Assessment : End-of-Unit Assessment

Problem 1

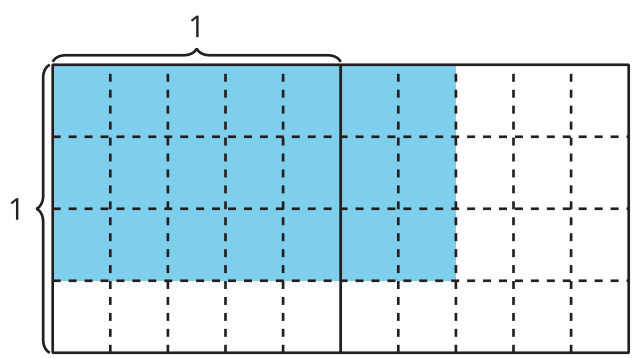
Students examine an area diagram showing a product of two non-unit fractions. Each true statement is essential to an understanding of the area model for finding a product of fractions.

- A explains why the denominator of a product of unit fractions can be taken as the product of their denominators.
- C interprets the area diagram as representing a product of fractions.
- E describes how to find the area.

Students may select B if they do not pay attention to the fact that the unit in the picture is a full square. They may select D if they are not careful about the numerator and denominator of the product.

Statement

Select **all** statements that are true about the diagram.



A. The area of each small shaded piece is $\frac{1}{4} \times \frac{1}{5}$ square unit.

B. The area of the shaded region is 21 square units.

C. The area of the shaded region is $\frac{3}{4} \times \frac{7}{5}$ square units.

D. The area of the shaded region is $\frac{20}{21}$ square units.

E. The area of the shaded region is $\frac{21}{20}$ square units.

Solution

["A", "C", "E"]

Aligned Standards

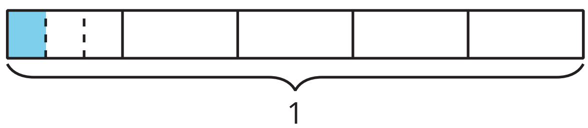
5.NF.B.4.b

Problem 2

Students identify expressions representing a tape diagram using both multiplication and division. Students may select A if they see the 5 equal parts in the whole and one of 3 equal parts shaded but apply the wrong operation to $\frac{1}{3}$ and $\frac{1}{5}$. They may select D if they see one of 3 equal parts shaded but do not identify the whole in the situation. They may select E if they see 7 parts, one of which is shaded, but do not identify that the parts are unequal.

Statement

Select **all** expressions that represent the shaded region.



A. $\frac{1}{3} + \frac{1}{5}$

B. $\frac{1}{5} \div 3$

C. $\frac{1}{3} \times \frac{1}{5}$

D. $\frac{1}{3}$

E. $\frac{1}{7}$

Solution

["B", "C"]

Aligned Standards

5.NF.B.4.a, 5.NF.B.7.a, 5.OA.A.2

Problem 3

Students match quotients of a whole number and a unit fraction with their values. All of the expressions use the same digits so that students are encouraged to think about the value of the expression rather than just look for a particular digit appearing in the value. Students can choose the correct answers by listing the expressions and values in terms of increasing size rather than calculating the values.

Statement

Match each expression with its value.

1. $5 \div \frac{1}{3}$	1. $\frac{1}{150}$
2. $\frac{1}{3} \div 5$	2. $\frac{1}{15}$

3. $\frac{1}{30} \div 5$
4. $5 \div \frac{1}{30}$

3. 15
4. 150

Solution

- A: 3
- B: 2
- C: 1
- D: 4

Aligned Standards

5.NF.B.7.a, 5.NF.B.7.b

Problem 4

Students divide a whole number by a unit fraction in a “how many in one group” situation. To solve the problem, students may write an expression, $440 \div \frac{1}{4}$, or they might draw a tape diagram as shown in the solution, or they might use a number line.

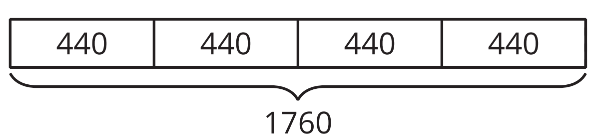
Students may see 440 and the fraction $\frac{1}{4}$ and be tempted to multiply these two numbers and give an answer of 110. This would be how far it is around $\frac{1}{4}$ of a 440 meter race track.

Statement

440 meters is $\frac{1}{4}$ of the way around the race track. How far is it around the whole race track? Explain or show your reasoning.

Solution

1,760 meters. Sample response: the tape diagram shows 440 is $\frac{1}{4}$ of 1,760.



