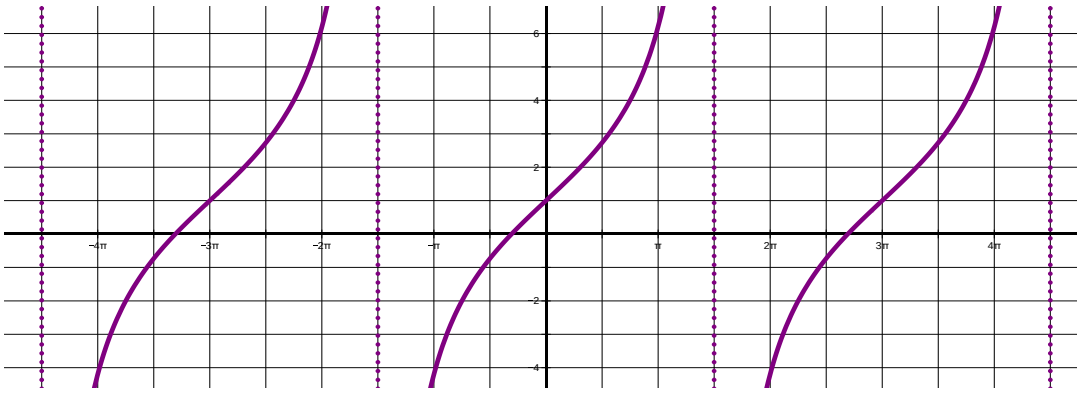
Use the figure to the right for problems 25 & 26.



25. Write a function in the form of $y = a\sin(bx - c) + d$ for the graph above.

26. Write a function in the form of $y = a\cos(bx - c) + d$ for the graph above.

Use the figure to the right for problems 27 & 28.



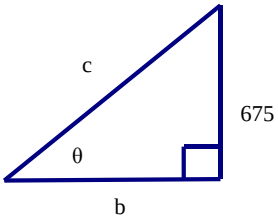
27. Write a function in the form of $y = a\tan(bx - c) + d$ for the graph above.

28. Write a function in the form of $y = a\cot(bx - c) + d$ for the graph above.

Draw a diagram and solve.

29. A 12-foot ladder makes an angle of $50°$ with the ground as it leans against a house. How far up the house does the ladder reach?

30. The cable supporting as ski lift rises 3 feet for each 8 feet of horizontal length. The top of the cable is fastened 675 feet above the cable's lowest point. Find the lengths b and c and the angle of elevation.



31. An airplane is flying east at a constant altitude of 28,000 meters. The pilot spots a ship at an angle of depression of $18.5°$. After 73 seconds the angle of depression is $38.4°$. Find the speed of the plane.

32. A ship leaves port at 20 miles per hour with a heading of $S44°W$. There is a warning buoy 5 miles directly north of port. What is the bearing of the warning buoy as seen from the ship after 5.5 hours.

33. At a distance of 56 feet from the base of a flagpole, the angle of elevation to the top of the flag that is 3.1 feet tall is $25.6°$. The angle of elevation to the bottom of the flag is $22.9°$. The pole extends 1 foot above the flag. Find the height of the pole.

34. An energy company uses one wellhead to drill several exploratory wells as different angles. They strike oil when they have drilled 2879 feet along an angle of depression of $44°$. Find the depth of the oil deposit.

35. A hiker travels 3.9 miles per hour at a heading of $S21°E$ from a ranger's station. After 3.5 hours, how far south and how far east is the hiker from the station?

Evaluate.

36. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

37. arctan(1)

38. sin(arcsin 0.7)

39. tan(arctan35)

40. $\arccos\left(\cos\frac{7\pi}{2}\right)$

41. $\arcsin\left(\sin\frac{3\pi}{4}\right)$

42. $\tan\left(\arccos\frac{24}{25}\right)$

43. $\csc\left(\arctan\frac{4}{x}\right)$

44. $\sec\left(\arctan\frac{2x}{\sqrt{1+4x^2}}\right)$

Graph the following functions.

