Problem of the Month: *Miles of Tiles*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Miles of Tiles*, students are engaged in tasks that involve puzzles of number relationships, equations, and simultaneous constraints. The mathematical topics that underlie this POM are measurement, number sentences, area models, variables, inverse operations, equations, quadratics, factoring, and simultaneous systems.

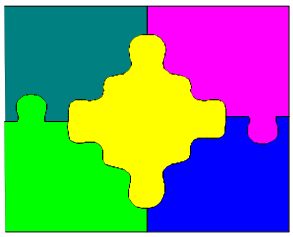
In the first levels of the POM, students are presented with a picture frame and a set of tiles of differing lengths; the task is to find different arrangements of the set of tiles to make a decorative frame. Their task involves finding number sentences that will sum to the region surrounding the picture. In Level B, students are presented

with an order 4 Magic Square. Students need to use number sense and a guess-and-check method to determine an arrangement of number tiles that makes the puzzle a magic square. In Level C, students are presented with a patio made of uniform rectangular tiles. Only one dimension of the rectangular patio is given and the task is to determine the dimensions of the uniform tile. This situation can be translated into systems of constraints with equal numbers of unknowns. In Level D, students are given a situation that involves a set of two non-commensurate square tiles. A rectangular tile is constructed with the length, the size of the larger tile, and the width the size of the smaller tile. Students are asked to create rectangular designs using 6 large tiles, 4 small tiles, and an unlimited number of rectangles. The task involves developing an understanding of an area model and its dimensions. Algebraic reasoning is central to this task. In the final level, students are asked to generalize the principles of the area model and its dimensions in Level D. Students will need to use quadratic relationships as well as products and factors to generalize the mathematics.

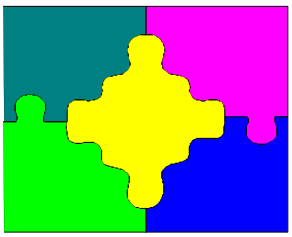
Mathematical Concepts

Algebra is the cornerstone of secondary mathematics. Algebraic thinking is taught in primary grades, with the foundations of algebra taught usually by the end of middle school. Even though the term algebraic thinking is routinely used, it cannot be simply defined. The underpinnings of algebra involve abstractions and language. There are several resources that define the most important concepts in algebra. One resource is *Fostering Algebraic Thinking.*

