Warm Up

Use the Distance Formula to find the distance, to the nearest tenth, between each pair of points.

1. $A(6, 2)$ and $D(-3, -2)$ 9.8

2. $C(4, 5)$ and $D(0, 2)$ 5

3. $V(8, 1)$ and $W(3, 6)$ 7.1

4. Fill in the table of values for the equation $y = x - 14$.

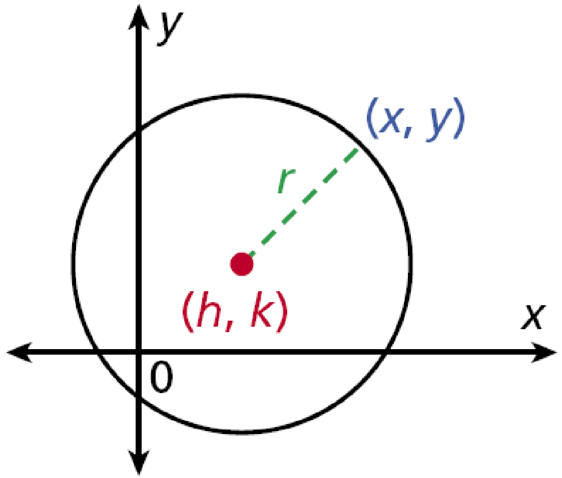
x	−1	0	1	2
y	−15	−14	−13	−12

Objectives

Write equations and graph circles in the coordinate plane.

Use the equation and graph of a circle to solve problems.

The equation of a circle is based on the Distance Formula and the fact that all points on a circle are equidistant from the center.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ *Distance Formula*

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$ *Substitute the given values.*

$$r^2 = (x - h)^2 + (y - k)^2$$ *Square both sides.*

Theorem 11-7-1 **Equation of a Circle**

The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

Example 1A: Writing the Equation of a Circle

Write the equation of each circle.

⊙*J* with center *J* (2, 2) and radius 4

$(x - h)^2 + (y - k)^2 = r^2$ *Equation of a circle*

$(x - 2)^2 + (y - 2)^2 = 4^2$ *Substitute 2 for h, 2 for k, and 4 for r.*

$(x - 2)^2 + (y - 2)^2 = 16$ *Simplify.*

Example 1B: Writing the Equation of a Circle

Write the equation of each circle.

⊙*K* that passes through *J*(6, 4) and has center *K*(1, –8)

$$r = \sqrt{(6 - 1)^2 + (4 - (-8))^2}$$ *Distance formula.*

$$= \sqrt{169} = 13$$ *Simplify.*

$$(x - 1)^2 + (y - (-8))^2 = 13^2$$ *Substitute 1 for h, –8 for k, and 13 for r.*

$$(x - 1)^2 + (y + 8)^2 = 169$$ *Simplify.*

If you are given the equation of a circle, you can graph the circle by making a table or by identifying its center and radius.

Example 2A: Graphing a Circle

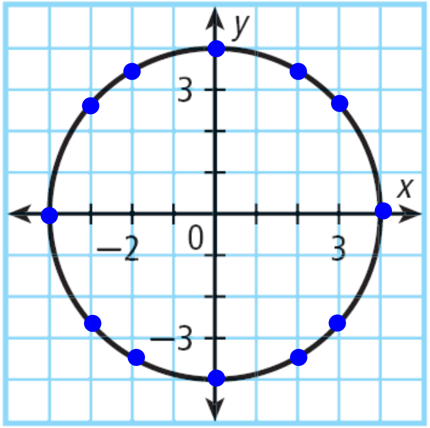
Graph $x^2 + y^2 = 16$.

Step 1 Make a table of values.

Since the radius is $\sqrt{16}$, or 4, use ±4 and use the values between for *x*-values.

x	−4	−3	−2	0	2	3	4
y	0	±2.6	±3.5	±4	±3.5	±2.6	0

Step 2 Plot the points and connect them to form a circle.

