The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." POMs may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to solve problems is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially as the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

#### **Overview**

In the Problem of the Month, *Got Your Number*, students use number properties, number operations, organized lists, counting methods, and probability to solve problems. The mathematical topics that underlie the POM are number sense, number properties, addition, subtraction, multiplication, division, fractions, representations of rational numbers, counting principles, systematic charting, probability, and generalizations.

In Level A of the POM, students are presented with a problem that involves summing three values. Their task involves picking three cards from five that were dealt in order to make a sum close to 20. The task is designed as a game for two

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students with the goal of keeping the difference from 20 as small as possible. Students conclude the game by describing their strategy. In Level B, their task involves picking four cards from six that were dealt in order to make two two-digit numbers whose sum is close to 100. The task is designed as a game for students with the goal of keeping the total difference from 100 as small as possible. In Level C. students pick four one-digit numbers from a stack of cards. They also spin a spinner that identifies one of the four basic operations. They are asked to determine the equation that produces the smallest result using those four digits and applying the identified operation to form two fractions. Level D involves a game between two students. Each student has to create the largest possible three-digit number. When a card is picked the student must place it in either the ones, tens, or hundreds place. Once a card is placed, it can't be moved. Students play independently and can't see one another's cards until the end when they compare their numbers to determine which number is greater. In Level E, students are asked to generalize their findings from Level D. Students are asked to predict and justify the best strategy for winning the game.

# **Got Your Number**

# Level A

Carol and Melissa are playing a game. They have a deck of 36 cards with just the numbers 1 through 9. After they mix up the cards, they put them into a pile. Below are the rules:

- Deal five number cards to each player.
- Use any three of your cards.
- Pick three numbers that add to a number near 20.
- Write a number sentence with your three cards and the total that is near 20.
- Find your score. Your score is the difference between your total and 20.
- For example, if you picked the cards 6, 9, 7, 6 + 9 + 7 = 22. So your total is 22. To find your score, subtract 20 from 22. 22 20 = 2.
- Shuffle the cards and play another round.

Play the game seven times. At the end of the game, sum all seven scores for each player. The player with the lowest total is the winner.

Got Your Number

# Level B

Sandy and Sally are playing a game. They have a deck of 36 cards with just the numbers 1 through 9. After they mix up the cards, they put them into a pile. Below are the rules:

- Deal six cards to each player.
- Select any four of your cards to make two numbers. Each number would be a two-digit number.
- Arrange the numbers and then add them to get a sum as near 100 as possible.
- Once you have selected the two numbers and found the sum, write out the equations.
- Determine your score by finding the difference (distance) between your number and 100.
- Shuffle the cards and play another round.
- Play the game seven times. At the end of the game, sum all seven scores for each player. The player with the lowest total is the winner.

Explain the strategy you used to try to win the games. Explain why you chose that strategy.

Got Your Number

# Level C

Jake and Linda are playing a game. They have a deck of 36 cards with just the numbers 1 through 9. After they mix up the cards, they put them into a pile. Below are the rules:

- Deal four cards to each player.
- Spin the spinner to select an operation.
- Arrange the digits into two fractions, such that the result of that operation upon the two fractions will produce the smallest possible outcome.
- Once you have selected the two fractions and found the outcome, write out the equations.
- The calculated outcome becomes your score for that round.
- Shuffle the cards and play another round.
- Play the game seven times. At the end of the game, sum all seven scores for each player. The player with the lowest total is the winner.

Explain the strategy you used to try to win the games. Explain why you chose that strategy.

Got Your Number

# Level D

Jean and Ford are playing a game. They each have a deck of six cards with just the numbers 1 through 6. After they each mix up the cards they put them into one pile. The goal of the game is to make the largest three-digit number. The first player picks the top card and places the number in either the ones, tens or hundreds place. The second player then picks a card and has the same options. Once a card is put in a location, it cannot be moved. Continue playing until all the places are filled. When each has created a three-digit number, determine which player has the largest number. That player is the winner! Note: The players are not able to see the other player's cards or placement until all the cards are drawn and played.

Play the game several times. Keep track of the results.

Write a detailed strategy for this game. Given specific cards, explain where you would put that number to insure the best probability of winning. Be thorough in developing a strategy. For example, if you drew a 4 as your second card, where would you put it? Of course you would need to explain options depending on where your first card was placed.

Problem of the Month

Got Your Number

# Level E

In the game that Jean and Ford are playing, suppose you drew a 4 on your first turn. Explain where (hundreds, tens or ones) you would place that card in order to give you the best chance of winning the game. Justify your answer with a mathematical argument.

Suppose you drew a 3 on your first turn. Explain where (hundreds, tens or ones) you would place that card and justify your answer.