Aligned Standards

5.NF.B.7.b, 5.NF.B.7.c

Problem 5

Students find products of non-unit fractions and mixed numbers with no context. Students may make a drawing such as an area diagram, but this is not required and the complexity of the numbers makes this more challenging. Other items on the assessment dealing with area and tape diagrams assess students’ ability to work with diagrams.

Statement

Find the value of each product.

1. $\frac{3}{12} \times \frac{2}{5}$
2. $\frac{8}{6} \times \frac{10}{11}$
3. $4 \times 6\frac{9}{10}$
4. $7\frac{3}{5} \times 4$

Solution

1. $\frac{6}{60}$ or equivalent
2. $\frac{80}{66}$ or equivalent
3. $27\frac{6}{10}$ or equivalent
4. $30\frac{2}{5}$ or equivalent

Aligned Standards

5.NF.B.4

Problem 6

This item complements the assessment problem about the distance around the track whose solution involved finding the value of a whole number divided by a unit fraction. This situation involves dividing a unit fraction by a whole number. Students may draw a number line or a tape diagram to support their reasoning.

Statement

An apple weighs $\frac{1}{2}$ pound. Diego cuts the apple into 4 equal pieces. How many pounds does each piece of the apple weigh? Explain your reasoning.

Solution

$\frac{1}{8}$

Sample response: If I divide each $\frac{1}{2}$ pound into 4 equal pieces I get 8 equal pieces total in a pound so each one is $\frac{1}{8}$ pound.

Aligned Standards

5.NF.B.7.a, 5.NF.B.7.c

Problem 7

Students find the product of non-unit fractions within a context. Students may use a drawing such as an area diagram as shown in the sample solution but this is not required. Students may forget to include the unit of liters with the response.

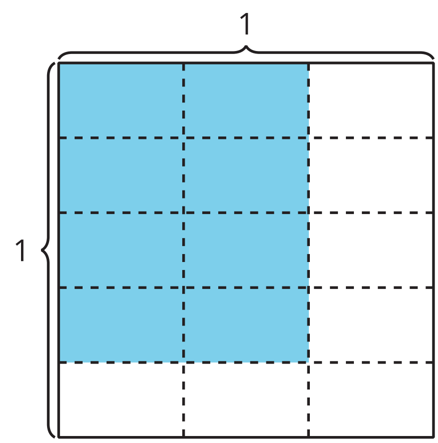
Statement

A container holds $\frac{4}{5}$ liter of water. During a hike, Jada drank $\frac{2}{3}$ of the water. How much water did Jada drink? Explain your reasoning.

Solution

$\frac{8}{15}$ liter or equivalent

Sample response: The square represents 1 liter of water and the shaded region represents $\frac{2}{3}$ of $\frac{4}{5}$ of a liter. The diagram shows that Jada drank $\frac{8}{15}$ of 1 liter.



Aligned Standards

5.NF.B.6

Problem 8

Students solve a multi-step problem involving area. They need to first find the area of each square tile, most likely either by fraction multiplication or by drawing a diagram. Then they analyze a common misconception, namely that when the side lengths of a square are multiplied by a factor the area of the square is also multiplied by that factor. Finally, they evaluate another product, this time of a fraction and a whole number. Note that the final answer for the area of the bathroom floor depends on the area of each tile and so student work here needs to be evaluated based on their answer for the area of each tile, assuming their solution method is to multiply that area by the number of tiles. In the same way, if students answer the second question incorrectly and then use this area to find the area of the bathroom floor, their work for the last question should be evaluated accordingly.

Statement

Each square tile on a bathroom floor measures $\frac{3}{2}$ feet by $\frac{3}{2}$ feet.

1. What is the area of each tile?
2. Mai says that the tiles have the same area as $\frac{3}{2}$ one-foot by one-foot tiles. Do you agree with Mai? Explain or show your reasoning.

3. The bathroom floor is covered by 12 of the $\frac{3}{2}$ feet by $\frac{3}{2}$ feet tiles. What is the area of the bathroom floor? Show or explain your reasoning.

Solution

1. Each tile has an area of $\frac{3}{2} \times \frac{3}{2}$ square feet or $\frac{9}{4}$ square feet.
2. I disagree with Mai. Sample response: A one foot by one foot tile has area 1 square foot so $\frac{3}{2}$ of them have area $\frac{3}{2}$ square feet. That's less than $\frac{9}{4}$ square feet.
3. $\frac{108}{4}$ square feet or 27 square feet. Sample response: There are 12 and each one has area $\frac{9}{4}$ square feet. That makes a total area of $12 \times \frac{9}{4}$ square feet. That's $\frac{108}{4}$ square feet or 27 square feet.

Aligned Standards

5.NF.B.6