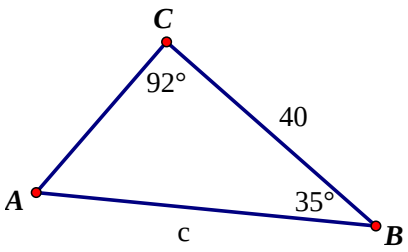
45. $y=\arctan(x-2)$

46. $y=\arccos(x+2)+\frac{\pi}{4}$

47. $y=\arcsin\frac{x}{2}$

CHAPTER 4 – PART 2

1. Find *c:*



2. Find c if A=31°, a = 11, and b =13.

3. Solve the triangle: *B*=32°, *C*= 25°, and *a* = 18.

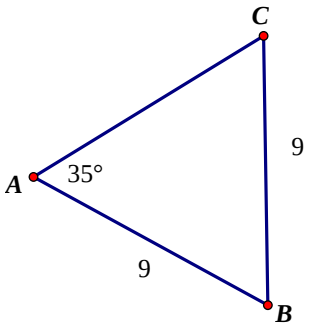
4. Find the area of the triangles: a) A = 39°, *a* = 13.3, and *b* = 13.3 b) *B* = 65°11', *a* = 5 and *c* =2

5. A pole 85 feet tall is standing at the bottom of a hill side that slopes up at an angle of elevation of 52°. A guy wire has an angle of elevation of 24° from the top of the pole to the hillside. Find the distance from the base of the pole to the guy wire's point of attachment on the hill.

6. A loading dock ramp that is 18 feet long rises at an angle of 15° from the horizon. Due to new design specifications, a longer ramp is to be used so that the angle is reduced to 8°. How much farther out from the dock will that put the foot of the ramp?

7. Two Coast Guard stations located 75 miles apart on a north-south line each receive a radio signal from a ship at sea. From the northernmost station, the ship's bearing is S 65° E. From the other station, the ship's bearing is N 20° E. How far is the ship from the northernmost station?

8. Find the third side of the triangle:



9. Use the law of cosines to solve triangle ABC given: a=11, b = 16, c =15

10. Use the law of cosines to solve triangle ABC given: A = 42°, b =3, and c = 9

11. Find the area of the triangle: a) equilateral triangle with perimeter of 39 b) a = 23.5, b = 23.5, and c = 26.4

CHAPTER 5

Verify the following equations using trigonometric identities:

1. $\cos(\pi-\theta)+\sin(\pi+\theta)=0$

2. $(\sin x+\cos x)^2=1+\sin 2x$

3. $\dfrac{\sin^2 x}{1-\cos x}=\dfrac{1+\sec x}{\sec x}$

4. $\dfrac{1+\cos x}{\sin x}+\dfrac{\sin x}{1+\cos x}=2\csc x$

5. $\sin(3\theta)=3\sin\theta-4\sin^3\theta$

Simplify the following using identities:

6. $\cos^2 x\sin^2 x-\cos^2 x$

7. $\sin^2 x+\sin^2 x\cot^2 x$

Evaluate the following:

8. Find the remaining trig functions given $\csc x=\sqrt{17}$ and $\tan x=-\dfrac{1}{4}$

9. Find the exact value of $\cos 345^\circ$.

10. Find the exact value of sin2x given $\sin x=-\dfrac{1}{11}$ and $\pi<x<\dfrac{3\pi}{2}$

11. Find the exact values of sin2x, cos2x, and tan2x given $\sin x=-\dfrac{5}{13}$ and $\dfrac{3\pi}{2}<x<2\pi$

12. Find the exact value of $\tan\dfrac{x}{2}$ given $\sin x=\dfrac{27}{45}$ and θ is in quadrant 1.

13. Find the exact value of $\sin\dfrac{x}{2}$ given $\tan x=\dfrac{48}{55}$ and θ is in quadrant 3.

14. Find the exact value of sine, cosine, and tangent of the angle $\frac{\pi}{12}$.

15. Find $\cos(A+B)$ given $\sin A=\frac{3}{7};\ \cos B=-\frac{5}{8};\ \frac{\pi}{2}<A\le\pi$ and $\pi<B\le\frac{3\pi}{2}$

16. Find $\tan(u-v)$ given that $\sin u=\frac{3}{5}$ and $\cos v=-\frac{5}{13}$ and both *u* and *v* are in quadrant 2.

Find <u>ALL SOLUTIONS</u> for the following trig equations:

