Problem of the Month: *Measuring Mammals*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Measuring Mammals,* students use algebraic thinking to solve problems involving proportional relationships, measurement, variables, and simultaneous equations. The mathematical topics that underlie this POM are linear measurement, proportional reasoning, scale factors, scale, ratios, variables, functions, inverse variation, and algebraic reasoning.

In the first levels of the POM, students view three drawings of giraffes. Their task is to determine the relational size of one giraffe to another. In Level B, students are given a picture and must use proportional reasoning to determine the linear dimension of an enlarged picture. Then the students are asked to find the inverse

relationship: given an enlarged measurement, what was the original size? In Level C, the students are presented with view tubes. The purpose is for the students to determine the relationship between the tubes themselves and the size of the objects they view. In Level D, students analyze the relationship between different-sized view tubes. The students investigate the relationship of objects viewed when a dimension of a tube is altered. In the final level of the POM students are presented with a situation in the wild where they need to use their developed knowledge of proportional reasoning, inverse variation, and algebraic thinking in order to find the height of an animal in the field.

Mathematical Concepts

Algebra is the cornerstone of secondary mathematics. Algebraic thinking is taught in primary grades with the foundations of algebra taught usually by the end of middle school. Even though the term algebraic thinking is routinely used, it cannot be simply defined. The underpinnings of algebra involve abstractions and language. There are several resources that define the most important concepts in algebra. One resource is *Fostering Algebraic Thinking.*

Problem of the Month
Measuring Mammals

Level A:

1. Examine the two giraffes named George and Geoff. Determine which giraffe is taller than the other. Explain the difference in the size of the giraffes. How did you determine your answer?

George

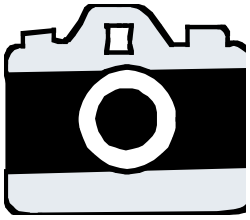
Geoff

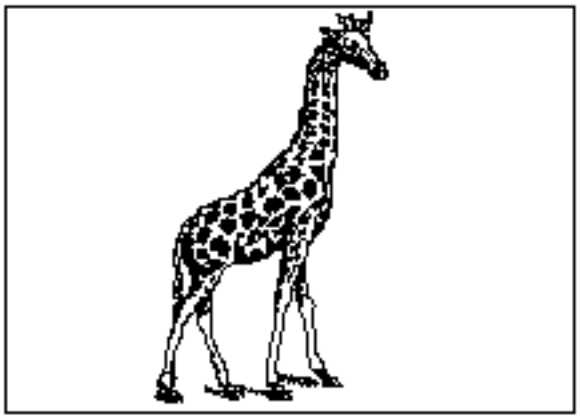
2. This is young Gerry. How much taller is George than Gerry? Explain how you found your answer.

Gerry

Level B:

1. You used your camera to take a picture of a giraffe at the zoo. Below is the picture of the giraffe you photographed. The camera shop makes pictures into posters 12 times the size of the picture. How tall will the giraffe be in the poster?





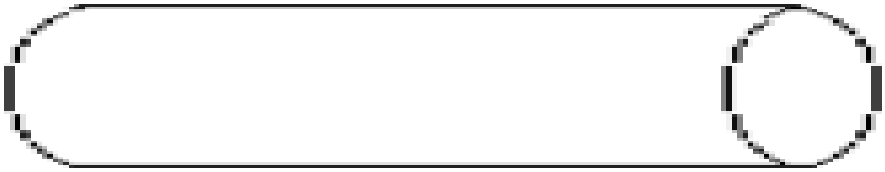
2. Suppose an elephant stands 32 inches tall in another poster that the camera shop made for you. How tall was the elephant in the original photograph?

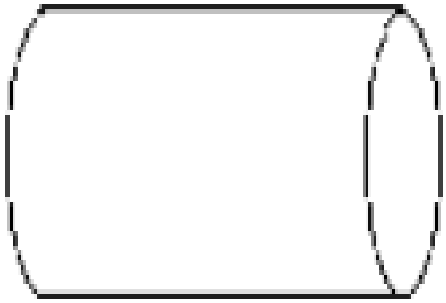
Explain your answer.

Level C:

You are a naturalist. You have heard that other naturalists are using scopes to calculate the size of animals in the wild. By viewing the animal through a scope, the height of an animal can be found.