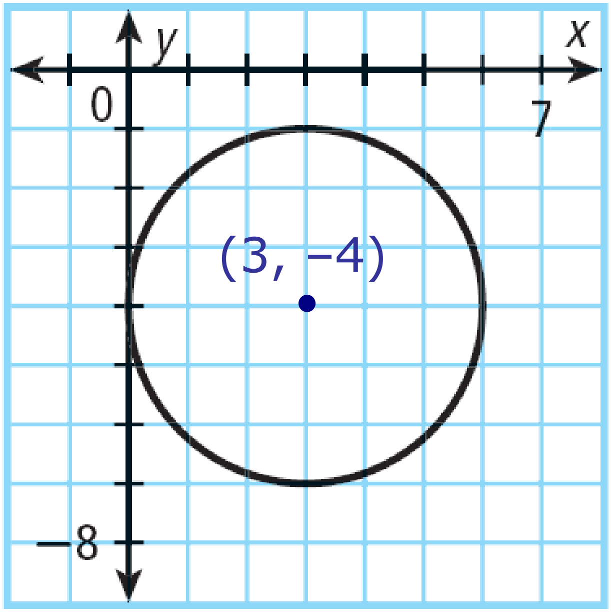


Example 2B: Graphing a Circle

Graph $(x - 3)^2 + (y + 4)^2 = 9$.

The equation of the given circle can be written as $(x - 3)^2 + (y - (-4))^2 = 3^2$.

So $h = 3$, $k = -4$, and $r = 3$.



The center is (3, –4) and the radius is 3. Plot the point (3, –4). Then graph a circle having this center and radius 3.

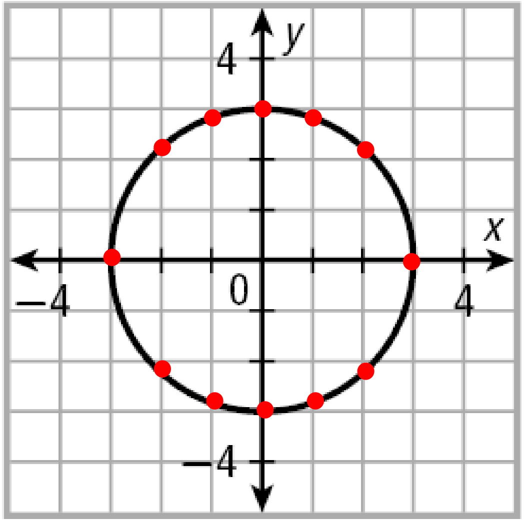
Check It Out! Example 2a

Graph $x^2 + y^2 = 9$.

Since the radius is $\sqrt{9}$, or 3, use ±3 and use the values between for *x*-values.

x	3	2	1	0	–1	–2	–3
y	0	±2.2	± 2.8	± 3	± 2.8	± 2.2	0

Step 2 Plot the points and connect them to form a circle.



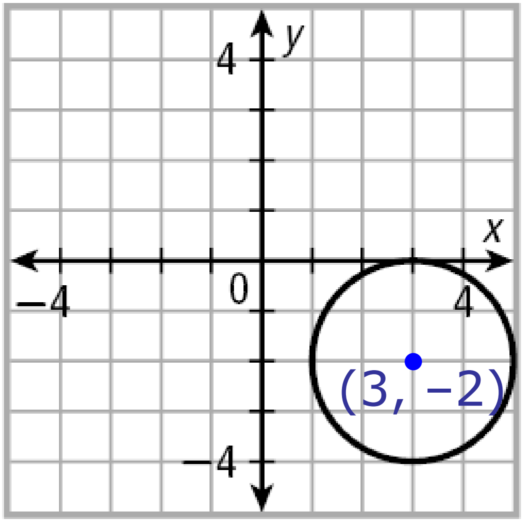
Check It Out! Example 2b

Graph $(x - 3)^2 + (y + 2)^2 = 4$.

The equation of the given circle can be written as $(x - 3)^2 + (y - (-2))^2 = 2^2$.

So $h = 3$, $k = -2$, and $r = 2$.

The center is (3, –2) and the radius is 2. Plot the point (3, –2). Then graph a circle having this center and radius 2.

