Orange School District

Mathematics

Curriculum Guide – Grade 6

2010 EDITION

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PHILOSOPHY

The philosophy upon which the Mathematics Curriculum Guide is to encourage and support the enjoyment of learning mathematics, as a way to make sense of the world in students' everyday lives. Mathematics is everywhere, from the practicalities of counting, to find easier ways of organizing numbers and data to model and represent daily life experiences. Mathematics involves other disciplines, and is a way in which ideas are communicated, such as in tables and graphs.

Mathematics is developmental by nature. Therefore it is important that should any concerns arise related to mathematics understanding, that this is communicated with the student's teacher as soon as possible. There are varied approaches used to teach and learn mathematics, which is referred to as a balanced mathematics approach. This includes traditional algorithms to approaching the study of mathematics that have been used for many years, along with newer and varied approaches, to provide multiple representations to model solving a problem.

The study of mathematics provides pathways to higher level thinking skills. As students learn mathematics, specialized terminology assist their development. This enables students to not only learn mathematics in a routine way, but to enable them to become problem solvers in novel situations, able to draw on a repertoire of skills and approaches.

We hope these beliefs will assist students to develop their understanding to use mathematics to make meaning, as well as to promote their critical thinking and development as lifelong learners. The goals are to promote problem-solving, and communication, to foster an understanding of the world, that has a conceptual foundation in the study of mathematics.

Vision

In Orange, we recognize that each student is unique and that the purpose of education is to enable every student to acquire the learning skills necessary to compete in the global community. It is essential that we provide a rigorous, high-quality Mathematics curriculum that allows each student's talents and abilities to be developed to their full potential.

Purpose

The Curriculum Guide was prepared by teachers and administrators with input from consultants who have expertise in Mathematics. Students and parents are welcome to read, review, and ask questions about the curriculum, to understand what they and their children are learning.

The Mathematics Curriculum Guide is based on an alignment with the New Jersey Core Content Curriculum Standards, and the Common Core State Standards which are a national set of shared standards which adopted by over 30 states. It is also based on national standards shared through the National Council of Teachers of Mathematics, which develops agreed upon content at each grade level.

Content was designed with a student development perspective across each grade, as well as a vertical articulation, with spirals learning upward, based on the foundation that is developed.

Mathematic Process Goals

In setting mathematical goals for a school curriculum, the choice of content topics must be accompanied by an analysis of the kinds of thinking students will be able to demonstrate upon completion of the curriculum. The text below describes the eleven key mathematical processes developed in all the main content strands used in the Mathematics program.

Counting
Determining the number of elements in finite data sets, trees, graphs, or combinations by application of mental computation, estimation, counting principles, calculators and computers, and formal algorithms

Visualizing
Recognizing and describing shape, size, and position of one-, two-, and three-dimensional objects and their images under transformations; interpreting graphical representations of data, functions, relations, and symbolic expressions

Comparing
Describing relationships among quantities and shapes using concepts such as equality and inequality, order of magnitude, proportion, congruence, similarity, parallelism, perpendicularity, symmetry, and rates of growth or change

Estimating
Determining reasonableness of answers; using "benchmarks" to estimate measures; using various strategies to approximate a calculation and to compare estimates

Measuring
Assigning numbers as measures of geometric objects and probabilities of events; choosing appropriate measures in a decision-making problem, choosing appropriate units or scales and making approximate measurements or applying formal rules to find measures

Modeling
Constructing, making inferences from, and interpreting concrete, symbolic, graphic, verbal, and algorithmic models of quantitative, visual, statistical, probabilistic, and algebraic relationships in problem situations; translating information from one model to another

Reasoning
Bringing to any problem situation the disposition and ability to observe, experiment, analyze, abstract, induce, deduce, extend, generalize, relate, and manipulate in order to find solutions or prove conjectures involving interesting and important patterns

Connecting
Identifying ways in which problems, situations, and mathematical ideas are interrelated and applying knowledge gained in solving one problem to other problems

Representing
Moving flexibly among graphic, numeric, symbolic, and verbal representations and recognizing the importance of having various representations of information in a situation

Using Tools
Selecting and intelligently using calculators, computers, drawing tools, and physical models to represent, simulate, and manipulate patterns and relationships in problem settings

Becoming Mathematicians
Having the disposition and imagination to inquire, investigate, tinker, dream, conjecture, invent, and communicate with others about mathematical ideas

Phases of Instruction

Problem-centered teaching opens the mathematics classroom to exploring, conjecturing, reasoning, and communicating. For this model of instruction, there are three phases: Launch, Explore, and Summarize.