17. $9\tan x+8\sqrt{3}=17\tan x$

18. $10\cos x-5\sqrt{2}=0$

19. $\sin 2x+\frac{\sqrt{2}}{2}=0$

Find solutions for each equation in the interval $[0,2\pi)$:

20. $3\cot^2 x-9=0$

21. $\tan^2 x-\sec x=-1$

22. $3\sec^2\frac{x}{2}-4\sec\frac{x}{2}-4=0$

23. $4\cos 3x-2\sqrt{3}=0$

<u>CHAPTER 6</u>

Perform the indicated operations if possible:

1. $\begin{bmatrix}4&6&-8\\-2&5&0\end{bmatrix}-\begin{bmatrix}18&3&12\\1&0&5\end{bmatrix}$

2. $2\begin{bmatrix}5&-2\\-3&4\\0&6\end{bmatrix}+\begin{bmatrix}1&7\\0&-4\\-5&5\end{bmatrix}$

3. $\begin{bmatrix}1&2\\3&-1\\0&4\end{bmatrix}\begin{bmatrix}1&0&0&-1\\-1&1&1&0\end{bmatrix}$

Use Matrices and Gaussian Elimination to Solve

4. $\begin{cases}-x+8y=10\\2x-y=-5\end{cases}$

5. $\begin{cases}x+y-2z=7\\-x+4y+3z=2\\2x-3y+2z=-2\end{cases}$

6. $\begin{cases}8x+3y=40\\16x-6y=41\end{cases}$

7. $\begin{cases}\frac{1}{3}x+\frac{1}{3}z=5\\2y-4z=18\\2x-y+4z=1\end{cases}$

Find the determinant

8. $\begin{vmatrix} 3 & 5 \\ 1 & 4 \end{vmatrix}$

9. $\begin{vmatrix} 2 & -5 & 3 \\ 0 & 8 & 1 \\ -5 & 4 & 0 \end{vmatrix}$

10. $\begin{vmatrix} -9 & 3 & 0 \\ 6 & -6 & 0 \\ -3 & -24 & -3 \end{vmatrix}$

11. $\begin{vmatrix} 12 & 18 & 6 \\ 0 & 30 & -12 \\ 0 & 0 & -12 \end{vmatrix}$

Solve the system of equations using Cramer's Rule

12. $\begin{cases} x-3y-2z=9 \\ 3x+2y+6z=20 \\ 4x-y+3z=25 \end{cases}$

13. $\begin{cases} -7x-y=8 \\ x+y=0 \end{cases}$

CHAPTER 7

For each conic re-write into standard form, sketch the graph and then provide the important information.

Circle: center and radius
Parabola: vertex, focus, directrix, axis of symmetry
Ellipse: center, vertices, co-vertices, foci, and eccentricity
Hyperbola: center, vertices, foci, and equations of asymptotes

1. $-y^2+x+12y-28=0$

2. $4x^2+4y^2+24x-12y-19=0$

3. $9x^2+16y^2-36x-80y-8=0$

4. $x^2-40=-y^2$

5. $-25x^2+16y^2-400=0$

6. $25x^2+9y^2-150x+36y+36=0$

7. $3x^2-18x+y+32=0$

8. $x^2-16y^2-2x-128y-271=0$

Use the information provided to write the standard form equation of each circle.

9. The endpoints of the diameter are (13, 5) and (-3,-5).

10. The center is at (9, 5) and passes through the point (16,-2).

11. The center lies on the y-axis and is tangent to the x-axis and the line $y=10$.

Use the information provided to write the standard form equation of each parabola.

12. The vertex is (-7,-3) and the focus is $\left(-7,-\frac{23}{8}\right)$

13. The focus is at $\left(-\frac{13}{4},0\right)$ and the directrix is $x=-\frac{19}{4}$

Use the information provided to write the standard form equation of each ellipse.

14. Vertices: $(7, 9), (7, -11)$
Foci: $(7, -1 + 4\sqrt{6}), (7, -1 - 4\sqrt{6})$

15. Foci: $(-5 + \sqrt{91}, 7), (-5 - \sqrt{91}, 7)$
Endpoints of minor axis: $(-5, 10), (-5, 4)$

Use the information provided to write the standard form equation of each hyperbola.