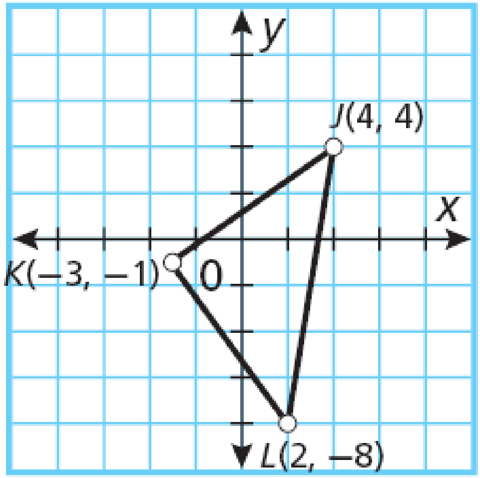


Example 3: Radio Application

An amateur radio operator wants to build a radio antenna near his home without using his house as a bracing point. He uses three poles to brace the antenna. The poles are to be inserted in the ground at three points equidistant from the antenna located at *J*(4, 4), *K*(–3, –1), and *L*(2, –8). What are the coordinates of the base of the antenna?

Step 1 Plot the three given points.

Step 2 Connect *J*, *K*, and *L* to form a triangle.



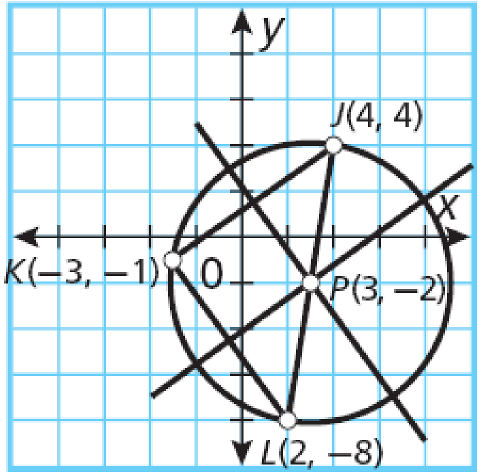
Example 3 Continued

Step 3 Find a point that is equidistant from the three points by constructing the perpendicular bisectors of two of the sides of ΔJKL.

The perpendicular bisectors of the sides of ΔJKL intersect at a point that is equidistant from *J*, *K*, and *L*.

The intersection of the perpendicular bisectors is *P* (3, –2). *P* is the center of the circle that passes through *J*, *K*, and *L*.

The base of the antenna is at *P* (3, –2).

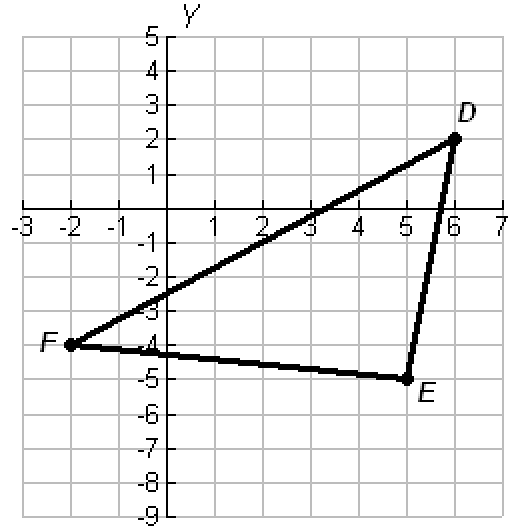


Check It Out! Example 3

What if…? Suppose the coordinates of the three cities in Example 3 (p. 801) are *D*(6, 2) , *E*(5, –5), and *F*(-2, -4). What would be the location of the weather station?

Step 1 Plot the three given points.

Step 2 Connect *D*, *E*, and *F* to form a triangle.

