

A Story of Units®

Eureka Math™

Grade 3, Module 3

Teacher Edition

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Grade 3 • Module 3

Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

OVERVIEW

This 25-day module builds directly on students' work with multiplication and division in Module 1. At this point, Module 1 instruction coupled with fluency practice in Module 2 has students well on their way to meeting the Grade 3 fluency expectation for multiplying and dividing within 100 (**3.OA.7**). Module 3 extends the study of factors from 2, 3, 4, 5, and 10 to include all units from 0 to 10, as well as multiples of 10 within 100. Similar to the organization of Module 1, the introduction of new factors in Module 3 spreads across topics. This allows students to build fluency with facts involving a particular unit before moving on. The factors are sequenced to facilitate systematic instruction with increasingly sophisticated strategies and patterns.

Topic A begins by revisiting the commutative property. Students study familiar facts from Module 1 to identify known facts using units of 6, 7, 8, and 9 (**3.OA.5**, **3.OA.7**). They realize that they already know more than half of their facts by recognizing, for example, that if they know 2×8 , they also know 8×2 through commutativity. This begins a study of arithmetic patterns that becomes an increasingly prominent theme in the module (**3.OA.9**). The subsequent lesson carries this study a step further; students apply the commutative property to relate 5×8 and 8×5 and then add one more group of 8 to solve 6×8 and, by extension, 8×6 . The final lesson in this topic builds fluency with familiar multiplication and division facts, preparing students for the work ahead by introducing the use of a letter to represent the unknown in various positions (**3.OA.3**, **3.OA.4**).

Topic B introduces units of 6 and 7, factors that are well suited to Level 2 skip-counting strategies and to the Level 3 distributive property strategy, already familiar from Module 1. Students learn to compose up to and then over the next ten. For example, to solve a fact using units of 7, they might count 7, 14, and then mentally add $14 + 6 + 1$ to make 21. This strategy previews the associative property using addition and illuminates patterns as students apply count-bys to solve problems. In the next lesson, students apply the distributive property (familiar from Module 1) as a strategy to multiply and divide. They decompose larger unknown facts into smaller known facts to solve. For example, $48 \div 6$ becomes $(30 \div 6) + (18 \div 6)$, or $5 + 3$ (**3.OA.5**, **3.OA.7**). Topic B's final lesson emphasizes word problems, providing opportunities to analyze and model. Students apply the skill of using a letter to represent the unknown in various positions within multiplication and division problems (**3.OA.3**, **3.OA.4**, **3.OA.7**).

Topic C anticipates the formal introduction of the associative property with a lesson focused on making use of structure to problem solve. Students learn the conventional order for performing operations when parentheses are and are not present in an equation (**3.OA.8**). With this student knowledge in place, the associative property emerges in the next lessons as a strategy to multiply using units up to 8 (**3.OA.5**). Units of 6 and 8 are particularly useful for presenting this Level 3 strategy. Rewriting 6 as 2×3 or 8 as 2×4 makes shifts in grouping readily apparent (see example on next page) and also utilizes the familiar factors 2,

3, and 4 as students learn the new material. The following strategy may be used to solve a problem like 8×5 :

$$8 \times 5 = (4 \times 2) \times 5$$

$$8 \times 5 = 4 \times (2 \times 5)$$

$$8 \times 5 = 4 \times 10$$

In the final lesson of Topic C, students relate division to multiplication using units up to 8. They understand division as both a quantity divided into equal groups and an unknown factor problem for which—given the large size of units—skip-counting to solve can be more efficient than dividing (**3.OA.3**, **3.OA.4**, **3.OA.7**).

Topic D introduces units of 9 over three days, with students exploring a variety of arithmetic patterns that become engaging strategies for quickly learning facts with automaticity (**3.OA.3**, **3.OA.7**, **3.OA.9**). Nines are placed late in the module so that students have enough experience with multiplication and division to recognize, analyze, and apply the rich patterns found in the manipulation of units of 9. As with other topics, the sequence ends with interpreting the unknown factor to solve multiplication and division problems (**3.OA.3**, **3.OA.4**, **3.OA.5**, **3.OA.7**).

In Topic E, students begin by working with facts using units of 0 and 1. From a procedural standpoint, these are simple facts that require little time for students to master; however, understanding the concept of nothing (zero) is more complex, particularly as it relates to division. This unique combination of simple and complex explains the late introduction of 0 and 1 in the sequence of factors. Students study the results of multiplying and dividing with units of 0 and 1 to identify relationships and patterns (**3.OA.7**, **3.OA.9**).

The topic closes with a lesson devoted to two-step problems involving all four operations (**3.OA.8**). In this lesson, students work with equations involving unknown quantities and apply the rounding skills learned in Module 2 to make estimations that help them assess the reasonableness of their solutions (**3.OA.8**).

In Topic F, students multiply by multiples of 10 (**3.NBT.3**). To solve a fact like 2×30 , they first model the basic fact 2×3 on the place value chart. Place value understanding helps them to notice that the product shifts one place value to the left when multiplied by 10: 2×3 tens can be found by simply locating the same basic fact in the tens column.

hundreds	tens	ones
		○○○
		○○○
		$2 \times 3 = 6$

hundreds	tens	ones
	○○○	
	○○○	
	$2 \times 3 \text{ tens} = 6 \text{ tens}$	
	$6 \text{ tens} = 60$	

In the subsequent lesson, place value understanding becomes more abstract as students model place value strategies using the associative property (**3.NBT.3**, **3.OA.5**). $2 \times 30 = 2 \times (3 \times 10) = (2 \times 3) \times 10$. The final lesson focuses on solving two-step word problems involving multiples of 10 and equations with unknown quantities (**3.OA.8**). As in the final lesson of Topic E, students estimate to assess the reasonableness of their solutions (**3.OA.8**).

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions.

Omit Lessons 6 and 10. Both lessons involve using the distributive property with multiplication and division, a recurring objective in Module 3. Within later distributive property lessons, incorporate units of 6 and 7.

Omit Lesson 11, a problem solving lesson involving multiplication and division. Lesson 11 shares an objective with Lesson 15 and is also similar to Lesson 7.

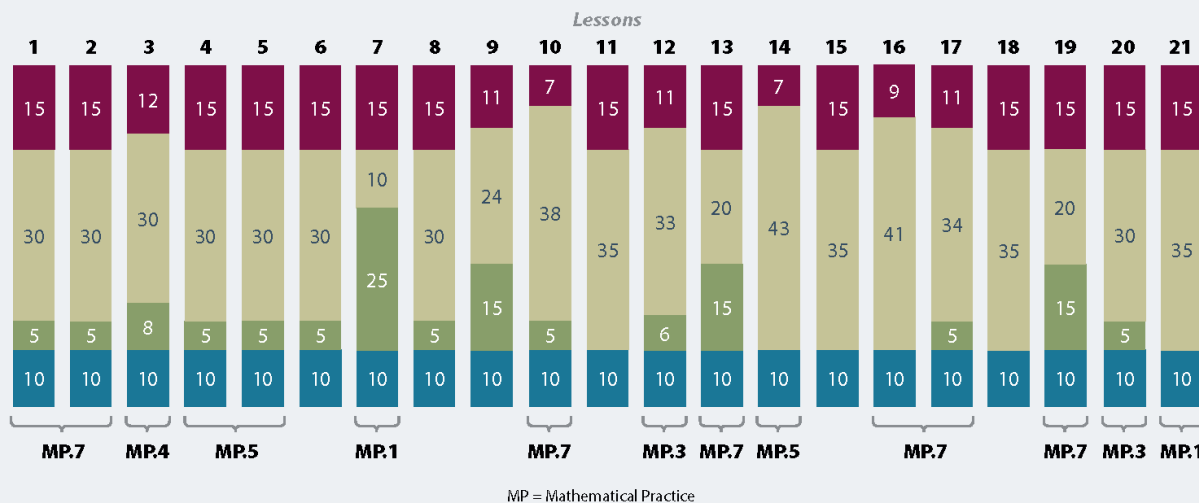
Omit Lesson 13. Study its essential understandings, and embed them into the delivery of Lesson 14's Concept Development. Modify Lesson 14 by omitting Part 1 of the Concept Development, a part which relies on the foundation of Lesson 13.



Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

- Fluency Practice
- Concept Development
- Application Problems
- Student Debrief



Focus Grade Level Standards

Represent and solve problems involving multiplication and division.¹

- 3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)
- 3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$.*

Understand properties of multiplication and the relationship between multiplication and division.²

- 3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

Multiply and divide within 100.³

- 3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.⁴

- 3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

¹The balance of this cluster is addressed in Module 1.

²The balance of this cluster is addressed in Module 1.

³From this point forward, fluency practice with multiplication and division facts is part of the students' on-going experience.

⁴After being fully taught in Module 3, this standard (as well as 3.OA.3) continues to be practiced throughout the remainder of the school year.

- 3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)⁵

- 3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Foundational Standards

- 2.OA.3** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
- 2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
- 2.NBT.2** Count within 1000; skip-count by 5s, 10s, and 100s.
- 3.OA.1** Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*
- 3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.*
- 3.OA.6** Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

Focus Standards for Mathematical Practice

- MP.1** **Make sense of problems and persevere in solving them.** Students engage in exploratory lessons to discover and interpret patterns, and they apply their observations to solving multi-step word problems involving all four operations.
- MP.3** **Construct viable arguments and critique the reasoning of others.** As students compare solution strategies, they construct arguments and critique the reasoning of their peers. This practice is particularly exemplified in daily Application Problems and problem-solving specific lessons in which students share and explain their work with one another.
- MP.4** **Model with mathematics.** Students use arrays, tape diagrams, and equations to represent word problem situations.

⁵The balance of this cluster is addressed in Module 2.

MP.5 Use appropriate tools strategically. Students analyze problems and select the appropriate tools and pathways to solutions. This is particularly evident as students select problem-solving strategies and use arithmetic properties as simplifying strategies when appropriate.

MP.7 Look for and make use of structure. In this module, patterns emerge as tools for problem solving. For example, students make use of structure as they utilize the distributive property to establish the $9 = 10 - 1$ pattern, or when they check the solution to a fact using units of 9 by making sure the sum of the digits in the product adds up to 9. They make use of the relationship between multiplication and division as they determine unknown factors and interpret their meanings.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives	Days
3.OA.4 3.OA.5 3.OA.7 3.OA.9 3.OA.1 3.OA.2 3.OA.3 3.OA.6	A The Properties of Multiplication and Division Lesson 1: Study commutativity to find known facts of 6, 7, 8, and 9. Lesson 2: Apply the distributive and commutative properties to relate multiplication facts $5 \times n + n$ to $6 \times n$ and $n \times 6$ where n is the size of the unit. Lesson 3: Multiply and divide with familiar facts using a letter to represent the unknown.	3
3.OA.3 3.OA.4 3.OA.5 3.OA.7 3.OA.1 3.OA.2 3.OA.6	B Multiplication and Division Using Units of 6 and 7 Lesson 4: Count by units of 6 to multiply and divide using number bonds to decompose. Lesson 5: Count by units of 7 to multiply and divide using number bonds to decompose. Lesson 6: Use the distributive property as a strategy to multiply and divide using units of 6 and 7. Lesson 7: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7.	4
3.OA.3 3.OA.4 3.OA.5 3.OA.7 3.OA.1 3.OA.2 3.OA.6 3.OA.8	C Multiplication and Division Using Units up to 8 Lesson 8: Understand the function of parentheses and apply to solving problems. Lesson 9: Model the associative property as a strategy to multiply. Lesson 10: Use the distributive property as a strategy to multiply and divide. Lesson 11: Interpret the unknown in multiplication and division to model and solve problems.	4



Standards	Topics and Objectives	Days
	Mid-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
3.OA.3 3.OA.4 3.OA.5 3.OA.7 3.OA.9 3.OA.1 3.OA.2 3.OA.6	D Multiplication and Division Using Units of 9 Lesson 12: Apply the distributive property and the fact $9 = 10 - 1$ as a strategy to multiply. Lessons 13–14: Identify and use arithmetic patterns to multiply. Lesson 15: Interpret the unknown in multiplication and division to model and solve problems.	4
3.OA.3 3.OA.7 3.OA.8 3.OA.9 3.OA.1 3.OA.2 3.OA.4 3.OA.6	E Analysis of Patterns and Problem Solving Including Units of 0 and 1 Lesson 16: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division. Lesson 17: Identify patterns in multiplication and division facts using the multiplication table. Lesson 18: Solve two-step word problems involving all four operations and assess the reasonableness of solutions.	3
3.OA.5 3.OA.8 3.OA.9 3.NBT.3 3.OA.1	F Multiplication of Single-Digit Factors and Multiples of 10 Lesson 19: Multiply by multiples of 10 using the place value chart. Lesson 20: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where n and m are less than 10) to multiply by multiples of 10. Lesson 21: Solve two-step word problems involving multiplying single-digit factors and multiples of 10.	3
	End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days		25

Terminology

New or Recently Introduced Terms

- Multiple (specifically with reference to naming multiples of 9 and 10, e.g., 20, 30, 40, etc.)
- Product (the quantity resulting from multiplying two or more numbers together)

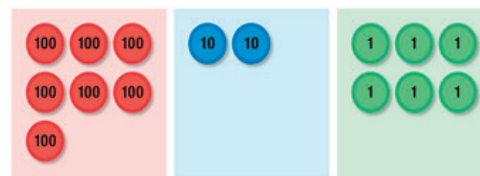
Familiar Terms and Symbols⁶

- Array (a set of numbers or objects that follow a specific pattern)
- Commutative property (e.g., $2 \times 3 = 3 \times 2$)
- Distribute (with reference to the distributive property; e.g., in $12 \times 3 = (10 \times 3) + (2 \times 3)$, the 3 is the multiplier for each part of the decomposition)
- Divide, division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., $15 \div 5 = 3$)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group, and the other is a multiplier that indicates the number of groups)
- Equation (a statement that two expressions are equal, e.g., $3 \times 4 = 12$)
- Even number (a whole number whose last digit is 0, 2, 4, 6, or 8)
- Expression (a number, or any combination of sums, differences, products, or divisions of numbers that evaluates to a number, e.g., 8×3 , $15 \div 3$)
- Factors (numbers that are multiplied to obtain a product)
- Multiply, multiplication (an operation showing how many times a number is added to itself, e.g., $5 \times 3 = 15$)
- Number bond (model used to show part–part–whole relationships)
- Number sentence (an equation or inequality for which both expressions are numerical and can be evaluated to a single number, e.g., $21 > 7 \times 2$, $5 \div 5 = 1$)
- Odd number (a number that is not even)
- Ones, twos, threes, etc. (units of one, two, or three)
- Parentheses (the symbols () used around a fact or numbers within an equation, expression, or number sentence)
- Quotient (the answer when one number is divided by another)
- Row, column (in reference to rectangular arrays)
- Tape diagram (a method for modeling problems)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the *missing* factor or quantity in multiplication or division)
- Value (how much)

⁶These are terms and symbols students have used or seen previously.