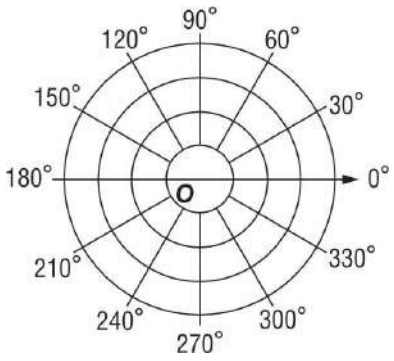
16. Vertices: $(-2, -4 + \sqrt{110}), (-2, -4 - \sqrt{110})$
Foci: $(-2, -4 + \sqrt{190}), (-2, -4 - \sqrt{190})$

17. Foci: $(3 + \sqrt{89}, 6), (3 - \sqrt{89}, 6)$
Asymptotes: $y = \frac{8}{5}x + \frac{6}{5}$
$y = -\frac{8}{5}x + \frac{54}{5}$

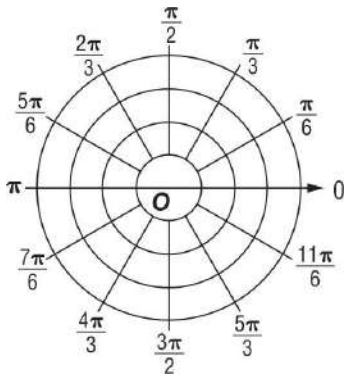
CHAPTER 9

Graph each point on a polar grid.

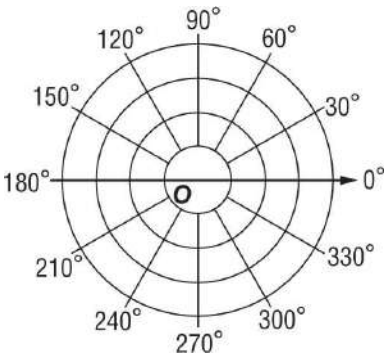
1. (2.5, 0°)



2. $\left(-2, \frac{\pi}{4}\right)$

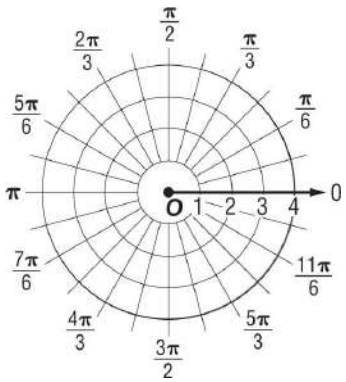


3. (–1, –30°)

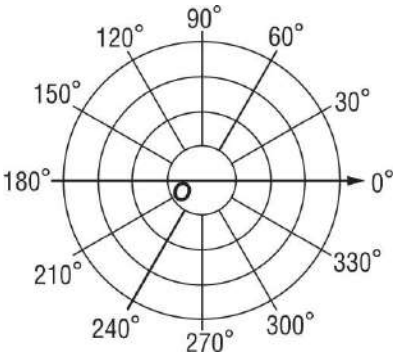


Graph each polar equation.

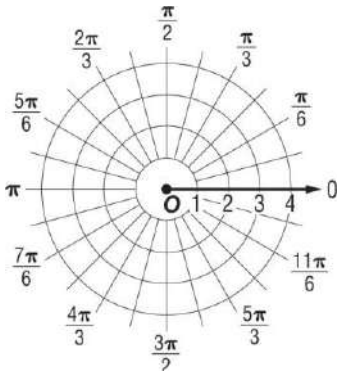
4. $r = 3$



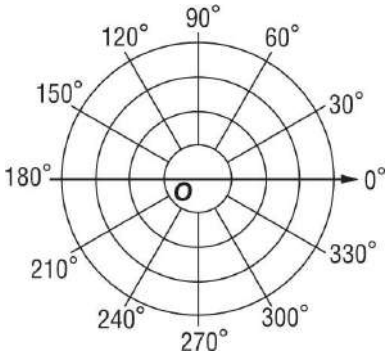
5. $\theta = 60°$



6. $r = 4$



7. LANDSCAPING A landscape architect has created a blueprint for the landscape design at a new building being constructed at a retirement community.



a. The architect has placed a gazebo at (3, –135°). Graph this point.

b. The design calls for a bench at (–4, 85°) and a pond at (1, 105°). Find the distance in feet between the pond and the bench.