You know that scopes come in different sizes. You think that by using the scopes along with mathematics, you can determine the actual heights of objects that you see through the scopes.



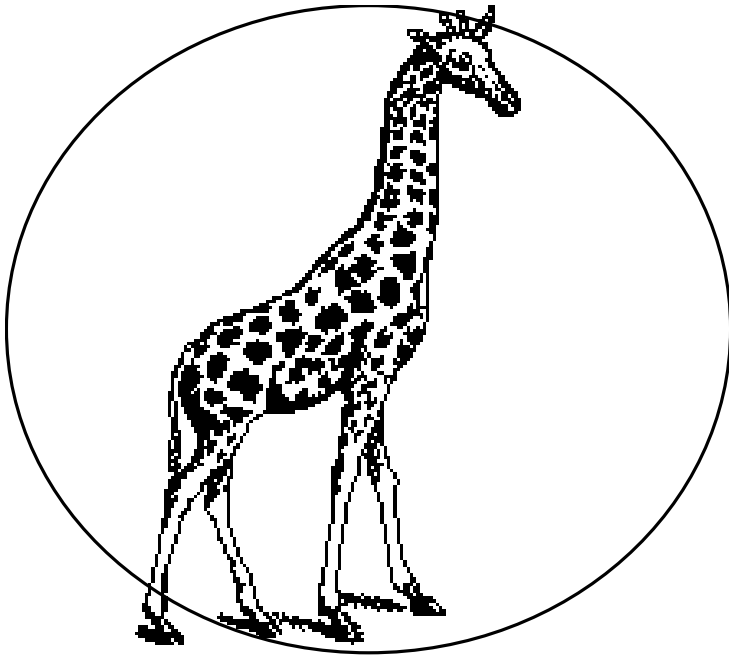


Experiment with various-sized scopes to determine what you can see at different distances with different scopes.

- How does the size of a particular scope affect the animal's image?
- Is the size of the image you see affected by how far away you are from an object?
- How does the distance from an object compare to the height of the image?
- Describe a type of experiment that you could conduct to answer these questions.

Select a scope of a particular size that you have used in your experiment. Suppose you were 60 feet away from a giraffe and the animal's image exactly filled the viewer. How tall is the giraffe?

Using mathematics, describe the relationship between the distance a viewer is from an object and the height of the object in the scope you used. Explain how that relates to the size of the scope.



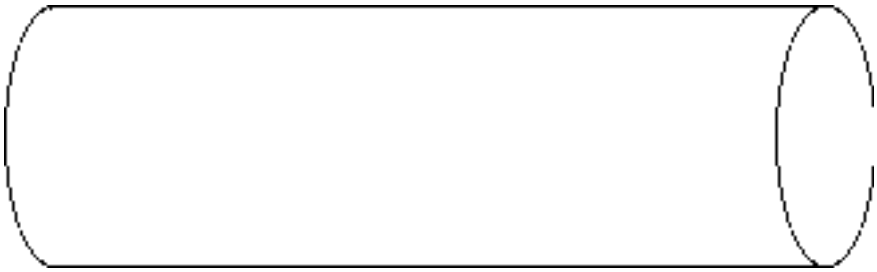
Level D:

You have four scopes of different sizes, labeled accordingly.

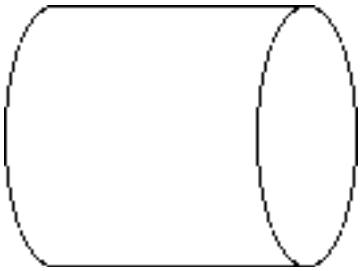
TYPE A

TYPE B
same size hole as type A but longer in length.

TYPE C
larger size hole as type A and B but same length as type B.



TYPE D
same size hole as type C and same length as type A.



You are standing fifty feet away from an animal. Looking through scope TYPE A, the animal's image fills the scope's opening exactly.

• Explain what you need to do to look through a TYPE B scope and see the full image of the same animal fill the opening of that view tube?

• Explain what you need to do to look through a TYPE D scope and see the full image of the same animal fill the opening of that view tube?