Suggested Tools and Representations

- Array
- Number bond (model used to show part–part–whole relationships)
- Place value disks (pictured at right)
- Tape diagram (a method for modeling problems)



Scaffolds⁷

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units*, please refer to “How to Implement *A Story of Units*.”

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	3.OA.3 3.OA.4 3.OA.5 3.OA.7 3.OA.9
End-of-Module Assessment Task	After Topic F	Constructed response and timed fluency with rubric	3.OA.3 3.OA.4 3.OA.5 3.OA.7 3.OA.8 3.OA.9 3.NBT.3

⁷Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.



Topic A

The Properties of Multiplication and Division

3.OA.4, 3.OA.5, 3.OA.7, 3.OA.9, 3.OA.1, 3.OA.2, 3.OA.3, 3.OA.6

Focus Standards:	3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$</i>
	3.OA.5	Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i>
	3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
	3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i>
Instructional Days:	3	
Coherence -Links from:	G2–M6	Foundations of Multiplication and Division
	G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
-Links to:	G3–M4	Multiplication and Area
	G4–M3	Multi-Digit Multiplication and Division

In Lesson 1, students study the commutativity of familiar Module 1 facts that use units of 2, 3, 4, 5, and 10 to discover facts that they already know using units of 6, 7, 8, and 9. For example, students recognize that if they know $3 \times 6 = 18$, then they also know $6 \times 3 = 18$. They write out familiar facts and those known through commutativity, organizing them in rows and columns to form the beginning of a table through which arithmetic patterns become visible. Students finish this lesson encouraged about the work to come after seeing that they already know more than half of their facts.

In Lesson 2, students apply commutativity in conjunction with the $n + 1$ strategy to solve unknown facts. For example, students relate 5×8 and 8×5 and then add one more group of 8 to solve 6×8 and, by extension, 8×6 . Adding one more group to a known fact in order to find an unknown fact continues to bridge student understanding in Module 1 and Module 3 as students are reminded of their prior work with the distributive property.

Lesson 3 introduces using a letter to represent the unknown in various positions within multiplication and division problems. In Module 1, students represented the unknown on tape diagrams, and occasionally in equations, using a question mark. This lesson uses familiar facts to introduce the new abstraction of using a letter as a placeholder.

A Teaching Sequence Toward Mastery of The Properties of Multiplication and Division

Objective 1: Study commutativity to find known facts of 6, 7, 8, and 9.
(Lesson 1)

Objective 2: Apply the distributive and commutative properties to relate multiplication facts $5 \times n + n$ to $6 \times n$ and $n \times 6$ where n is the size of the unit.
(Lesson 2)

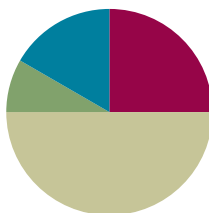
Objective 3: Multiply and divide with familiar facts using a letter to represent the unknown.
(Lesson 3)

Lesson 1

Objective: Study commutativity to find known facts of 6, 7, 8, and 9.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Sprint: Mixed Multiplication **3.OA.7** (9 minutes)
- Group Counting **3.OA.1** (3 minutes)
- Commutative Property of Multiplication **3.OA.5** (3 minutes)

Sprint: Mixed Multiplication (9 minutes)

Materials: (S) Mixed Multiplication Sprint

Note: This Sprint reviews familiar multiplication facts from Module 1 and prepares students for today's lesson on using commutativity with known facts to find unknown facts.

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes, sevens, eights, and nines in this activity anticipates multiplication using those units later in the module.

Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Group Counting in Module 3 no longer explicitly includes twos, threes, fours, and fives. However, consider including those units if the class has not yet mastered those facts.

Whisper/talking, hum/talking, or think/talking by threes and fours can also work as a scaffold to build fluency with sixes and eights.

Commutative Property of Multiplication (3 minutes)

Materials: (S) Personal white board

Note: This activity reviews the commutative property from Module 1 and anticipates its use in today's lesson.

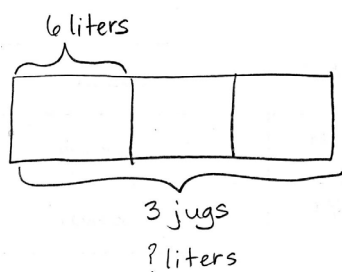
T: (Project array with 3 groups of 2 circles.) Write two multiplication sentences and two division sentences for this array.

S: (Write $3 \times 2 = 6$, $2 \times 3 = 6$, $6 \div 2 = 3$, and $6 \div 3 = 2$.)

Continue with the following suggested sequence: 2 groups of 9, 3 groups of 7, and 5 groups of 8.

Application Problem (5 minutes)

Geri brings 3 water jugs to her soccer game to share with teammates. Each jug contains 6 liters of water. How many liters of water does Geri bring?



$3 \times 6 = 18$
Geri brings 18 liters
of water for her
team.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Extend for students working above grade level with a related word problem with larger factors.

For example, "Kelly drinks 3 liters of water each day. How many liters of water does she drink in a week?"

Note: This problem reviews multiplication using units of three. It leads into the discussion of commutativity in the Concept Development.

Concept Development (30 minutes)

Materials: (S) Personal white board, Problem Set

Part 1: Explore commutativity as it relates to multiplication.

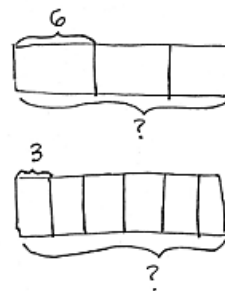
Draw or project the tape diagrams shown to the right.

T: Talk to your partner. Which tape diagram represents the Application Problem? How do you know? (Allow time for discussion.)

T: Draw both tape diagrams on your personal white board. Write a multiplication sentence for each. (Allow time for students to work and finish.)

T: How are the multiplication sentences related?

S: They use the same numbers. \rightarrow Both have a product of 18. \rightarrow They use the same factors but in a different order. The product is the same.



MP.7

- T: This is an example of the commutative property that we studied in Module 1. What does this property tell us about the product and its factors?
- S: Even if the order of the factors changes, the product stays the same!
- T: Earlier in the year, we learned our threes, including 3×6 . If we know 3×6 , what other fact do we know?
- S: 6×3 .
- T: What is the product of both 3×6 and 6×3 ?
- S: 18.
- T: To show that 3×6 and 6×3 equal the same amount, we can write $3 \times 6 = 6 \times 3$. (Model.)
- T: Using commutativity as a strategy, we know many more facts than just the ones we've practiced!

**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Review the commutative property by exploring arrays—concrete or pictorial. Review 3 twos is 2 threes, for example, by 6 students standing in 2 rows of 3, and then 3 rows of 2.

When drawing the array, use color to differentiate 6 threes from 3 sixes.

Continue with the following suggested sequence:

- $2 \times 7 = 7 \times 2$
- 5 eights = 8 fives
- 4 nines = 9 fours

Part 2: Use the multiplication chart to find known facts through commutativity.

- T: Problem 1(a) on your Problem Set shows a multiplication chart. The shaded numbers along the left column and the top are factors. The numbers inside the chart are products. Each un-shaded box represents the product of one multiplication fact. Find the total number of facts on your multiplication chart. (Allow time for students to count.) How many facts are on the chart?
- S: 100 facts.
- T: Let's use the chart to locate the product of 3 and 6. Put your finger on the row labeled 3, and slide it to the right until it's also in the column labeled 6. The number in the square where the row and column meet is the product, which has been done for you. Using the chart, what is the product of 3 and 6?
- S: 18.
- T: Let's now locate the product of 6 and 3. Find the square where the row for 6 and the column for 3 meet. Use commutativity to write the product of 6 and 3 in that square on your chart.
- S: (Write 18.)
- T: We can use commutativity to solve many new facts and fill in the products on the chart. On the chart, write the products for all the facts that we've already studied. Then, fill in those you can solve using commutativity. (Allow time for students to work.)
- T: Shade in the facts you completed. (Allow time for students to work.) How many are left to learn?
- S: 16.
- T: Look carefully at those 16 facts. Are there any that you will be able to solve using the commutative property once you know one?

- S: Yes! There are 12 facts that we can use the commutative property to solve. That means we only need to know 6 of them.
- T: Really, there are only 10 new facts to learn before you know all the facts up to 10×10 .

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the purposeful sequencing of the Problem Set guide selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Consider assigning incomplete problems for homework or at another time during the day.

Student Debrief (10 minutes)

Lesson Objective: Study commutativity to find known facts of 6, 7, 8, and 9.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How did commutativity help you solve more facts than you thought you knew in Problem 1(a)?
- Invite students to share their processes for finding the multiplication facts for the array in Problem 2.

Name Gina Date _____

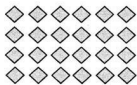
1. a. Solve. Shade in the multiplication facts for sixes, sevens, eights, and nines that you already know. Then shade in the facts for sixes, sevens, eights, and nines that you can solve using the commutative property.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

b. Complete the chart. Each bag contains 7 apples.

Number of bags	2	3	4	5	6
Total number of apples	14	21	28	35	42

2. Use the array to write 2 different multiplication sentences.


 $24 = 4 \times 6$
 $24 = 6 \times 4$

3. Complete the equations.

a. 2 sevens = 7 twos
= 14

b. 3 sixes = 6 threes
= 18

c. 10 eights = 8 tens
= 80

d. $4 \times$ 6 = 6×4
= 24

e. $8 \times 5 =$ 5 $\times 8$
= 40

f. 4 $\times 7 = 7 \times$ 4
= 28

g. $3 \times 9 = 10$ threes $-$ 1 three
= 27

h. 10 fours $-$ 1 four = 9 $\times 4$
= 36

i. $8 \times 4 = 5$ fours $+$ 3 fours
= 32

j. 5 fives $+$ 1 five = 6×5
= 30

k. 5 threes $+$ 2 threes = 7 \times 3
= 21

l. 5 twos $+$ 5 twos = 10 twos
= 20

- In Problems 3(a), 3(b), and 3(c), what do you notice about the words and numbers on each side of the equal sign? How are they related?
- How did you know to subtract 1 three in Problem 3(g)? What would that problem look like rewritten as an equation?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

English language learners and others benefit from reviewing *commutative property* and *commutativity* during the Debrief. Allow students to explain the property to a partner in their first language, and/or record the term with an example in a personal math dictionary.

A

Number Correct: _____

Mixed Multiplication

1.	$2 \times 1 =$	
2.	$2 \times 2 =$	
3.	$2 \times 3 =$	
4.	$4 \times 1 =$	
5.	$4 \times 2 =$	
6.	$4 \times 3 =$	
7.	$1 \times 6 =$	
8.	$2 \times 6 =$	
9.	$1 \times 8 =$	
10.	$2 \times 8 =$	
11.	$3 \times 1 =$	
12.	$3 \times 2 =$	
13.	$3 \times 3 =$	
14.	$5 \times 1 =$	
15.	$5 \times 2 =$	
16.	$5 \times 3 =$	
17.	$1 \times 7 =$	
18.	$2 \times 7 =$	
19.	$1 \times 9 =$	
20.	$2 \times 9 =$	
21.	$2 \times 5 =$	
22.	$2 \times 6 =$	

23.	$2 \times 7 =$	
24.	$5 \times 5 =$	
25.	$5 \times 6 =$	
26.	$5 \times 7 =$	
27.	$4 \times 5 =$	
28.	$4 \times 6 =$	
29.	$4 \times 7 =$	
30.	$3 \times 5 =$	
31.	$3 \times 6 =$	
32.	$3 \times 7 =$	
33.	$2 \times 7 =$	
34.	$2 \times 8 =$	
35.	$2 \times 9 =$	
36.	$5 \times 7 =$	
37.	$5 \times 8 =$	
38.	$5 \times 9 =$	
39.	$4 \times 7 =$	
40.	$4 \times 8 =$	
41.	$4 \times 9 =$	
42.	$3 \times 7 =$	
43.	$3 \times 8 =$	
44.	$3 \times 9 =$	

B

Number Correct: _____

Improvement: _____

Mixed Multiplication

1.	$5 \times 1 =$	
2.	$5 \times 2 =$	
3.	$5 \times 3 =$	
4.	$3 \times 1 =$	
5.	$3 \times 2 =$	
6.	$3 \times 3 =$	
7.	$1 \times 7 =$	
8.	$2 \times 7 =$	
9.	$1 \times 9 =$	
10.	$2 \times 9 =$	
11.	$2 \times 1 =$	
12.	$2 \times 2 =$	
13.	$2 \times 3 =$	
14.	$4 \times 1 =$	
15.	$4 \times 2 =$	
16.	$4 \times 3 =$	
17.	$1 \times 6 =$	
18.	$2 \times 6 =$	
19.	$1 \times 8 =$	
20.	$2 \times 8 =$	
21.	$5 \times 5 =$	
22.	$5 \times 6 =$	

23.	$5 \times 7 =$	
24.	$2 \times 5 =$	
25.	$2 \times 6 =$	
26.	$2 \times 7 =$	
27.	$3 \times 5 =$	
28.	$3 \times 6 =$	
29.	$3 \times 7 =$	
30.	$4 \times 5 =$	
31.	$4 \times 6 =$	
32.	$4 \times 7 =$	
33.	$5 \times 7 =$	
34.	$5 \times 8 =$	
35.	$5 \times 9 =$	
36.	$2 \times 7 =$	
37.	$2 \times 8 =$	
38.	$2 \times 9 =$	
39.	$3 \times 7 =$	
40.	$3 \times 8 =$	
41.	$3 \times 9 =$	
42.	$4 \times 7 =$	
43.	$4 \times 8 =$	
44.	$4 \times 9 =$	

Name _____

Date _____

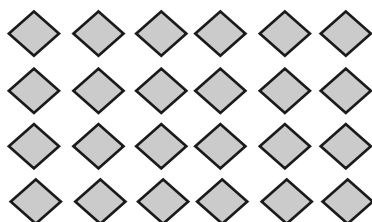
1. a. Solve. Shade in the multiplication facts that you already know. Then, shade in the facts for sixes, sevens, eights, and nines that you can solve using the commutative property.

