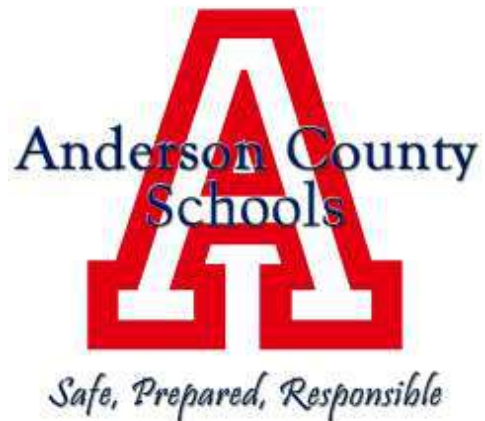# First Grade - Mathematics

Kentucky Core Academic Standards with Targets

Student Friendly Targets

Pacing Guide

College and Career Readiness Anchor Standards for Math

The K-5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to eight mathematical practices: 1) Make sense of problems and persevere in solving them, 2) Reason abstractly and quantitatively, 3) Construct viable arguments and critique the reasoning of others, 4) Model with mathematics, 5) Use appropriate tools strategically, 6) Attend to precision, 7) Look for and make use of structure, and 8) Look for express regularity in repeated reasoning.
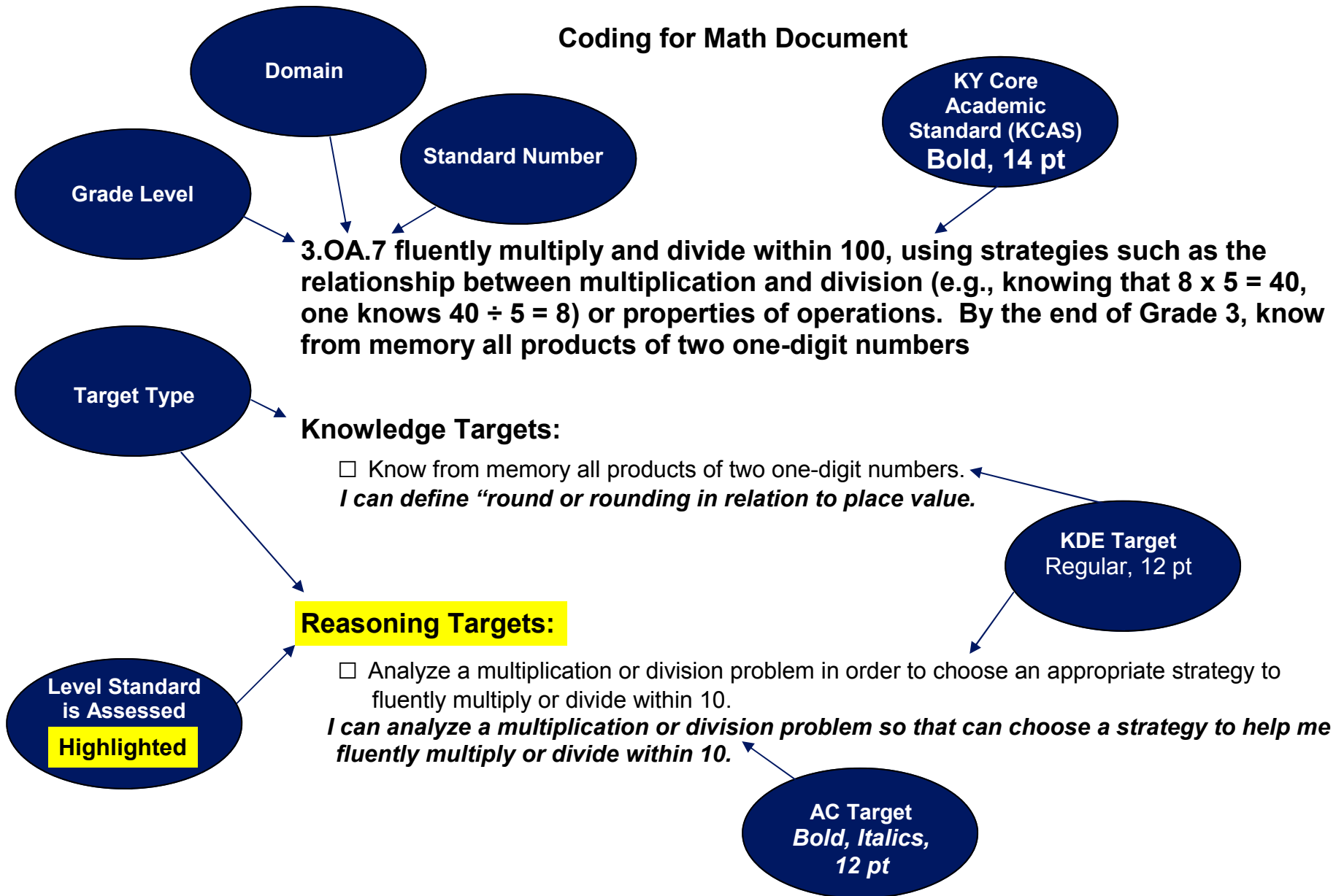
Mathematics is divided into five domains: 1) Counting and Cardinality (CC), 2) Operations and Algebraic Thinking (OA), 3) Number and Operations in Base Ten (NBT), 4) Measurement and Data (MD), and 5) Geometry (G).

Development of Pacing Guide

During the summer 2011, Anderson County teachers and administrators developed learning targets for each of the Kentucky Core Content Standards. In winter 2012, curriculum resource teachers verified the congruency of the standards and targets and recommended revisions. Teachers refined the work and began planning the development of common assessments to ensure students learn the intended curriculum. Anderson County Schools would like to thank each of our outstanding teachers and administrators who contributed to this important math curriculum project. Special thanks to Robin Arnzen, Stephanie Barnes, Traci Beasley, Julie Bowen, Tony Calvert, Linda Dadisman, Amanda Ellis, Leslie Fields, Amy Gritton, Lauren Hamel, Linda Hill, Sharon Jackman, Lesley Johnson, Steve Karsner, Chris Kidwell, Joel Maude, Melissa Montgomery, Matt Ogden, Kim Penn, Wayne Reese, Monica Rice, Chrystal Rowland, Kim Ruble, Jennifer Sallee, Amy Satterly, Krista Sawyer, Francine Sloan, Jeanna Slusher, Shayla Smith, T.J. Spivey, Rebecca Stevens, Emily Thacker, Lori Wells, Shannon Wells, Tim Wells, and Jamie White. Thanks also to Tony Calvert (EBW), Brian Edwards (ACHS), and Alex Hunter (ACMS) for providing comments to the work.

North Carolina State Board of Education created a most helpful document entitled "Common Core Instructional Support Tools - Unpacking Standards". The document answers the question "What do the standards mean that a student must know and be able to do?" The "unpacking" is included in our "What Does This Standard Mean?" section. The complete North Carolina document can be found at http://www.dpi.state.nc.us/docs/acre/standards/common-core-tools/unpacking/math/1st.pdf

Coding for Math Document

Domain

Standard Number

Grade Level

KY Core Academic Standard (KCAS) Bold, 14 pt

3.OA.7 fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers

Target Type

Knowledge Targets:

☐ Know from memory all products of two one-digit numbers.
I can define "round or rounding in relation to place value.

**KDE Target
Regular, 12 pt**

Reasoning Targets:

**Level Standard is Assessed
Highlighted**

☐ Analyze a multiplication or division problem in order to choose an appropriate strategy to fluently multiply or divide within 10.
I can analyze a multiplication or division problem so that can choose a strategy to help me fluently multiply or divide within 10.

**AC Target
Bold, Italics,
12 pt**

Anderson County Elementary
Pacing Guide

Math
Grade 1

Numbers to 10
Adding and Subtracting to/from 10
Graphing

Standard	What Does This Standard Mean?	Dates Taught
1.NBT.1 Count to 120 (10), starting at any number less than 120 (10). In this range, read and write numerals and represent a number of objects with a written numeral. **Knowledge Targets:** ☐ Write numerals up to 120. *I can write numbers to 10, starting at any number less than 10.* **Reasoning Targets:** ☐ Represent a number of objects up to 120 with a written numeral. *I can show numbers 0-10 using objects* **Performance Skills Targets:** ☐ Count (saying the number sequence) to 120, starting at any number less than 120. *I can count numbers to 10, starting at any number less than 10. (Underpinning)*	First Grade students rote count forward to 120 by counting on from any number less than 120. First graders develop accurate counting strategies that build on the understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after). In addition, first grade students read and write numerals to represent a given amount. As first graders learn to understand that the position of each digit in a number impacts the quantity of the number, they become more aware of the order of the digits when they write numbers. For example, a student may write "17" and mean "71". Through teacher demonstration, opportunities to "find mistakes", and questioning by the teacher ("I am reading this and it says seventeen. Did you mean seventeen or seventy-one? How can you change the number so that it reads seventy-one?"), students become precise as they write numbers to 120.	First Nine Weeks

☐ **Read the numerals up to 120.**
I can read numbers to 10, starting at any number less than 10.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Knowledge Targets:
☐ Use symbol for an unknown number in an addition or subtraction problem within 20.
I can solve addition and subtraction problems with missing parts/addends. (within 10.) (Underpinning)

<mark>**Reasoning Targets:**</mark>
☐ Solve word problems using addition and subtraction within 20.
 I can solve addition and subtraction word problems (within 10).
☐ Interpret situations to solve word problems with unknowns in all positions within 20 using addition and subtraction.
I can solve addition and subtraction word problems with missing parts/addends.
☐ Determine appropriate representations for solving word problems involving different situations using addition and subtraction.
I can decide whether to add or subtract in a word problem.