Problem of the Month

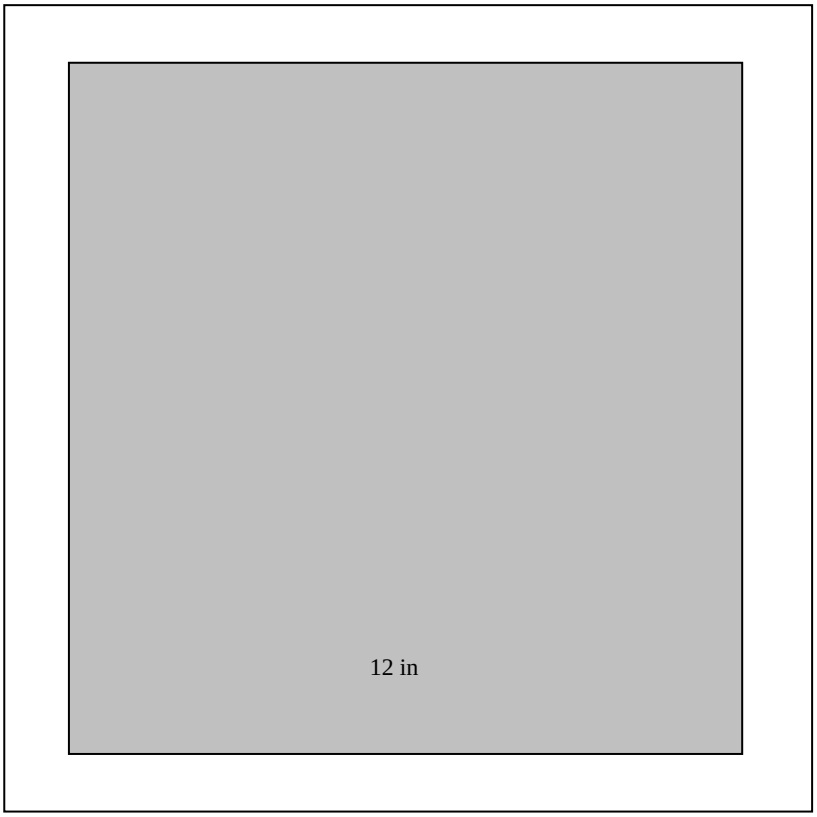


Miles of Tiles

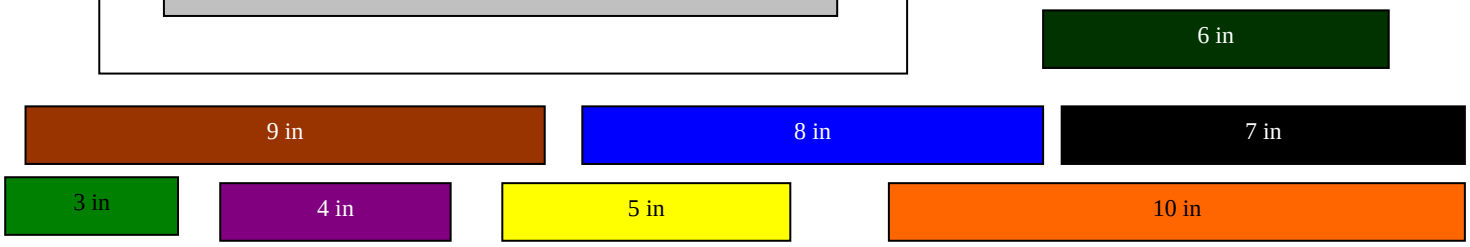


Level A:

You have a picture frame. You would like to decorate the frame by gluing tiles on it. The frame is a square shape.



The frame is 1 inch wide all around. The inside of the frame is a 12 by 12 inch square. The outside of the frame is a 14 by 14 inch square.
There are 8 tiles, each a different length (3, 4, 5, 6, 7, 8, 9 and 10 inches).

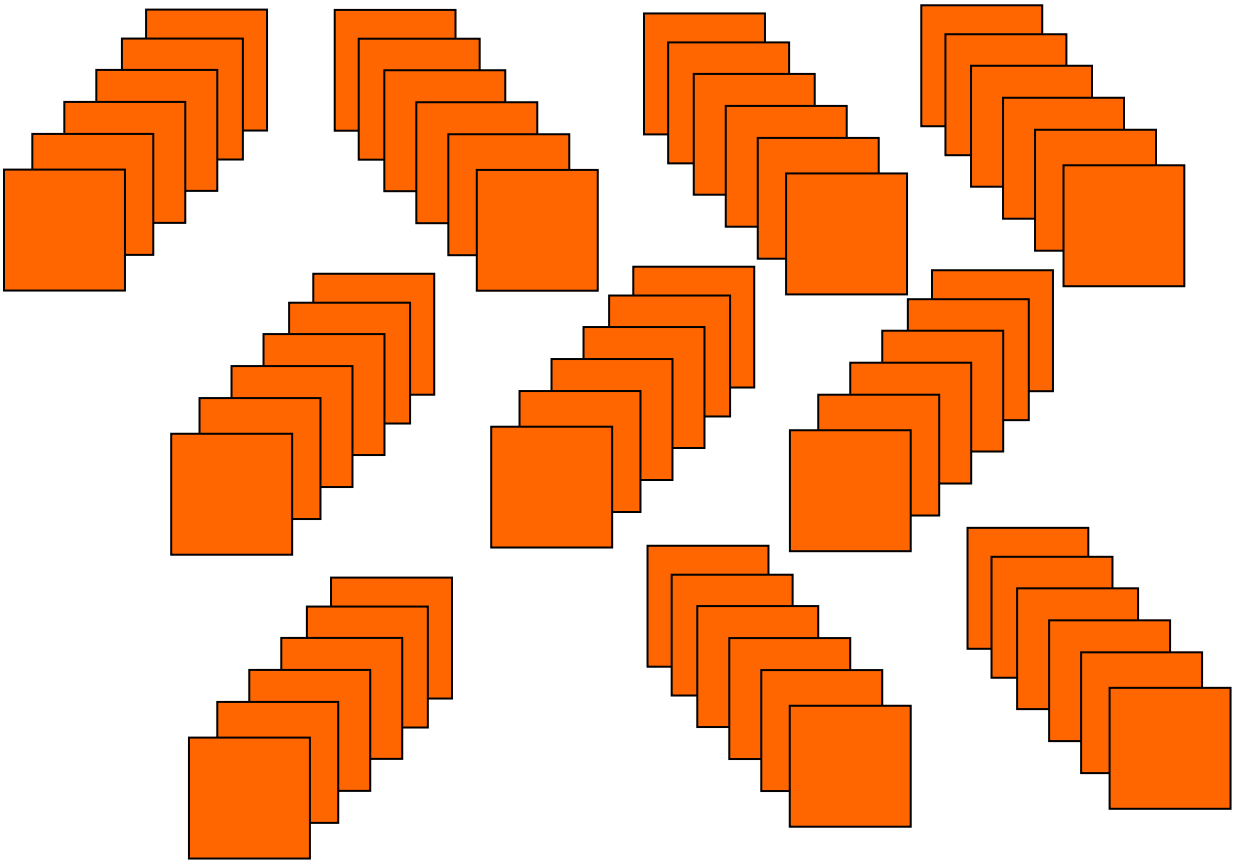


Determine how the tiles should be placed in order to cover the frame with tiles. How many different arrangements can you make? Explain how you found your answers.