×	1	2	3	4	5	6	7	8	9	10
1		2	3							
2		4		8				16		
3						18				
4					20					
5										50
6		12								
7										
8										
9										
10										

- b. Complete the chart. Each bag contains 7 apples.

Number of Bags	2		4	5	
Total Number of Apples		21			42

2. Use the array to write two different multiplication sentences.



$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

3. Complete the equations.

a. 2 sevens = _____ twos

= $\frac{14}{\quad}$

g. $3 \times 9 = 10$ threes – _____ three

= _____

b. 3 _____ = 6 threes

= _____

h. 10 fours – 1 four = _____ \times 4

= _____

c. 10 eights = 8 _____

= _____

i. $8 \times 4 = 5$ fours + _____ fours

= _____

d. $4 \times$ _____ = 6×4

= _____

j. _____ fives + 1 five = 6×5

= _____

e. $8 \times 5 =$ _____ $\times 8$

= _____

k. 5 threes + 2 threes = _____ \times _____

= _____

f. _____ $\times 7 = 7 \times$ _____

= $\frac{28}{\quad}$

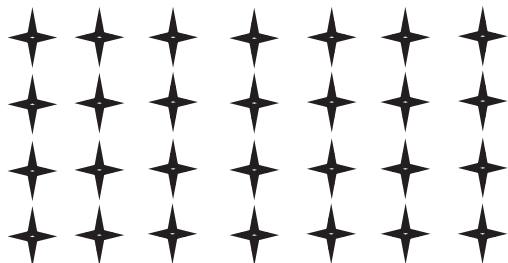
l. _____ twos + _____ twos = 10 twos

= _____

Name _____

Date _____

1. Use the array to write two different multiplication facts.



_____ = _____ × _____

_____ = _____ × _____

2. Karen says, "If I know $3 \times 8 = 24$, then I know the answer to 8×3 ." Explain why this is true.

Name _____

Date _____

1. Complete the charts below.

a. A tricycle has 3 wheels.

Number of Tricycles	3		5		7
Total Number of Wheels		12		18	

b. A tiger has 4 legs.

Number of Tigers			7	8	9
Total Number of Legs	20	24			

c. A package has 5 erasers.

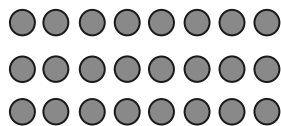
Number of Packages	6				10
Total Number of Erasers		35	40	45	

2. Write two multiplication facts for each array.



_____ = _____ × _____

_____ = _____ × _____



_____ = _____ × _____

_____ = _____ × _____

3. Match the expressions.

3×6

7 threes

3 sevens

2×10

2 eights

9×5

5×9

8×2

10 twos

6×3

4. Complete the equations.

a. 2 sixes = _____ twos
= 12

d. $4 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \times 4$
= 28

b. _____ $\times 6 = 6$ threes
= _____

e. 5 twos + 2 twos = _____ \times _____
= _____

c. $4 \times 8 = \underline{\hspace{1cm}} \times 4$
= _____

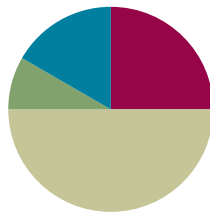
f. _____ fives + 1 five = 6×5
= _____

Lesson 2

Objective: Apply the distributive and commutative properties to relate multiplication facts $5 \times n + n$ to $6 \times n$ and $n \times 6$ where n is the size of the unit.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Sprint: Use the Commutative Property to Multiply **3.OA.5** (9 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Make Ten **3.OA.5** (2 minutes)

Sprint: Use the Commutative Property to Multiply (9 minutes)

Materials: (S) Use the Commutative Property to Multiply Sprint

Note: This Sprint reviews Lesson 1.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes, sevens, eights, and nines in this activity anticipates multiplication using those units later in the module. Focusing on the mentioned transitions bolsters student understanding of the distributive property of multiplication.

Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60, emphasizing the 30 to 36 transition
- Sevens to 70, emphasizing the 35 to 42 transition
- Eights to 80, emphasizing the 40 to 48 transition
- Nines to 90, emphasizing the 45 to 54 transition

Make Ten (2 minutes)

Note: This fluency activity prepares students for the skip-counting strategies used to multiply units of 6 and 7 in Lessons 4 and 5.

T: (Write $9 + \underline{\quad} = 10$.) Say the unknown addend.

S: 1.

Continue with the following suggested sequence: $1 + \underline{\quad} = 10$, $5 + \underline{\quad} = 10$, $8 + \underline{\quad} = 10$, $2 + \underline{\quad} = 10$, $6 + \underline{\quad} = 10$, $7 + \underline{\quad} = 10$, $4 + \underline{\quad} = 10$, and $3 + \underline{\quad} = 10$.

Application Problem (5 minutes)

Jocelyn says 7 fives has the same answer as 3 sevens + 2 sevens. Is she correct? Explain why or why not.

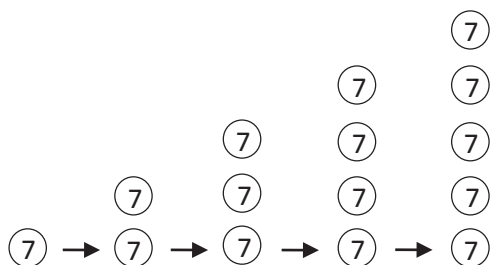
Note: This problem reviews the commutative property from Lesson 1 and also previews the first fact used in the Concept Development to ensure all students' automaticity with the answer.

Jocelyn is correct.
3 sevens + 2 sevens is 5 sevens.
Using the commutative property, we know that 5 sevens is equal to 7 fives. If we write it as an equation, it would look like this: $5 \times 7 = 7 \times 5$ and the answer to both facts is 35.

Concept Development (30 minutes)

Materials: (S) Personal white board

T: (Draw 1 circle with a 7 inside.) This circle represents 1 unit of 7. As I draw circles, count the sevens with me. (Draw circles one on top of the other until you make one column of 5 circles.)



S: 1 seven, 2 sevens, 3 sevens, 4 sevens, 5 sevens.

T: Whisper the multiplication fact for 5 sevens.

S: 5×7 .

T: Use commutativity to name a related fact.

S: 7×5 .

T: What are 5 sevens, 5×7 , and 7×5 all equal to?

S: 35.

MP.7



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

This lesson moves from pictorial representations in the vignette to abstract in the examples suggested for continued guided practice. Depending on the needs of the class, start out concretely. Have students manipulate cubes rather than draw or look at dots. As they become comfortable with the material, transition to pictorial circles, and, if appropriate, eventually to an abstract example.

MP.7

- T: Let's use our familiar fives facts to find facts we haven't learned yet. (Draw a dot above the first 5 dots in another color, shown right.) What is 5 sevens + 1 seven?
- S: 6 sevens.
- T: (Write $35 + 7$.) Tell your partner how this expression shows the total of 6 sevens.
- S: 35 is the total of 5 sevens, and 7 is the total of 1 seven. $\rightarrow 35 + 7$ shows 5 sevens + 1 seven in number form. \rightarrow It's the break apart and distribute strategy we learned before! The dots show 6 sevens broken into 5 sevens and 1 seven because we know those facts, and they're easy!
- T: What is the total of 6 sevens?
- S: 42.
- T: On your personal white board, use commutativity to write the two multiplication facts we just solved.
- S: (Write 6×7 and 7×6 .)
- T: Compare 5×7 and 6×7 . What is the difference between them?
- S: 6×7 has one more group of 7 than 5×7 . \rightarrow That's what the teacher showed with the dots, 5 sevens and 6 sevens.
- T: By noticing that 6×7 is only 1 more group of 7 than 5×7 , we used the total of 5×7 to help us make an easy addition problem to find 6×7 .

7
7
7
7
7
7



NOTES ON MULTIPLE MEANS OF REPRESENTATION:




Problem 1 of the Problem Set reviews 6×7 used in the vignette using blocks. Although the blocks were not used in the lesson, it is familiar enough to feel friendly for students and provides an opportunity to discuss the difference in models during the Debrief.

Continue with the following suggested sequence. Use the model of the dots as necessary, changing the value of 1 dot to match the problem.

- 5×9 to find 6×9 and 9×6
- 5×6 to find 6×6

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Name: <u>Gina</u> Date: _____	
1. Each  has a value of 7.	
	Unit form: 5 <u>sevens</u>
	Facts: $5 \times 7 = 7 \times 5$
	Total = <u>35</u>
	Unit form: 6 sevens = <u>5</u> sevens + <u>1</u> seven
	$= 35 + 7$
	$= 42$
	Facts: $6 \times 7 = 42$
	$7 \times 6 = 42$

Student Debrief (10 minutes)

Lesson Objective: Apply the distributive and commutative properties to relate multiplication facts $5 \times n + n$ to $6 \times n$ and $n \times 6$ where n is the size of the unit.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What pattern did you notice between Problems 1 and 2?
- Explain to your partner how one fact can help you solve two new facts.
- Explain why you used multiplication or division to solve Problem 4. How does a division sentence in this problem relate to a multiplication sentence?
- How does the strategy we learned today relate to the break apart and distribute strategy we studied in Module 1?
- How might you use the strategy we practiced today to solve other problems? For example, how might you use 5×7 to help you solve 7×7 ?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

2) a. Each dot has a value of 8.

Unit form: 5 eights

Facts: $5 \times \underline{8} = \underline{8} \times 5$

Total: 40

b. Use the fact above to find 8×6 . Show your work using pictures, numbers or words.

6 eights { 1 eight
5 eights

6 eights = 5 eights + 1 eight
= $40 + 8$
= 48

Since I know $6 \times 8 = 48$ by adding 1 more eight to 5 eights, I also know that $8 \times 6 = 48$ because of commutativity.

3. An author writes 9 pages of her book each week. How many pages does she write in 7 weeks? Use a five fact to solve.

5 nines { 1 nine
4 nines } 2 nines

$5 \times 9 = 45$ $2 \times 9 = 18$

$7 \times 9 = (5 \times 9) + (2 \times 9)$
= $45 + 18$
= 63

She writes 63 pages in 7 weeks.

4. Mrs. Gonzalez buys a total of 32 crayons for her classroom. Each pack contains 8 crayons. How many packs of crayons does Mrs. Gonzalez buy?

32
8
How many 8s?

$32 \div 8 = 4$
Mrs. Gonzalez buys 4 packs of crayons.

5. Hannah has \$500. She buys a camera for \$435 and 4 other items for \$9 each. Now Hannah wants to buy speakers for \$50. Does she have enough money to buy the speakers? Explain.

$4 \times \$9 = \36

$\$435$
+ 36
471

$\begin{array}{r} 471 \\ - 471 \\ \hline 29 \end{array}$

Hannah has \$29 left. She does not have enough money to buy the speakers.

A

Number Correct: _____

Use the Commutative Property to Multiply

1.	$2 \times 2 =$	
2.	$2 \times 3 =$	
3.	$3 \times 2 =$	
4.	$2 \times 4 =$	
5.	$4 \times 2 =$	
6.	$2 \times 5 =$	
7.	$5 \times 2 =$	
8.	$2 \times 6 =$	
9.	$6 \times 2 =$	
10.	$2 \times 7 =$	
11.	$7 \times 2 =$	
12.	$2 \times 8 =$	
13.	$8 \times 2 =$	
14.	$2 \times 9 =$	
15.	$9 \times 2 =$	
16.	$2 \times 10 =$	
17.	$10 \times 2 =$	
18.	$5 \times 3 =$	
19.	$3 \times 5 =$	
20.	$5 \times 4 =$	
21.	$4 \times 5 =$	
22.	$5 \times 5 =$	

23.	$5 \times 6 =$	
24.	$6 \times 5 =$	
25.	$5 \times 7 =$	
26.	$7 \times 5 =$	
27.	$5 \times 8 =$	
28.	$8 \times 5 =$	
29.	$5 \times 9 =$	
30.	$9 \times 5 =$	
31.	$5 \times 10 =$	
32.	$10 \times 5 =$	
33.	$3 \times 3 =$	
34.	$3 \times 4 =$	
35.	$4 \times 3 =$	
36.	$3 \times 6 =$	
37.	$6 \times 3 =$	
38.	$3 \times 7 =$	
39.	$7 \times 3 =$	
40.	$3 \times 8 =$	
41.	$8 \times 3 =$	
42.	$3 \times 9 =$	
43.	$9 \times 3 =$	
44.	$4 \times 4 =$	

B

Number Correct: _____

Improvement: _____

Use the Commutative Property to Multiply

1.	$5 \times 2 =$	
2.	$2 \times 5 =$	
3.	$5 \times 3 =$	
4.	$3 \times 5 =$	
5.	$5 \times 4 =$	
6.	$4 \times 5 =$	
7.	$5 \times 5 =$	
8.	$5 \times 6 =$	
9.	$6 \times 5 =$	
10.	$5 \times 7 =$	
11.	$7 \times 5 =$	
12.	$5 \times 8 =$	
13.	$8 \times 5 =$	
14.	$5 \times 9 =$	
15.	$9 \times 5 =$	
16.	$5 \times 10 =$	
17.	$10 \times 5 =$	
18.	$2 \times 2 =$	
19.	$2 \times 3 =$	
20.	$3 \times 2 =$	
21.	$2 \times 4 =$	
22.	$4 \times 2 =$	

23.	$6 \times 2 =$	
24.	$2 \times 6 =$	
25.	$2 \times 7 =$	
26.	$7 \times 2 =$	
27.	$2 \times 8 =$	
28.	$8 \times 2 =$	
29.	$2 \times 9 =$	
30.	$9 \times 2 =$	
31.	$2 \times 10 =$	
32.	$10 \times 2 =$	
33.	$3 \times 3 =$	
34.	$3 \times 4 =$	
35.	$4 \times 3 =$	
36.	$3 \times 6 =$	
37.	$6 \times 3 =$	
38.	$3 \times 7 =$	
39.	$7 \times 3 =$	
40.	$3 \times 8 =$	
41.	$8 \times 3 =$	
42.	$3 \times 9 =$	
43.	$9 \times 3 =$	
44.	$4 \times 4 =$	

Name _____

Date _____

1. Each  has a value of 7.



Unit form: 5 _____

Facts: $5 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \times 5$

Total = _____



Unit form: 6 sevens = _____ sevens + _____ seven

= $35 + \underline{\hspace{1cm}}$

= _____

Facts: _____ \times _____ = __________ \times _____ = _____

2. a. Each dot has a value of 8



Unit form: 5 _____



Facts: $5 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \times 5$



Total = _____

- b. Use the fact above to find 8×6 . Show your work using pictures, numbers, or words.

