Problem of the Month
Through the Grapevine

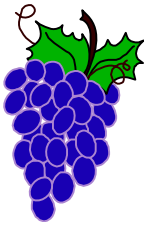
The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:
In the Problem of the Month *Through the Grapevine,* students use mathematical concepts of statistics including measures of center and representations. In addition students use proportional reasoning, analyze trends in data sets, and design their own survey. The mathematical topics that underlie this POM are measures of center, samples, proportional reasoning, scatterplots, the interpretation of data sets, and designing and carrying out a survey and data analysis.

In the first levels of the POM, students collect data from raisin boxes and examine data sets to find the most and the least, generate graphs, and make predictions. Continuing through the levels, students find the mean average as a typical sample size and use that information to make predictions in situations involving scale. Students find measures of center and use proportional reasoning. In level D, a scatterplot is presented. Students are asked to draw a line that best fits the data and use that line to make predictions and to describe trend. In the final levels of the POM, students are to design their own survey, collect data, analyze the results, and draw conclusions.

Problem of the Month



Through the Grapevine

Level A:

Examine several packages of individual raisin boxes.

Count how many raisins are in each box.

Make a table of the number of raisins in each box.

Draw a graph of the number of raisins in the boxes from your table.

Which box had the most raisins?

Which box had the least raisins?

Why do you think there is a different number of raisins in each box?

How many raisins would probably be in the next box you open? Explain why you think it would be that number.

Level B:

Find the mean (average) number of raisins in your sample.

Suppose each student in your class ate one individual box of raisins for lunch. How many raisins would be eaten? Explain how you figured it out.

Suppose, on average, each student in your class ate one individual box of raisins each week throughout the school year. Approximately how many raisins would be eaten? Explain how you found your estimate.

A recipe for oatmeal cookies calls for two individual boxes of raisins to be used. The recipe makes one dozen cookies. Approximately how many raisins would be in a single cookie from that recipe? Explain your solution.

Explain why knowing the mean (average) number of raisins in a box is important to know.

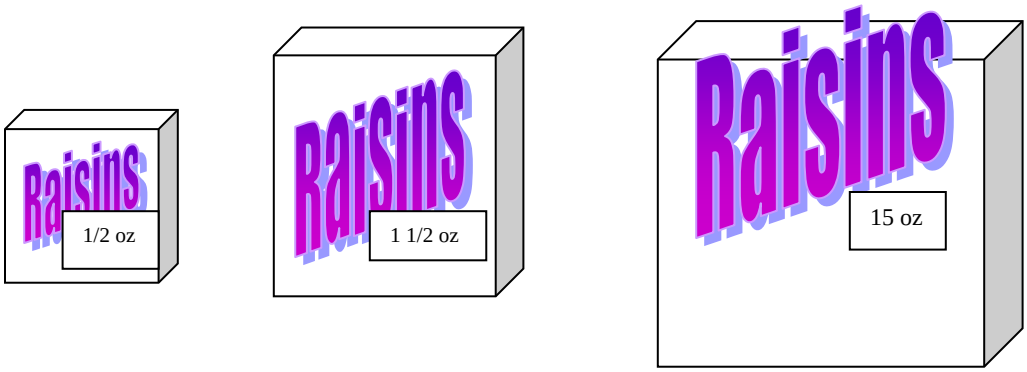
Level C:

What is the median value of raisins in a box from your sample?

What was the mode of your raisin box sample?

What is the range of your raisin box sample?

Describe a reason why someone might want to use the median rather than the mean as the "average" number of raisins in a box.



