Problem of the Month: *Slice and Dice*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month, *Slice and Dice,* students investigate classic number theory problems. They explore number operations, organized lists and counting methods to solve problems. The mathematical topics that underlie this POM are knowledge of number sense, addition, subtraction, division, factors and divisibility, counting principles, systematic charting and closed-form equations. The mathematics that includes counting principles and systematic charting is often referred to as discrete mathematics. In addition students use spatial visualization and model with geometric figures. The mathematics includes optimizing and deriving generalizations. Students determine maximums, minimums and explicit functions to model and generalize geometric situations.

In the first level of the POM, students are asked to find the minimum number of cuts to create equal slices of bread for a family. Their task involves finding the least number of cuts to make bread slices for the seven members of the family. In level B, students are asked to find the minimum number of cuts to create equal slices of pumpkin bread for 14 people. Students then explain how to determine the minimum number of cuts for any number of people. In level C, students are asked investigate how to create pens to house pigs so no two pigs are together. The pigs are in a large rectangular pen that needs to be partitioned by fences. The students are asked to determine the greatest number of pens that can be formed by building six fences. In level D, the students investigate how twelve hot dogs can be shared equally among seven friends. Students then are asked how eight hot dogs are shared equally among eighteen friends. Students are to model and generalize the outcomes of sharing equally given any number of the hot dogs and any number of friends. In level E, students investigate points on a circle and the cords that connect the points. The students are asked to find the greatest number of regions into which a circle can be divided given n number of points on the circle. Students determine a generalization given any number of points.

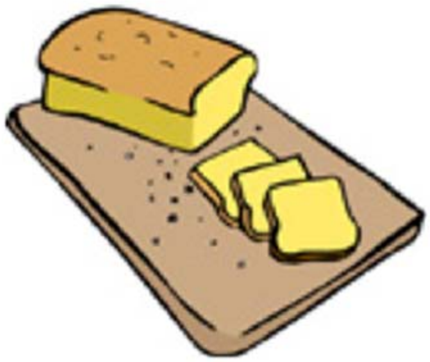
Mathematical Concepts

The big idea of this POM is number theory – the investigation of our real number system and its operations. Number theory is one of the oldest branches of pure mathematics, and one of the largest. Of course, it concerns questions about numbers, usually meaning whole numbers or rational numbers (fractions). Elementary number theory involves divisibility among integers -- the Euclidean algorithm (and thus the existence of greatest common divisors), elementary properties of primes (the unique factorization theorem, the infinitude of primes), etc. Tools for solving some number theory problems draw on ideas from discrete mathematics. A second big idea is geometry including spatial visualization and partitioning polygons. Students use optimization strategies to determine either maximums or minimums. A third big idea is finding generalizations and explicit functions. This requires students to recognize repeated reasoning to find rules, equations or functions and to model the geometric situation.

Problem of the Month
Slice & Dice

Level A

My mother bakes the best tasting banana bread in loaf pans. We have 7 people in our family. We all want as much of the bread as we can have. What is the least number of cuts my mother needs to make so everyone gets the same-sized slice from the whole loaf?



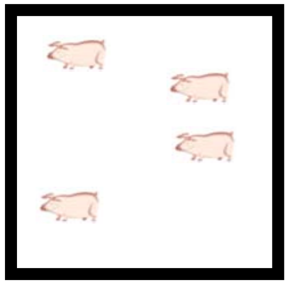
Explain your answer.

Level B
I am having a Halloween party and mom is making pumpkin bread in one very long loaf. There will be 14 people at my party. How many cuts will my mother need to make to give each of us the same amount? Find the least number of cuts that needs to be made.

Explain the least number of cuts needed to share the same-sized slices for any number of people who might attend a party? How do you know?

Level C

Peggy and Penny own a pig farm. Lately the pigs in the pen have been getting into fights and stealing each other's food. They own a large, square fenced-in area. They decide to build straight fences inside the fenced-in area in order to isolate each pig into its own individual pen.



The fenced-in area has 4 pigs. What is the minimum number of straight fences needed to divide the 4 pigs such that each pig has its own pen? Show your solution.

Suppose Peggy and Penny can build three straight fences. What number of pigs could they individually pen? Demonstrate how the fences could be individually divided and how many pigs can be accommodated. Justify your solutions.

Explain what number of pigs can be accommodated if they built 6 straight fences? What is the largest number of pigs that can have an individual pen?

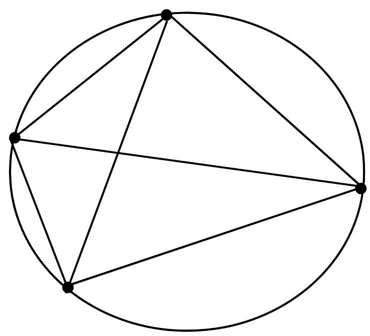
Level D
I would like to share 12 hotdogs among seven friends so all of the hotdogs are eaten and everyone gets the same amount. How much will each friend get? What is the minimum number of cuts needed to divide the hotdogs?

I would like to share 8 hotdogs among 18 friends so all of the hotdogs are eaten and everyone gets the same amount. How much will each friend get? What is the minimum number of cuts needed to divide the hotdogs?