3. An author writes 9 pages of her book each week. How many pages does she write in 7 weeks?
Use a fives fact to solve.
-
4. Mrs. Gonzalez buys a total of 32 crayons for her classroom. Each pack contains 8 crayons. How many packs of crayons does Mrs. Gonzalez buy?
-
5. Hannah has \$500. She buys a camera for \$435 and 4 other items for \$9 each. Now Hannah wants to buy speakers for \$50. Does she have enough money to buy the speakers? Explain.

Name _____

Date _____

Use a fives fact to help you solve 7×6 . Show your work using pictures, numbers, or words.

Name _____

Date _____

1. Each  has a value of 9.



Unit form: _____

Facts: $5 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \times 5$

Total = _____



Unit form: 6 nines = _____ nines + _____ nine

= $45 + \underline{\hspace{1cm}}$

= _____

Facts: _____ \times _____ = __________ \times _____ = _____

2. There are 6 blades on each windmill. How many total blades are on 7 windmills? Use a fives fact to solve.

-
3. Juanita organizes her magazines into 3 equal piles. She has a total of 18 magazines. How many magazines are in each pile?

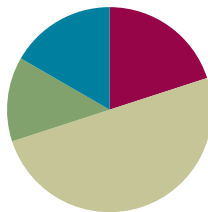
-
4. Markuo spends \$27 on some plants. Each plant costs \$9. How many plants does he buy?

Lesson 3

Objective: Multiply and divide with familiar facts using a letter to represent the unknown.

Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(8 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (12 minutes)

- Familiar Facts **3.OA.4** (5 minutes)
- Multiply Using the Distributive Property **3.OA.5** (5 minutes)
- Make Ten **3.OA.5** (2 minutes)

Familiar Facts (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the relationship between multiplication and division from Grade 3 Module 1 in anticipation of today's lesson.

T: (Write $5 \times 3 = \underline{\quad}$.) Say the multiplication sentence.

S: $5 \times 3 = 15$.

T: (Write $5 \times 3 = 15$. To the right, write $15 \div 3 = \underline{\quad}$.) On your personal white board, write the division sentence.

S: (Write $15 \div 3 = 5$.)

Repeat process for 4×3 and 7×2 .

T: (Write $\underline{\quad} \times 2 = 10$.) Say the unknown factor.

S: 5.

T: (Write $10 \div 2 = \underline{\quad}$.) On your board, write the division sentence.

S: (Write $10 \div 2 = 5$.)

Repeat process for $___ \times 3 = 6$ and $___ \times 2 = 16$.

T: (Write $20 = ___ \times 10$.) Say the unknown factor.

S: 2.

T: (Write $20 \div 10 = ___$.) On your board, write the division sentence.

S: (Write $20 \div 10 = 2$.)

Repeat process for $18 = ___ \times 3$ and $45 = ___ \times 5$.

Multiply Using the Distributive Property (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the $n + 1$ strategy from Lesson 2.

T: (Project a 6×9 array, covering the sixth row of 9.)
How many groups of 9 are there?

S: 5.

T: Let's find how many are in the array counting by fives.
(Point as students count.)

S: 5, 10, 15, 20, 25, 30, 35, 40, 45.

T: Let's find how many are in the array counting by nines.
(Point as students count.)

S: 9, 18, 27, 36, 45.

T: Write two multiplication sentences for this array.

S: (Write $9 \times 5 = 45$ and $5 \times 9 = 45$.)

T: (Reveal the sixth row of 9.) How many groups of 9 are there now?

S: 6.

T: Add 1 more group of 9 to 45. (Write $45 + 9 = ______$.) On your board, write the addition sentence.

S: (Write $45 + 9 = 54$.)

T: On your board, write two multiplication sentences for this array.

S: (Write $9 \times 6 = 54$ and $6 \times 9 = 54$.)

Continue with the following suggested sequence: $5 \times 8 \rightarrow 6 \times 8$, $5 \times 7 \rightarrow 6 \times 7$, and $5 \times 6 \rightarrow 6 \times 6$.

Make Ten (2 minutes)

Note: This fluency activity prepares students for the skip-counting strategies used to multiply units of 6 and 7 in Lessons 4 and 5.

T: I'll say a number between 0 and 10. You say the number that you add to it to make ten. 9.

S: 1.

Continue with the following suggested sequence: 8, 7, 6, 5, 9, 1, 8, 2, 7, 3, 6, 4, 8, 4, 7, 3, 6, 1, 2, 5, and 9.

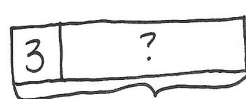


NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Use color to customize the presentation of the *Multiply Using the Distributive Property* fluency activity. Using a different color for each row of 9 may help students count groups of 9. The various colors can additionally help students interpret each column of the array as fives.

Application Problem (8 minutes)

Twenty-four people line up to use the canoes at the park. Three people are assigned to each canoe. How many canoes are used?



24 people
? canoes

$$? \times 3 = 24$$

$$? = 8$$

The people use
8 canoes.

Note: Students may choose to model either as division or as multiplication. In the Concept Development, this problem provides a context for using a letter to represent the unknown.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Challenge above grade level learners by adding complexity to the Application Problem.

For example: Twenty-eight people lined up for canoes. Three people are assigned to each canoe. Create at least two solutions to make it possible for those people who don't make a threesome to go canoeing.

Concept Development (30 minutes)

Materials: (S) Personal white board

Problem 1: Use a letter to represent the unknown in multiplication.

T: (Show a student's tape diagram and equation for the Application Problem, or use the example work above.) This is Student A's work on the Application Problem. What do the question marks in her work represent?

S: The unknown. → The number of canoes that are used.

T: We can use a letter to represent the unknown value instead of a question mark. For this problem, we might choose letter *c* to help us express that the unknown stands for how many *canoes* are used in the problem. How can using a letter to express the unknown value change the way we model and solve?

S: There will be *c*'s where the question marks were on the tape diagram and in the equation. I don't think it changes the way you solve though.

T: Let's confirm your thinking. On your personal white board, solve the Application Problem using the letter *c* to express the unknown on your model and in your equation. Solve, and then compare with your work on the Application Problem.

S: (Solve and compare; possible work to the right.)

T: In a complete sentence, what is the value of *c*?

S: The value of *c* is 8.

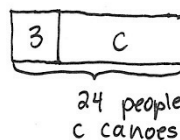
T: How many canoes do the people use?

S: The people use 8 canoes.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Clarify unknowns for English language learners and others by pre-teaching using a few simple equations with letters, such as $2 + 2 = h$; $h = 4$. Or, use the *Familiar Facts* fluency in Lesson 4.



$$c \times 3 = 24$$

$$c = 8$$

The people use
8 canoes.

Problem 2: Use a letter to represent the unknown in division.

Project or show the following problem: Twenty-one students are grouped in threes to go on a field trip. How many groups of students are there?

- T: Read the problem with your partner. Then, whisper what the unknown represents.
- S: (Read problem.) The unknown represents the number of groups.
- T: Before we solve, talk with your partner about which letter you might choose to express the unknown on your model and in your equation.
- S: We could use s for students. \rightarrow I think n reminds me that we are looking for the number of groups. $\rightarrow g$ is best because it stands for *groups*.
- T: Model the problem, and write an equation to solve. Let's use the letter g to express the unknown.
- S: (Model and write $21 \div 3 = g$.)
- T: In a complete sentence, tell the value of g .
- S: The value of g is 7.
- T: How many groups of students are there?
- S: There are 7 groups of students.

Continue with the following suggested sequence to show unknowns in various positions:

- $24 = 4 \times r$
- $5 = 50 \div m$
- $27 \div b = 3$
- $d \div 6 = 3$

Call attention to how the unknown is written, for example, $n = 14$. Students should emulate this in their work.

Depending on time, extend the lesson by assigning each student (or pair of students) a letter of the alphabet. Task them with writing a simple word problem in which their assigned letter represents the unknown. They first solve their own problem and then exchange with another student to solve a new one.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

**NOTES ON CHOOSING VARIABLES:**

Point out that some letters may potentially be confused with other symbols. Letters s , o , l , x , and t , respectively, resemble 5, 0, 1, the multiplication symbol (\times), and the addition symbol ($+$). Encourage students to use other letters if possible.

**NOTES ON NUMBER CHOICES:**

This lesson intentionally uses known facts from Module 1. Ideally, students are fairly automatic with these facts so that the focus stays on naming the unknown, represented by the letter, rather than on performing the calculation.

Student Debrief (10 minutes)

Lesson Objective: Multiply and divide with familiar facts using a letter to represent the unknown.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Explain to your partner how you solved Problem 3. (Review division as both an *unknown factor* and an *equal groups* problem.)
- Tell your partner the steps you took to model and solve Problem 4. (This problem likely posed the greatest challenge.)
- Why is using a letter to represent the unknown more helpful than using a question mark?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Gina Date _____

1. Each equation contains a letter representing the unknown. Find the value of the unknowns, and then write the letters that match the answers to solve the riddle.

$5 \times 4 = e$ $e = 20$ $21 \div 3 = \ell$ $\ell = 7$
 $24 \div i = 4$ $i = 6$ $21 = c \times 7$ $c = 3$
 $32 = s \times 8$ $s = 4$ $t + 10 = 7$ $t = -70$
 $8 = 80 \div n$ $n = 10$ $4 = 36 \div k$ $k = 9$
 $4 = 36 \div k$ $k = 9$ $b = 2$ $24 \div b = 12$
 $8 = a \div 3$ $a = 24$ $h = 5$ $35 = 7 \times h$

Which tables do you NOT have to learn?

k	i	s	e	l	c	t	a	b	e	s		
9	6	70	3	5	20	10	70	24	2	7	20	4

2. Lonna buys 3 t-shirts for \$8 each.

a. What is the total amount Lonna spends on 3 t-shirts? Use the letter m to represent the total amount of money Lonna spends, and then solve the problem.

$3 \times \$8 = m$
 $m = \$24$
 Lonna spends \$24 on t-shirts.

b. If Lonna hands the cashier 3 ten dollar bills, how much change will she receive? Use the letter c in an equation to represent the change, and then find the value of c .

$3 \times \$10 = \30
 $\$30 - \$24 = c$
 $c = \$6$
 Lonna will get \$6 in change.

3. Miss Potts used a total of 28 cups of flour to bake some bread. She used 4 cups of flour for each loaf of bread. How many loaves of bread did she bake? Represent the problem using multiplication and division sentences and a letter for the unknown. Then, solve the problem.

$4 \times n = 28$
 $28 \div 4 = n$
 $n = 7$
 Miss Potts made 7 loaves of bread.

4. At a table tennis tournament, two games went on for a total of 32 minutes. One game took 12 minutes longer than the other. How long did it take to complete each game? Use letters to represent the unknowns. Solve the problem.

$32 - 12 = n$
 $n = 20$
 $20 \div 2 = g$
 $g = 10$
 One game took 10 minutes.
 $10 + 12 = m$
 $m = 22$
 The other game took 22 minutes.

CHALLENGE!

Name _____

Date _____

1. Each equation contains a letter representing the unknown. Find the value of the unknowns, and then write the letters that match the answers to solve the riddle.

$5 \times 4 = e$

$e = \underline{\quad}$

$21 \div 3 = \ell$

$\ell = \underline{\quad}$

$24 \div i = 4$

$i = \underline{\quad}$

$32 = s \times 8$

$s = \underline{\quad}$

$21 = c \times 7$

$c = \underline{\quad}$

$8 = 80 \div n$

$n = \underline{\quad}$

$t \div 10 = 7$

$t = \underline{\quad}$

$4 = 36 \div k$

$k = \underline{\quad}$

$b = \underline{\quad}$

$24 \div b = 12$

$8 = a \div 3$

$a = \underline{\quad}$

$h = \underline{\quad}$

$35 = 7 \times h$

Which tables do you NOT have to learn?

9

6

70

3

5

20

10

70

24

2

7

20

4

2. Lonna buys 3 t-shirts for \$8 each.
- What is the total amount Lonna spends on 3 t-shirts? Use the letter m to represent the total amount of money Lonna spends, and then solve the problem.
 - If Lonna hands the cashier 3 ten dollar bills, how much change will she receive? Use the letter c in an equation to represent the change, and then find the value of c .

3. Miss Potts used a total of 28 cups of flour to bake some bread. She used 4 cups of flour for each loaf of bread. How many loaves of bread did she bake? Represent the problem using multiplication and division sentences and a letter for the unknown. Then, solve the problem.

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

4. At a table tennis tournament, two games went on for a total of 32 minutes. One game took 12 minutes longer than the other. How long did it take to complete each game? Use letters to represent the unknowns. Solve the problem.

**CHALLENGE!**

Name _____

Date _____

Find the value of the unknown in Problems 1–4.

1. $z = 5 \times 9$

$z = \underline{\hspace{2cm}}$

2. $30 \div 6 = v$

$v = \underline{\hspace{2cm}}$

3. $8 \times w = 24$

$w = \underline{\hspace{2cm}}$

4. $y \div 4 = 7$

$y = \underline{\hspace{2cm}}$

5. Mr. Strand waters his rose bushes for a total of 15 minutes. He waters each rose bush for 3 minutes. How many rose bushes does Mr. Strand water? Represent the problem using multiplication and division sentences and a letter for the unknown. Then, solve the problem.