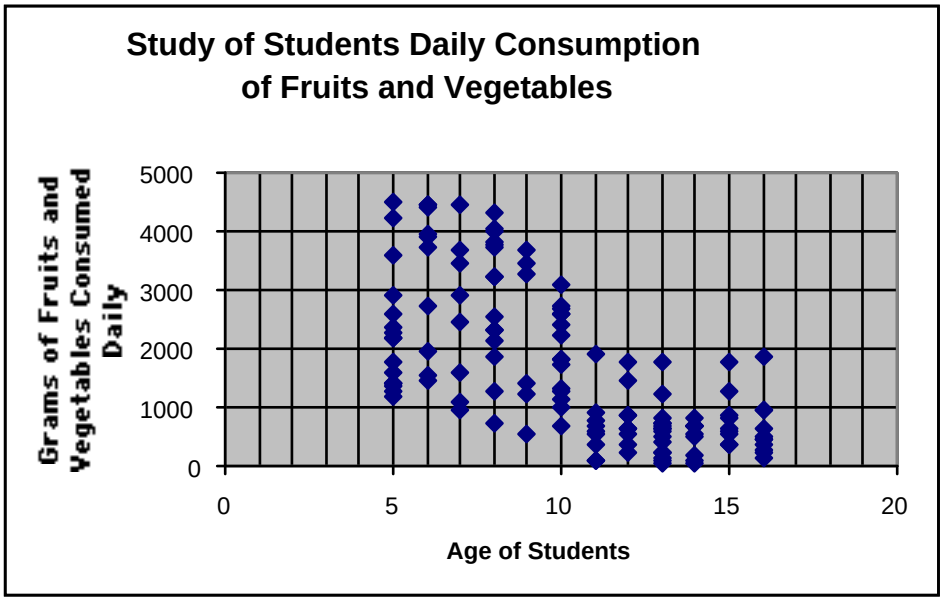
Individual size boxes are 1/2 ounce. Regular boxes are 1 1/2 ounces. Family size boxes are 15 ounces.

How many raisins would you estimate are in a Regular size box? Explain.

How many raisins would you estimate are in a Family size box? Explain.

Level D:

A study conducted by the American College of Nutrition concluded that a minimum of two servings of raisins a day may prevent some forms of cancer. Other studies claim that eating five servings of fruits and vegetables (2000 grams) per day will reduce the chance of getting certain cancers by 35%. Unfortunately, several studies show that many teens do not eat enough fruits and vegetables.



The graph shown above is a study of students and how many fruits and vegetables they eat each day.

Using the data from the study, describe how many grams of fruits and vegetables a typical 14-year-old eats each day.

What is the median amount of grams of fruits and vegetables the 6-year-olds eat each day, according to the study?

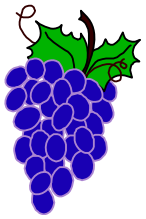
Draw a straight line that best fits the data in the graph.

Write a description of what the graph shows. What does the graph tell you about the relationship between the age of the students and the amount of grams of fruits and vegetables they eat?

Level E:

Design and conduct a survey of your own to determine how many servings of fruits and vegetables the people in your sample eat each day. Select a sample population of at least 15 individuals. Track the sample's eating habits for about a week. Collect the data from your study. Analyze your data using statistical tools. Report your findings using statistical measures and graphs. State the conclusions of your study and whether they are consistent with other studies.

Problem of the Month



Through the Grapevine

Primary Version Level A

Materials: Individual raisin boxes (1/2 ounces), enough for one box per student.

Discussion on the rug: (Teacher holds up an individual box of raisins.) "How many raisins do you think this box holds? Why did you make that guess?" (Teacher solicits answers from students.) "Do you think each box holds exactly the same number of raisins? Why or why not?" (Teacher solicits answers from students and then states that the class will investigate this question.)

In small groups: (Each student has an individual box of raisins. Each student counts the number of raisins in their box. They might count twice to double check. The teacher solicits the counts from each student. The teacher lists the counts and maybe creates a chart like a line plot to illustrate them. The teacher asks the following questions.)
"Which box had the most raisins?
Which box had the least raisins?
Why do you think there are different number of raisins in each box?
How many raisins would probably be in the next box you open? Explain why you think it would be that number."
(At the end of the investigation have students either discuss or dictate a response to the summary questions above.)