What is the relationship between the number of hotdogs and the number of friends? Predict the minimum number of cuts given any number of hotdogs and any number of friends. Explain you reasoning.

Level E

Suppose that we have 4 points on a circle. We connect each point to each of the other points with a line segment. The greatest number of regions into which the circle can be divided is 8 regions. See the example below.



Four Points
Eight Regions

Imagine that we have ***n*** points on a circle. We connect each spot to each of the other points with a line segment. What is the greatest number of regions into which the circle can be divided? Determine a generalization given any number of points.

Problem of the Month
Slice & Dice

Primary Version Level A

Materials: For the teacher - Picture 1 of a cut loaf of bread and Picture 2 of an uncut loaf of bread. For each student - paper and pencil.

Discussion on the rug: Teacher shows Picture 1 of the bread with some slices cut. **"Look at this picture can someone tell us what they see?"** Students volunteer what they see. Teacher asks, **"How are slices made?"** Students respond. Teacher asks, **"How many times do you think the bread was cut?"** Teacher calls on students to explain. Teacher shows Picture 2 of the full loaf. **"If two people wanted to share this whole loaf of bread, how many times would they need to it?"** Class explores these ideas. After some individual think time, then pair share, the teacher asks, **"Who can explain the number times the bread needed to be cut? Please show me."** Teacher invites students to share.

In small groups: Students work in pairs. Teacher passes out Picture 2 of the whole loaf to each pair. The teacher says to the class, **"Four friends want to share this loaf of bread. How many times will your mother need to cut to get the pieces you need?"**

At the end of the investigation have students either discuss or dictate a response to this summary question: **"Tell me how many times the mother cut the bread to make the pieces for the four friends. Tell me how you figured it out."**

Picture 1

Picture 2

Problem of the Month
Slice and Dice
Task Description – Level A
This task challenges students to use geometric objects and number concepts. The students are asked to find the minimum number of cuts to create equal slices of bread for a family. Their task involves finding the least number of cuts to make bread slices for the seven members of the family.
Common Core State Standards Math - Content Standards
<u>**Operations and Algebraic Thinking**</u> **Represent and solve problems involving addition and subtraction.** 1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. <u>**Geometry**</u> **Reason with shapes and their attributes.** 2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.2 Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Problem of the Month
Slice and Dice
Task Description – Level B
This task challenges students to use geometric objects and number concepts. The students are asked to find the minimum number of cuts to create equal slices of pumpkin bread for 14 people. Students then explain how to determine the minimum number of cuts for any number of people.
Common Core State Standards Math - Content Standards
<u>Operations and Algebraic Thinking</u> **Represent and solve problems involving addition and subtraction.** 2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem**.** **Operations and Algebraic Thinking 3.OA** **Solve problems involving the four operations, and identify and explain patterns in arithmetic**. 8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding **<u>Geometry</u>** **Reason with shapes and their attributes.** 3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month
Slice and Dice
Task Description – Level C
This task challenges a student to investigate a problem involving geometric shapes and number patterns. The students are asked to investigate how to create pens to house pigs so no two pigs are together. The pigs are in a large rectangular pen that needs to be partitioned by fences. The students are asked to determine the greatest number of pens that can be formed by building six fences.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 4.G.1.Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **<u>Operations and Algebraic Thinking</u>** **Use the four operations with whole numbers to solve problems.** 4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Generate and analyze patterns.** 4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month:
Slice and Dice
Task Description – Level D
This task challenges students to investigate a problem involving number patterns. The students investigate how twelve hot dogs can be shared equally among seven friends. Students then are asked how eight hot dogs are shared equally among eighteen friends. Students are to model and generalize the outcomes of sharing equally given any number of hot dogs with any number of friends.
Common Core State Standards Math - Content Standards
<u>Operations and Algebra</u> **Gain familiarity with factors and multiples.** 4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of the factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is a prime or composite. **<u>Expressions and Equations</u>** **Represent and analyze quantitative relationships between dependent and independent variables.** 6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variable using graphs and tables, and relate these to the equations. **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
Common Core State Standards Math – Standards of Mathematical Practice
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month
Slice and Dice
Task Description – Level E
This task challenges students to investigate problems involving geometric patterns and finding generalizations. The students investigate points on a circle and the cords that connect the points. The students are asked to find the greatest number of regions into which a circle can be divided given n number of points on the circle. Students determine a generalization given any number of points.
Common Core State Standards Math - Content Standards
<u>**High School – Functions – Building Functions**</u> **Build a function that models relationship between two quantities.** F-BF.1 Write a function that describes a relationship between two quantities. **Interpret expressions for functions in terms of the situation they model.** F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.
Common Core State Standards Math – Standards of Mathematical Practice
MP.7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(\mathbf{x} – \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation (y -2)/(x-1) = 3. Noticing the regularity in the way terms cancel when expanding (x-1)(x+1), $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
Slice and Dice
Task Description – Primary Level
This task challenges students to use geometric objects and number concepts. The students are asked to find the minimum number of cuts to create equal slices of bread for four people. Their task involves finding the number of cuts to make 4 bread slices.
Common Core State Standards Math - Content Standards
Operations and Algebraic Thinking **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **Geometry** **Reason with shapes and their attributes.** 1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(\mathbf{x} – \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.