• Suppose TYPE C scope is 3 times as long in length as TYPE A and the diameter of the opening of TYPE C is twice the diameter of TYPE A. Exactly where would you need to stand to see the full image of the same animal fill the opening of that TYPE C scope?

• You are a tourist on safari on an African plain. There is an elephant standing still a distance away across the river. The current of the river is swift and you cannot cross it. You have in your possession a piece of paper that can be rolled into various-sized view tubes, a short measuring tape, a pen and calculator. You would like to determine the height of the elephant. Explain to another tourist how you were able to accurately determine the elephant's height.

Level E:

You are out in the wild. You have a short measuring stick with metric calibrations. You see a giraffe in the distance. You hold the measuring stick up, like a painter might with a brush. Your arm is extended in front of you and you sight the animal and determine a height measure on the calibrated stick. You move back ten feet and make a similar sighting with a new measurement. The giraffe has not moved.

Explain how you could determine the actual height of the animal. Explain the relationship between the measurements. Write an algebraic formula for calculating the height of an object given the process described. Explain why it works.

Problem of the Month
Measuring Mammals

Primary Version Level A

Materials: A picture of a giraffes for each student, long strips of paper

Discussion on the rug: Teacher shows the giraffes. **"What do we call these animals? Who can read the names of these animals? Which animal do you think is tallest? How would you know for sure?"**

In small groups: Each group has a paper of giraffes titled *Who Is Tallest*? Paper strips are on a material table that students might select to use.

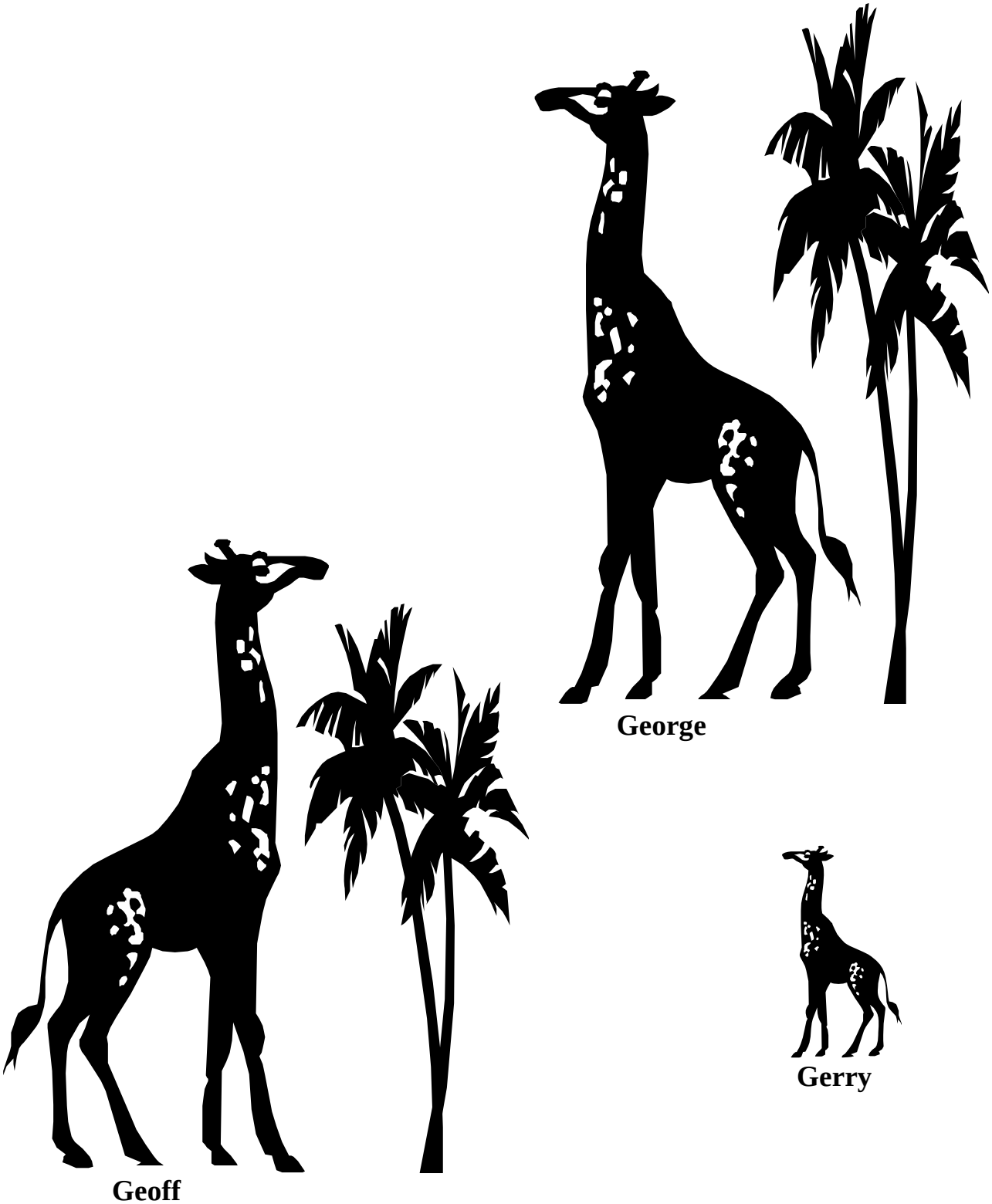
Teacher asks the following questions, only going on to the next question if students have success.