Justify your strategy proving why you would place any given card in any given location. Your justification should be complete and provide a valid argument for where each card should be placed given any situation.

Problem of the Month

Got Your Number

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# **Got Your Number**

## **Primary Version Level A**

**Materials:** The deck of cards (1-9) for each pair.

**Discussion on the rug:** The teacher starts a discussion about the number 10. "Why is the number 10 an important number?" The teacher invites ideas from the class.

"We are going to play a fun game today. It is called <u>Make Ten</u>." The teacher demonstrates how to play the game with two players. "We play in pairs with a deck of number cards. Each player picks seven cards from the deck. You look at your cards and find two cards that add-up (count up) to 10. For example: 8 and 2 make 10. If you can make ten, then put those two cards together in a 10-pairs pile and then pick two more cards. If you can't make ten, then say 'pass' and pick a new card. Switch turns with each other until all the cards on the deck are picked and all the pairs that make ten are found. We will examine our 10-pairs pile all together."

**In small groups:** Each group plays the game until the deck is used and all pairs of ten are made. Have students look over and count up how many 10-pairs they made. After the game, the teacher asks the class to list the 10-pairs that the students made.

The teacher asks the following questions:

"Suppose you are playing this game with a new friend. Explain to your friend how you play the game and which cards you need to put together to make 10-pairs."

At the end of the investigation have students either discuss or dictate a response to the summary question.

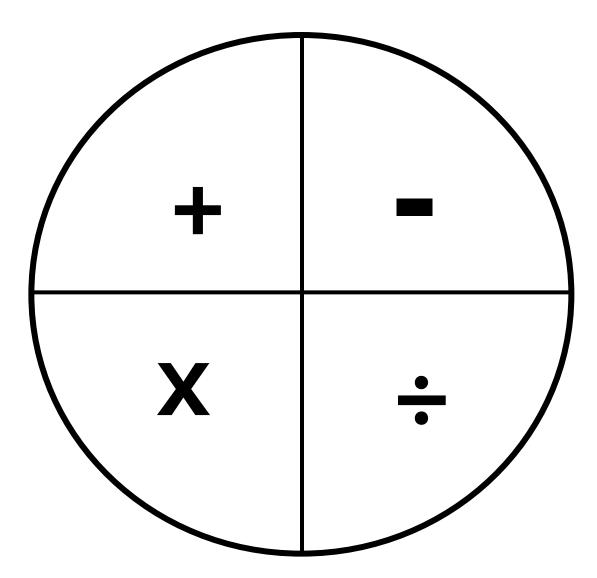
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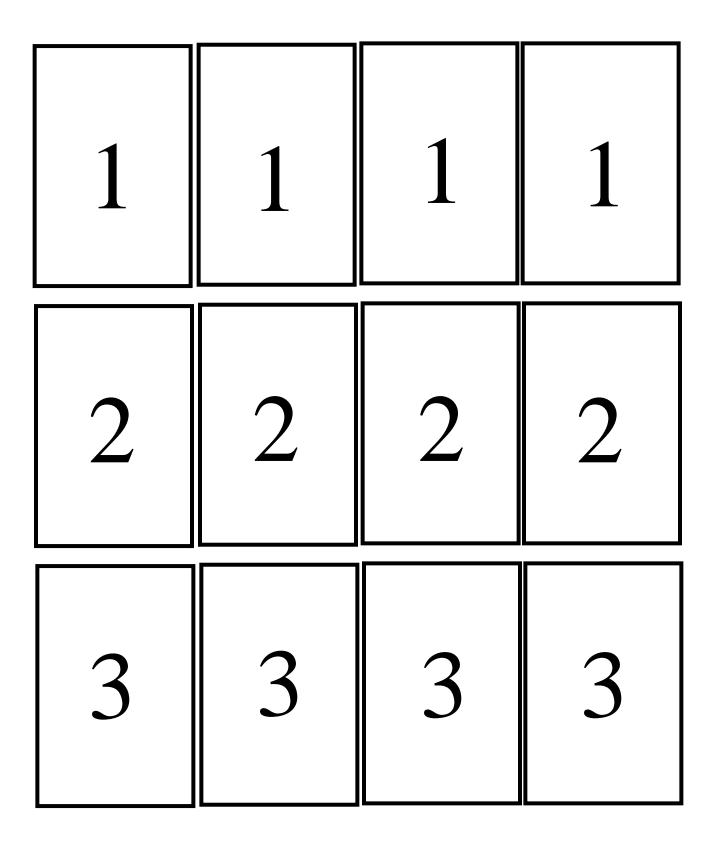
# **Operation Spinner**