Level B:

You work for a puzzle company and your job is to write the solutions to the puzzles that have been designed.

You have been assigned to determine the solution to the puzzle called *Transforming Tiles*. The puzzle provides 60 tiles. The object is to arrange all 60 tiles into a rectangle.



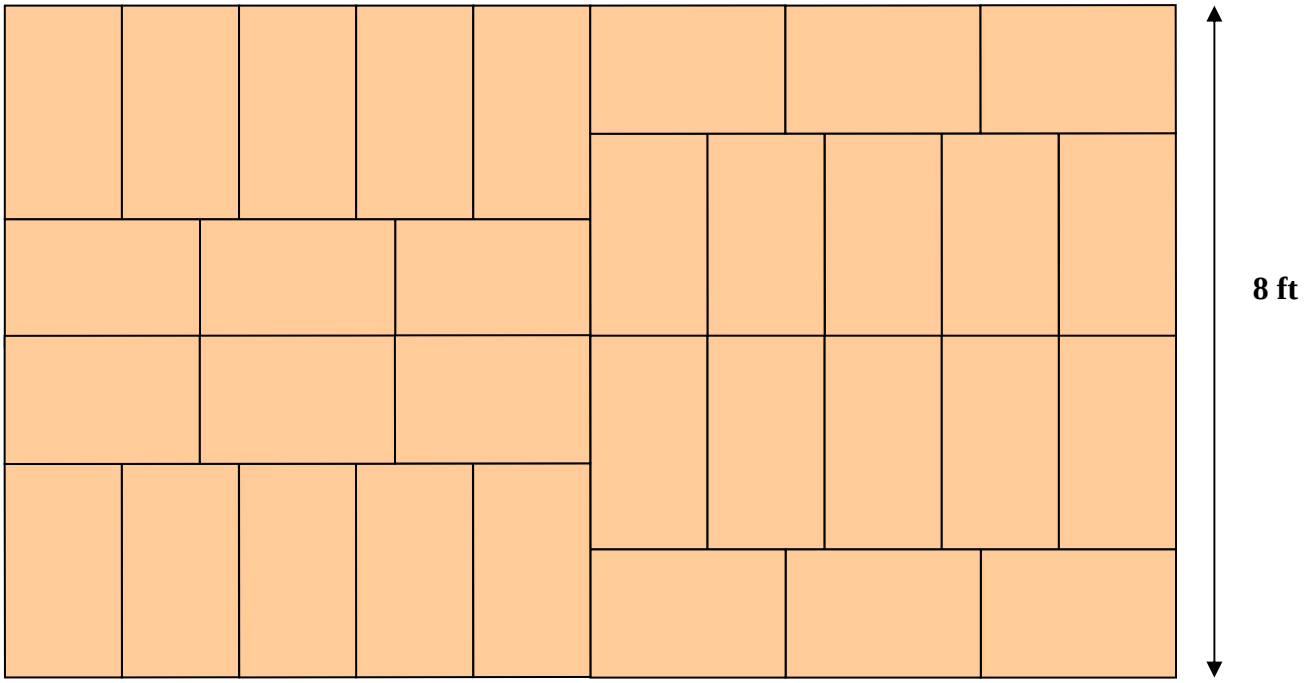
Determine all the arrangements of the tiles that can be constructed into a rectangle. Name the arranged rectangle by the number of rows and the number of columns of tiles. Explain how you found your solution.

How many possible arrangements are there that fit the conditions?

How do you know that you have found all the arrangements?

Level C:

You work for a tile company that makes tiles for patios. A customer sent you the following picture of his patio. He said the patio is made up of the same tiles, positioned either vertically or horizontally. He says he wants to replace three tiles that are cracked. He didn't tell you the dimensions of the tile itself but did tell you that the width of the patio was 8 ft.

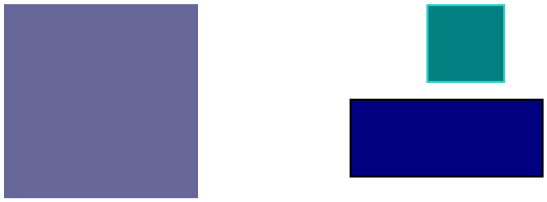


Determine the dimensions of a tile in the patio.

Explain how you found your solution.

Level D:

You work for a tile manufacturing company. The company has over-stocked a certain set of tiles. There are three different tiles in this particular set. One is a large square tile, the second is a small square tile and the third is a rectangle. The length of the rectangle is the same length as one side of the large square. The width of the rectangle is the same size as one side of the small square.



This happens to work out to be good news. A set of these tiles can be arranged into rectangular configurations to create nice tile patterns.

Find all the rectangular configurations that can be made using 6 large squares and 4 small squares along with a certain number of rectangles.

How many rectangles are needed to make a rectangular configuration? Explain how you know.

Illustrate all the different configurations that can be created.