Launch

In the first phase, the teacher launches the problem with the whole class. This involves helping students understand the problem setting, the mathematical context, and the challenge. The following questions can help the teacher prepare for the launch:

- What are students expected to do?
- What do the students need to know to understand the context of the story and the challenge of the problem?
- What difficulties can I foresee for students?
- How can I keep from giving away too much of the problem solution?

The launch phase is also the time when the teacher introduces new ideas, clarifies definitions, reviews old concepts, and connects the problem to past experiences of the students. It is critical that, while giving students a clear picture of what is expected, the teacher leaves the potential of the task intact. He or she must be careful to not tell too much and consequently lower the challenge of the task to something routine, or to cut off the rich array of strategies that may evolve from a more open launch of the problem.

Explore

The nature of the problem suggests whether students work individually, in pairs, in small groups, or occasionally as a whole class to solve the problem during the explore phase. The Teacher's Guide suggests an appropriate grouping. As students work, they gather data, share ideas, look for patterns, make conjectures, and develop problem-solving strategies.

It is inevitable that students will exhibit variation in their progress. The teacher's role during this phase is to move about the classroom, observing individual performance and encouraging on-task behavior. The teacher helps students persevere in their work by asking appropriate questions and providing confirmation and redirection where needed. For students who are interested in and capable of deeper investigation, the teacher may provide extra questions related to the problem. These questions are called Going Further and are provided in the explore discussion in the Teacher's Guide. Suggestions for helping students who may be struggling are also provided in the Teacher's Guide. The explore part of the instruction is an appropriate place to attend to differentiated learning.

The following questions can help the teacher prepare for the explore phase:

- How will I organize the students to explore this problem? (Individuals? Pairs? Groups? Whole class?)
- What materials will students need?
- How should students record and report their work?
- What different strategies can I anticipate they might use?

- What questions can I ask to encourage student conversation, thinking, and learning?
- What questions can I ask to focus their thinking if they become frustrated or off-task?
- What questions can I ask to challenge students if the initial question is "answered"?

As the teacher moves about the classroom during the explore, she or he should attend to the following questions:

- What difficulties are students having?
- How can I help without giving away the solution?
- What strategies are students using? Are they correct?
- How will I use these strategies during the summary?

Summarize

It is during the summary that the teacher guides the students to reach the mathematical goals of the problem and to connect their new understanding to prior mathematical goals and problems in the unit. The summarize phase of instruction begins when most students have gathered sufficient data or made sufficient progress toward solving the problem. In this phase, students present and discuss their solutions as well as the strategies they used to approach the problem, organize the data, and find the solution. During the discussion, the teacher helps students enhance their conceptual understanding of the mathematics in the problem and guides them in refining their strategies into efficient, effective, generalizable problem-solving techniques or algorithms.

Although the summary discussion is led by the teacher, students play a significant role. Ideally, they should pose conjectures, question each other, offer alternatives, provide reasons, refine their strategies and conjectures, and make connections. As a result of the discussion, students should become more skillful at using the ideas and techniques that come out of the experience with the problem.

If it is appropriate, the summary can end by posing a problem or two that checks students' understanding of the mathematical goal(s) that have been developed at this point in time. Check for Understanding questions occur occasionally in the summary in the Teacher's Guide. These questions help the teacher to assess the degree to which students are developing their mathematical knowledge. The following questions can help the teacher prepare for the summary:

- How can I help the students make sense of and appreciate the variety of methods that may be used?
- How can I orchestrate the discussion so that students summarize their thinking about the problem?
- What questions can guide the discussion?
- What concepts or strategies need to be emphasized?
- What ideas do not need closure at this time?
- What definitions or strategies do we need to generalize?

Target Goals

Number and Operation Goals

Number Sense

- Use numbers in various forms to solve problems (6, 7, 8)
- Understand and use large numbers, including in exponential and scientific notation (6, 7, 8)
- Reason proportionally in a variety of contexts using geometric and numerical reasoning, including scaling and solving proportions (6, 7, 8)
- Compare numbers in a variety of ways, including differences, rates, ratios, and percents and choose when each comparison is appropriate (6, 7, 8)
- Order positive and/or negative rational numbers (6, 7, 8)
- Express rational numbers in equivalent forms (6)
- Make estimates and use benchmarks (6, 7, 8)

Operations and Algorithms

- Develop understanding and skill with all four arithmetic operations on fractions and decimals (6)
- Develop understanding and skill in solving a variety of percent problems (6)
- Use the order of operations to write, evaluate, and simplify numerical expressions (7, 8)
- Develop fluency with paper and pencil computation, calculator use, mental calculation, estimation; and choose among these when solving problems (6, 7)

Properties

- Understand the multiplicative structure of numbers, including the concepts of prime and composite numbers, evens, odds, and prime factorizations (6)
- Use the commutative and distributive properties to write equivalent numerical expressions (7, 8)