First grade students extend their experiences in Kindergarten by working with numbers to 20 to solve a new type of problem situation: Compare (See **Table 1** at end of document for examples of all problem types). In a Compare situation, two amounts are compared to find "How many more" or "How many less".

Problem Type: Compare		
Difference Unknown	Bigger Unknown	Smaller Unknown
"How many more?" version. Lucy has 7 apples. Julie has 9 apples. How many more apples does Julie have than Lucy?	"More" version suggests operation. Julie has 2 more apples than Lucy. Lucy has 7 apples. How many apples does Julie have?	"Fewer" version suggests operation. Lucy has 2 fewer apples than Julie. Julie has 9 apples. How many apples does Lucy have?
"How many fewer?" version Lucy as 7 applies. Julie has 9 apples. How many fewer apples does Lucy have than Julie? $7 + \square = 9$ $9 - 7 = \square$	"Fewer" version suggest wrong operation Lucy has 2 fewer apples than Julie. Lucy has apples. How many apples does Julie have? $7 + 2 = \diamond$	"More" version suggests wrong operation * Julie has 2 more apples than Lucy. Julie has 9 apples. How many apples does Lucy have? $9 - 2 = \square$ $\square + 2 = 9$ *Mastery at 2nd gr

Compare problems are more complex than those introduced in Kindergarten. In order to solve compare problem types, 1st graders must think about a quantity that is not physically

present and must conceptualize that amount. In addition, the language of "how many more" often become lost or not heard with the language of 'who has more'. With rich experiences that encourage students to match problems with objects and drawings can help students master these challenges.

NOTE: Although 1st grade students should have experiences solving and discussing all 12 problem types located in Table 1, they are not expected to master all types by the end of 1st grade due to the high language and conceptual demands of some of the problem types. Please see Table 1 at the end of this document for problem types that 1st grade students are expected to master by the end of 1st grade.

First graders also extend the sophistication of the methods they used in Kindergarten (counting) to add and subtract within this larger range. Now, first graders use the methods of counting on, making ten, and doubles +/-1 or +/- 2 to solve problems.

Example: Nine bunnies were sitting on the grass. Some more bunnies hopped there. Now, there are 13 bunnies on the grass. How many bunnies hopped over there?	
Counting On Method	Student: Niinnneeee….holding a finger for each next number counted 10, 11, 12, 13. Holding up her four fingers, 4! 4 bunnies hopped over there."
Example: Eight red applies and 6 green apples are on the tree. How many apples are on the tree?	
Making Tens Method	Student: I broke up 6 into 2 and 4. Then, I took the 2 and added it to the 8. That's 10. Then I add the 4 to the 10. That's 14. So there are 14 apples on the tree.
Example: Thirteen apples are on the table. Six of them are red and the rest are green. How many apples are green?	
Doubles +/- 1 or 2	Student: I know that 6 and 6 is 12. So 6 + 7 is 13. There are 7 green apples.

In order for students to read and use equations to represent their thinking, they need extensive experiences with addition and subtraction situations in order to connect the experiences with symbols (+, -, =) and equations (5 = 3 + 2). In Kindergarten, students demonstrated the understanding of how objects can be joined (addition) and separated (subtraction) by representing addition and subtraction situations using objects, pictures and words. In 1st grade, student ex tend this understanding of addition and subtraction situations to use the addition symbol (+) to represent joining situation, the subtraction symbol (-) to represent separating situation, and the equal sign (=) to represent a relationship regarding quantity between one side of the equation and the other.

1.OA.3 Apply properties of operations as strategies to add and subtract.3 Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

Knowledge Targets:
- ☐ Explain how properties of operation strategies work.
- *I can explain how properties of operation strategies work.*
- *I can use "turn around facts" to solve addition problems. (Underpinning)*

- ☐ Apply strategies using properties of operations to solve addition and subtraction problems.
- *I can use properties of operations strategies to solve addition and subtraction problems.*
- *I can use "turn around" facts to solve addition problems.*
- *I can make 10 when adding 3 numbers.*

Elementary students often believe that there are hundreds of isolated addition and subtraction facts to be mastered. However, when students understand the commutative and associative properties, they are able to use relationships between and among numbers to solve problems. First Grade students apply properties of operations as strategies to add and subtract. Students do not use the formal terms "commutative" and "associative". Rather, they use the understandings of the commutative and associative property to solve problems.

Commutative Property of Addition	Associate Property of Addition
The order of the addends does not change the sum. For example, if 8 + 2 = 10 is known, then 2 + 8 = 10 is also known.	The grouping of the 3 or more addends does not affect the sum. For example, when adding 2 + 6 + 4, the sum from adding the first two numbers first (2 + 6) and then the third number (4) is the same as if the second and third numbers are added first) 6 + 4 equals 10 and add those two numbers first before adding 2. Regardless of the

order, the sum remains 12.

Students use mathematical tools and representations (e.g., cubes, counters, number balance, number line, 100 chart) to model these ideas).

Commutative Property Examples:

Cubes
A student uses 2 colors of cues to make as many different combinations of 8 as possible. When recording the combinations, the student records that 3 green cubes and 5 blue cubes equals 8 cubes in all. In addition, the student notices that 5 green cubes and 3 blue cubes also equals 8 cubes.
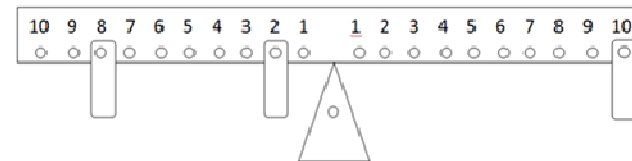
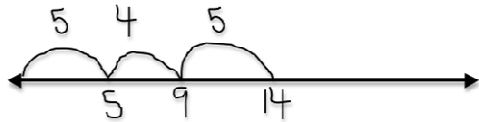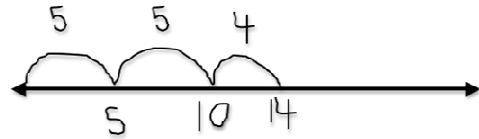$$3 + 5 = 8$$

$$5 + 3 = 8$$

Number Balance
A student uses a number balance to investigate the commutative property. "If 8 and 2 equals 10, then I think that if I put a weight on 2 first this time and then on 8, it'll also be 10"

Associative Property Examples:

Number Line $\square$ = 5 + 4+ 5
Student A: First I jumped to 5. Then I jumped 4 more, so I landed on 9. Then I jumped 5 more and landed on 14.

Student B: 1 got 14 too, but I did it a different way. First I jumped to 5. Then, I jumped 5 again. That's 10. Then I jumped 4 more. See, 14.

Mental Math There are 9 red jelly beans, 7 green jelly beans, and 3 black jelly beans. How many jelly beans are there in all?

Student: "I know that 7 + 3 is 10. And 10 and 9 is 19. There are 19 jelly beans.

1.OA.4 Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.

Knowledge Targets:
- ☐ Identify the unknown in a subtraction problem.
- *I can find the missing part in a subtraction problem. (Underpinning)*

<mark>**Reasoning Targets:**</mark>
- ☐ Solve subtraction problems to find the missing addend.
- I can *relate addition to subtraction using fact families.*
- ☐ Explain the relationship of addition and subtraction.
- *I can solve subtraction problems to find the missing*

First Graders often find subtraction facts more difficult to learn than addition facts. By understanding the relationship between addition and subtraction, First Graders are able to use various strategies described below to solve subtraction problems.

For Sums to 10

*Think-Addition:

Think-Addition uses known addition facts to solve for the unknown part or quantity within a problem. When students use this strategy, they think, "What goes with this part to make the total?" The think-addition strategy is particularly helpful for subtraction facts with sums of 10 or less and can be used for sixty-four of the 100 subtraction facts. Therefore, in order for think-addition to be an effective strategy, students must have mastered addition facts first.

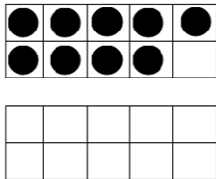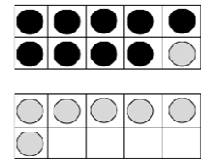|	
addend.	For example, when working with the problem 9 - 5 = ☐, First Graders think "Five and what makes nine?", rather than relying on a counting approach in which the student counts 9, counts off 5, and then counts what's left. When subtraction is presented in a way that encourages students to think using addition, they use known addition facts to solve a problem. Example: 10 – 2 = ☐ Student: "2 and what make 10? I know that 8 and 2 make 10. So, 10 – 2 = 8." For Sums Greater than 10 The 36 facts that have sums greater than 10 are often considered the most difficult for students to master. Many students will solve these particular facts with Think-Addition (described above), while other students may use other strategies described below, depending on the fact. Regardless of the strategy used, all strategies focus on the relationship between addition and subtraction and often use 10 as a benchmark number. *Build Up Through 10: This strategy is particularly helpful when one of the numbers to be subtracted is 8 or 9. Using 10 as a bridge, either 1 or 2 are added to make 10, and then the remaining amount is added for the final sum. Example: 15 – 9 = ☐ Student A: "I'll start with 9. I need one more to make 10. Then, I need 5 more to make 15. That's 1 and 5- so it's 6. 15 – 9 = 6." Student B: "I put 9 counters on the 10 frame. Just looking at it I can tell that I need 1 more to get to 10. Then I need 5 more to get to 15. So, I need 6 counters."	