1. "Which giraffe is tallest? How can you check to know for sure?"

2. "Gerry is a young giraffe. How much taller is George than Gerry? How do you know? Show how you figured it out."

At the end of the investigation have students either draw a picture to represent their solution or dictate a response to the summary question.

Who Is Tallest?



Problem of the Month

Measuring Mammals

Task Description – Level A

Given a drawing of three giraffes, the student is challenged to determine the relational size of giraffes – one to another. A student will use measurement and the transitive property to work on this task.

Common Core State Standards Math - Content Standards

Measurement and Data

Describe and compare measureable attributes.

K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Measure lengths indirectly and by iterating length units.

1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Measure and estimate lengths in standard units.

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Problem of the Month

Measuring Mammals

Task Description – Level B

This task challenges a student to determine the linear dimension of an enlarged picture from a given picture. The student must also find an inverse relationship when given the enlarged measurement – what was the original size? The student will use proportional reasoning and a given picture to work on this task.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Measuring Mammals

Task Description – Level C

This task challenges a student to determine the relationship between the tubes themselves and the sizes of the objects they view. The student will use proportions and actual viewing tubes to work on this task.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Expressions and Equations

Reason about and solve one-variable equations and inequalities.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Solve real-life and mathematical problems using numerical algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.EE.4.a Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently.

Functions

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Common Core State Standards Math – Standards of Mathematical Practice

MP. 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Problem of the Month:

Measuring Mammals

Task Description – Level D

This task challenges a student to analyze the relationship between different-sized view tubes. The student will investigate the relationship of objects viewed when a dimension of a tube is altered. The student will use proportions and linear functions to work on this task.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Expressions and Equations

Reason about and solve one-variable equations and inequalities.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Solve real-life and mathematical problems using numerical algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.EE.4.a Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently.

Functions

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

High School – Algebra – Creating Equations

Create equations that describe numbers or relationships.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; …

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context.

Common Core State Standards Math – Standards of Mathematical Practice

MP. 2 Reason abstractly and quantitatively.

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Problem of the Month

Measuring Mammals

Task Description – Level E

This task challenges a student to use his or her developed knowledge of proportional reasoning, inverse variation, and algebraic thinking in order to find the height of an animal in a field in the wild.

Common Core State Standards Math - Content Standards

Functions

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

High School – Algebra - Creating Equations

Create equations that describe numbers or relationships.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; …

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context.

High School – Functions - Building Functions

Build new functions from existing functions.

F-BF.4 Find inverse functions.

F-BF.4.b (+) Verify by composition that one function is the inverse of another.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Measuring Mammals

Task Description – Primary Level

This task challenges a student to work with the measurement of height or length using pictures of giraffes. A discussion is held so that a student would understand what a giraffe looks like and how to determine which giraffe is taller. Paper strips are provided for students to use when determining which giraffe is taller than another. Additionally, a third smaller giraffe is added to the mix, and the teacher asks the class how much taller is the tallest giraffe than the smallest. Student are asked how they know and to show how their solution was determined.

Common Core State Standards Math - Content Standards

Measurement and Data

Describe and compare measureable attributes.

K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Measure lengths indirectly and by iterating length units.

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