Data and Probability Goals

Formulating Questions

- Formulate questions that can be answered through data collection and analysis (6, 7, 8)
- Design data collection strategies to gather data to answer these questions (6, 7, 8)
- Design experiments and simulations to test hypotheses about probability situations (8)

Data Collection

- Carry out data collection strategies to answer questions (6, 7, 8)
- Distinguish between samples and populations (8)
- Characterize samples as representative or non- representative, as random (8)
- Use these characterizations to evaluate the quality of the collected data (8)

Data Analysis

- Organize, analyze, and interpret data to make predictions, construct arguments, and make decisions (6, 7)
- Use measures of center and spread to describe and to compare data sets (6, 7)
- Be able to read, create, and choose data representations, including bar graphs, line plots, coordinate graphs, box and whisker plots, histograms, and stem and leaf plots (6, 7)
- Informally evaluate the significance of differences between sets of data (7, 8)
- Use information from samples to draw conclusions about populations (8)

Probability

- Distinguish between theoretical and experimental probabilities and understand the relationship between them
- Use probability concepts to make decisions (6)
- Find and interpret expected value (7)
- Compute and compare the chances of various outcomes, including two-stage outcomes (7)

Geometry and Measurement Goals

Shapes and Their Properties

- Generate important examples of angles, lines, and two- and three-dimensional shapes (6)
- Categorize, define, and relate figures in a variety of representations (6, 7)
- Understand principles governing the construction of shapes with reasons why certain shapes serve special purposes(e.g. triangles for trusses) (6)
- Build and visualize three-dimensional figures from various two-dimensional representations and vice versa
- Recognize and use shapes and their properties to make mathematical arguments and to solve problems
- Use the Pythagorean Theorem and properties of special triangles (e.g. isosceles right triangles) to solve problems (8)
- Use a coordinate grid to describe and investigate relationships among shapes (7, 8)
- Recognize and use standard, essential geometric vocabulary (6, 7, 8)

Transformations-Symmetry, Similarity, and Congruence

- Recognize line, rotational, and translational symmetries and use them to solve problems (6, 8)
- Use scale factor and ratios to create similar figures or determine whether two or more shapes are similar or congruent (7)
- Predict ways that similarity and congruence transformations affect lengths, angle measures, perimeters, areas, volume, and orientation (7, 8)
- Investigate the effects of combining one or more transformations of a shape (8)
- Identify and use congruent triangles and/or quadrilaterals to solve problems about shapes and measurement
- Use properties of similar figures to solve problems about shapes and measurement (7)
- Use a coordinate grid to explore and verify similarity and congruence relationships (7, 8)

Measurement

- Understand what it means to measure an attribute of a figure or a phenomenon (6)
- Estimate and measure angles, line segments, areas, and volumes using tools and formulas (6, 7)
- Relate angle measure and side lengths to the shape of a polygon (6)
- Find area and perimeter of rectangles, parallelograms, triangles, circles, and irregular figures (7)
- Find surface area and volume of rectangular solids, cylinders, prisms, cones, and pyramids and find the volume of spheres (7)
- Relate units within and between the customary and metric systems (6, 7)
- Use ratios and proportions to derive indirect measurements (7)
- Use measurement concepts to solve problems (6, 7, 8)

Geometric Connections

- Use geometric concepts to build understanding of concepts in other areas of mathematics (6, 7, 8)
- Connect geometric concepts to concepts in other areas of mathematics (6, 7, 8)

Algebra Goals

Patterns of Change-Functions

- Identify and use variables to describe relationships between quantitative variables in order to solve problems or make decisions (7, 8)
- Recognize and distinguish among patterns of change associated with linear, inverse, exponential and quadratic functions (7, 8)

Representation

- Construct tables, graphs, symbolic expressions and verbal descriptions and use them to describe and predict patterns of change in variables (7, 8)
- Move easily among tables, graphs, symbolic expressions, and verbal descriptions (7, 8)
- Describe the advantages and disadvantages of each representation and use these descriptions to make choices when solving problems (7, 8)
- Use linear, inverse, exponential and quadratic equations and inequalities as mathematical models of situations involving variables (7, 8)

Symbolic Reasoning

- Connect equations to problem situations (7, 8)
- Connect solving equations in one variable to finding specific values of functions (8)
- Solve linear equations and inequalities and simple quadratic equations using symbolic methods (7, 8)
- Find equivalent forms of many kinds of equations, including factoring simple quadratic equations (7, 8)
- Use the distributive and commutative properties to write equivalent expressions and equations (8)
- Solve systems of linear equations (8)
- Solve systems of linear inequalities by graphing (8)

Connected Mathematics

Organization of Student Units

Connected Mathematics provides eight student units for each grade. One additional unit is offered to allow flexibility in meeting state standards and meeting the PA Anchors. Each unit has an organizational structure around an important mathematical idea or cluster of related ideas such as area and perimeter, operations on fractions, ratio and proportion, linear relationship or quadratic relationships.