*Back Down Through 10
This strategy uses take-away and 10 as a bridge. Students take away an amount to make 10, and then take away the rest. It is helpful for facts where the ones digit of the two-digit number is close to the number being subtracted.

Example: 16 – 7 = ☐

Student A: "I'll start with 16 and take off 6. That makes 10. I'll take one more off and that makes 9. 16 – 7 = 9."

Student B: "I used 16 counters to fill one ten frame completely and most of the other one. Then, I can take these 6 off from the 2nd ten frame. Then, I'll take one more from the first ten frame. That leaves 9 on the ten frame."

*Van de Walle and Lovin, 2006

| **1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).**
Knowledge Targets:
☐ Know how to count on and count back.
I can count on and count back (to/from 10).

<mark>**Reasoning Targets:**</mark>
☐ Explain how counting on and counting back relate to addition and subtraction.
I can explain counting on and counting back to relate addition and subtraction. | When solving addition and subtraction problems to 20, First Graders often use counting strategies, such as counting all, counting on, and counting back, before fully developing the essential strategy of using 10 as a benchmark number. Once students have developed counting strategies to solve addition and subtraction problems, it is very important to move students towards strategies that focus on composing and decomposing number using ten as a benchmark number, as discussed in 1.OA.6, particularly since counting becomes a hindrance when working with larger numbers. By the end of First Grade, students are expected to use the strategy of 10 to solve problems.

Counting All: Students count all objects to determine the total | |

amount.

Counting On & Counting Back: Students hold a "start number" in their head and count on/back from that number.

Example: 15 + 2 = E

Counting All	Counting On
The student counts out fifteen counters. The student adds two more counters starting at 1 (1, 2, 3, 4,…14, 15, 16, 17) to find the total amount.	Holding 15 in her head, the student holds up one finger and says 15, then holds up another finger and says 17. The student knows that 15 + 2 is 17, since she counted on 2 using her fingers.

Example: 12 - 3 = E

Counting All	Counting Back
The student counts out twelve counters. To determine the final amount, the student counts each one (1, 2, 3, 4, 5, 6, 7, 8, 9) to find the final amount.	Keeping 12 in his head, the student counts backwards, "11" as he holds up one finger; says "10" as he holds up a second finger; says "9" as he holds up a third finger. Seeing that he has counted back 3 since he is holding up 3 fingers, the student states that 12 – 3 = 9.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows

In Second Grade, students use various strategies to add and subtract. Second Graders are use strategies that make sense to them, and through their use they internalize faces and develop fluency. When students are able to demonstrate fluency within 10, they are accurate (answer correctly), efficient (within 4-5 seconds) and flexible (use strategies such as decomposing numbers to make ten). Students need efficient strategies in order for facts to become fluent.

Example: Sam has 8 red marbles and 7 green marbles. How

12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Knowledge Targets:
- ☐ Add fluently within 10.

I can add fluently to 10. (Underpinning)
I can subtract fluently from 10.

- ☐ Apply strategies to add and subtract within 20.

I can apply strategies to add and subtract within 10.

many marbles does Sam have in all?

Making 10 and Decomposing a Number	Creating an Easier Problem with Known Sums
I know that 8 plus 2 is 10, so I broke up (decomposed) the 7 up into a 2 and a 5. First I added 8 and 2 to get 10, and then added the 5 to get 15. 7 = 2 + 5 8 + 2 = 10 10 + 5 = 15	I broke up (decomposed) 8 into 7 and 1. I know that and 7 is 14. I added 1 more to get 15. 8 = 7 + 1 7 + 7 = 14 14 + 1 = 15

Example: There were 14 birds in the tree. 6 flew away. How many birds are in the tree now?

Back Down Through Ten	Relationship between Addition and Subtraction
I know that 14 minus 4 is 10. So I broke the 6 up into 4 and a 2. 14 minus 4 is 10. Then I took away 2 more to get 8. 6 = 4 + 2 14 – 4 = 10 10 - 2 = 8	I thought, '6 and what makes 14?' I know that 6 plus 6 is 12 and two more is 14. That's 8 altogether. So, that means that 14 minus 6 is 8. 6 + 8 = 14 14 - 6 = 8

Numbers to 40
Addition and Subtraction to/from 40

1.NBT.1 Count to 120 (40), starting at any number less than 120 (40). In this range, read and write numerals and represent a number of objects with a written numeral.	First Grade students rote count forward to 120 by counting on from any number less than 120. First graders develop accurate counting strategies that build on the understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after). In addition, first grade students read and write numerals to represent a given amount.	Second Nine Weeks

Knowledge Targets:
☐ Write numerals up to 120.
I can write numbers to 40, starting at any number less than 40.

Reasoning Targets:
☐ Represent a number of objects up to 120 with a written numeral.
I can show numbers 0-40 using objects

<mark>Performance Skills Targets:</mark>
☐ Count (saying the number sequence) to 120, starting at any number less than 120.
☐ Read the numerals up to 120.
I can count numbers to 40, starting at any number less than 40.
I can read numbers to 40, starting at any number less than 40.

As first graders learn to understand that the position of each digit in a number impacts the quantity of the number, they become more aware of the order of the digits when they write numbers. For example, a student may write "17" and mean "71". Through teacher demonstration, opportunities to "find mistakes", and questioning by the teacher ("I am reading this and it says seventeen. Did you mean seventeen or seventy-one? How can you change the number so that it reads seventy-one?"), students become precise as they write numbers to 120.

1.NBT.2abc Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
a. 10 can be thought of as a bundle of ten ones — called a "ten."
b. The numbers from 11 to 19 are composed of a ten and a one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Knowledge Targets:
☐ Explain what each digit of a two-digit number represents.
I can explain what each digit of a two-digit number

First Grade students are introduced to the idea that a bundle of ten ones is called "a ten". This is known as unitizing. When First Grade students unitize a group of ten ones as a whole unit ("a ten"), they are able to count groups as though they were individual objects. For example, 4 trains of ten cubes each have a value of 10 and would be counted as 40 rather than as 4. This is a monumental shift in thinking, and can often be challenging for young children to consider a group of something as "one" when all previous experiences have been counting single objects. This is the foundation of the place value system and requires time and rich experiences with concrete manipulatives to develop.

A student's ability to conserve number is an important aspect of this standard. It is not obvious to young children that 42

represents.
☐ Identify a bundle of 10 ones as a "ten".
I can show a group of 10 ones as a ten.

<mark>Reasoning Targets:</mark>
☐ Represent numbers 11 to 19 as composed of a ten and correct number of ones.
*I can show numbers (11-19) as a group of ten and ones.*

☐ Represent the numbers 20, 30, 40, 50, 60, 70, 80, and 90 as composed of the correct number of tens. (Underpinning)
I can show decade numbers as a group of tens. (Underpinning)

cubes is the same amount as 4 tens and 2 left-overs. It is also not obvious that 42 could also be composed of 2 groups of 10 and 22 leftovers. Therefore, first graders require ample time grouping proportional objects (e.g., cubes, beans, beads, ten-frames) to make groups of ten, rather than using pre-grouped materials (e.g., base ten blocks, pre-made bean sticks) that have to be "traded" or are non-proportional (e.g., money).

Example: 42 cubes can be grouped many different ways and still remain a total of 42 cubes.
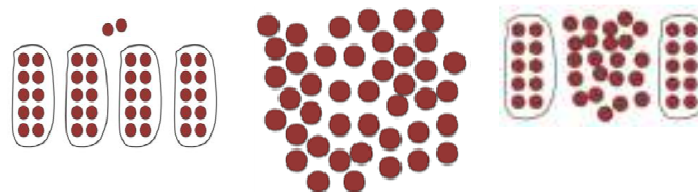
"We want children to construct the idea that all of these are the same and that the sameness is clearly evident by virtue of the groupings of ten. Groupings by tens is not just a rule that is followed but that any grouping by tens, including all or some of the singles, can help tell how many." (Van de Walle & Lovin, p. 124)

As children build this understanding of grouping, they move through several stages: Counting By Ones; Counting by Groups & Singles; and Counting by Tens and Ones.

Counting By Ones: At first, even though First Graders will have grouped objects into tens and left-overs, they rely on counting all of the individual cubes by ones to determine the final amount. It is seen as the only way to determine how many.
Example:
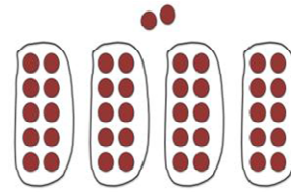
Teacher: How many counters do you have?

Student: 1, 2, 3, 4,…41, 42. I have 42 counters.