Explain how you know that you have found all possible rectangular configurations.

Level E:

Your boss is proud of how you handled the over-stocked tile problem. Your boss wants to know your secret. So the boss has asked you to address these questions.

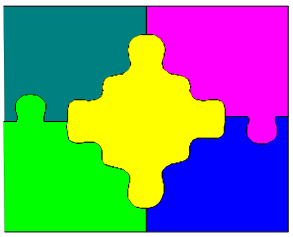
First, determine whether or not you can make rectangular configurations given any number of large and small squares. In these configurations you can use as many rectangles as needed. If so, how would you proceed? If not, how would you know for sure and what else might you do?

Secondly, if you were given a specific number of large squares, small squares and rectangles, determine if you could always make a rectangular configuration from those tiles? If so, how would you proceed? If not, how would you know for sure?

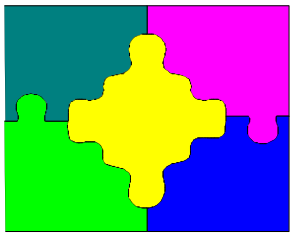
Write a memo to the boss that would address the questions listed above.

Remember your tiling career hangs in the balance, so you need to use mathematics to explain your reasoning.

Problem of the Month



Miles of Tiles



Primary Version Level A

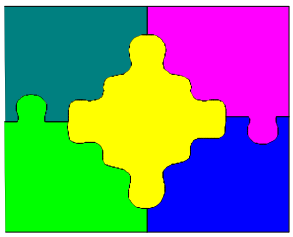
Materials: Set of Cuisenaire Rods per group, paper frame and paper tiles

Discussion on the rug: Teacher holds up the picture of the empty picture frame. **"We want to make this frame colorful. How might we make the frame colorful?"** Teacher solicits answers from students. **"Those are good answers, but I was thinking we could use color tiles."** Teacher hold up some tiles. **"How could we use these tiles to color the frame**?" Students demonstrate. **"What do we need to be careful about in placing these tiles?"** Teacher questions students to clarify that the tiles must not overlap or gap in order to fit in the space provided.

In small groups: Each group has an empty picture frame and a set of tiles. The teacher states, **"We want to make the frame colorful. Try to put the different tiles around the frame. Can you make them fit? We don't want white area showing (gaps). We need to be careful not to cover other tiles (overlap)."**

Students use rods to guess and check. Once they have found a solution they may glue the paper tiles down on the paper frame. Once the class has completed their frames, process the activity by looking at student work and asking, **"How are the frames alike? Different? How do we know whether the tiles fit? How many different frames do we have? What parts of the frames are alike? Why? Explain how to design your frame."**

At the end of the investigation have students either discuss or dictate a response to this summary question.



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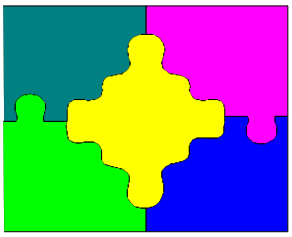