- **Unit Opener:** Three questions that reflect the major mathematical goals are posed. The questions are meant to pique curiosity, point to ideas to be investigated and draw students into the unit. The questions will be encountered throughout the unit either as problems to be explored in class or in homework.

- **Mathematical Highlights:** A set of goals that preview the important ideas and ways of thinking developed in the unit.

- **Investigations:** The core of the unit where students develop the conceptual understanding, reasoning, and procedural skills. Each investigation builds toward the mathematical goals. Each unit is composed of three to five investigations with the following key elements:

- **Problem:** The problem is a three phase Instructional Model consisting of the **Launch, the Explore, and Summary.**

1. **Launch** (Typically whole group, however, depending on the difficulty, teacher may have students work on problems individually prior to discussion).

   a. **Getting .Ready** is a feature of the launch that occurs before each problem.
   b. **The Launch** is the introduction of new ideas.
   c. **Did You Know** present interesting facts related to the context of the investigation.
   d. Clarification of mathematical definitions occurs during the Launch.
   e. Launch time should encompass a review of old concepts.
   f. The teacher utilizes this phase to connect problems to past experiences.
   g. Questioning is critical during this phase the teacher must be careful not to devolve too much information or to cut off the rich array of strategies that may occur during the discussion.

2. **Explore** (This phase ca be individual, pairs, or small group)

a. Explore is the best place for differentiated learning to occur (see differentiated learning below).

b. Teacher questioning is critical during explore and utilization of **Going Further.**

c. As students solve the Problems, they uncover important mathematical processes.

3. **Summary** (This phase is whole group and individual)

a. The **Summary** occurs at the end of each Problem and when most students have gathered sufficient progress toward solving the Problem.

b. Students present and discuss solutions as well as strategies.

c. Students pose conjectures, question others, synthesize information, look for generalities, offer alternatives, provide reasons, refine strategies and conjectures and make connections.

d. **Applications – Connections – Extensions (ACE)** is a set of exercises to be used as Homework intended as an opportunity for students to think further about ideas in the lesson. **Applications** help students solidify their understanding by providing practice with ideas and strategies that were in the **Investigation; Connections** is a powerful learning strategy to connect new knowledge to prior learning, and/or "real world problems"; **Extensions** exercises that may provide an opportunity to think beyond what is covered in the **Problem.**

- **Mathematical Reflections:** At the conclusion of each Investigation students are asked to reflect upon and write about what they have learned. Students are provided questions to help organize their thoughts and summarize important concepts and strategies.
- **Unit Project:** At lease four units at each grade level include projects. Projects are open-ended tasks that present opportunities for students to engage in independent work as well as to demonstrate their broad understanding of the unit.
- **Looking Back and Looking Ahead:** This feature provides a review of big ideas and connections in the unit. It includes problems that allow students to demonstrate their understanding, explain their reasoning, summarizing and connecting what they have learned across units.
- **Differentiation Strategies:**

A. **Extensions (advanced proficient students**) are exercises that may provide a challenge for students to think beyond what is covered in the problem as well as foreshadows mathematics in future units or the pursuit of interesting applications.

B. **Special Needs Handbook (special needs students**) contains a wealth of samples of accommodations materials. Other accommodations should come directly from the individual student I.E.P. The curriculum of Connected Mathematics Program is already embedded with strategies that research as well as practitioners indicate are beneficial to special needs students i.e., calculator practice, utilization of manipulatives (special needs students need concrete representational to develop their understandings),and finally real life problems (pedagogical techniques repeatedly stressed in reaching special needs students). Use of ACE and Looking Back and Looking Ahead.

C. **Ranas, pulgas y cubos pintados Relaciones Cuadraticas (second language learners)** The Connected Mathematics Program recommends the following strategies for second language learners: effective questioning, cooperative groups, support vocabulary development, bilingual vocabulary chart, practice out loud, graphic organizers, rebus techniques, rubrics, and diagram code techniques. It is imperative to learn about students' home countries, languages, and previous educational experiences; value students' differences as resources; and stay connected with families.

D. **Websites Addressing Differentiation**

http://www.op97.k12.il.us/lincoln/teacher/differentiation/index.himl

http://connectedmath.msu.edu/conferences/users/usertalk09.

Common Core Standards » Mathematics » Grade 6

Ratios & Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

- 1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
- 2. Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
- 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

- 1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? Compute fluently with multi-digit numbers and find common factors and multiples.*

Compute fluently with multi-digit numbers and find common factors and multiples.

- 2. Fluently divide multi-digit numbers using the standard algorithm.