Problem of the Month

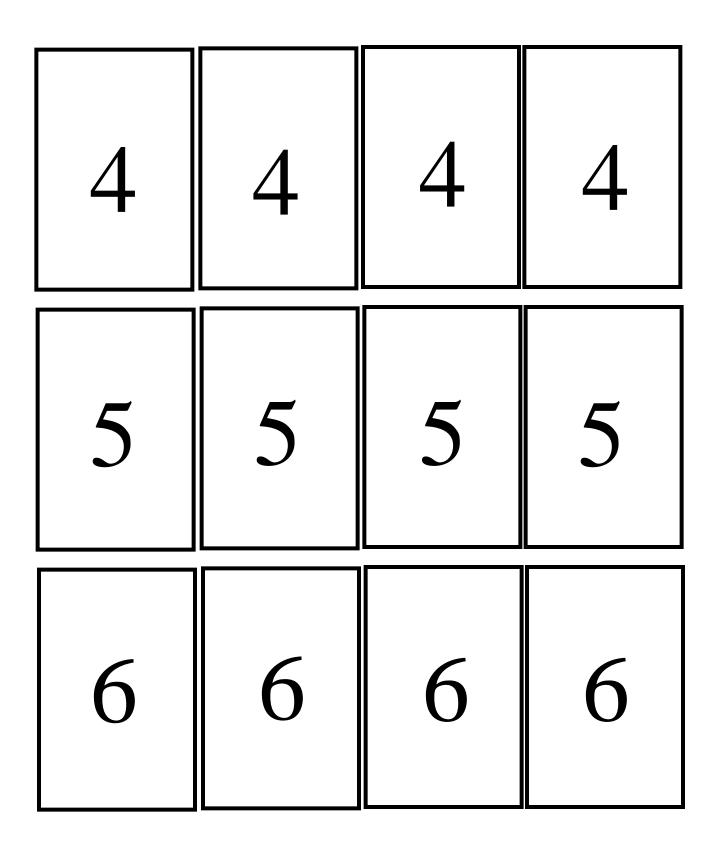
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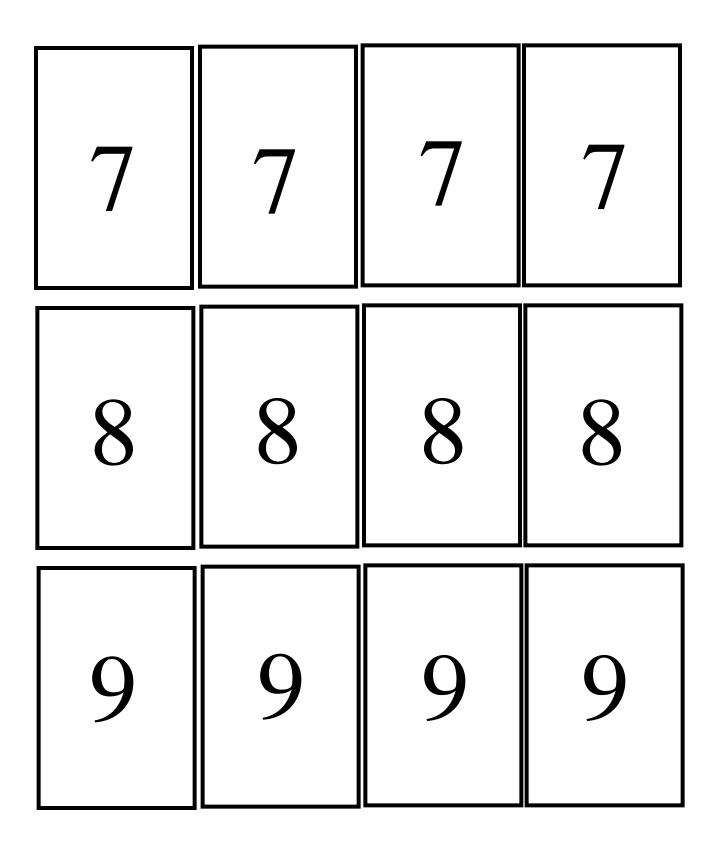
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#### Task Description – Level A

This task challenges students to estimate using addition in order to choose 3 cards out of 5 that make a sum close to 20. Students need to compare options to 20 before making the final choice. Students need to add accurately and then compare the sums to 20 using subtraction to get a score.

#### Common Core State Standards Math - Content Standards

#### **Operations and Algebraic Thinking**

#### Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction with 100 to solve one- and two-step problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

#### Add and subtract within 20.

2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one- digit numbers.

#### Common Core State Standards Math – Standards of Mathematical Practice MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see 5 – 3(x - y)<sup>2</sup> as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

#### Task Description – Level B

This task challenges students to choose 4 cards from 6 to make 2 two-digit numbers that will add close to 100. Students must use place-value knowledge to estimate and make their choices. Students must then be able to accurately use comparison subtraction to find the distance from 100. Finally students use place-value understanding to generalize the situation by describing a strategy for choosing and arranging the cards to form the two-digit numbers.