8. GOLFING A golf ball is hit and lands in tall grass. From one position, the golfer surveys the grassy area defined by $-\frac{7\pi}{12} \geq \theta \geq -\frac{17\pi}{12}$ and $6 \geq r \geq 0$, where r is measured in feet. Find the area of the region.

Find the rectangular coordinates for each point with the given polar coordinates.

1. (6, 120°)

2. (–4, 45°)

3. $\left(4, \frac{\pi}{6}\right)$

Find two pairs of polar coordinates for each point with the given rectangular coordinates if $0 \leq \theta < 2\pi$.

4. (2, 2)

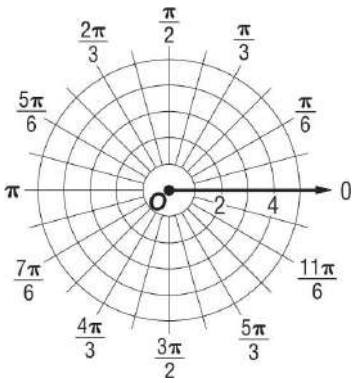
5. (2, –3)

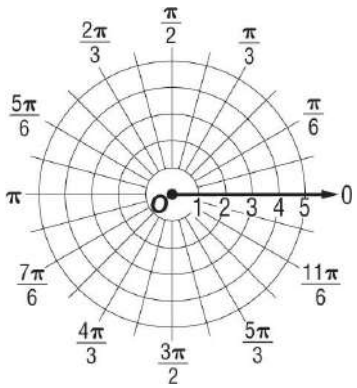
6. $(-3, \sqrt{3})$

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

7. $x^2 + y^2 = 9$

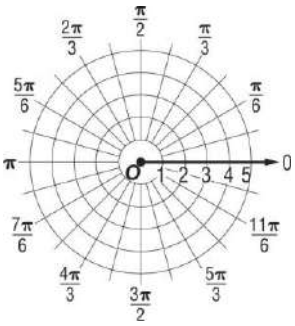
8. $y = 3$



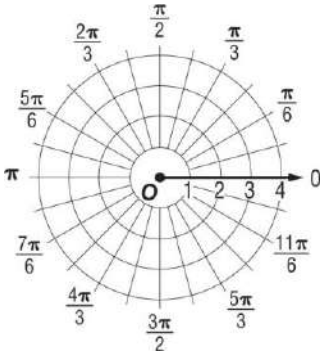


Graph each equation by plotting points.

1. $r = 2 \sin \theta$

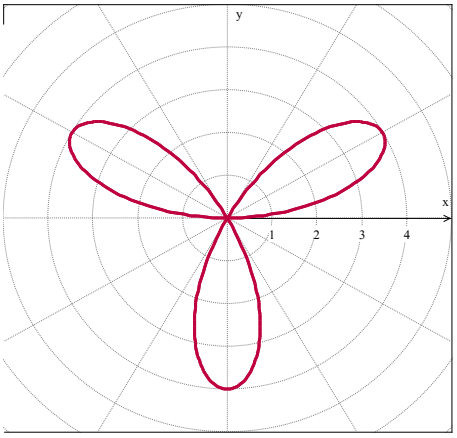


2. $r = 4 \cos \theta$

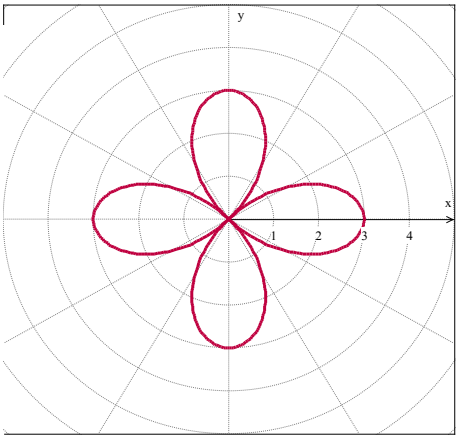


Write an equation for each graph.

3.



4.



5.