1.6

Midpoint and Distance in the Coordinate plane

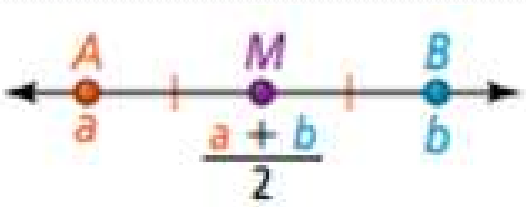
In a video game, two ancient structures shoot light beams toward each other to form a time portal. The portal forms exactly halfway between the two structures. Your character is on the grid shown as a blue dot. How do you direct your character to the portal? Explain how you found your answer.

Down ½ square and left 7 squares. This appears to be half-way between the two red dots.

Midpoint Formulas

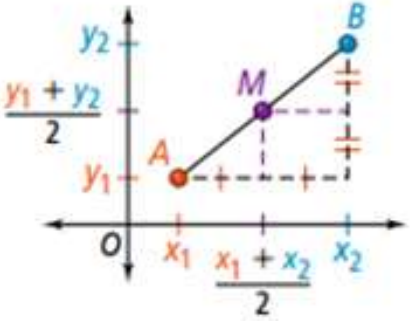
- On a Number Line, the midpoint is the **average** of the coordinates of the **endpoints**.

The coordinate of the midpoint M of $\overline{AB}$ is $\frac{a+b}{2}$.



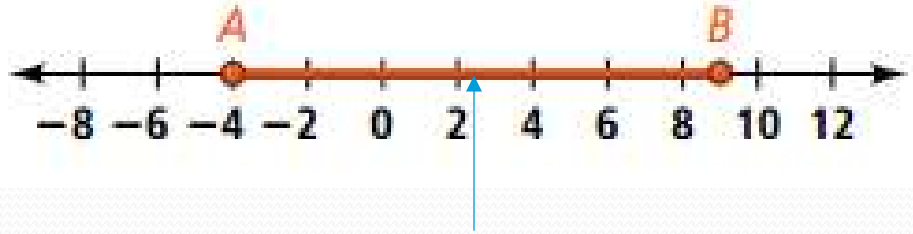
- In the Coordinate Plane, the coordinates of the midpoint are the average of the **x-coordinates** and the average of the **y-coordinates** of the **endpoints**.

Given $\overline{AB}$ where $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the midpoint of $\overline{AB}$ are $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



Examples:

1. $\overline{AB}$ **has endpoints at** -4 **and 9. What is the coordinate of its midpoint?**



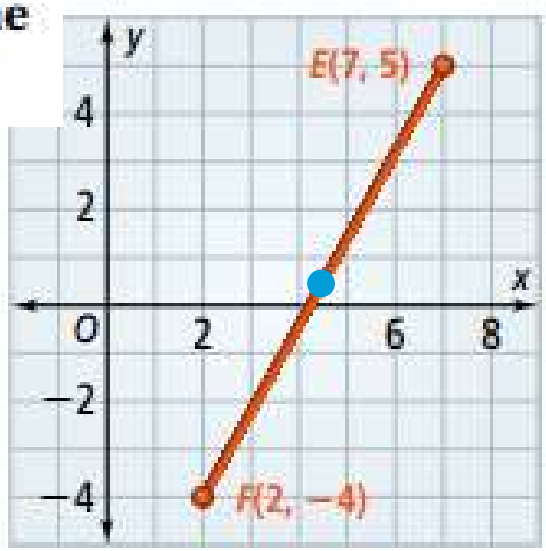
$$\frac{a+b}{2} = \frac{9+-4}{2} = 2.5$$

2. $\overline{EF}$ **has endpoints** $E(7, 5)$ **and** $F(2, -4)$**. What are the coordinates of its midpoint** M**?**

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{7+2}{2}, \frac{5+-4}{2}\right)$$

$$= (4.5, 0.5)$$



You Try!

Find the coordinate of the midpoint of the segment with the given endpoints.

1. 3 and 5

$$\frac{3+5}{2} = 4$$

2. -7 and 4

$$\frac{-7+4}{2} =$$

-1.5

Find the coordinates of the midpoint of $\overline{AB}$.