- 3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

- 4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2). Apply and extend previous understandings of numbers to the system of rational numbers.*

Apply and extend previous understandings of numbers to the system of rational numbers.

- 5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

- 6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.

- Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

- Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

- 7. Understand ordering and absolute value of rational numbers.

- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

- Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.*

- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*

- Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

- 1. Write and evaluate numerical expressions involving whole-number exponents.
- 2. Write, read, and evaluate expressions in which letters stand for numbers.
- Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
- Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*
- 3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
- 4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. Reason about and solve one-variable equations and inequalities.*

Reason about and solve one-variable equations and inequalities.

- 5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
- 8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using

graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

- 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- 2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- 3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability

Develop understanding of statistical variability.

- 1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
- 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
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Summarize and describe distributions.

- 4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- 5. Summarize numerical data sets in relation to their context, such as by:
- Reporting the number of observations.
- Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
- Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Key Elements	**Mathematical Content**	***By the end of 6th grade students will be able to:***
## *Number and Operation Goals* **Numbers, Number Systems and Number Relationships** ***Prime Time*** • Factors and Multiples • Classifying Numbers by the Sum of the Proper Factors • The Multiplicative Identity • Finding Near Perfect Numbers • Formal Proofs About Even and Odd Numbers • The Fundamental Theorem of Arithmetic • Common Factors and Common Multiples • The Least Common Multiples • The Relationship of Factor Pairs to the Square Root of the Number • Finding Prime Factorizations • Classifying Number 1 • Finding Greater Prime Numbers • Using Prime Factorizations to Find the Greatest Common Factor and the Least Common Multiple	## *Number and Operation Goals* **Number Sense** • Use numbers in various forms to solve problems • Understand and use large numbers, including in exponential and scientific notation • Reason proportionally in a variety of contexts using geometric and numerical reasoning, including scaling and solving proportions (• Compare numbers in a variety of ways, including differences, rates, ratios, and percents and choose when each comparison is appropriate • Order positive and/or negative rational numbers • Express rational numbers in equivalent forms • Make estimates and use benchmarks **Operations and Algorithms** Develop understanding and skill with all four arithmetic operations on fractions and decimals Develop understanding and skill in solving a variety of percent problems Use the order of operations to write, evaluate, and simplify numerical expressions Develop fluency with paper and pencil computation, calculator use, mental calculation, estimation; and choose	• Understand relationships among factors, multiples, divisors, and products • Recognize and use properties of prime and composite numbers, even and odd numbers, and square numbers • Develop strategies for finding factors and multiples, least common multiples, and greatest common factors • Recognize and use the fact that every whole number greater than one can be written in exactly one way as a product of prime numbers • Develop ways to model situations involving fractions, decimals, and percents to reason about situations • Understand and use equivalent fractions <> • Recognize and use properties of prime and composite numbers, even and odd numbers, and square numbers • Use rectangles to represent the factor pairs of numbers • Develop strategies for finding factors and multiples, least common multiples, and greatest common factors • Recognize and use the fact that every whole number greater than one can be written in exactly one way as a product of prime numbers • Use factors and multiples to solve problems and to explain some numerical facts of everyday life • Develop a variety of strategies for solving problems-building models, making lists and tables,

• The Relationship Between the Greatest Common Factor and the Least Common Multiple • Applied Problems ***Bits and Pieces I*** • Interpreting the Notation for a Fraction • Interpretations of Fractions o Fractions as Parts of a Whole o Fractions as Measures or Quantities o Fractions as Indicated Divisions o Fractions as Decimals o Fractions as Percents • Models of Fractions, Decimals, and Percents o Fraction-Strip Models o Number-Line Models o Partition Models o Grid-Area Models o Percent Bar Models • Equivalence of Fractions • Fraction Benchmarks • Fractions Between Fractions • Place Value Notation for Decimal Fractions • Ordering Decimals ***Bits and Pieces II*** • Writing Number Sentences	among these when solving problems **Fractions and Percents** • Compare and order fractions and percent representations • Move flexibly among fraction and decimals • Use physical models and drawings to help reason about a situation • Use context to help reason about a situation • Use estimation to understand a situation Rectangles can be used to represent the factor pairs of numbers Factors and multiples can be used to solve problems and to explain some numerical facts of everyday life A variety of strategies for solving problems-building models, making lists and tables, drawing diagrams, and solving simpler problems can be developed. ***Factors*** Numbers can be used in various forms to solve problems Understand and use large numbers, including in exponential and scientific notation We can reason proportionally in a variety of contexts using geometric and numerical reasoning, including scaling and	drawing diagrams, and solving simpler problems • Build an understanding of fractions, decimals, and percents and the relationships between and among these concepts and their representations • Develop ways to model situations involving fractions, decimals, and percents • Understand and use equivalent fractions to reason about situations • Compare and order fractions • Move flexibly among fraction, decimal, and percent representations • Use benchmarks such as 0, 1/2, and 1 to help estimate the size of a number or sum • Develop and use benchmarks that relate different forms of representations of rational numbers (for example, 50% can be represented as 0.5) • Use physical models and drawings to help reason about a situation • Look for patterns and describe how to continue the pattern • Use context to help reason about a situation • Use estimation to understand a situation • •