Counting By Groups and Singles: While students are able to group objects into collections of ten and now tell how many groups of tens and left-overs there are, they still rely on counting by ones to determine the final amount. They are unable to use the groups and left-overs to determine how many.
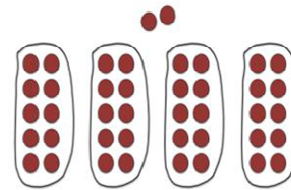Example:

Teacher: How many counters do you have?

Student: I have 4 groups of 10 and 2 left-overs.

Teacher: Does that help you know how many? How many do you have?

Student: Let me see. 1, 2, 3, 4,5, …41, 42. I have 42 counters.

Counting by Tens and Ones: Students are able to group objects into ten and ones, tell how many groups and left-overs there are, and now use that information to tell how many. Ex: "I have 3 groups of ten and 4 left-overs. That means that there are 34 cubes in all." Occasionally, as this stage is becoming fully developed, first graders rely on counting by ones to "really" know that there are 34, even though they may have just counted the total by groups and left-overs.
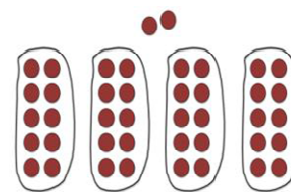Example:

Teacher: How many counters do you have?

Student: I have 4 groups of 10 and 2 left-overs.

Teacher: Does that help you know how many? How many doe you have?

Student: Yes, that means that I have 42 counters.

Teacher: Are you sure?

Student: Um. Let me count just to make sure… 1, 2, 3, 4,…41, 42. Yes, I was right. There are 42 counters.

b. First Grade students extend their work from Kindergarten when they composed and decomposed numbers from 11 to 19 into ten ones and some further ones. In Kindergarten, everything was thought of as individual units: "ones". In First Grade, students are asked to unitize those ten individual ones as a whole unit: "*one* ten". Students in first grade explore the idea that the teen numbers (11 to 19) can be expressed as *one* ten and some leftover ones. Ample experiences with a variety of groupable materials (e.g., links, beans, beads) and ten frames help students develop this concept.

Example: Here is a pile of 12 cubes. Do you have enough to make a ten? Would you have any leftover? If so, how many leftovers would you have?

Student A
I filled a ten frame to make one ten and had two counters left over. I had enough to make a ten with some leftover. The number 12 has 1 ten and 2 ones.

Student B
I counted out 12 cubes. I had enough to make 10. I now have 1 ten and 2 cubes left over. So the number 12 has 1 ten and 2 ones.

In addition, when learning about forming groups of 10, First Grade students learn that a numeral can stand for many different amounts, depending on its position or place in a number. This is an important realization as young children begin to work through reversals of digits, particularly in the teen numbers.

Example: Comparing 19 to 91

19
91

Teacher: Are these numbers the same or different?
Students: different.
Teacher: who do you think so?
Students: Even though they both have a one and a nine, the top one is nineteen. The bottom one is ninety-one.
Teacher: Is that true some of the time, or all of the time? How do you know?

c. First Grade students apply their understanding of groups of ten as stated in 1.NBT.2b to decade numbers (e.g. 10, 20, 30, 40). As they work with groupable objects, first grade students understand that 10, 20, 30…80, 90 are comprised of a certain amount of groups of tens with none left-over.

| **1.NBT.3 Compare two-digit numbers based on meanings of the tens and one digits recording the results of comparisons with the symbols <,=,>.** | First Grade students use their understanding of groups and order of digits to compare two numbers by examining the amount of tens and ones in each number. After numerous experiences verbally comparing two sets of objects using comparison vocabulary (e.g., 42 is more than 31. 23 is less than 52, 61 is the same amount as 61.), first grade students | |

Knowledge Targets:
- ☐ Identify the value of each digit represented in the two-digit number.
  I can identify the value of the digits in the tens and ones place.
- ☐ **Know what each symbol represents >, <, and =.**
  I can know the meaning of the symbols <,=,> (Underpinning)

- ☐ Compare two two-digit numbers based on meanings of the tens and ones digits.
  I can compare two 2-digit numbers using tens and ones with <, =,> symbols.
- ☐ Use >, =, and < symbols to record the results of comparisons.
  I can use < = > symbols to record the results of comparisons.

connect the vocabulary to the symbols: greater than (>), less than (<), equal to (=).

Example: Compare these two numbers. 42 __ 45

Student A	Student B
42 has 4 tens and 2 ones. 45 has 4 tens and 5 ones. They have the same number of tens, but 45 has more ones than 42. So, 42 is less than 45. 42 < 45	42 is less than 45. I know this because when I count up I say 42 before I say 45. 42 < 45 This says 42 is less than 45.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Knowledge Targets:
- ☐ Use symbol for an unknown number in an addition or subtraction problem within 20.
  I can solve addition and subtraction problems with missing parts/addends. (within 10.) (Underpinning)

First grade students extend their experiences in Kindergarten by working with numbers to 20 to solve a new type of problem situation: Compare (See **Table 1** at end of document for examples of all problem types). In a Compare situation, two amounts are compared to find "How many more" or "How many less".

Problem Type: Compare		
Difference Unknown	Bigger Unknown	Smaller Unknown
"How many more?" version.	"More" version suggests operation.	"Fewer" version suggests operation.
Lucy has 7 apples. Julie has 9 apples. How many more apples does Julie have than Lucy?	Julie has 2 more apples than Lucy. Lucy has 7 apples. How many apples does Julie have?	Lucy has 2 fewer apples than Julie. Julie has 9 apples. How many apples does Lucy have?

☐ Solve word problems using addition and subtraction within 20.

I can solve addition and subtraction word problems (within 10).

☐ Interpret situations to solve word problems with unknowns in all positions within 20 using addition and subtraction.

I can solve addition and subtraction word problems with missing parts/addends.

☐ Determine appropriate representations for solving word problems involving different situations using addition and subtraction.

I can decide whether to add or subtract in a word problem.

"How many fewer?" version	"Fewer" version suggest wrong operation	"More" version suggests wrong operation *
Lucy as 7 applies. Julie has 9 apples. How many fewer apples does Lucy have than Julie? $7 + \square = 9$ $9 - 7 = \square$	Lucy has 2 fewer apples than Julie. Lucy has ___ apples. How many apples does Julie have? $7 + 2 = \varhexstar$	Julie has 2 more apples than Lucy. Julie has 9 apples. How many apples does Lucy have? $9 - 2 = \square$ $\square + 2 = 9$ *Mastery at 2nd gr

Compare problems are more complex than those introduced in Kindergarten. In order to solve compare problem types, 1st graders must think about a quantity that is not physically present and must conceptualize that amount. In addition, the language of "how many more" often become lost or not heard with the language of 'who has more'. With rich experiences that encourage students to match problems with objects and drawings can help students master these challenges.

NOTE: Although 1st grade students should have experiences solving and discussing all 12 problem types located in Table 1, they are not expected to master all types by the end of 1st grade due to the high language and conceptual demands of some of the problem types. Please see Table 1 at the end of this document for problem types that 1st grade students are expected to master by the end of 1st grade.

First graders also extend the sophistication of the methods they used in Kindergarten (counting) to add and subtract within this larger range. Now, first graders use the methods of counting on, making ten, and doubles +/-1 or +/- 2 to solve problems.

Example: Nine bunnies were sitting on the grass. Some more bunnies hopped there. Now, there are 13 bunnies on the grass. How many bunnies hopped over there?

Counting On Method	Student:

	Niinnneeee….holding a finger for each next number counted 10, 11, 12, 13. Holding up her four fingers, 4! 4 bunnies hopped over there."
Example: Eight red applies and 6 green apples are on the tree. How many apples are on the tree?	
Making Tens Method	Student: I broke up 6 into 2 and 4. Then, I took the 2 and added it to the 8. That's 10. Then I add the 4 to the 10. That's 14. So there are 14 apples on the tree.
Example: Thirteen apples are on the table. Six of them are red and the rest are green. How many apples are green?	
Doubles +/- 1 or 2	Student: I know that 6 and 6 is 12. So 6 + 7 is 13. There are 7 green apples.

In order for students to read and use equations to represent their thinking, they need extensive experiences with addition and subtraction situations in order to connect the experiences with symbols (+, -, =) and equations (5 = 3 + 2). In Kindergarten, students demonstrated the understanding of how objects can be joined (addition) and separated (subtraction) by representing addition and subtraction situations using objects, pictures and words. In 1st grade, student ex tend this understanding of addition and subtraction situations to use the addition symbol (+) to represent joining situation, the subtraction symbol (-) to represent separating situation, and the equal sign (=) to represent a relationship regarding quantity between one side of the equation and the other.

| **1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.** | First Grade students solve multi-step word problems by adding (joining) three numbers whose sum is less than or equal to 20, using a variety of mathematical representations.