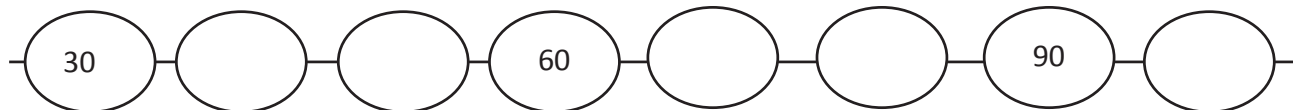
$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Name _____

Date _____

1. a. Complete the pattern.



- b. Find the value of the unknown.

$10 \times 2 = d$ $d = \underline{20}$

$10 \times 6 = w$ $w = \underline{\hspace{2cm}}$

$3 \times 10 = e$ $e = \underline{\hspace{2cm}}$

$10 \times 7 = n$ $n = \underline{\hspace{2cm}}$

$f = 4 \times 10$ $f = \underline{\hspace{2cm}}$

$g = 8 \times 10$ $g = \underline{\hspace{2cm}}$

$p = 5 \times 10$ $p = \underline{\hspace{2cm}}$

2. Each equation contains a letter representing the unknown. Find the value of the unknown.

$8 \div 2 = n$	$n = \underline{\hspace{2cm}}$
$3 \times a = 12$	$a = \underline{\hspace{2cm}}$
$p \times 8 = 40$	$p = \underline{\hspace{2cm}}$
$18 \div 6 = c$	$c = \underline{\hspace{2cm}}$
$d \times 4 = 24$	$d = \underline{\hspace{2cm}}$
$h \div 7 = 5$	$h = \underline{\hspace{2cm}}$
$6 \times 3 = f$	$f = \underline{\hspace{2cm}}$
$32 \div y = 4$	$y = \underline{\hspace{2cm}}$

3. Pedro buys 4 books at the fair for \$7 each.
- What is the total amount Pedro spends on 4 books? Use the letter b to represent the total amount Pedro spends, and then solve the problem.
 - Pedro hands the cashier 3 ten dollar bills. How much change will he receive? Write an equation to solve. Use the letter c to represent the unknown.
4. On field day, the first-grade dash is 25 meters long. The third-grade dash is twice the distance of the first-grade dash. How long is the third-grade dash? Use a letter to represent the unknown and solve.



Topic B

Multiplication and Division Using Units of 6 and 7

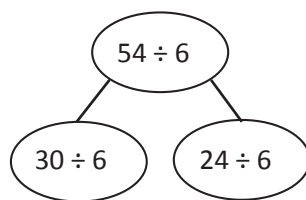
3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7, 3.OA.1, 3.OA.2, 3.OA.6

Focus Standards:	3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
	3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$</i>
	3.OA.5	Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i>
	3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
Instructional Days:	4	
Coherence -Links from:	G2–M3	Place Value, Counting, and Comparison of Numbers to 1,000
	G2–M6	Foundations of Multiplication and Division
	G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
	-Links to:	G3–M4 Multiplication and Area
		G4–M3 Multi-Digit Multiplication and Division
		G4–M5 Fraction Equivalence, Ordering, and Operations
		G4–M7 Exploring Measurement with Multiplication

In Lessons 4 and 5, students count by sixes and sevens, composing up to and then over the next ten. For example, students might count 6, 12, 18, and then mentally add $18 + 2 + 4$ to make 24. This skip-counting method utilizes make ten strategies from Grades 1 and 2. Initially, students use number bonds to decompose and identify appropriate number pairs. In the example above, 18 needs 2 more to make 20. The next six can be decomposed as 2 and 4. Eventually, students are able to use mental math as they manipulate numbers and skip-count to multiply. Although a formal introduction to the associative property comes in Topic C, these lessons preview the concept using addition:

- $6 + 6 = 6 + 4 + 2$
- $18 + 6 = 18 + 2 + 4$
- $36 + 6 = 36 + 4 + 2$
- $48 + 6 = 48 + 2 + 4$

Lesson 6 builds on Lesson 2 with a formal re-introduction of the distributive property using the $5 + n$ pattern to multiply and divide. Students understand that multiples of 6 can be thought of as $(5 + 1) \times n$ to make 5 and 1 more groups, or 6 groups of n . Similarly, multiples of 7 can be thought of as $(5 + 2) \times n$ to make 5 and 2 more groups, or 7 groups of n . In division, students decompose the dividend using a multiple of 5 and then add the quotients of the smaller division facts to find the quotient of the larger unknown division fact. For example:



$$\begin{aligned}
 54 \div 6 &= (30 \div 6) + (24 \div 6) \\
 &= 5 + 4 \\
 &= 9
 \end{aligned}$$

Use of the $5 + n$ pattern as a strategy builds on concepts in Lessons 2, 4, and 5. It also facilitates mental math, particularly using units of 6.

In Lesson 7, students use tape diagrams to analyze multiplication and division word problems and to determine the unknown. This is the first time they solve problems using new units, with a letter to represent the unknown.

A Teaching Sequence Toward Mastery of Multiplication and Division Using Units of 6 and 7

Objective 1: Count by units of 6 to multiply and divide using number bonds to decompose. (Lesson 4)

Objective 2: Count by units of 7 to multiply and divide using number bonds to decompose. (Lesson 5)

Objective 3: Use the distributive property as a strategy to multiply and divide using units of 6 and 7. (Lesson 6)

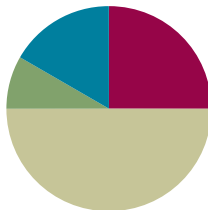
Objective 4: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7. (Lesson 7)

Lesson 4

Objective: Count by units of 6 to multiply and divide using number bonds to decompose.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Group Counting **3.OA.1** (4 minutes)
- Familiar Facts **3.OA.4** (4 minutes)
- Multiply Using the Distributive Property **3.OA.5** (4 minutes)
- Make Ten Game **3.OA.5** (3 minutes)

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes and sevens prepares students for multiplication using those units in this topic. Group counting by eights and nines anticipates multiplication using those units later in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

Familiar Facts (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the relationship between multiplication and division from Module 1 and uses a letter to represent the unknown from Lesson 3 in this module, Grade 3 Module 3.

T: (Write $6 \times 2 = a$.) On your personal white board, write the value of a .

S: (Write $a = 12$.)

T: (Write $6 \times 2 = 12$. To the right, write $12 \div 6 = b$.) On your board, write the value of b .

S: (Write $b = 2$.)

Repeat the process for the following: $7 \times 3 = c$, $21 \div 7 = d$, $e \times 4 = 24$, $24 \div 4 = f$, $g \times 2 = 18$, $18 \div 2 = h$, $16 = i \times 2$, $16 \div 8 = j$, $45 = 5 \times k$, and $45 \div 9 = m$.

Multiply Using the Distributive Property (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the $n + 1$ strategy from Lesson 2.

T: (Project a 6×6 array, covering the sixth row of 6.) How many groups of 6 are there?

S: 5.

T: Let's find how many are in the array counting by fives. (Point as students count.)

S: 5, 10, 15, 20, 25, 30.

T: Let's find how many are in the array counting by sixes. (Point as students count.)

S: 6, 12, 18, 24, 30.

T: Write two multiplication sentences for this array.

S: (Write $6 \times 5 = 30$ and $5 \times 6 = 30$.)

T: (Reveal the sixth row of 6.) How many groups of 6 are there now?

S: 6.

T: Add 1 more group of 6 to 30. (Write $30 + 6 = \underline{\hspace{1cm}}$.) On your board, write the addition sentence.

S: (Write $30 + 6 = 36$.)

T: On your board, write a multiplication sentence for this array.

S: (Write $6 \times 6 = 36$.)

Continue with the following suggested sequence: $5 \times 8 \rightarrow 6 \times 8$, $5 \times 7 \rightarrow 6 \times 7$, and $5 \times 9 \rightarrow 6 \times 9$.

Make Ten Game (3 minutes)

Materials: (S) Set of playing cards numbered 1–9

Note: This fluency activity prepares students for today's Concept Development.

Students play in pairs. Each pair has a set of 9 cards, each with a number 1–9.

T: (Write $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = 10$.) Spread the cards out in front of you.

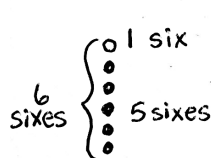
T: Put your hands behind your back. I'll write a number in the first blank. When you know the number that belongs in the second blank, touch the card that shows the number. The first person to touch the card keeps it. Whoever has the most cards at the end wins. (Write $8 + \underline{\hspace{1cm}} = 10$.)

S: (Touch the 2 card. The first to touch it keeps the card.)

Continue with the following suggested sequence: 5, 2, 7, 1, 4, 3, and 6; students replace cards: 1, 5, 3, 2, 4, 7, and 6; students replace cards: 4, 7, 3, 9, and 6.

Application Problem (5 minutes)

Marshall puts 6 pictures on each of the 6 pages in his photo album. How many pictures does he put in the photo album in all?



$$6 \text{ sixes} = 5 \text{ sixes} + 1 \text{ six}$$

$$= 30 + 6$$

$$= 36$$

Marshall puts 36 pictures in his photo album.

Note: This problem is designed to review the Lesson 2 concept of applying the distributive property to relate multiplication by $n + 1$ to multiplication by n .



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

To encourage students to apply the distributive property, adjust the numbers in the Application Problem. For example, a student above grade level may mentally solve 6 times 6 yet choose to use the distributive property to multiply 6 times 13.

Concept Development (30 minutes)

Materials: (S) Personal white board

Part 1: Use number bonds to decompose and skip-count using units of 6.

T: Some of you may have skip-counted by six to get the answer to Marshall's problem. When we're skip-counting by six, how do we get the next number in our sequence?

S: Add 6.

T: Like this? (Write $6 + 6 = 12$.)

T: Think back to our Fluency Practice today. What number should I add to 6 to make 10?

S: 4.

T: Write my equation on your board. Then, draw a number bond to break apart the second six, showing how to solve using a make ten strategy. (Draw the number bond as shown.)

T: 6 plus 4 equals?

S: 10.

T: Write it next to $6 + 6$. (Write the equation as shown.)

T: 10 plus 2 equals...?

S: 12.

T: Write that under $6 + 4 = 10$. (Write the equation as shown.)

T: So, what is 6 plus 6?

S: 12.

$$\begin{array}{r} 6 + 6 = 12 \\ \wedge \\ 4 \quad 2 \end{array}$$

$$\begin{array}{r} 6 + 4 = 10 \\ 10 + 2 = 12 \end{array}$$



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Learners differ in their ability to process information. Help students organize board notes to support their understanding. For example, consider labeling, writing out, illustrating, or putting to music the steps of the make ten strategy.

Have students repeat the process for the next two minutes to see how high they can count by six:

$12 + 6 = 18$, $18 + 6 = 24$, $24 + 6 = 30$, $30 + 6 = 36$, $36 + 6 = 42$, $42 + 6 = 48$, $48 + 6 = 54$, and $54 + 6 = 60$.

T: What patterns did you notice counting by six?

S: Sometimes we broke apart the six to complete the ten, as in $18 + 6$. Other times, we broke apart the two-digit number to add to the ones, as in $12 + 6$, and sometimes to complete the ten, as in $24 + 6$.

T: How did adding 6 and 6 help you add 36 and 6?

S: You add it the same way by breaking the six into 4 and 2 to complete the ten. → You can also think of $36 + 6$ as 3 tens more than $6 + 6$.

T: How did adding 18 and 6 help you add 48 and 6?

S: We broke apart the six into 2 and 4 to complete the ten. → $48 + 6$ is also just 3 tens more than $18 + 6$.

MP.5

T: Why is a make ten strategy with number bonds helpful for counting by sixes?

S: Adding sixes is harder than adding with tens.

T: What other count-by is it helpful for? Is it helpful for the fives?

S: No, because adding by fives is easy to do mentally.

T: Is it helpful for sevens?

S: I think so. Sevens are hard to skip-count by, too.

T: Now that we have discussed how the make ten strategy makes skip-counting sixes more efficient, let's try it out to solve multiplication and division problems.

Part 2: Use skip-counting by sixes to solve multiplication and division problems.

T: Skip-count by six 10 times. Write the count-by sequence on your board. You can also record your addition on your board.

Sample Student Board of Count-By:

6	12	18	24	30	36	42	48	54	60
	4+2		2+4			4+2		2+4	

T: What is the last number in the sequence?

S: 60.

T: 60 is the same as how many sixes?

S: 10 sixes.

$$\begin{array}{r}
 12 + 6 = 18 \\
 \begin{array}{r}
 10 \quad 2 \\
 \diagup \quad \diagdown
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 2 + 6 = 8 \\
 10 + 8 = 18
 \end{array}$$

$$\begin{array}{r}
 18 + 6 = 24 \\
 \begin{array}{r}
 20 \quad 4 \\
 \diagup \quad \diagdown
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 18 + 2 = 20 \\
 20 + 4 = 24
 \end{array}$$

$$\begin{array}{r}
 24 + 6 = 30 \\
 \begin{array}{r}
 20 \quad 4 \\
 \diagup \quad \diagdown
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 4 + 6 = 10 \\
 20 + 10 = 30
 \end{array}$$

$$30 + 6 = 36$$

$$\begin{array}{r}
 36 + 6 = 42 \\
 \begin{array}{r}
 40 \quad 2 \\
 \diagup \quad \diagdown
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 36 + 4 = 40 \\
 40 + 2 = 42
 \end{array}$$

$$\begin{array}{r}
 42 + 6 = 48 \\
 \begin{array}{r}
 40 \quad 2 \\
 \diagup \quad \diagdown
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 2 + 6 = 8 \\
 40 + 8 = 48
 \end{array}$$

$$\begin{array}{r}
 48 + 6 = 54 \\
 \begin{array}{r}
 50 \quad 4 \\
 \diagup \quad \diagdown
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 48 + 2 = 50 \\
 50 + 4 = 54
 \end{array}$$

$$\begin{array}{r}
 54 + 6 = 60 \\
 \begin{array}{r}
 50 \quad 4 \\
 \diagup \quad \diagdown
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 4 + 6 = 10 \\
 50 + 10 = 60
 \end{array}$$

- T: Tell a partner what multiplication problem we just solved, and then write it on your board.
- S: 6 times 10 equals 60. (Write $6 \times 10 = 60$.)
- T: We just used our count-by six sequence to help us find that 6 times 10 equals 60.
- T: We can also use skip-counting to solve division problems. Write the last number in the sequence on your board, followed by a division symbol.
- T: What did we count by to get to 60?
- S: Sixes!
- T: Write 6 after the division symbol, followed by an equal sign.
- T: How many times did we count by six to get to 60?
- S: 10 times!
- T: Write 10 as the answer to this division problem. Read your equation to a partner.
- T: Turn and talk to a partner. What do you notice about the multiplication and division problems we solved?
- S: They use the same numbers. → The division fact uses the same numbers as the multiplication fact, just in a different order. → 60 divided by 6 equals 10 is the related division fact for 6 times 10 equals 60.
- T: That's right. They are related facts. Now, you have learned another strategy to solve multiplication and division facts with sixes!