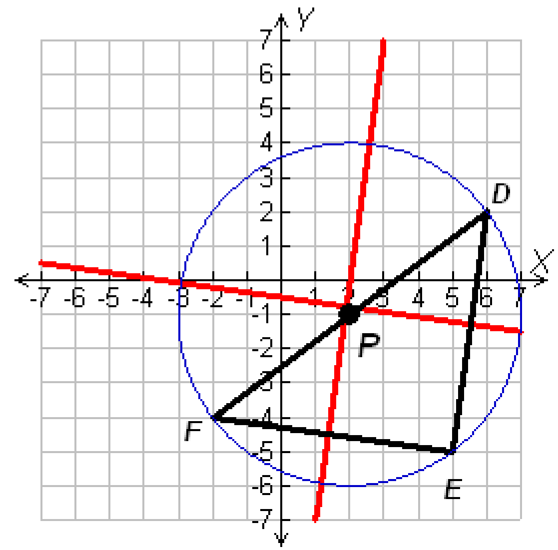


Check It Out! Example 3 Continued

Step 3 Find a point that is equidistant from the three points by constructing the perpendicular bisectors of two of the sides of Δ*DEF.*

The perpendicular bisectors of the sides of Δ*DEF* intersect at a point that is equidistant from *D*, *E*, and *F*.

The intersection of the perpendicular bisectors is *P*(2, –1). *P* is the center of the circle that passes through *D*, *E*, and *F*.



The base of the antenna is at *P*(2, –1).

Lesson Quiz: Part I

Write the equation of each circle.

1. ⊙L with center L (–5, –6) and radius 9

$(x + 5)^2 + (y + 6)^2 = 81$

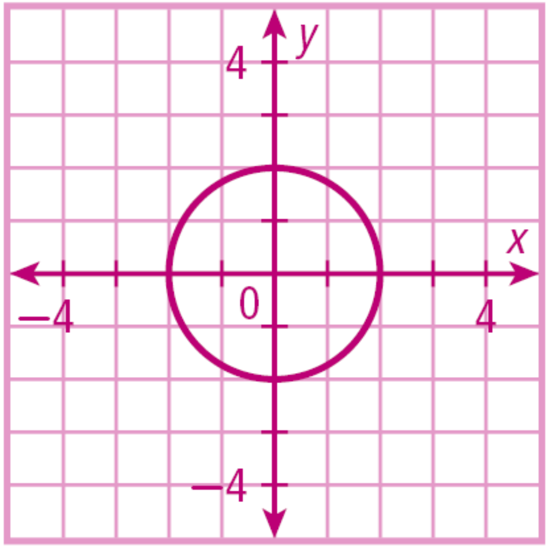
2. ⊙D that passes through (–2, –1) and has center D(2, –4)

$(x - 2)^2 + (y + 4)^2 = 25$

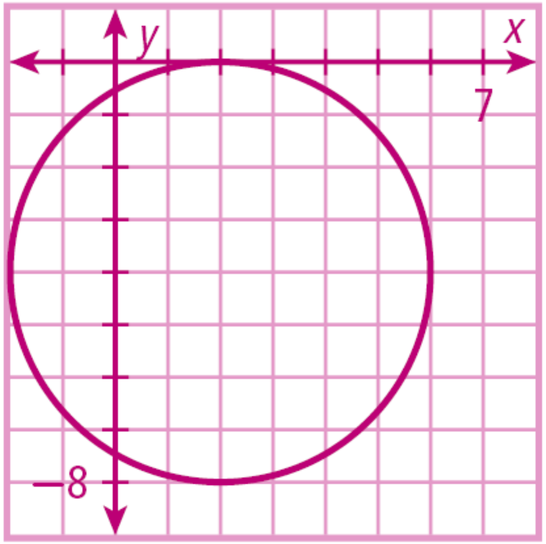
Lesson Quiz: Part II

Graph each equation.

3. $x^2 + y^2 = 4$



4. $(x - 2)^2 + (y + 4)^2 = 16$



Lesson Quiz: Part III

5. A carpenter is planning to build a circular gazebo that requires the center of the structure to be equidistant from three support columns located at $E(-2, -4)$, $F(-2, 6)$, and $G(10, 2)$.

What are the coordinates for the location of the center of the gazebo?

$(3, 1)$