Example: **Mrs. Smith has 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies does Mrs. Smith have?** | |

Knowledge Targets:
☐ Know how to add three whole numbers whose sum is less than or equal to 20.
   *I can add 3 numbers with a sum of 20 or less.*

☐ Solve word problems involving addition of three whole numbers whose sum is less than or equal to 20.
   I can solve word problems with 3 addends having a sum of 20 or less.
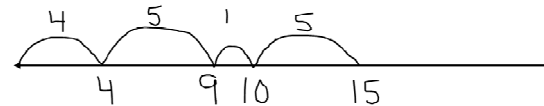
Student 1:
I put 4 counters on the Ten Frame for the oatmeal raisin cookies. Then, I put 5 different color counters on the ten frame for the chocolate chip cookies. Then, I put another 6 color counters out for the gingerbread cookies. Only one of the gingerbread cookies fit, so I had 5 leftover. Ten and five more makes 15 cookies. Mrs. Smith has 15 cookies.

$4 + 5 + 6 = ☼$
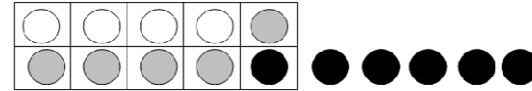
Student 2:
I used a number line. First I jumped to 4, and then I jumped 5 more. That's 9. I broke up 6 into 1 and 5 so I could jump 1 to make 10. Then, I jumped 5 more and got 15. Mrs. Smith has 15 cookies.

$4 + 5 + 6 = ☼$

Student 3:
I wrote: $4 + 5 + 6 = ☐$. I know that 4 and 6 equals 10, so the oatmeal raisin and gingerbread equals 10 cookies. Then I added the 5 chocolate chip cookies. 10 and 5 is 15. So, Mrs. Smith has 15 cookies.

1.OA.3 Apply properties of operations as strategies to add and subtract.3 Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

Elementary students often believe that there are hundreds of isolated addition and subtraction facts to be mastered. However, when students understand the commutative and associative properties, they are able to use relationships between and among numbers to solve problems. First Grade students apply properties of operations as strategies to add and subtract. Students do not use the formal terms "commutative" and "associative". Rather, they use the understandings of the commutative and associative property

Knowledge Targets:

☐ Explain how properties of operation strategies work.
I can explain how properties of operation strategies work.
I can use "turn around facts" to solve addition problems. (Underpinning)

<mark>Reasoning Targets:</mark>

☐ Apply strategies using properties of operations to solve addition and subtraction problems.
I can use properties of operations strategies to solve addition and subtraction problems.
I can use "turn around" facts to solve addition problems.
I can make 10 when adding 3 numbers.

to solve problems.

Commutative Property of Addition	Associate Property of Addition
The order of the addends does not change the sum. For example, if 8 + 2 = 10 is known, then 2 + 8 = 10 is also known.	The grouping of the 3 or more addends does not affect the sum. For example, when adding 2 + 6 + 4, the sum from adding the first two numbers first (2 + 6) and then the third number (4) is the same as if the second and third numbers are added first) 6 + 4 equals 10 and add those two numbers first before adding 2. Regardless of the order, the sum remains 12.

Students use mathematical tools and representations (e.g., cubes, counters, number balance, number line, 100 chart) to model these ideas).

Commutative Property Examples:

Cubes
A student uses 2 colors of cues to make as many different combinations of 8 as possible. When recording the combinations, the student records that 3 green cubes and 5 blue cubes equals 8 cubes in all. In addition, the student notices that 5 green cubes and 3 blue cubes also equals 8 cubes.

3 + 5 = 8

5 + 3 = 8

Number Balance
A student uses a number balance to investigate the commutative property. "If 8 and 2 equals 10, then I think that if I put a weight on 2 first this time and then on 8, it'll also be 10"

Associative Property Examples:

Number Line $\square$ = 5 + 4+ 5
Student A: First I jumped to 5. Then I jumped 4 more, so I landed on 9. Then I jumped 5 more and landed on 14.

Student B: 1 got 14 too, but I did it a different way. First I jumped to 5. Then, I jumped 5 again. That's 10. Then I jumped 4 more. See, 14.

Mental Math There are 9 red jelly beans, 7 green jelly beans, and 3 black jelly beans. How many jelly beans are there in all?

Student: "I know that 7 + 3 is 10. And 10 and 9 is 19. There are 19 jelly beans.

| **1.OA.4 Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.** | First Graders often find subtraction facts more difficult to learn than addition facts. By understanding the relationship between addition and subtraction, First Graders are able to use various strategies described below to solve subtraction problems. | |

Knowledge Targets:

☐ Identify the unknown in a subtraction problem.
I can find the missing part in a subtraction problem. (Underpinning)

<mark>Reasoning Targets:</mark>

☐ Solve subtraction problems to find the missing addend.
I can *relate addition to subtraction using fact families.*
☐ Explain the relationship of addition and subtraction.
I can solve subtraction problems to find the missing addend.

For Sums to 10

*Think-Addition:

Think-Addition uses known addition facts to solve for the unknown part or quantity within a problem. When students use this strategy, they think, "What goes with this part to make the total?" The think-addition strategy is particularly helpful for subtraction facts with sums of 10 or less and can be used for sixty-four of the 100 subtraction facts. Therefore, in order for think-addition to be an effective strategy, students must have mastered addition facts first.

For example, when working with the problem 9 - 5 = ☐, First Graders think "Five and what makes nine?", rather than relying on a counting approach in which the student counts 9, counts off 5, and then counts what's left. When subtraction is presented in a way that encourages students to think using addition, they use known addition facts to solve a problem.

Example: 10 – 2 = ☐

Student: "2 and what make 10? I know that 8 and 2 make 10. So, 10 – 2 = 8."

For Sums Greater than 10

The 36 facts that have sums greater than 10 are often considered the most difficult for students to master. Many students will solve these particular facts with Think-Addition (described above), while other students may use other strategies described below, depending on the fact. Regardless of the strategy used, all strategies focus on the relationship between addition and subtraction and often use 10 as a benchmark number.

*Build Up Through 10:
This strategy is particularly helpful when one of the numbers to be subtracted is 8 or 9. Using 10 as a bridge, either 1 or 2 are added to make 10, and then the remaining amount is added for the final sum.

Example: 15 – 9 = ☐

Student A: "I'll start with 9. I need one more to make 10. Then, I need 5 more to make 15. That's 1 and 5- so it's 6. 15 – 9 = 6."

Student B: "I put 9 counters on the 10 frame. Just looking at it I can tell that I need 1 more to get to 10. Then I need 5 more to get to 15. So, I need 6 counters."

*Back Down Through 10

This strategy uses take-away and 10 as a bridge. Students take away an amount to make 10, and then take away the rest. It is helpful for facts where the ones digit of the two-digit number is close to the number being subtracted.

Example: 16 – 7 = ☐

Student A: "I'll start with 16 and take off 6. That makes 10. I'll take one more off and that makes 9. 16 – 7 = 9."

Student B: "I used 16 counters to fill one ten frame completely and most of the other one. Then, I can take these 6 off from the 2nd ten frame. Then, I'll take one more from the first ten frame. That leaves 9 on the ten frame."

*Van de Walle and Lovin, 2006

| **1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).**
Knowledge Targets:
☐ Know how to count on and count back. | When solving addition and subtraction problems to 20, First Graders often use counting strategies, such as counting all, counting on, and counting back, before fully developing the essential strategy of using 10 as a benchmark number. Once students have developed counting strategies to solve addition and subtraction problems, it is very important to move | |

I can count on and count back (to/from 10).

<mark>Reasoning Targets:</mark>

☐ Explain how counting on and counting back relate to addition and subtraction.

I can explain counting on and counting back to relate addition and subtraction.

students towards strategies that focus on composing and decomposing number using ten as a benchmark number, as discussed in 1.OA.6, particularly since counting becomes a hindrance when working with larger numbers. By the end of First Grade, students are expected to use the strategy of 10 to solve problems.

Counting All: Students count all objects to determine the total amount.

Counting On & Counting Back: Students hold a "start number" in their head and count on/back from that number.

Example: 15 + 2 = E

Counting All	Counting On
The student counts out fifteen counters. The student adds two more counters starting at 1 (1, 2, 3, 4,…14, 15, 16, 17) to find the total amount.	Holding 15 in her head, the student holds up one finger and says 15, then holds up another finger and says 17. The student knows that 15 + 2 is 17, since she counted on 2 using her fingers.

Example: 12 - 3 = E

Counting All	Counting Back
The student counts out twelve counters. To determine the final amount, the student counts each one (1, 2, 3, 4, 5, 6, 7, 8, 9) to find the final amount.	Keeping 12 in his head, the student counts backwards, "11" as he holds up one finger; says "10" as he holds up a second finger; says "9" as he holds up a third finger. Seeing that he has counted back 3 since he is holding up 3 fingers, the student states that 12 – 3 = 9.

1.OA.6 Add and subtract within 20, demonstrating

In Second Grade, students use various strategies to add and subtract. Second Graders are use strategies that make sense

fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Knowledge Targets:
  □ Add fluently within 10.
  I can add fluently to 10. (Underpinning)
  I can subtract fluently from 10.

<mark>**Reasoning Targets:**</mark>
  □ Apply strategies to add and subtract within 20.
  I can apply strategies to add and subtract within 10.

to them, and through their use they internalize faces and develop fluency. When students are able to demonstrate fluency within 10, they are accurate (answer correctly), efficient (within 4-5 seconds) and flexible (use strategies such as decomposing numbers to make ten). Students need efficient strategies in order for facts to become fluent.