Problem of the Month

Through the Grapevine

Task Description – Level A

This task challenges a student to examine a set of data from real life. Students must count and compare numbers of raisins, identifying most and least. Students are asked to think about weight and variation in a context.

Common Core State Standards Math - Content Standards

Measurement and Data

Represent and interpret data.

2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represented a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.

Statistics and Probability

Develop understanding of statistical variability.

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by the center, spread, and overall shape.

Summarize and describe distributions.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

6.SP.5 Summarize numerical data sets in relations to their context, such as by:

a. Reporting the number of observations
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Through the Grapevine

Task Description – Level B

This task challenges a student to work with mean and use proportional reasoning to scale up and scale down.

Common Core State Standards Math - Content Standards

Number and Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NBT. 6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent rations, tape diagrams, double number line diagrams, or equations.

Statistics and Probability

Develop understanding of statistical variability.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by the center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all the values with a single numbers, while a measure of variation describes how the values vary with a single number.

Summarize and describe distributions.

6. SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

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MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Through the Grapevine

Task Description – Level C

This task challenges a student to use measures of center to describe variations in data and decide which one best describes the data set. Students use their measures to make predictions about scaling the size of the box of raisins using proportional reasoning.

Common Core State Standards Math - Content Standards

Statistics and Probability

Develop understanding of statistical variability.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by the center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all the values with a single numbers, while a measure of variation describes how the values vary with a single number.

Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Draw informal comparative inferences about two populations.

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Problem of the Month

Through the Grapevine

Task Description – Level D

This task challenges a student to analyze bivariate data on a scatterplot to find median and line of best fit, and describe trends in the data. Students are asked to use various data tools to make inferences about the data.

Common Core State Standards Math - Content Standards

Statistics and Probability

Develop understanding of statistical variability.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by the center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all the values with a single numbers, while a measure of variation describes how the values vary with a single number.

Summarize and describe distributions.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Use random sampling to draw inferences about a population.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Draw informal comparative inferences about two populations.

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Investigate patterns of association in bivariate data.

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

High School – Statistics and Probability – Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables.

S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data, use functions fitted to data to solve problems in the context of the data.

Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Through the Grapevine

Task Description – Level E

This task challenges a student to design a survey to gather data on fruit and vegetable consumption. Students are asked to use a variety of statistical tools to analyze and describe the data and interpret the meaning of the measures in terms of the survey questions.

Common Core State Standards Math - Content Standards

Statistics and Probability

Develop understanding of statistical variability.

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and account for it in the answers.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by the center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all the values with a single numbers, while a measure of variation describes how the values vary with a single number.

Summarize and describe distributions.

6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Use random sampling to draw inferences about a population.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Draw informal comparative inferences about two populations.

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Investigate patterns of association in bivariate data.

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

<u>High School – Statistics and Probability – Interpreting Categorical and Quantitative Data</u>
Summarize, represent, and interpret data on two categorical and quantitative variables.
S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data, use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Problem of the Month

Through the Grapevine

Task Description – Primary Level

This task challenges a student to use estimation and counting to find the number of raisins in a small box. Students think about whether all boxes will hold the same amount or why there might be variation in the boxes. Students compare their results by thinking about most and least and use information to make predictions.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
c. Understand that each successive number name refers to a quantity that is one larger.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration.

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g. by using matching and counting strategies.

Number and Operations in Base Ten

Understand place value.

1.NBT.2 Understand that two digits of a two-digit number present amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones – called a "ten".
b. The numbers from11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones.
c. The numbers 10,20,30,40,50,60,70,80,90 refer to one, two three, four, five, six, seven, eight, or nine tens and (and 0 ones).

1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of the comparisons with the symbols >, =, and <.

Measurement and Data

Represent and interpret data.

2.MD.9 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together and , take-apart, and compare problems using information presented in a bar graph.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative

relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.