3. A(6,7) , B(4,3)

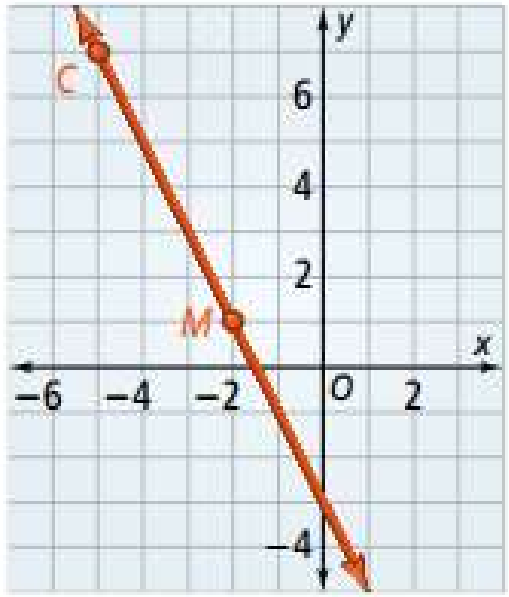
$$\left(\frac{6+4}{2}, \frac{7+3}{2}\right) = (5, 5)$$

4. A (14,-2), B (7, -8)

$$\left(\frac{14+7}{2}, \frac{-2+-8}{2}\right) = (10.5, -5)$$

Example:

3. The midpoint of $\overline{CD}$ is $M(-2, 1)$. One endpoint is $C(-5, 7)$. What are the coordinates of the other endpoint D?



Work Backwards!!!

M= $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(-2,1)= $\left(\frac{-5 + x_2}{2}, \frac{7 + y_2}{2}\right)$

Split it up into x and y equations, and solve!

$2 \cdot -2 = \frac{-5 + x_2}{2} \cdot 2$

Multiply each side by 2.

$-4 = -5 + x_2$

$+5 \quad +5$

$1 = x_2$

$2 \cdot 1 = \frac{7 + y_2}{2} \cdot 2$

$2 = 7 + y_2$

$-7 \quad -7$

$-5 = x_2$

The other endpoint is (1, -5).

You Try!

The coordinates of point Y are given. The midpoint of $\overline{XY}$ is $(3, -5)$. Find the coordinates of point X.

5. Y(0,2)

$(3, -5) = \left(\frac{0 + x_2}{2}, \frac{2 + y_2}{2}\right)$

$6 = 0 + x_2$ $\quad -10 = 2 + y_2$

$6 = x_2$ $\quad -12 = y_2$

$(6, -12)$

6. Y(-10,5)

$(3, -5) = \left(\frac{-10 + x_2}{2}, \frac{5 + y_2}{2}\right)$

$6 = -10 + x_2$ $\quad -10 = 5 + y_2$

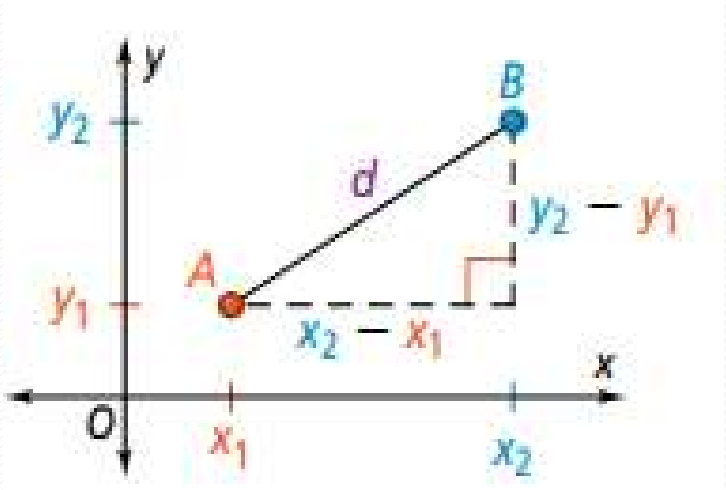
$16 = x_2$ $\quad -15 = y_2$

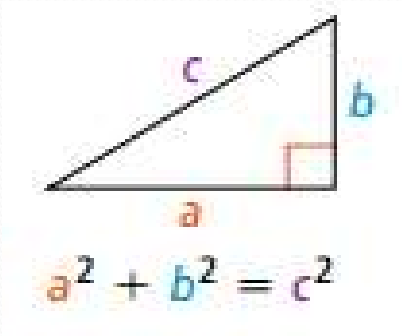
$(16, -15)$

The Distance Formula

- The **Distance between two points** (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$





- The distance formula is based on the Pythagorean Theorem, $a^2 + b^2 = c^2$, a formula for the three sides of a right triangle.