Example: Sam has 8 red marbles and 7 green marbles. How many marbles does Sam have in all?

Making 10 and Decomposing a Number	Creating an Easier Problem with Known Sums
I know that 8 plus 2 is 10, so I broke up (decomposed) the 7 up into a 2 and a 5. First I added 8 and 2 to get 10, and then added the 5 to get 15. 7 = 2 + 5 8 + 2 = 10 10 + 5 = 15	I broke up (decomposed) 8 into 7 and 1. I know that and 7 is 14. I added 1 more to get 15. 8 = 7 + 1 7 + 7 = 14 14 + 1 = 15

Example: There were 14 birds in the tree. 6 flew away. How many birds are in the tree now?

Back Down Through Ten	Relationship between Addition and Subtraction
I know that 14 minus 4 is 10. So I broke the 6 up into 4 and a 2. 14 minus 4 is 10. Then I took away 2 more to get 8. 6 = 4 + 2 14 – 4 = 10 10 - 2 = 8	I thought, '6 and what makes 14?' I know that 6 plus 6 is 12 and two more is 14. That's 8 altogether. So, that means that 14 minus 6 is 8. 6 + 8 = 14 14 - 6 = 8

Numbers to 100
Adding to 100
Subtracting from 100
Time

1.NBT.1 Count to 120 (100), starting at any number less than 120 (100). In this range, read and write numerals and represent a number of objects with a written numeral. **Knowledge Targets:** ☐ Write numerals up to 120. *I can write numbers to 100, starting at any number less than 100.* **Reasoning Targets:** ☐ Represent a number of objects up to 120 with a written numeral. *I can show numbers 0-100 using objects* <mark>**Performance Skills Targets:**</mark> ☐ Count (saying the number sequence) to 120, starting at any number less than 120. *I can count numbers to 100, starting at any number less than 100.* ☐ Read the numerals up to 120. *I can read numbers to 100, starting at any number less than 100.*	First Grade students rote count forward to 120 by counting on from any number less than 120. First graders develop accurate counting strategies that build on the understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after). In addition, first grade students read and write numerals to represent a given amount. As first graders learn to understand that the position of each digit in a number impacts the quantity of the number, they become more aware of the order of the digits when they write numbers. For example, a student may write "17" and mean "71". Through teacher demonstration, opportunities to "find mistakes", and questioning by the teacher ("I am reading this and it says seventeen. Did you mean seventeen or seventy-one? How can you change the number so that it reads seventy-one?"), students become precise as they write numbers to 120.	Third Nine Weeks
1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?*	In order to determine whether an equation is true or false, First Grade students must first understand the meaning of the equal sign. This is developed as students in Kindergarten and First Grade solve numerous joining and separating situations with mathematical tools, rather than symbols. Once the concepts of joining, separating, and "the same	

6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

Knowledge Targets:
- ☐ Explain the meaning of an equal sign (the quantity on each side of the equality symbol is the same.
  - *I can explain the meaning of an equal sign. (Underpinning)*

<mark>Reasoning Targets:</mark>
- ☐ Compare the values on each side of an equal sign.
  - *I can compare the values on each side of an equal sign. (Underpinning)*
- ☐ Determine if the equation is true or false.
  - *I can determine if an equation is true or false.*

amount/quantity as" are developed concretely, First Graders are ready to connect these experiences to the corresponding symbols (+, -, =). Thus, students learn that the equal sign does not mean "the answer comes next", but that the symbol signifies an equivalent relationship that the left side 'has the same value as' the right side of the equation.

When students understand that an equation needs to "balance", with equal quantities on both sides of the equal sign, they understand various representations of equations, such as:
- an operation on the left side of the equal sign and the answer on the right side (5 + 8 = 13)
- an operation on the right side of the equal sign and the answer on the left side (13 = 5 + 8)
- numbers on both sides of the equal sign (6 = 6)
- operations on both sides of the equal sign (5 + 2 = 4 + 3).

Once students understand the meaning of the equal sign, they are able to determine if an equation is true (9 = 9) or false (9 = 8).

1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = – 3, 6 + 6 = ?.*

Knowledge Targets:
- ☐ Recognize part-part-whole relationships of three whole numbers.
  Example:

  ☐ + 5 = 8

  5 = ☐ – 3   In each instance the 3 and 5 represent the parts and the 9 would be representative of the whole.
  - *I can identify the part-part-whole relationships of three whole numbers. (Underpinning)*

<mark>Reasoning Targets:</mark>

First Graders use their understanding of and strategies related to addition and subtraction as described in 1.OA.4 and 1.OA.6 to solve equations with an unknown. Rather than symbols, the unknown symbols are boxes or pictures.

Example: Five cookies were on the table. I ate some cookies. Then there were 3 cookies. How many cookies did I eat?

Student A: What goes with 3 to make 5? 3 and 2 is 5. So, 2 cookies were eaten.
Student B: Fiiivee, four, three (*holding up 1 finger for each count*). 2 cookies were eaten (*showing 2 fingers*).
Student C: We ended with 3 cookies. Threeeee, four, five (*holding up 1 finger for each count*). 2 cookies were eaten (*showing 2 fingers*).

☐ Determine the missing value in an addition or subtraction equation by using a variety of strategies. **I can determine the missing value/unknown in an addition or subtraction equation using a variety of strategies.**	
1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **Knowledge Targets:** ☐ Identify the value of each digit of a number within 100. *I can identify the value of each digit of a number within 100.* ☐ Decompose any number within one hundred into ten(s) and one(s). *I can break down numbers into tens and ones within 100.* <mark>**Reasoning Targets:**</mark> ☐ Choose an appropriate strategy for solving an addition or subtraction problem within 100. *I can choose a strategy for solving an addition or subtraction problem within 100.* ☐ Relate the chosen strategy (using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction) to a written method (equation) and explain the reasoning used.	First grade students use concrete materials, models, drawings and place value strategies to add within 100. They do so by being flexible with numbers as they use the base-ten system to solve problems. The standard algorithm of carrying or borrowing is not an expectation nor a focus in First Grade. Students are not expected to fluently add and subtract whole numbers using standard algorithms until the end of Fourth Grade. Example: 24 red apples and 8 green apples are on the table. How many apples are on the table? Student A: I used ten frames. I put 24 chips on 3 ten frames. Then, I counted out 8 more chips. 6 of them filled up the third ten frame. That meant I had 2 left over. 3 tens and 2 left over. That's 32. So, there are 32 apples on the table. **24 + 6 = 30** **30 + 2 = 32** Student B: I used an open number line. I started at 24. I knew that I needed 6 more jumps to get to 30. So, I broke apart 8 into 6 and 2. I took 6 jumps to land on 30 and then 2 more. I landed on 32. So, there are 32 apples on the table. **24 + 6 = 30** **30 + 2 = 32**

I can explain the reason why I chose a strategy to solve addition and subtraction problems. **(Underpinning)**

☐ Use composition and decomposition of tens when necessary to add and subtract within 100.

I can break apart or put together tens to add and subtract within 100.
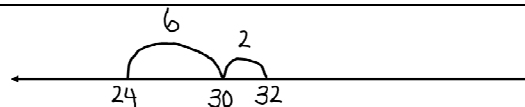
Student C:

I turned 8 into 10 by adding 2 because it's easier to add. So, 24 and ten more is 34.

But, since I added 2 extra, I had to take them off again. 34 minus 2 is 32. There are 32 apples on the table.

8 + 2 = 10

24 + 10 = 34

34 - 2 = 32

Example: 63 apples are in the basket. Mary put 20 more apples in the basket. How many apples are in the basket?

Student A:

I used ten frames. I picked out 6 filled ten frames. That's 60. I got the ten frame with 3 on it. That's 63. Then, I picked one more filled ten frame for part of the 20 that Mary put in. That made 73. Then, I got one more filled ten frame to make the rest of the 20 apples from Mary. That's 83. So, there are 83 apples in the basket.
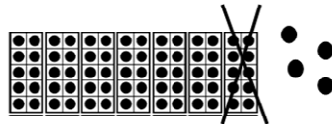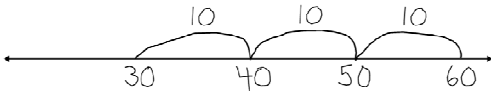
63 + 10 = 73

73 + 10 = 83
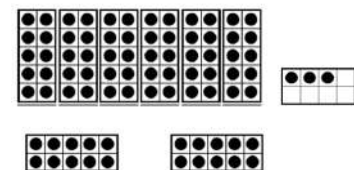
Student B:

I used a hundreds chart. I started at 63 and jumped down one row to 73. That means I moved 10 spaces. Then, I jumped down one more row (that's another 10 spaces) and landed on 83. So, there are 83 apples in the basket.

63 + 10 = 73

73 + 10 = 83

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	(63)	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	(83)	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student C:
I knew that 10 more than 63 is 73. And 10 more than 73 is 83. So, there are 83 apples in the basket.

63 + 10 = 73

73 + 10 = 83

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

Knowledge Targets:
- ☐ Identify the value of each digit in a number within 100.

I can identify the value of each digit of a number within 100.

- ☐ Apply knowledge of place value to mentally add or subtract 10 to/from a given two digit number.