• Developing Algorithms • Estimation • Addition and Subtraction • Multiplication • Developing the Multiplication Algorithm • Using Distribution as a Strategy to Multiply Fractions • Division • Understanding Division as an Operation • Developing a Division Algorithm • Multiplying by the Denominator and Dividing by the Numerator • Multiplying by the Reciprocal • Relating Multiplication and Division • Inverse Relationships • Use benchmarks and other strategies to estimate the reasonableness of results of operations with fractions • Develop ways to model sums, differences, products, and quotients with areas, strips, and number lines • Use estimates and exact solutions to make decisions • Use knowledge of fractions and equivalence of fractions to develop algorithms for adding, subtracting, multiplying and dividing fractions • Recognize when addition, subtraction, multiplication, or division is the	solving proportions **Properties** Understand the multiplicative structure of numbers, including the concepts of prime and composite numbers, evens, odds, and prime factorizations • Use benchmarks such as 0, 1/2, and 1 to help estimate the size of a number or sum • Develop and use benchmarks that relate different forms of representations of rational numbers (for example, 50% can be represented as 0.5) • Look for patterns and describe how to continue the pattern	•

appropriate operation to solve a problem • Write fact families to show the inverse relationship between addition and subtraction, and between multiplication and division • Solve problems using operations on fractions		• • • • •
• **Geometry and Measurement** ***Shapes and Designs* (Geometry)** • Explore parallel lines and angles created by lines intersecting parallel lines • Find patterns that help determine angle sums of polygons • Reason about and solve problems involving shapes ***Covering and Surrounding*** **Shapes and Designs** • Polygon • Sorting Shapes • Symmetries of Shapes • Tessellations • Angle Measures • Angles of a Polygon • Angles and Parallel Lines • Sums of the Angles of Polygons • Interior Angles of Regular Polygons	**Geometry and Measurement Goals** **Shapes and Their Properties** • Generate important examples of angles, lines, and two- and three-dimensional shapes • Categorize, define, and relate figures in a variety of representations • Understand principles governing the construction of shapes with reasons why certain shapes serve special purposes(e.g. triangles for trusses) • Build and visualize three-dimensional figures from various two-dimensional representations and vice versa • Recognize and use shapes and their properties to make mathematical arguments and to solve problems • Use the Pythagorean Theorem and properties of special triangles (e.g. isosceles right triangles) to solve problems • Use a coordinate grid to describe and investigate relationships among shapes • Recognize and use standard, essential geometric vocabulary **Transformations-Symmetry, Similarity, and Congruence** • Recognize line, rotational, and translational symmetries and use them to solve problems • Use scale factor and ratios to create similar figures	**Geometry and Measurement** • ***GEOMETRY*** • Build on knowledge of operations on fractions and whole numbers • Develop and use benchmarks and other strategies to estimate the answers to computations with decimals • Develop meaning of and algorithms for operations with decimals • Use the relationship between decimals and fractions to develop and understand why decimal algorithms work • Use the place value interpretation of decimals to make sense of short-cut algorithms for operations • Generalize number patterns to help make sense of decimal operations • Choose between addition, subtraction, multiplication or division as an appropriate operation to use to

- Exterior Angles of Regular Polygons
- Exploring Side Lengths of Polygons

Covering and Surrounding

- The Measurement Process
- Measuring Perimeter and Area
- Area and Perimeter of Rectangles
- Parentheses and Order of Operations
- Area of Triangles
- Area of Parallelograms
- Area and Circumference of Circles
- Estimating Perimeter and Areas of Irregular Figures
- Accuracy and Error
- Relationships Between Shapes and Size – Maximum and Minimum
- Fixed Area
- Fixed Perimeter

- Understand area and relate area to covering a figure
- Understand perimeter and relate perimeter to surrounding a figure
- Develop strategies for finding areas and perimeters of rectangular shapes and non- rectangular shapes
- Discover relationships between perimeter and area, including that each can vary while the other stays fixed
- Understand how the areas of simple geometric figures relate to each other (e.g. the area of a parallelogram is twice the area

or determine whether two or more shapes are similar or congruent

- Predict ways that similarity and congruence transformations affect lengths, angle measures, perimeters, areas, volume, and orientation
- Investigate the effects of combining one or more transformations of a shape
- Identify and use congruent triangles and/or quadrilaterals to solve problems about shapes and measurement
- Use properties of similar figures to solve problems about shapes and measurement (7)
- Use a coordinate grid to explore and verify similarity and congruence relationships (7, 8)