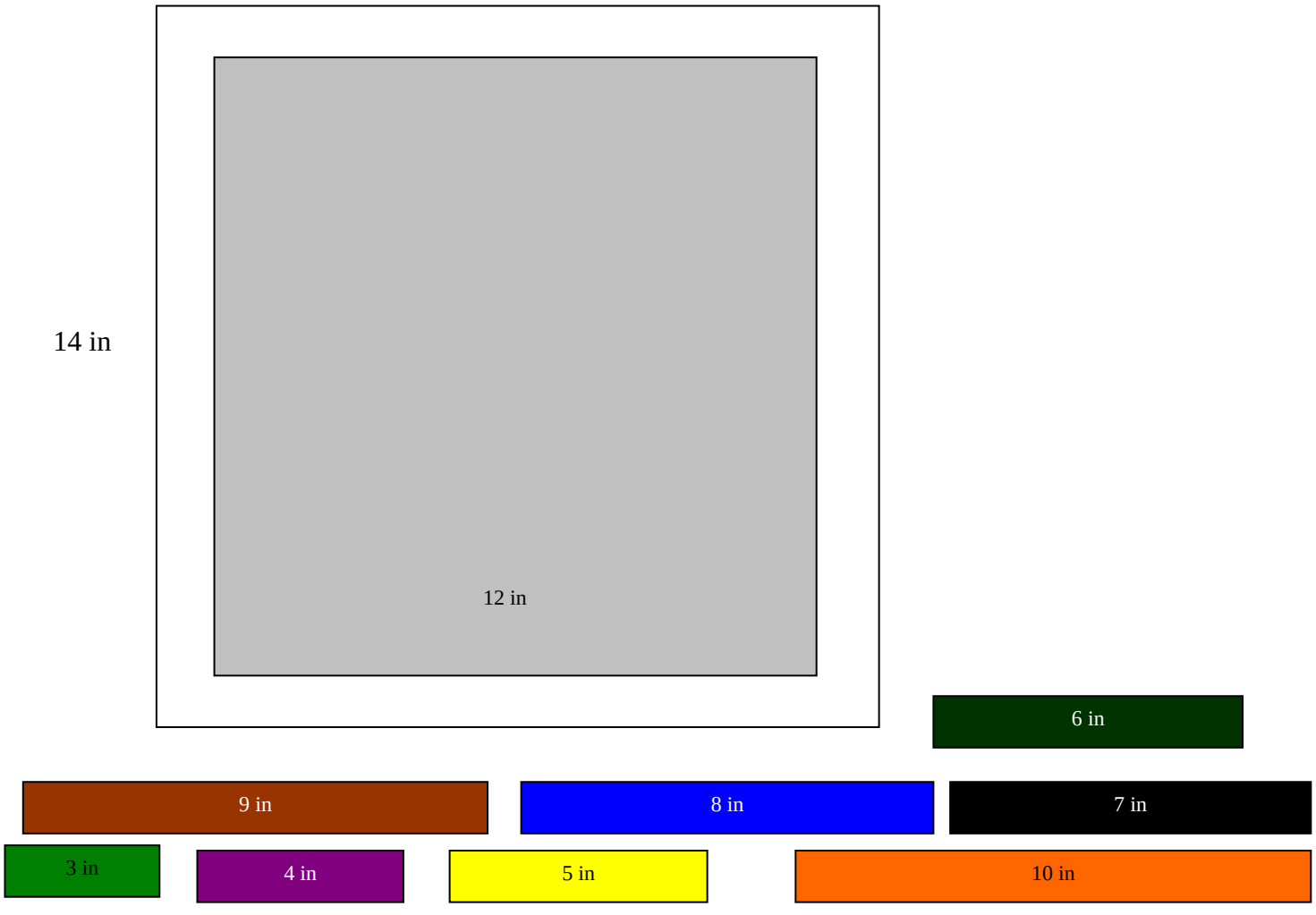

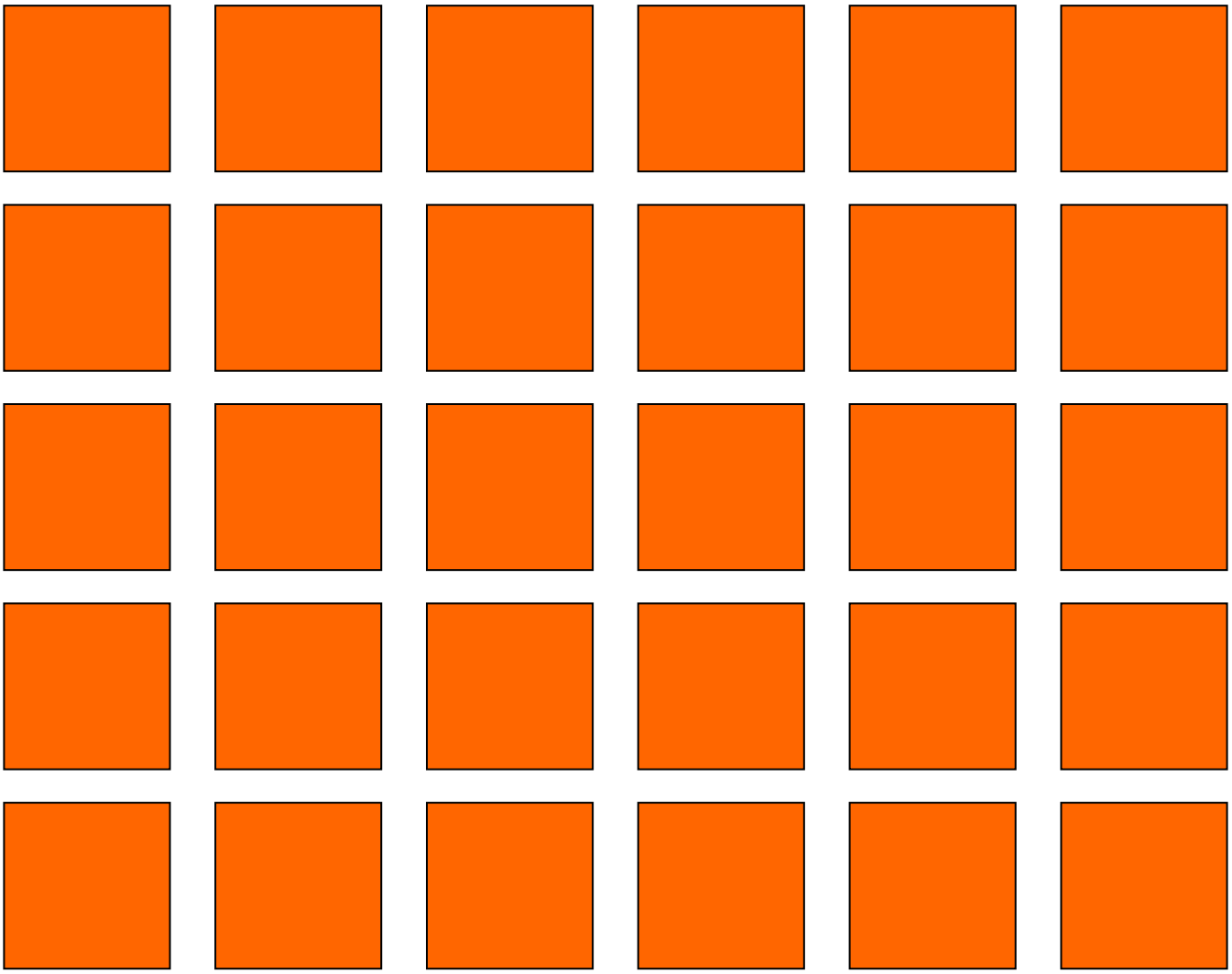

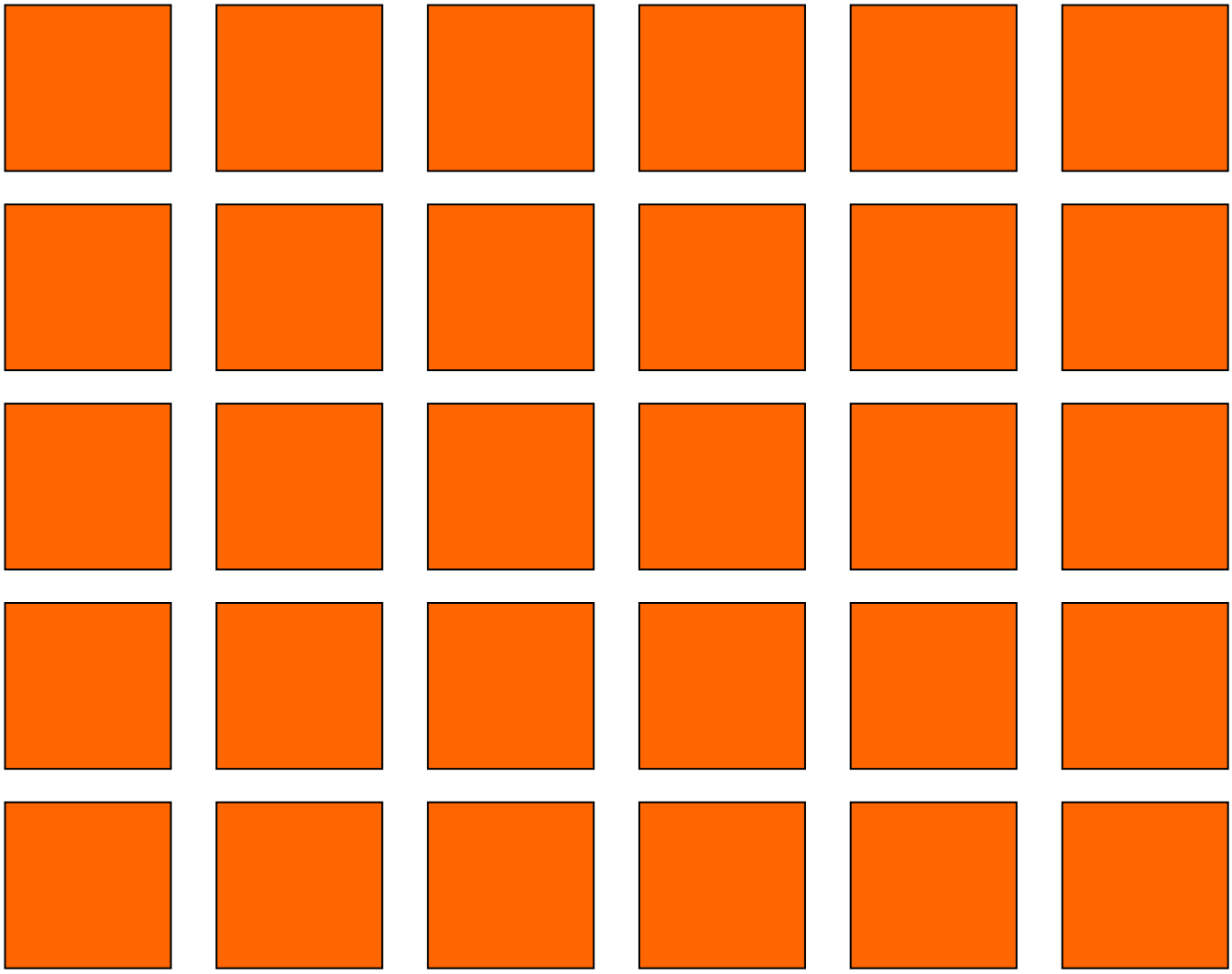

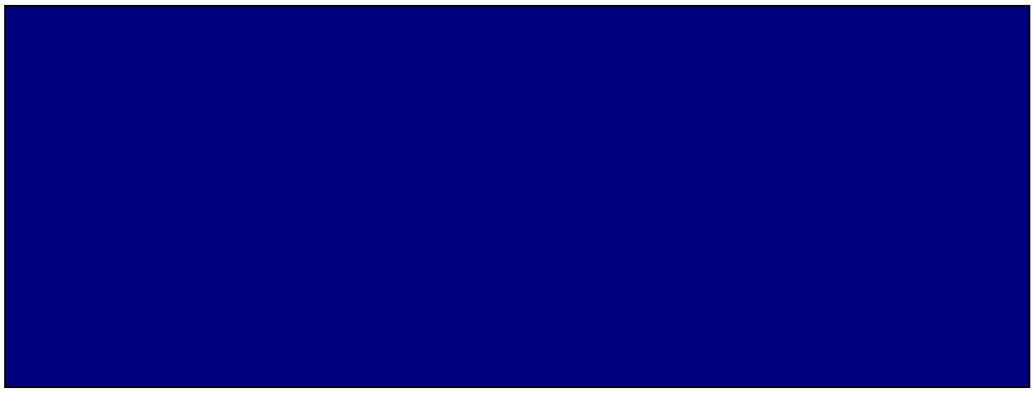

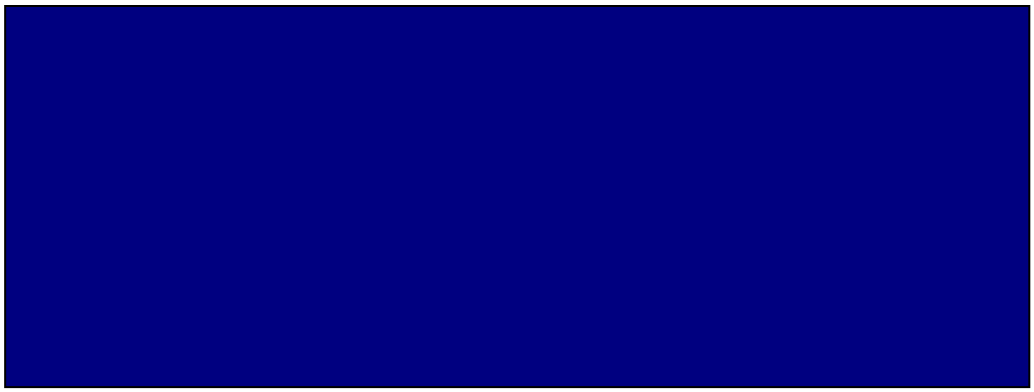

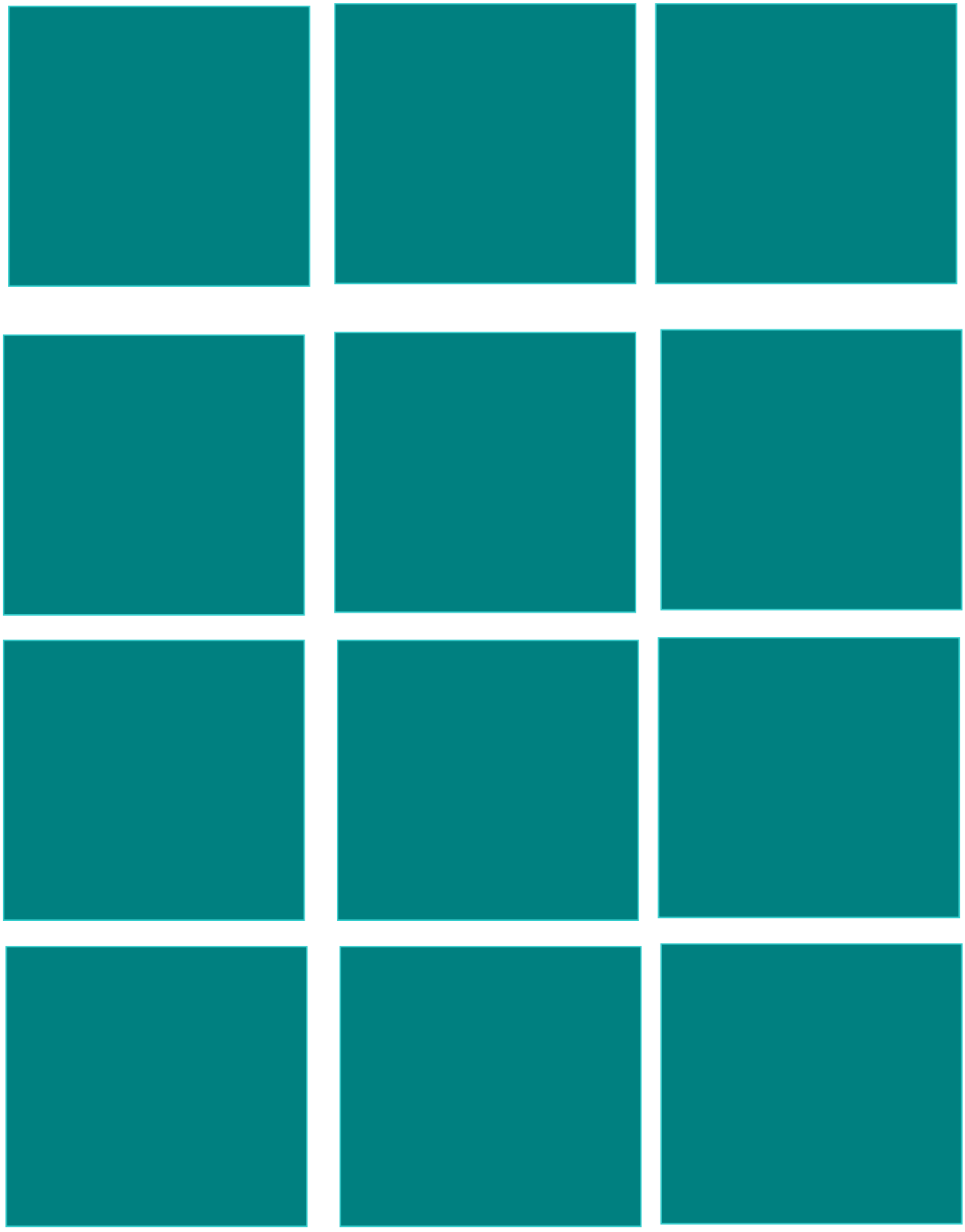

Problem of the Month

Miles of Tiles

Task Description – Level A

This task challenges a student to work with a picture frame and a set of tiles of differing lengths to find different arrangements of the set of tiles to make a decorative picture frame. The task involves finding number sentences that sum to the region surrounding the picture frame.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Reason with shapes and their attributes.

1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Problem of the Month

Miles of Tiles

Task Description – Level B

This task challenges a student to determine an arrangement of a given number of tiles to make a level four Magic Square. Students will use number sense and guess-and-check methods to help them solve this task.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Number and Operations

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Miles of Tiles

Task Description – Level C