#### Common Core State Standards Math - Content Standards

Number and Operations in Base Ten

#### Use place value understanding and properties of operations to add and subtract.

2.NBT. 5 fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT. 6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

#### Common Core State Standards Math - Standards of Mathematical Practice

#### MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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#### Task Description – Level C

This task challenges students to take four digits and make them into two fractions that yield the lowest result when subjected to a given operation. Students must have an understanding of equivalence to compare their results to those of their partners. Students might choose to use common denominators and equivalent fractions or students might choose to change results to decimals to make the comparisons. Finally, students must reflect on how the operations affect the answers and explain a strategy for composing the fractions. They must also decide if order matters in how the fractions are placed in the problem.

#### Common Core State Standards Math - Content Standards

#### **Number and Operations - Fractions**

#### Extend understanding of fraction equivalence and ordering.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g. by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize the results of the comparisons with symbols >, =, or <, and justify the conclusions, e.g. by using a visual fraction model. **Use equivalent fractions as a strategy to add and subtract fractions.** 

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

# Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

Number and Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.7 Add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition an subtraction; relate the strategy to a written method and explain the reasoning used.

#### The Number System

# Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g. by using visual fraction models and equations to represent the problem.

#### Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

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#### Got Your Number

#### Task Description - Level D

This task challenges students to play a game of chance to form a 3-digit number by drawing cards from a deck with digits 1 to 6. After each draw the student must decide whether to place the card in the ones, tens, or hundreds place. Once a number is placed, it can't be moved. Students must then compare numbers to decide which is larger. After playing the game a few times, students must think about probabilities to determine a detailed strategy for playing the game.

#### Common Core State Standards Math - Content Standards

#### Number and Operations in Base Ten

Understand place value.

2.NBT.4 Compare two three-digit numbers based on meanings of hundreds, tens, and ones digits using >, =, < symbols to record the results of comparisons.

#### Statistics and Probability

#### Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicates greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of discrepancy.

7..SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation. 7.SP.8.a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

7SP.8.b Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., rolling double sixes), identify the outcomes in the sample space, which compose the event.

#### <u>High School – Statistics and Probability – Using Probability to Make Decisions</u>

Use probability to evaluate outcomes of decisions.

S-MD.7 Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).

#### High School – Statistics and Probability – Conditional Probability and the Rules of Probability

**Understand independence and conditional probability and use them to interpret data.** S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of

the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").

S-CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.6 Find the conditional probability of A given B as the fraction B's outcomes that also belong to A, and interpret the answer in terms of the model.

S-CP.8 Apply the general Multiplication Rule in a uniform probability model, P9A and B) =

P(A)P(BIA)=P(B)P(AIB), and interpret the answer in terms of the model.

Common Core State Standards Math – Standards of Mathematical Practice

#### MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to

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compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.4 Model with mathematics.** 

# Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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#### **Got Your Number**

#### Task Description – Level E

This task challenges students to think about probability in the context of a place-value game and use mathematics to justify a strategy for playing the game. The students must first analyze specific choice points within the game to find the probabilities for the events. Then students must use their analysis of the probabilities to develop a full strategy for the game.

#### Common Core State Standards Math - Content Standards

#### Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicates greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability.

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7.SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation. 7.SP.8a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

7SP.8b Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., rolling double sixes), identify the outcomes in the sample space, which compose the event.

<u>High School – Statistics and Probability – Using Probability to Make Decisions</u> Use probability to evaluate outcomes of decisions.

S-MD.7 Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game.)

#### <u>High School – Statistics and Probability – Conditional Probability and the Rules of Probability</u> Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.6 Find the conditional probability of A given B as the fraction B's outcomes that also belong to A, and interpret the answer in terms of the model.

S-CP.8 Apply the general Multiplication Rule in a uniform probability model, P9A and B) =

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Common Core State Standards Math – Standards of Mathematical Practice

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analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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#### **Got Your Number**

#### Task Description – Primary Level

This task challenges students to think about the importance of the number 10. Students use drawing, counting, and addition to find combinations of two number cards that make exactly 10.

Common Core State Standards Math - Content Standards

#### **Operations and Algebraic Thinking**

# Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

K.OA.2 Solve addition and subtraction word problems and add and subtract with 10, e.g. by using objects or drawings to represent the problem.

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g. by using objects or drawings, and record the answer with a drawing or equation.

#### Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknown number to represent the problem.

#### Add and subtract within 20.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction with 10. Use strategies such as counting on;; making ten, decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums by creating the known equivalent.

#### Common Core State Standards Math – Standards of Mathematical Practice

#### MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.