Measurement

- Understand what it means to measure an attribute of a figure or a phenomenon
- Estimate and measure angles, line segments, areas, and volumes using tools and formulas
- Relate angle measure and side lengths to the shape of a polygon
- Find area and perimeter of rectangles, parallelograms, triangles, circles, and irregular figures
- Find surface area and volume of rectangular solids, cylinders, prisms, cones, and pyramids and find the volume of spheres
- Relate units within and between the customary and metric systems
- Use ratios and proportions to derive indirect measurements
- Use measurement concepts to solve problems

Geometric Connections

- Use geometric concepts to build understanding of concepts in other areas of mathematics (6, 7, 8)
- Connect geometric concepts to concepts in other areas of mathematics (6, 7, 8)

solve a problem

- Understand that decimals are often associated with measurements in real world situations
- Solve problems using operations on decimals
- Use understanding of operations and the meaning of percents to solve percent problems of the form a% of b equals c for any one of the variables a, b, or c.
- Create and interpret circle graphs
- Estimate the size of any angle using reference to a right angle and other benchmark angles
- Use an angle ruler for making more accurate angle measurements
- Determine which polygons fit together to cover a flat surface and why
- Explain the property of triangles that makes them useful as a stable structure for building
- Find that the sum of any two side lengths of a triangle is greater than the third side length
- Find that the sum of any three side lengths of a quadrilateral is greater than the fourth side length
- Understand some important properties of polygons and recognize polygonal shapes both in and out of the classroom

Shapes and Designs **(Geometry)**

of a triangle with the same base and height) • Develop formulas and procedures-stated in words and/or symbols-for finding areas and perimeters of rectangles, parallelograms, triangles, and circles • Develop techniques for estimating the area and perimeter of an irregular figure • Recognize situations in which measuring perimeter or area will help answer practical questions • Decimal Multiplication and Division • Decimal Estimation • Developing Algorithms for Computing With Decimals • Decimal Forms of Rational Numbers • Percents • Working Backwards • Circle Graphs	• Explore parallel lines and angles created by lines intersecting parallel lines • Investigate the symmetries of a shape-rotational or reflectional •	• Understand some important properties of polygons and recognize polygonal shapes both in and out of the classroom • Investigate the symmetries of a shape-rotational or reflectional • Estimate the size of any angle using reference to a right angle and other benchmark angles • Use an angle ruler for making more accurate angle measurements • Find patterns that help determine angle sums of polygons • Determine which polygons fit together to cover a flat surface and why • Explain the property of triangles that makes them useful as a stable structure for building • Find that the sum of any two side lengths of a triangle is greater than the third side length • Find that the sum of any three side lengths of a quadrilateral is greater than the fourth side length • Reason about and solve problems involving shapes • Understand perimeter and relate perimeter to surrounding a figure • Develop strategies for finding areas and perimeters of rectangular shapes and non- rectangular shapes • Understand how the areas of simple geometric figures relate to each other (e.g. the area of a parallelogram is twice the area of a triangle with the same base and height) • Develop formulas and procedures-stated in words and/or symbols-for finding areas and perimeters of rectangles, parallelograms, triangles, and circles • Develop techniques for estimating the area and perimeter of an irregular figure • Recognize situations in which measuring perimeter or area will help answer practical questions
Data and Probability ***How Likely Is It?* (Probability)**	• • ***Data and Probability Goals***	