I can use place value to mentally add or subtract 10 to/from a given two-digit number.
- ☐ Explain how to mentally find 10 more or 10 less than the given two-digit number.

I can explain how I used place value to mentally add or subtract 10 to/from a two-digit number.

First Graders build on their work with tens and ones by mentally adding ten more and ten less than any number less than 100. First graders are not expected to compute differences of two-digit numbers other than multiples of ten. Ample experiences with ten frames and the hundreds chart help students use the patterns found in the tens place to solve such problems mentally.

Example: There are 74 birds in the park. 10 birds fly away. How many birds are in the park now?

Student A
I thought about a 100s board. I started at 74. Then, because 10 birds flew away, I moved back one row for a total of 10 spaces. I landed on 64. So, there are 64 birds left in the park.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	(74)	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	Student B I pictured 7 ten frames and 4 left over in my head. Since 10 birds flew away, I took one of the ten frames away. That left 6 ten frames and 4 left over. So, there are 64 birds left in the park. Student C I know that 10 less than 74 is 64. So there are 64 birds in the park.	
1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Knowledge Targets:** ☐ Identify the value of each digit of a number within 100. *I can identify the value of each digit of a number within 100.* ==**Reasoning Targets:**== ☐ Subtract multiples of 10 in the range of 10-90 from multiples of 10 in the range of 10-90 (positive or zero differences). *I can subtract multiples of 10 within 10-90.* ☐ Choose appropriate strategy (concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction) for solving subtraction problems with multiples of 10.	First Grade students use concrete models, drawings and place value strategies to subtract multiples of 10 from decade numbers (e.g., 30, 40, 50). They often use similar strategies as discussed in 1.OA.4. Example: There are 60 students in the gym. 30 students leave. How many students are still in the gym? Student A I used a number line. I started at 60 and moved back 3 jumps of 10 and landed on 30. There are 30 students left. **60 - 10 = 50** **50 – 10 = 40** **40 – 10 = 30** Student B I used a hundreds chart and started at 60. I moved back 3 rows of ten to land on 30. There are 30 students left.	

I can choose a strategy for solving subtraction problems with multiples of 10. (Underpinning) ☐ Relate the chosen strategy to a written method. *I can write and explain the chosen strategy.*		

	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18	19	20
	21	22	23	24	25	26	27	28	29	30
	31	32	33	34	35	36	37	38	39	40
	41	42	43	44	45	46	47	48	49	50
	51	52	53	54	55	56	57	58	59	60
	61	62	63	64	65	66	67	68	69	70
	71	72	73	74	75	76	77	78	79	80
	81	82	83	84	85	86	87	88	89	90
	91	92	93	94	95	96	97	98	99	100

$60 - 10 = 50$

$50 - 10 = 40$

$40 - 10 = 30$

Student C
I thought, "30 and what makes 60?". I know 3 and 3 is 6. So, I thought that 30 and 30 makes 60. There are 30 students still in the gym.

$30 + 30 = 60$

1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.

Knowledge Targets:
☐ Recognize that analog and digital clocks are objects that measure time.
I can identify the tools that measure time. (Underpinning)
☐ Know hour hand and minute hand and distinguish between the two.
I can identify hour and minute hand. (Underpinning)

Reasoning Targets:
☐ Determine where the minute hand must be when the time is to the hour (o'clock).
I can tell where the minute hand must be when the time is to the hour (o'clock). (Underpinning)
☐ Determine where the minute hand must be when the time is to the half hour (thirty).
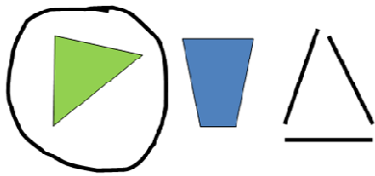*I can tell where the minute had must be when the time is to the half hour (30).*

For young children, reading a clock can be a difficult skill to learn. In particular, they must understand the differences between the two hands on the clock and the functions of these hands. By carefully watching and talking about a clock with only the hour hand, First Graders notice when the hour hand is directly pointing at a number, or when it is slightly ahead/behind a number. In addition, using language, such as "about 5 o'clock" and "a little bit past 6 o'clock", and "almost 8 o'clock" helps children begin to read an hour clock with some accuracy. Through rich experiences, First Grade students read both analog (numbers and hands) and digital clocks, orally tell the time, and write the time to the hour and half-hour.
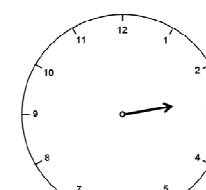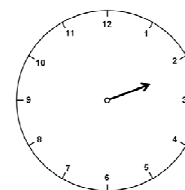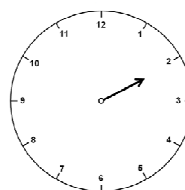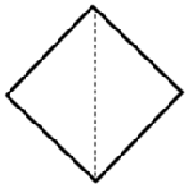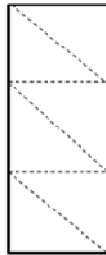
All of these clocks indicate the hour of "two", although they look slightly different. This is an important idea for students as they learn to tell time.

- ☐ Tell/write the time to the hour and half hour correctly using analog and digital clocks-for instance when it is 3:30 the hour hand is between the 3 and the 4; the minute hand is on the 6.

I can tell/write time to the hour and half-hour correctly using analog and digital clocks.

Measurement
Geometry
Fractions

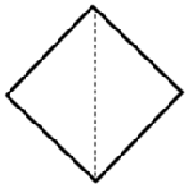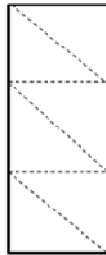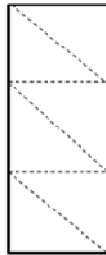|		
1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object. **Knowledge Targets:** ☐ Identify the measurement known as the length of an object. ***I can identify length of an object. (Underpinning)*** ☐ Directly compare the length of three objects. ***I can compare lengths of three objects.*** <mark>**Reasoning Targets:**</mark> ☐ Order three objects by length. ***I can order three objects by length.*** ☐ Compare the lengths of two objects indirectly by using a third object to compare them (e.g., if the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C.) ***I can compare the length of two objects using a third object to compare them.***	First Grade students typically measure and order objects by the length of each of the objects. Sometimes, the lengths are not known, but the relationships between the three objects are known. This concept is known as the transitivity principle for indirect measurement. Example: The snake handler is trying to put the snakes in order- from shortest to longest. She knows that the red snake is longer than the green snake. She also knows that the green snake is longer than the blue snake. What order should she put the snakes? Student: Ok. I know that the red snake is longer than the green snake and the blue snake because, since it's longer than the green, that means that it's also longer than the blue snake. So the longest snake is the red snake. I also know that the green snake and red snake are both longer than the blue snake. So, the blue snake is the shortest snake. That means that the green snake is the medium sized snake. <table><tr><td colspan="3" align="center">Snakes</td></tr><tr><td>Shortest</td><td align="center">←——————→</td><td>Longest</td></tr><tr><td>blue</td><td>green</td><td>red</td></tr></table> First Grade students indirectly measure objects by comparing	Fourth Nine Weeks

	the length of two objects by using a third object as a measuring tool. This concept is referred to as transitivity. Example: Which is longer: the height of the bookshelf or the height of a desk? Student A: I used a pencil to measure the height of the bookshelf and it was 6 pencils long. I used the same pencil to measure the height of the desk and the desk was 4 pencils long. Therefore, the bookshelf is taller than the desk. Student B: I used a book to measure the bookshelf and it was 3 books long. I used the same book to measure the height of the desk and it was a little less than 2 books long. Therefore, the bookshelf is taller than the desk.	
1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **Knowledge Targets:** ☐ Knows to use the same size non-standard objects as iterated (repeating) units. *I can know to use the same size non-standard objects to measure. (Underpinning)* ☐ Know that length can be measured with various units. *I can tell that length can be measured in different units. (Underpinning)* **Reasoning Targets:** ☐ Compare a smaller unit of measurement to a larger object. *I can compare a smaller unit of measurement to a larger object. (Underpinning)*	First Graders use non-standard objects to measure objects which help students focus on the attribute being measured. A non-standard object also lends itself to future discussions regarding the need for a standard unit. First Grade students use multiple copies of one object to measure the length larger object. Through numerous experiences and careful questioning by the teacher, students will recognize the importance of careful measuring so that there are not any gaps or overlaps in order to get an accurate measurement. This concept is a foundational building block for the concept of area in 3rd Grade. Example: How long is the pencil, using paper clips to measure? Student: I carefully placed paper clips end to end. The pencil is 5 paper clips long. I thought it would take about 6 paperclips.	

☐ Determine the length of the measured object to be the number of smaller iterating (repeating) objects that equal its length.
I can find the length of an object to using smaller repeated objects. (Underpinning)

☐ Demonstrate the measurement of an object using non-standard units (e.g. paper clips, Unifix cubes, etc.) by laying the units of measurement end to end with no gaps or overlaps.
I can measure an object using non-standard units by laying the units of measurement end to end with no gaps or overlaps.

1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

Knowledge Targets:
☐ Identify defining attributes of shapes.
I can name different attributes of shapes. (Underpinning)
☐ Identify non-defining attributes of shapes.
I can name non-defining attributes of shapes. (Underpinning)

Reasoning Targets:
☐ Distinguish between (compare/contrast) defining and non-defining attributes of shapes.
I can compare and contrast defining and non-defining attributes of shapes.