This task challenges a student to use systems of equations. A student is given a picture of a patio made of uniform rectangular tiles. Only one dimension of a rectangular patio made with these uniform tiles is given and a student is asked to determine the dimensions of the uniform tile. This situation can be translated into systems of constraints with equal numbers of unknowns.

Common Core State Standards Math - Content Standards

Expressions and Equations

Reason about and solve one-variable equations and inequalities.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

8.EE.8.c Solve real-world and mathematical problems leading to two linear equations in two variables.

High School – Algebra - Creating Equations

Create equations that describe numbers or relationships.

A-CED.2 Create equations in two or more variables to represent relationships between quantities…

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month:

Miles of Tiles

Task Description – Level D

This task challenges a student to make a table and use guess-and-check. A student uses a set of three different-sized tiles. The length of the rectangular tile matches the side of the large square. The width of the rectangular tile matches the side of the small square. A student must create all the possible configurations which can be made using 6 large squares, 4 small squares, and an unlimited number of rectangles.

Common Core State Standards Math - Content Standards

Measurement and Data

Represent and interpret data.

1.MD.4 Organize, represent, and interpret data with up to three categories; …

Geometry

Analyze, compare, create and compose shapes.

K.G.6 Compose simple shapes to form larger shapes.

Reason with shapes and their attributes.

1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Miles of Tiles

Task Description – Level E

This task challenges a student to use products and factors as well as quadratic relationships. A student is asked to generalize the principles of the area model and its dimensions in Level D.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Expressions and Equations

Reason about and solve one-variable equations and inequalities.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Functions

Define, evaluate, and compare functions.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. …

High School – Algebra - Creating Equations

Create equations that describe numbers or relationships.

A-CED.2 Create equations in two or more variables to represent relationships between quantities…

High School – Algebra - Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-SSE.3.a Factor a quadratic expression to reveal the zeros of the function it defines.

High School – Functions - Building Functions

F-BF.1 Write a function that describes a relationship between two quantities.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.5 Use appropriate tools strategically.

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Problem of the Month

Miles of Tiles

Task Description – Primary Level

This task challenges a student to work with a picture frame and a set of tiles of differing lengths to find different arrangements of tiles to make a decorative picture frame. The task involves finding number sentences that sum to the region surrounding the picture frame. A student is asked to use paper Cuisenaire Rods to make the frame. A discussion is first held so that a student would understand that the frame cannot have gaps or overlaps of the different-colored strips of Cuisenaire Rods. At the end of the task, the whole class has a discussion on how the frames are alike and different. At the end of the investigation a student will dictate a response to the question, "Explain how to design your frame."

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Reason with shapes and their attributes.

1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.

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