Continue with the following suggested sequence to help students develop strategies for and learn the following facts. Refer students to the times table chart in Lesson 1 to focus their attention on the 16 new facts:

- 6×6
- 6×7
- 6×8
- 6×9

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Name Gina Date _____

1. Count-by 6 to fill in the blanks. Match each number in the count-by with its multiplication fact.

Student Debrief (10 minutes)

Lesson Objective: Count by units of 6 to multiply and divide using number bonds to decompose.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- With a partner, list the related division facts for each number in the skip-counting sequence in Problem 1.
- What other multiplication and division problems can you solve with the sequence you already have in Problem 2?
- How was using the skip-counting strategy in Problem 4 different from the other problems?
- How does your method of adding 18 and 6 help you add 18 tens and 6 tens, and \$480 and \$60?
- How did the Application Problem connect to today's lesson?
- How did the Fluency Practice help prepare you for the lesson?

Exit Ticket (3 minutes)

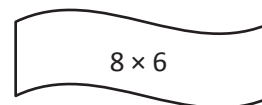
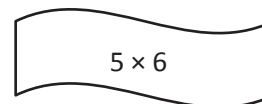
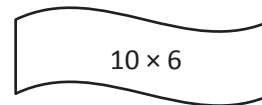
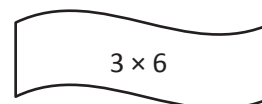
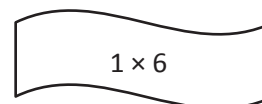
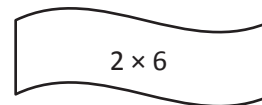
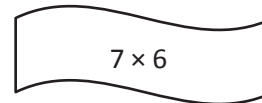
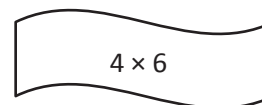
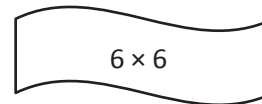
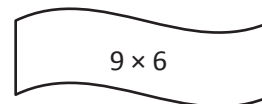
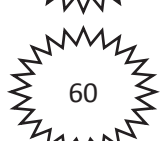
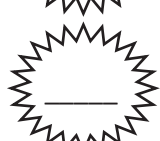
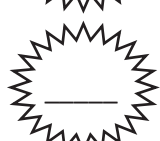
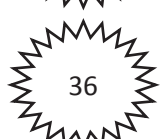
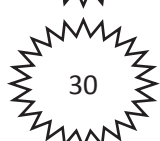
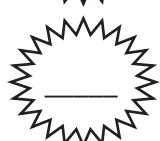
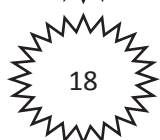
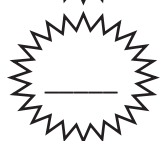
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

<p>2. Count by six to fill in the blanks below.</p> <p>6, <u>12</u>, <u>18</u>, <u>24</u></p> <p>Complete the multiplication equation that represents the final number in your count-by.</p> <p>$6 \times \underline{4} = \underline{24}$</p> <p>Complete the division equation that represents your count-by.</p> <p>$\underline{24} \div 6 = \underline{4}$</p>	<p>3. Count by six to fill in the blanks below.</p> <p>6, <u>12</u>, <u>18</u>, <u>24</u>, <u>30</u>, <u>36</u>, <u>42</u></p> <p>Complete the multiplication equation that represents the final number in your count-by.</p> <p>$6 \times \underline{7} = \underline{42}$</p> <p>Complete the division equation that represents your count-by.</p> <p>$\underline{42} \div 6 = \underline{7}$</p>
<p>4. Mrs. Byrne's class skip-counts by six for a group counting activity. When she points up, they count up by six and when she points down, they count down by six. The arrows show when she changes direction.</p> <p>a. Fill in the blanks below to show the group counting answers.</p> <p> \uparrow 0, 6, <u>12</u>, 18, <u>24</u> \downarrow <u>18</u>, 12 \uparrow <u>18</u>, 24, 30, <u>36</u> \downarrow 30, 24, <u>18</u> \uparrow 24, <u>30</u>, 36, <u>42</u>, 48 </p> <p>b. Mrs. Byrne says the last number that the class counts is the product of 6 and another number. Write a multiplication sentence and a division sentence to show she's right.</p> <p> $6 \times \underline{8} = 48$ $48 \div 6 = \underline{8}$ </p>	
<p>5. Julie counts by six to solve 6×7. She says the answer is 36. Is she right? Explain your answer.</p> <p> <u>6, 12, 18, 24, 30, 36, 42</u> No, Julie is not right. I counted by six 7 times and I got 42. So, $6 \times 7 = 42$, not 36. </p>	

Name _____

Date _____

1. Skip-count by six to fill in the blanks. Match each number in the count-by with its multiplication fact.



2. Count by six to fill in the blanks below.

6, _____, _____, _____

Complete the multiplication equation that represents the final number in your count-by.

$$6 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Complete the division equation that represents your count-by.

$$\underline{\hspace{2cm}} \div 6 = \underline{\hspace{2cm}}$$

3. Count by six to fill in the blanks below.

6, _____, _____, _____, _____, _____, _____

Complete the multiplication equation that represents the final number in your count-by.

$$6 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Complete the division equation that represents your count-by.

$$\underline{\hspace{2cm}} \div 6 = \underline{\hspace{2cm}}$$

4. Mrs. Byrne's class skip-counts by six for a group counting activity. When she points up, they count up by six, and when she points down, they count down by six. The arrows show when she changes direction.
- a. Fill in the blanks below to show the group counting answers.

↑ 0, 6, _____, 18, _____ ↓ _____, 12 ↑ _____, 24, 30, _____ ↓ 30, 24, _____ ↑ 24, _____, 36, _____, 48

- b. Mrs. Byrne says the last number that the class counts is the product of 6 and another number. Write a multiplication sentence and a division sentence to show she's right.

$$6 \times \underline{\hspace{2cm}} = 48$$

$$48 \div 6 = \underline{\hspace{2cm}}$$

5. Julie counts by six to solve 6×7 . She says the answer is 36. Is she right? Explain your answer.

Name _____

Date _____

1. Sylvia solves 6×9 by adding $48 + 6$. Show how Sylvia breaks apart and bonds her numbers to complete the ten. Then, solve.

2. Skip-count by six to solve the following:

a. $8 \times 6 =$ _____

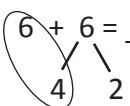
b. $54 \div 6 =$ _____

Name _____

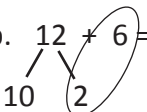
Date _____

1. Use number bonds to help you skip-count by six by either making a ten or adding to the ones.


a. $6 + 6 = \underline{10} + \underline{2} = \underline{\quad}$



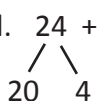
b. $12 + 6 = \underline{10} + \underline{8} = \underline{\quad}$



c. $18 + 6 = \underline{\quad} + \underline{\quad} = \underline{\quad}$




d. $24 + 6 = \underline{\quad} + \underline{\quad} = \underline{\quad}$



e. $30 + 6 = \underline{\quad}$

f. $36 + 6 = \underline{\quad} + \underline{\quad} = \underline{\quad}$



g. $42 + 6 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

h. $48 + 6 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

i. $54 + 6 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

2. Count by six to fill in the blanks below.

6, _____, _____, _____, _____

Complete the multiplication equation that represents the final number in your count-by.

$$6 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Complete the division equation that represents your count-by.

$$\underline{\hspace{2cm}} \div 6 = \underline{\hspace{2cm}}$$

3. Count by six to fill in the blanks below.

6, _____, _____, _____, _____, _____

Complete the multiplication equation that represents the final number in your count-by.

$$6 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Complete the division equation that represents your count-by.

$$\underline{\hspace{2cm}} \div 6 = \underline{\hspace{2cm}}$$

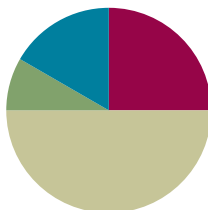
4. Count by six to solve $48 \div 6$. Show your work below.

Lesson 5

Objective: Count by units of 7 to multiply and divide using number bonds to decompose.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Multiply By 6 **3.OA.7** (7 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Make Seven Game **3.OA.5** (4 minutes)

Multiply By 6 (7 minutes)

Materials: (S) Multiply by 6 (1–5) (Pattern Sheet)

Note: This activity builds fluency with multiplication facts using units of six. It supports students knowing from memory all products of two one-digit numbers.

T: (Write $5 \times 6 = \underline{\quad}$.) Let's skip-count by sixes to find the answer. I'll raise a finger for each six. (Count with fingers to 5 as students count, and record the count-by sequence on the board.)

S: 6, 12, 18, 24, 30.

T: (Circle 30, and write $5 \times 6 = 30$ above it. Write $3 \times 6 = \underline{\quad}$.) Let's skip-count up by sixes again. (Count with fingers to 3 as students count.)

S: 6, 12, 18.

T: Let's see how we can skip-count down to find the answer, too. Start at 30 with 5 fingers, 1 for each six. (Count down with your fingers as students say numbers.)

S: 30 (5 fingers), 24 (4 fingers), 18 (3 fingers).

Repeat the process for 4×6 .

T: (Distribute the Multiply by 6 Pattern Sheet.) Let's practice multiplying by 6. Be sure to work left to right across the page.

Directions for administration of Multiply By Pattern Sheet:

- Distribute Multiply By Pattern Sheet.
- Allow a maximum of two minutes for students to complete as many problems as possible.
- Direct students to work left to right across the page.
- Encourage skip-counting strategies to solve unknown facts.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sevens prepares students for multiplication using units of seven in this lesson. Group counting by eights and nines anticipates multiplication using those units later in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sevens to 70
- Eights to 80
- Nines to 90

Make Seven Game (4 minutes)

Materials: (S) Set of playing cards numbered 1–6

Note: This activity prepares students for the skip-counting strategy used to multiply units of seven in today's lesson.

Students play in pairs. Each pair has a set of six cards, each with a number 1–6.

- T: (Write $___ + ___ = 7$.) Spread the cards out in front of you.
- T: Put your hands behind your back. I'll write a number in the first blank. When you know the number that belongs in the second blank, touch the card that shows the number. The first person to touch the card keeps it. Whoever has the most cards at the end wins. (Write $5 + ___ = 7$.)
- S: (Touch the 2 card. The first to touch it keeps the card.)

Continue with the following suggested sequence: 1, 4, 2, 3, and 6.

Application Problem (5 minutes)

Gracie draws 7 rows of stars. In each row, she draws 4 stars. How many stars does Gracie draw in all? Use a letter to represent the unknown and solve.

Note: This problem reviews the Grade 3 Module 1 concept of multiplying using units of four. It is used in the Concept Development to lead into skip-counting by sevens. Be sure to circulate and find a student's answer to use as an example in the Concept Development (find a student who counted by four 7 times to solve the problem).

★★★★	4
★★★★	8
★★★★	12
★★★★	16
★★★★	20
★★★★	24
★★★★	28

$7 \times 4 = g$
 $g = 28$
 Gracie draws
 28 stars.

Concept Development (30 minutes)

Materials: (S) Personal white board

Part 1: Use number bonds to decompose and make ten as a strategy for skip-counting units of 7.

- T: I noticed that Student A solved the Application Problem by skip-counting by four 7 times. Is there another count-by strategy that could be used to solve this problem?
- S: Skip-count by seven 4 times.
- T: Let's show that work on our boards. Write 7 on your board.
- T: How do we get the next number in our count?
- S: Add 7.
- T: Can we use a number bond to add 7 by making ten like we did with sixes?
- S: Yes, we can break apart 7 into 3 and 4, and then use the 3 to make ten with the first 7.
- T: Work with a partner to use number bonds to show how to count by seven 4 times.
- T: Check your work with mine. (Project work as shown.)
- T: What is the last number in the sequence when you count by seven 4 times?
- S: 28.
- T: Is the answer the same even though Student A counted by four 7 times?
- S: Yes, it's the same because we just switched the order of the factors. → The product is the same, but the order of the factors is different. It's the commutative property.
- T: Work with a partner to use number bonds to complete your sequence by counting by seven 10 times. (Circulate and check student work.)
- T: Everyone, at my signal, read your count by seven sequence.
- S: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

For English language learners, pre-teach and/or clarify unfamiliar math terms such as *sequence*, *row*, *factors*, *product*, *number bond*, *count by*, and *skip-count*.

Depending on each learner's needs, give explicit prompts for every step of the make ten strategy to count by seven 4 times. Alternatively, scaffold with a checklist or template.

Count by Seven 4 Times:

$$\begin{array}{r}
 0 + 7 = 7 \\
 7 + 7 = 14 \\
 \quad \wedge \\
 \quad 3 \ 4 \\
 14 + 7 = 21 \\
 \quad \wedge \\
 \quad 6 \ 1 \\
 21 + 7 = 28 \\
 \quad \wedge \\
 \quad 20 \ 1
 \end{array}$$

Remaining Count by Seven:

$$\begin{array}{r}
 28 + 7 = 35 \\
 \quad \wedge \\
 \quad 2 \ 5 \\
 35 + 7 = 42 \\
 \quad \wedge \\
 \quad 5 \ 2 \\
 42 + 7 = 49 \\
 \quad \wedge \\
 \quad 40 \ 2 \\
 49 + 7 = 56 \\
 \quad \wedge \\
 \quad 1 \ 6 \\
 56 + 7 = 63 \\
 \quad \wedge \\
 \quad 4 \ 3 \\
 63 + 7 = 70 \\
 \quad \wedge \\
 \quad 60 \ 3
 \end{array}$$

Part 2: Skip-count by seven to solve multiplication and division problems.