The Meaning of Probability Strategies for Finding Outcomes o Organized List o Tree Diagram Experimental vs. Theoretical Probability o Comment on the Likelihood of 20 Heads o Comment on Random o Comment on the Use of Outcomes, Results, and Events o Law of Large Numbers o Comment on Area and Angles in a Spinner o Comment on Experimental and Theoretical Probabilities in Genetics ˅ Using Probabilities to Make Predictions and Decisions **Data About Us** Different Types of Data o Numerical Data o Categorical Data Distribution Data Reduction o Standard Graphs Line Plot Frequency Bar Graph Stem-and-leaf plot Coordinate graph o Measures of Center o Measures of Variation ˅ Covariation	**Formulating Questions** • Formulate questions that can be answered through data collection and analysis • Design data collection strategies to gather data to answer these questions • Design experiments and simulations to test hypotheses about probability situations **Data Collection** • Carry out data collection strategies to answer questions • Distinguish between samples and populations • Characterize samples as representative or non-representative, as random • Use these characterizations to evaluate the quality of the collected data **Data Analysis** • Organize, analyze, and interpret data to make predictions, construct arguments, and make decisions • Use measures of center and spread to describe and to compare data sets • Be able to read, create, and choose data representations, including bar graphs, line plots, coordinate graphs, box and whisker plots, histograms, and stem and leaf plots • Informally evaluate the significance of differences between sets of data • Use information from samples to draw conclusions about populations **Probability** • Distinguish between theoretical and experimental probabilities and understand the relationship between them (6) • Use probability concepts to make decisions	***Data and Probability*** • Understand that probabilities are useful for predicting what will happen over the long run • Understand the concepts of equally likely and not-equally likely • Understand that fairness implies equally likely outcomes. • Understand that there are two ways to build probability models: by gathering data from experiments (experimental probability) and by analyzing the possible equally likely outcomes (theoretical probability) • Understand that experimental probabilities are better estimates of theoretical probabilities when they are based on larger numbers of trials • Develop strategies for finding both experimental and theoretical probabilities • Critically interpret statements of probability to make decisions or answer questions <>***Data About Us*** **(Data Analysis)** • Understand and use the process of data investigation by posing questions, collecting data, analyzing data distributions, and making interpretations to answer questions • Represent data distributions using line plots, bar graphs, stem-and-leaf plots, and coordinate graphs • Compute the mean, median, or mode and the range of the data • Distinguish between categorical data and numerical data and identify which graphs and statistics may be used to represent each kind of data • Make informed decisions about which graph or graphs and which of the measures of center (mean,

Algebra

Finding Patterns

Examine and formalize patterns and relationships in words, graphs, tables, and with symbols.

Explore the relationship between the number of sides of a polygon and the sum of the interior angles of the polygon. They develop a rule for calculating the sum of the interior angle measures of a polygon with N sides.

Estimate the area.

Develop formulas and procedures-stated in words and symbols-for finding areas and perimeters of rectangles, parallelograms, triangles, and circles.

Addition and subtraction are inverse operations and that multiplication and division are inverse operations. This is a fundamental idea in equation solving.

Use these ideas to find a missing factor or addend in a number sentence.

Represent and interpret graphs for the relationship between variables, such as the relationship between length of an arm span and height of a person, using words, tables, and graphs.

Patterns of Change-Functions

- Identify and use variables to describe

- Find and interpret expected value
- Compute and compare the chances of various outcomes, including two-stage outcomes

Algebra Goals

Patterns of Change-Functions

- Identify and use variables to describe relationships between quantitative variables in order to solve problems or make decisions
- Recognize and distinguish among patterns of change associated with linear, inverse, exponential and quadratic functions

Representation

- Construct tables, graphs, symbolic expressions and verbal descriptions and use them to describe and predict patterns of change in variables (7, 8)
- Move easily among tables, graphs, symbolic expressions, and verbal descriptions (7, 8)
- Describe the advantages and disadvantages of each representation and use these descriptions to make choices when solving problems (7, 8)
- Use linear, inverse, exponential and quadratic equations and inequalities as mathematical models of situations involving variables (7, 8)

Symbolic Reasoning

- Connect equations to problem situations (
- Connect solving equations in one variable to finding specific values of functions
- Solve linear equations and inequalities and simple quadratic equations using symbolic methods
- Find equivalent forms of many kinds of equations,

median, or mode) and range may be used to describe a data distribution

- Develop strategies for comparing data distributions

Algebraic Expressions

By the end of 6th grade students will be able to:

- Describe, extend and create numerical and geometric patterns using models.

- Generalize a pattern, relation or function in words, tables, and/or graphs.

- Analyze patterns and identify mathematical relationships to pose and solve problems.

***Patterns* (Algebra)**

- Recognize problem situations in which two or more quantitative variables are related to each other
- Identify quantitative variables in situations
- Describe patterns of change between two variables that are shown in words, tables and graphs of data
- Construct tables and graphs to display relations among variables
- Use algebraic symbols to write equations relating variables
- Use tables, graphs, and equations to solve problems

relationships between quantitative variables in order to solve problems or make decisions • Recognize and distinguish among patterns of change associated with linear, inverse, exponential and quadratic functions **Representation** • Construct tables, graphs, symbolic expressions and verbal descriptions and use them to describe and predict patterns of change in variables • Move easily among tables, graphs, symbolic expressions, and verbal descriptions • Describe the advantages and disadvantages of each representation and use these descriptions to make choices when solving problems • Use linear, inverse, exponential and quadratic equations and inequalities as mathematical models of situations involving variables **Symbolic Reasoning** • Connect equations to problem situations • Connect solving equations in one variable to finding specific values of functions	including factoring simple quadratic equations • Use the distributive and commutative properties to write equivalent expressions and equations • Solve systems of linear equations • Solve systems of linear inequalities by graphing	• • •