First Grade students use their knowledge of defining and non-defining attributes of shapes to identify, name, build and draw shapes (including triangles, squares, rectangles, and trapezoids). They understand that defining attributes are always-present features that classify a particular object (e.g., number of sides, angles, etc.). They also understand that non-defining attributes are features that may be present, but do not identify what the shape is called (e.g., color, size, orientation, etc.).

Example:
All triangles must be closed figures and have 3 sides. These are defining attributes.
Triangles can be different colors, sizes and be turned in different directions. These are non-defining attributes.

Student
I know that this shape is a triangle because it has 3 sides. It's also closed, not open.

☐ Build shapes to show defining attributes. ☐ Draw shapes to show defining attributes. ***I can build and draw shapes to show defining attributes.***	Student I used toothpicks to build a square. I know it's a square because it has 4 sides. And, all 4 sides are the same size. ☐

1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

Knowledge Targets:
  ☐ Know that shapes can be composed and decomposed to make new shapes.
I can tell that shapes can be put together and broken apart to make new shapes. (Underpinning)
  ☐ Describe properties of original and composite shapes.
I can describe properties of original and composite shapes. (Underpinning)

Reasoning Targets:
  ☐ Determine how the original and created composite shapes are alike and different.
I can determine how the original and created shapes are alike and different. (Underpinning)

<mark>Product Targets:</mark>
  ☐ Create composite shapes.
Create two-dimensional and three-dimensional shapes.
  ☐ Compose new shapes from a composite shape.
I can create composite shapes.
I can compose new shapes from a composite shape.

As first graders create composite shapes, a figure made up of two or more geometric shapes, they begin to see how shapes fit together to create different shapes. They also begin to notice shapes within an already existing shape. They may use such tools as pattern blocks, tangrams, attribute blocks, or virtual shapes to compose different shapes.

Example: What shapes can you create with triangles?

Student A: I made a square. I used 2 triangles.	Student B: I made a trapezoid. I used 4 triangles.	Student C: I made a tall skinny rectangle. I used 6 triangles.

1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths and quarters, and use the phrases half of, fourth of and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Knowledge Targets:
- ☐ Identify when shares are equal.
- *I can identify when parts are equal.*
- ☐ Describe equal shares using vocabulary: halves, fourths and quarters, half of, fourth of, and quarter of.
- *I can identify two and four equal shares.*
- ☐ Describe the whole as two of two or four of four equal shares.

I can describe equal shares using vocabulary: halves, fourths, quarters, half of, fourth of, quarter of. (Underpinning)
I can describe the number of parts that make a whole.

<mark>Reasoning Targets:</mark>
- ☐ Justify how dividing, (decomposing) a circle or rectangle into more equal shares creates smaller pieces.
- *I can explain the relationship of dividing a shape into more equal shares which creates smaller shares/pieces.*

First Graders begin to partition regions into equal shares using a context (e.g., cookies, pies, pizza). This is a foundational building block of fractions, which will be extended in future grades. Through ample experiences with multiple representations, students use the words, halves, fourths, and quarters, and the phrases half of, fourth of, and quarter of to describe their thinking and solutions. Working with the "the whole", students understand that "the whole" is composed of two halves, or four fourths or four quarters.

Example: How can you and a friend share equally (partition) this piece of paper so that you both have the same amount of paper to paint a picture.

Student 1	Student 2
I would split the paper right down the middle. That gives us 2 halves. I have half of the paper and my friend has the other half of the paper.	I would split it from the corner to corner (diagonally). She gets half of the paper and I get half of the paper. See, if we cut on the line, the parts are the same size.
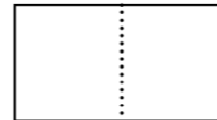
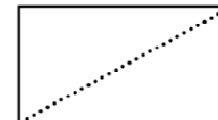|		
1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **Knowledge Targets:** ☐ Write numerals up to 120. *I can write numbers to 120, starting at any number less than 120.* **Reasoning Targets:** ☐ Represent a number of objects up to 120 with a written numeral. *I can show numbers 0-120 using objects* ==**Performance Skills Targets:**== ☐ Count (saying the number sequence) to 120, starting at any number less than 120. *I can count numbers to 120, starting at any number less than 120.* ☐ Read the numerals up to 120. *I can read numbers to 120, starting at any number less than 120.*	First Grade students rote count forward to 120 by counting on from any number less than 120. First graders develop accurate counting strategies that build on the understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after). In addition, first grade students read and write numerals to represent a given amount. As first graders learn to understand that the position of each digit in a number impacts the quantity of the number, they become more aware of the order of the digits when they write numbers. For example, a student may write "17" and mean "71". Through teacher demonstration, opportunities to "find mistakes", and questioning by the teacher ("I am reading this and it says seventeen. Did you mean seventeen or seventy-one? How can you change the number so that it reads seventy-one?"), students become precise as they write numbers to 120.	
1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **Knowledge Targets:** ☐ Recognize different methods to organize data. *I can identify different ways to organize data. (Underpinning)* ☐ **Recognize** different methods to represent data. *I can identify different ways to represent data.*	First Grade students collect and use categorical data (e.g., eye color, shoe size, age) to answer a question. The data collected are often organized in a chart or table. Once the data are collected, First Graders interpret the data to determine the answer to the question posed. They also describe the data noting particular aspects such as the total number of answers, which category had the most/least responses, and interesting differences/similarities between the categories. As the teacher provides numerous opportunities for students to create questions, determine up to 3 categories of possible responses, collect data, organize data, and interpret the results, First Graders build a solid foundation for future data representations (picture and bar graphs) in Second Grade.	

Reasoning Targets:

☐ Organize data with up to three categories.

I can organize up to 3 pieces of information/data.

☐ Represent data with up to three categories.

I can show up to 3 pieces of information/data using charts and graphs.

☐ Interpret data representation by asking and answering questions about the data.

I can read and interpret charts and graphs that show different information

I can answer and ask questions about information/data from charts and graphs.

Example: **Survey Station**

During Literacy Block, a group of students work at the Survey Station. Each student writes a question, creates up to 3 possible answers, and walks around the room collecting data from classmates. Each student then interprets the data and writes 2-4 sentences describing the results. When all of the students in the Survey Station have completed their own data collection, they each share with one another what they discovered. They ask clarifying questions of one another regarding the data, and make revisions as needed. They later share their results with the whole class.

Student: The question, "What is your favorite flavor of ice cream?" is posed and recorded. The categories chocolate, vanilla and strawberry are determined as anticipated responses and written down on the recording sheet. When asking each classmate about their favorite flavor, the student's name is written in the appropriate category. Once the data are collected, the student counts up the amounts for each category and records the amount. The student then analyzes the data by carefully looking at the data and writes 4 sentences about the data.

Name Barbara

What is your favorite flavor of ice cream?	
Chocolate	Amy Ethan Dylan Emma Ryan Elijah Ava Emily Aiden Brittany THoMAS Nathan	12
Vanilla	Sarah Maria Brian Katie KiTTy	5
Strawberry	Rodney Brandon Darrell Mia Tonya Jose	6

12 people liked chocolate. Chocolate has the most votes. Vanilla has 5 votes. 1 more vote and it can tie with strawberry.

Standards	Mathematical Practices
Students are expected to:	
1.MP.1. Make sense of problems and persevere in solving them.	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.
1.MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.
1.MP.3. Construct viable arguments and critique the reasoning of others.	First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask questions.
1.MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.
1.MP.5. Use appropriate tools strategically.	In first grade, students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first graders decide it might be best to use colored chips to model an addition problem.
1.MP.6. Attend to precision.	As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.
1.MP.7. Look for and make use of structure.	First graders begin to discern a pattern or structure. For instance, if students recognize *12 + 3 = 15,* then they also know *3 + 12 = 15. (Commutative property of addition.)* To add *4 + 6 + 4, the first two numbers can be added to make a ten, so 4 + 6 + 4 = 10 + 4 = 14.*
1.MP.8. Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract "ten" and multiples of "ten" they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves, "Does this make sense?"

Math Accountable Talk

Teach students to use one of the following when discussing each other's math work.

I agree with _____ because _____.

I like the way _____ used _____ because as his/her reader, it helps me _____.

I disagree with _____ because _____.

I got a different answer than _____ because _____.

I can add to _____'s thoughts: _____

I got the same answer as _____ but my strategy was different.

I have a question for _____.

I don't understand why _____ got the answer of _____ because _____.

Glossary

Table 1 Common addition and subtraction situations (adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp.32-33.)

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ **(K)**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ **(1st)**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ **(2nd)**
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ **(K)**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ **(1st)**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ **(2nd)**

	Total Unknown	Addend Unknown	Both Addends Unknown[2]
Put together/Take apart[3]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ **(K)**	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$ **(K)**	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$ **(1st)**

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[4]	("How many more?" version) Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? **(1st)** ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$ **(1st)**	(Version with "more") Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? **(1st)** (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$ **(1st)**	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? **(1st)** (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$ **(2nd)**

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, first grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the second grade year, including problem types from the previous year(s).

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