- T: Let's use our sequence to solve multiplication and division problems with seven. I am going to say a multiplication or division problem. Write the problem on your personal white board, and use your sequence to find the answer. At my signal, show your board.
- T: Let's do a practice one together. Turn and talk to a partner. How can you use your skip-counting sequence to solve 42 divided by 7?
- S: I can count 6 sevens in the sequence, which takes me to 42. So, 42 divided by 7 equals 6.
- T: Write the equation on your board.
- S: (Write $42 \div 7 = 6$.)
- T: At my signal, show me your board. (Signal.)
- S: (Show board.)
- T: Okay, here we go, next problem! 49 divided by 7 equals...? (After students work, signal.)


**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Some learners may prefer to use the distributive or commutative property to solve 7 times 6. Encourage their personal choices of efficient strategies. Challenge learners to present a multiplication fact they would solve using skip-counting. Ask, "How do you choose your strategy to solve?"

Continue with the following suggested sequence:





















- 7×6
- $7 \times e = 56$
- $f \div 7 = 9$

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Name Gina Date _____

1. Skip-count by seven to fill in the blanks in the fish bowls. Match each count-by to its multiplication expression. Then, use the multiplication equation to write the related division fact directly to the right.

		$42 \div 7 = 6$
		$21 \div 7 = 3$
		$56 \div 7 = 8$
		$49 \div 7 = 7$
		$7 \div 7 = 1$
		$70 \div 7 = 10$
		$63 \div 7 = 9$
		$28 \div 7 = 4$
		$14 \div 7 = 2$
		$35 \div 7 = 5$

Multiply.

$6 \times 1 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$

$6 \times 5 = \underline{\quad}$ $6 \times 1 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 1 = \underline{\quad}$

$6 \times 3 = \underline{\quad}$ $6 \times 1 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$ $6 \times 1 = \underline{\quad}$

$6 \times 5 = \underline{\quad}$ $6 \times 1 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$

$6 \times 2 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$

$6 \times 2 = \underline{\quad}$ $6 \times 1 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$

$6 \times 1 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$

$6 \times 4 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$

$6 \times 4 = \underline{\quad}$ $6 \times 1 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$

$6 \times 4 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$

$6 \times 4 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$ $6 \times 1 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$

$6 \times 2 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$

$6 \times 4 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$

$6 \times 5 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$

$6 \times 3 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$

multiply by 6 (1–5)

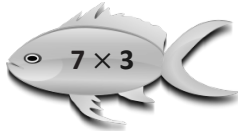
Name _____

Date _____

1. Skip-count by seven to fill in the blanks in the fish bowls. Match each count-by to its multiplication expression. Then, use the multiplication expression to write the related division fact directly to the right.



$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$



$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$



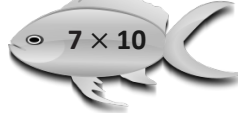
$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$



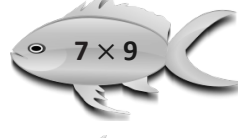
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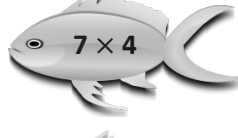
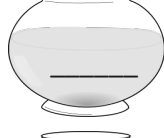
$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$



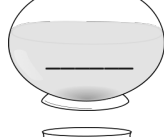
$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$



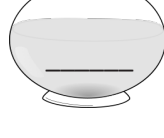
$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$



$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$



$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$



$$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$$

2. Complete the count-by seven sequence below. Then, write a multiplication equation and a division equation to represent each blank you filled in.

7, 14, _____, 28, _____, 42, _____, _____, 63, _____

a. _____ \times 7 = _____ _____ \div 7 = _____

b. _____ \times 7 = _____ _____ \div 7 = _____

c. _____ \times 7 = _____ _____ \div 7 = _____

d. _____ \times 7 = _____ _____ \div 7 = _____

e. _____ \times 7 = _____ _____ \div 7 = _____

-
3. Abe says $3 \times 7 = 21$ because 1 seven is 7, 2 sevens are 14, and 3 sevens are $14 + 6 + 1$, which equals 21. Why did Abe add 6 and 1 to 14 when he is counting by seven?

-
4. Molly says she can count by seven 6 times to solve 7×6 . James says he can count by six 7 times to solve this problem. Who is right? Explain your answer.

Name _____

Date _____

Complete the count-by seven sequence below. Then, write a multiplication equation and a division equation to represent each number in the sequence.

7, 14, _____, 28, _____, 42, _____, _____, 63, _____

a. _____ \times 7 = _____ _____ \div 7 = _____

b. _____ \times 7 = _____ _____ \div 7 = _____

c. _____ \times 7 = _____ _____ \div 7 = _____

d. _____ \times 7 = _____ _____ \div 7 = _____

e. _____ \times 7 = _____ _____ \div 7 = _____

f. _____ \times 7 = _____ _____ \div 7 = _____

g. _____ \times 7 = _____ _____ \div 7 = _____

h. _____ \times 7 = _____ _____ \div 7 = _____

i. _____ \times 7 = _____ _____ \div 7 = _____

j. _____ \times 7 = _____ _____ \div 7 = _____

Name _____

Date _____

1. Use number bonds to help you skip-count by seven by making ten or adding to the ones.

a. $7 + 7 = \underline{10} + \underline{4} = \underline{\quad}$

$\begin{array}{c} / \quad \backslash \\ 3 \quad 4 \end{array}$

b. $14 + 7 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

$\begin{array}{c} / \quad \backslash \\ 6 \quad 1 \end{array}$

c. $21 + 7 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

$\begin{array}{c} / \quad \backslash \\ 20 \quad 1 \end{array}$

d. $28 + 7 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

$\begin{array}{c} / \quad \backslash \\ 2 \quad 5 \end{array}$

e. $35 + 7 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

$\begin{array}{c} / \quad \backslash \\ 5 \quad 2 \end{array}$

f. $42 + 7 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

g. $49 + 7 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

h. $56 + 7 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

2. Skip-count by seven to fill in the blanks. Then, fill in the multiplication equation, and use it to write the related division fact directly to the right.

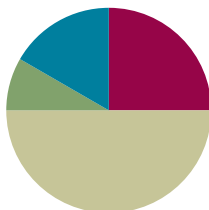
_____	$7 \times 10 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
_____	$7 \times 9 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
_____	$7 \times 8 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
<u>49</u>	$7 \times 7 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
_____	$7 \times 6 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
_____	$7 \times 5 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
<u>28</u>	$7 \times 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
_____	$7 \times 3 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
_____	$7 \times 2 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$
<u>7</u>	$7 \times 1 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div 7 = \underline{\hspace{2cm}}$

Lesson 6

Objective: Use the distributive property as a strategy to multiply and divide using units of 6 and 7.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Multiply By 6 **3.OA.7** (8 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Decompose Multiples of 6 and 7 **3.OA.5** (3 minutes)

Multiply By 6 (8 minutes)

Materials: (S) Multiply by 6 (6–10) (Pattern Sheet)

Note: This activity builds fluency with multiplication facts using units of six. It supports students knowing from memory all products of two one-digit numbers. See Lesson 5 for the directions for administering a Multiply By Pattern Sheet.

- T: (Write $7 \times 6 = \underline{\quad}$.) Let's skip-count up by sixes. I'll raise a finger for each six. (Count with fingers to 7 as students count.)
- S: 6, 12, 18, 24, 30, 36, 42.
- T: Let's skip-count by sixes starting at 30. Why is 30 a good place to start?
- S: It's a fact we already know, so we can use it to figure out a fact we don't know.
- T: Let's see how we can skip-count down to find the answer, too. Start at 60 with 10 fingers, 1 for each six. (Count down with your fingers as students say numbers.)
- S: 60 (10 fingers), 54 (9 fingers), 48 (8 fingers), 42 (7 fingers).

Continue with the following suggested sequence: 9×6 , 6×6 , and 8×6 .

- T: (Distribute Multiply by 6 Pattern Sheet.) Let's practice multiplying by 6. Be sure to work left to right across the page.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Multiply By 6 is carefully scaffolded to support student success. However, consider adjusting the activity to suit students' diverse needs. For example, focus on one skill, such as skip-counting down to solve. Or, have students choose and solve the three hardest facts using three different strategies.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sevens prepares students for multiplication using units of seven in this lesson. Group counting by eights and nines anticipates multiplication using those units later in the module. Direct students to count forward and backward, occasionally changing the direction of the count.

- Sevens to 70
- Eights to 80
- Nines to 90

Decompose Multiples of 6 and 7 (3 minutes)

Materials: (S) Personal white board

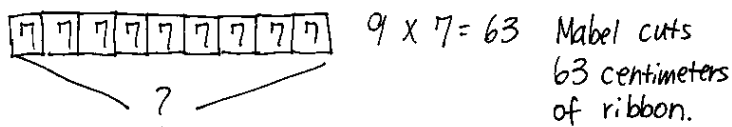
Note: This activity prepares students to use the distributive property with number bonds in today's lesson.

T: (Project a number bond with a whole of 48 and 12 as a part.) On your personal white board, fill in the unknown part in the number bond.

Continue with the following suggested sequence: a whole of 54 and 24 as a part, a whole of 49 and 14 as a part, and a whole of 63 and 21 as a part.

Application Problem (5 minutes)

Mabel cuts 9 pieces of ribbon for an art project. Each piece of ribbon is 7 centimeters long. What is the total length of the pieces of ribbon that Mabel cuts?



Note: This problem reviews multiplication using units of seven. It is the same problem that is used in the first example in the Concept Development. Here it is given a context, while in the Concept Development it is not because the focus shifts to using the distributive property.

Concept Development (30 minutes)

Materials: (S) Personal white board

Part 1: Apply the distributive property to multiply using units of 6 and 7.

T: We used 9×7 to solve the Application Problem. Say 9×7 in unit form.

S: 9 sevens.

T: Model 9×7 using a tape diagram.

Then, write the fact under the diagram.
(Allow students time to work.)

T: Recently, we used the break apart and distribute strategy to help solve larger multiplication facts. Discuss with your partner. How did we do that?

S: We broke apart a bigger fact into two smaller facts. → That made it easier to solve because we just added the products of the two smaller facts.

T: Breaking the bigger fact into 5 plus something helped us make those two smaller facts.
9 sevens can be broken into 5 sevens plus how many sevens?

S: 5 sevens plus 4 sevens.

T: Draw a dotted line separating the 5 sevens from 4 sevens on your tape diagram. Label the 5 sevens and 4 sevens on your tape diagram with multiplication facts.

S: (Draw line, label 5×7 and 4×7 .)

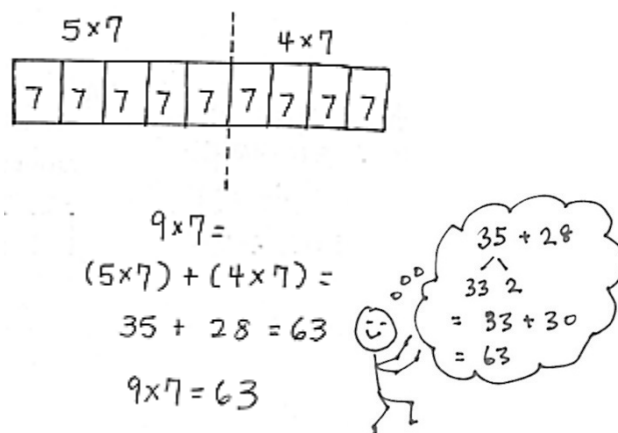
T: Let's use those facts to rewrite 5 sevens plus 4 sevens like this. (Write $(5 \times 7) + (4 \times 7)$.)
Remind your partner why this expression is the same as 9×7 .

S: It's the same because the 5 and the 4 together make 9. → And the 7 just got distributed to the 2 parts.

T: Now, solve. Check your work with your partner's.
(Allow students time to work.)

T: What is 9×7 ?

S: 63.



A NOTE ON MULTIPLE MEANS OF ENGAGEMENT:

Alternatively, challenge students working above grade level to use, compare, and present three different multiplication strategies to solve 8×6 , including the 5 plus something ($5 + n$) strategy.

Continue with the following suggested sequence:

- 8×6
- 8×7

Part 2: Use addition number bonds to apply the distributive property to divide using units of 6 and 7.

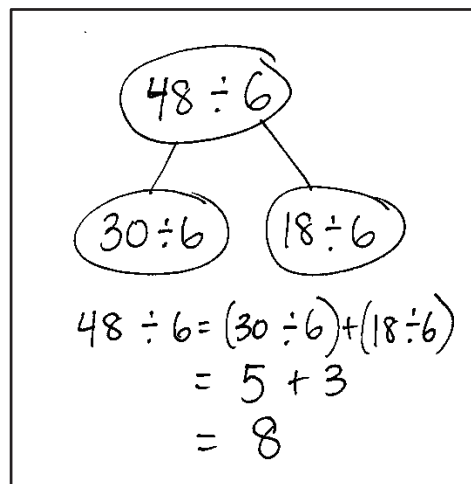
- T: We also used the break apart and distribute strategy earlier this year with arrays and division. Instead of using arrays today, let's use number bonds.
- T: Write $48 \div 6$ on your board and circle it.
- T: We need to break apart $48 \div 6$ into two smaller division expressions. Why would 30 make a good breaking point?
- S: $30 \div 6$ is an easy fives fact.
- T: Write and circle $30 \div 6$ as a part on your number bond.
- S: (Write and circle $30 \div 6$ as a part on the number bond.)
- T: We have $30 \div 6$ as one of our parts. What division expression do we need to write for the other part?
- S: $18 \div 6$.
- T: How do you know?
- S: 30 plus 18 equals 48. \rightarrow I know because we used 30, and we need 18 more to get to 48.
- T: Write and circle $18 \div 6$ as the other part.
- T: Let's show that work with an equation. Write $48 \div 6 = (30 \div 6) + (18 \div 6)$. Put parentheses around the two expressions to show that we solve these two division facts first.
- T: How can we use the quotients of these two division expressions to find the quotient of $48 \div 6$?
- S: Add the quotient of $30 \div 6$ and the quotient of $18 \div 6$.
- T: (Write addition sign as shown.) Add the two quotients to solve for $48 \div 6$.
- S: (Write $5 + 3 = 8$.)
- T: $48 \div 6$ is...?
- S: 8.
- T: Write the answer below your equation. This is a great problem to solve this way since adding to 30 is so easy. What is another 5 fact that results in an easy number?
- S: 5 times 8 is 40. 5 times 4 is 20. \rightarrow The even numbers!
- T: What fact would you like to try next?
- S: Let's do a big number divided by 8. \rightarrow 56 divided by 8.