Example:

4. What is the distance between $U(-7, 5)$ and $V(4, -3)$? Round to the nearest tenth.

Let $U(-7, 5)$ be (x_1, y_1) and $V(4, -3)$ be (x_2, y_2).

Use the Distance Formula.

Substitute.

Simplify within the parentheses.

Simplify.

Use a calculator.

To the nearest tenth, $UV =$ _____

You Try!

Find the distance between each pair of points. If necessary, round to the nearest tenth.

7. A(6,7) B(-1,7)

$$d = \sqrt{(-1-6)^2 + (7-7)^2}$$

$$d = \sqrt{49+0}$$

d = 7

8. E (-1,7) F(12,0)

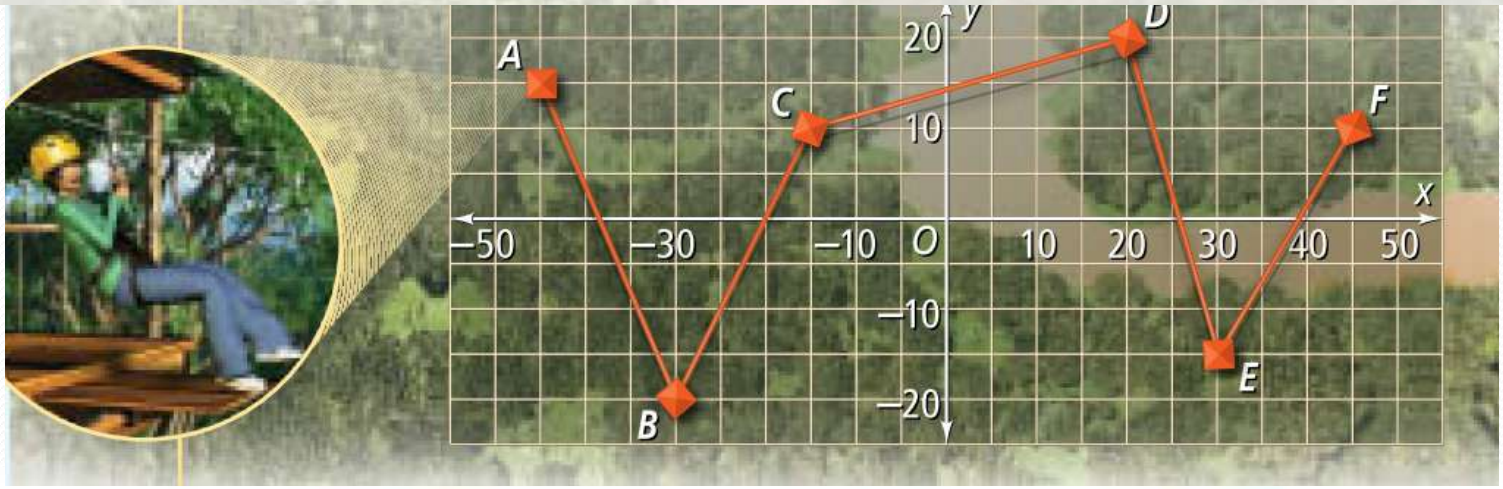
$$d = \sqrt{(12--1)^2 + (0-7)^2}$$

$$d = \sqrt{169+49} = \sqrt{218}$$

d ≈ 14.8

You Try!

9. **Recreation** On a zip-line course, you are harnessed to a cable that travels through the treetops. You start at Platform *A* and zip to each of the other platforms. How far do you travel from Platform *B* to Platform *C*? Each grid unit represents 5 m.



Let Platform $B(-30, -20)$ be (x_1, y_1) and Platform $C(-15, 10)$ be (x_2, y_2).

Use the Distance Formula.

Substitute.

Simplify.

You travel about 33.5 m. between platform B and C.

FIND THE ERROR!

Error Analysis Your friend calculates the distance between points $Q(1, 5)$ and $R(3, 8)$. What is his error?

$$d = \sqrt{(1 - 8)^2 + (5 - 3)^2}$$
$$= \sqrt{(-7)^2 + 2^2}$$
$$= \sqrt{49 + 4}$$
$$= \sqrt{53} \approx 7.3$$

Your friend did not subtract the x-coordinates and then the y-coordinates.

It should be: $d = \sqrt{(3 - 1)^2 + (8 - 5)^2} = \sqrt{4 + 9} \approx 3.6$