**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Clarify that the two smaller division expressions must be divided by 6 expressions. Emphasize the value of this strategy as a way to solve sixes facts by using familiar fives, fours, threes, or twos facts.

Alternatively, challenge students to present, compare, and justify the use of a different number bond for $48 \div 6$.

Sample Teacher Board



Repeat the process with $56 \div 8$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use the distributive property as a strategy to multiply and divide using units of 6 and 7.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What pattern did you notice in Problems 1(a) through 1(d)? What multiplication fact is used in all of these problems? How does this fact help you solve these problems?
- What division fact did you use to complete the number bond in Problem 3? Why?
- Show a partner your picture for Problem 4. How does your picture show the break apart and distribute strategy?
- What number bond did you use to solve Problem 5? Explain your choice. Explain why Kelly could not break apart $42 \div 7$ into $30 \div 7$ and $12 \div 7$.
- How does using the break apart and distribute strategy help you multiply and divide using known facts to find the answers to larger, unknown facts?

Name: Gina Date: _____

1. Label the tape diagrams. Then fill in the blanks below to make the statements true.

a) $6 \times 6 = 36$

$(5 \times 6) = 30$ $(1 \times 6) = 6$

$(6 \times 6) = (5 \times 6) + (1 \times 6)$
 $= 30 + 6$
 $= 36$

b) $7 \times 6 = 42$

$(5 \times 6) = 30$ $(2 \times 6) = 12$

$(7 \times 6) = (5 \times 6) + (2 \times 6)$
 $= 30 + 12$
 $= 42$

c) $8 \times 6 = 48$

$(5 \times 6) = 30$ $(3 \times 6) = 18$

$(8 \times 6) = (5 \times 6) + (3 \times 6)$
 $= 30 + 18$
 $= 48$

d) $9 \times 6 = 54$

$(5 \times 6) = 30$ $(4 \times 6) = 24$

$(9 \times 6) = (5 \times 6) + (4 \times 6)$
 $= 30 + 24$
 $= 54$

2. Break apart: 54 to solve $54 \div 6$.

$54 \div 6$

$30 \div 6$ $24 \div 6$

$54 \div 6 = (30 \div 6) + (24 \div 6)$
 $= 5 + 4$
 $= 9$

3. Break apart: 49 to solve $49 \div 7$.

$49 \div 7$

$35 \div 7$ $14 \div 7$

$49 \div 7 = (35 \div 7) + (14 \div 7)$
 $= 5 + 2$
 $= 7$

4. Robert says that he can solve 6×8 by thinking of it as $(5 \times 8) + 8$. Is he right? Draw a picture to help explain your answer.

$6 \times 8 = (5 \times 8) + (1 \times 8)$
 $= (5 \times 8) + 8$
 $= 40 + 8$
 $= 48$

Yes, Robert's thinking is correct. He is using the break apart and distribute strategy to solve.

5. Kelly solves $42 \div 7$ by using a number bond to break apart 42 into two parts. Show what her work might look like below.

$42 \div 7$

$35 \div 7$ $7 \div 7$

$42 \div 7 = (35 \div 7) + (7 \div 7)$
 $= 5 + 1$
 $= 6$

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Multiply.

$6 \times 1 = \underline{\quad}$ $6 \times 2 = \underline{\quad}$ $6 \times 3 = \underline{\quad}$ $6 \times 4 = \underline{\quad}$

$6 \times 5 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$ $6 \times 8 = \underline{\quad}$

$6 \times 9 = \underline{\quad}$ $6 \times 10 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$

$6 \times 5 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$ $6 \times 8 = \underline{\quad}$

$6 \times 5 = \underline{\quad}$ $6 \times 9 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$ $6 \times 10 = \underline{\quad}$

$6 \times 6 = \underline{\quad}$ $6 \times 5 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$

$6 \times 6 = \underline{\quad}$ $6 \times 8 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$ $6 \times 9 = \underline{\quad}$

$6 \times 6 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$

$6 \times 8 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$ $6 \times 9 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$

$6 \times 8 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$ $6 \times 8 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$

$6 \times 8 = \underline{\quad}$ $6 \times 9 = \underline{\quad}$ $6 \times 9 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$

$6 \times 9 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$ $6 \times 9 = \underline{\quad}$ $6 \times 8 = \underline{\quad}$

$6 \times 9 = \underline{\quad}$ $6 \times 8 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$ $6 \times 9 = \underline{\quad}$

$6 \times 7 = \underline{\quad}$ $6 \times 9 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$ $6 \times 8 = \underline{\quad}$

$6 \times 9 = \underline{\quad}$ $6 \times 7 = \underline{\quad}$ $6 \times 6 = \underline{\quad}$ $6 \times 8 = \underline{\quad}$

multiply by 6 (6–10)

Name _____

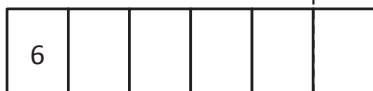
Date _____

1. Label the tape diagrams. Then, fill in the blanks below to make the statements true.

a. $6 \times 6 = \underline{\hspace{2cm}}$

$(5 \times 6) = \underline{\hspace{2cm}}$

$(\underline{\hspace{1cm}} \times 6) = \underline{\hspace{2cm}}$



$$\begin{aligned}
 (6 \times 6) &= (5 + 1) \times 6 \\
 &= (5 \times 6) + (1 \times 6) \\
 &= \underline{30} + \underline{\hspace{1cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

b. $7 \times 6 = \underline{\hspace{2cm}}$

$(5 \times 6) = \underline{\hspace{2cm}}$

$(\underline{\hspace{1cm}} \times 6) = \underline{\hspace{2cm}}$



$$\begin{aligned}
 (7 \times 6) &= (5 + 2) \times 6 \\
 &= (5 \times 6) + (2 \times 6) \\
 &= \underline{30} + \underline{\hspace{1cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

c. $8 \times 6 = \underline{\hspace{2cm}}$

$(5 \times 6) = \underline{\hspace{2cm}}$

$(\underline{\hspace{1cm}} \times 6) = \underline{\hspace{2cm}}$



$$\begin{aligned}
 8 \times 6 &= (5 + \underline{\hspace{1cm}}) \times 6 \\
 &= (5 \times 6) + (\underline{\hspace{1cm}} \times 6) \\
 &= \underline{30} + \underline{\hspace{1cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

d. $9 \times 6 = \underline{\hspace{2cm}}$

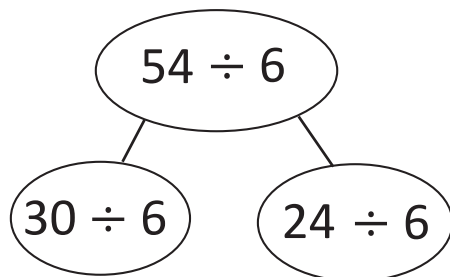
$(5 \times 6) = \underline{\hspace{2cm}}$

$(\underline{\hspace{1cm}} \times 6) = \underline{\hspace{2cm}}$



$$\begin{aligned}
 9 \times 6 &= (5 + \underline{\hspace{1cm}}) \times 6 \\
 &= (5 \times 6) + (\underline{\hspace{1cm}} \times 6) \\
 &= \underline{30} + \underline{\hspace{1cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

2. Break apart 54 to solve $54 \div 6$.

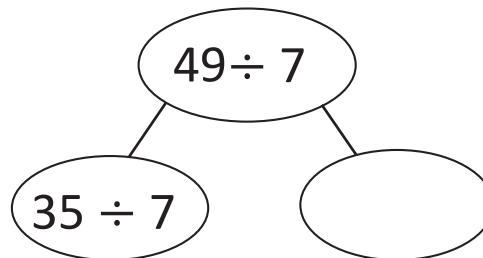


$$54 \div 6 = (30 \div 6) + (\underline{\hspace{2cm}} \div 6)$$

$$= 5 + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

3. Break apart 49 to solve $49 \div 7$.



$$49 \div 7 = (35 \div 7) + (\underline{\hspace{2cm}} \div 7)$$

$$= 5 + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

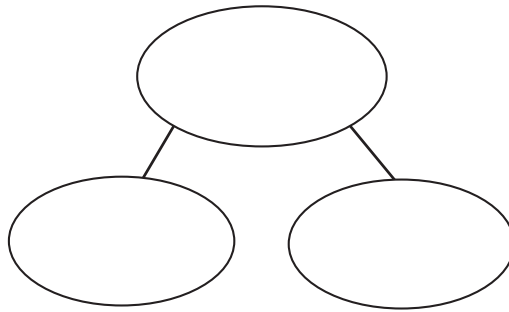
4. Robert says that he can solve 6×8 by thinking of it as $(5 \times 8) + 8$. Is he right? Draw a picture to help explain your answer.

5. Kelly solves $42 \div 7$ by using a number bond to break apart 42 into two parts. Show what her work might look like below.

Name _____

Date _____

1. A parking lot has space for 48 cars. Six cars can park in 1 row. Break apart 48 to find how many rows there are in the parking lot.



2. Malia solves 6×7 using $(5 \times 7) + 7$. Leonidas solves 6×7 using $(6 \times 5) + (6 \times 2)$. Who is correct? Draw a picture to help explain your answer.

Name _____

Date _____

1. Label the tape diagrams. Then, fill in the blanks below to make the statements true.

a. $6 \times 7 = \underline{\hspace{2cm}}$

$(5 \times 7) = \underline{\hspace{2cm}}$

$(\underline{\hspace{1cm}} \times 7) = \underline{\hspace{2cm}}$



$$\begin{aligned}
 (6 \times 7) &= (5 + 1) \times 7 \\
 &= (5 \times 7) + (1 \times 7) \\
 &= \underline{35} + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

b. $7 \times 7 = \underline{\hspace{2cm}}$

$(5 \times 7) = \underline{\hspace{2cm}}$

$(\underline{\hspace{1cm}} \times 7) = \underline{\hspace{2cm}}$



$$\begin{aligned}
 (7 \times 7) &= (5 + 2) \times 7 \\
 &= (5 \times 7) + (2 \times 7) \\
 &= \underline{35} + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

c. $8 \times 7 = \underline{\hspace{2cm}}$

$(5 \times 7) = \underline{\hspace{2cm}}$

$(\underline{\hspace{1cm}} \times 7) = \underline{\hspace{2cm}}$

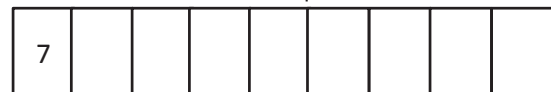


$$\begin{aligned}
 8 \times 7 &= (5 + \underline{\hspace{1cm}}) \times 7 \\
 &= (5 \times 7) + (\underline{\hspace{1cm}} \times 7) \\
 &= \underline{35} + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

d. $9 \times 7 = \underline{\hspace{2cm}}$

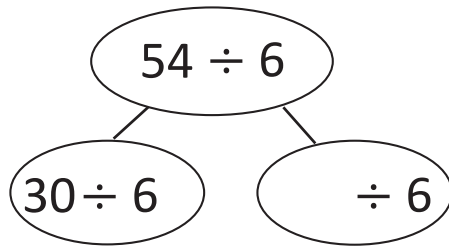
$(5 \times 7) = \underline{\hspace{2cm}}$

$(\underline{\hspace{1cm}} \times 7) = \underline{\hspace{2cm}}$



$$\begin{aligned}
 9 \times 7 &= (5 + \underline{\hspace{1cm}}) \times 7 \\
 &= (5 \times 7) + (\underline{\hspace{1cm}} \times 7) \\
 &= \underline{35} + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

2. Break apart 54 to solve $54 \div 6$.

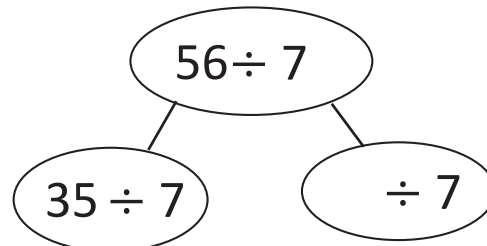


$$54 \div 6 = (30 \div 6) + (\quad \div 6)$$

$$= 5 + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

3. Break apart 56 to solve $56 \div 7$



$$56 \div 7 = (\quad \div \quad) + (\quad \div \quad)$$

$$= 5 + \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

4. Forty-two third grade students sit in 6 equal rows in the auditorium. How many students sit in each row? Show your thinking.

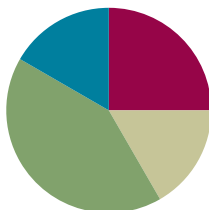
5. Ronaldo solves 7×6 by thinking of it as $(5 \times 7) + 7$. Is he correct? Explain Ronaldo's strategy.

Lesson 7

Objective: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Concept Development	(10 minutes)
■ Application Problems	(25 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Multiply By 7 **3.OA.7** (7 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Decompose the Multiplication Sentence **3.OA.5** (4 minutes)

Multiply By 7 (7 minutes)

Materials: (S) Multiply by 7 (1–5) (Pattern Sheet)

Note: This activity builds fluency with multiplication facts using units of seven. It supports students knowing from memory all products of two one-digit numbers. See Lesson 5 for the directions for administering a Multiply By Pattern Sheet.

T: (Write $5 \times 7 = \underline{\quad}$.) Let's skip-count by sevens to find the answer. I'll raise a finger for each seven. (Count with fingers to 5 as students count, and record the count-by sequence on the board.)

S: 7, 14, 21, 28, 35.

T: (Circle 35 and write $5 \times 7 = 35$ above it. Write $3 \times 7 = \underline{\quad}$.) Let's skip-count up by sevens again. (Count with fingers to 3 as students count.)

S: 7, 14, 21.

T: Let's see how we can skip-count down to find the answer, too. Start at 35 with 5 fingers, 1 for each seven. (Count down with fingers as students say numbers.)

S: 35 (5 fingers), 28 (4 fingers), 21 (3 fingers).

Repeat the process for 4×7 .

T: (Distribute the Multiply by 7 Pattern Sheet.) Let's practice multiplying by 7. Be sure to work left to right across the page.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes reviews multiplication using units of six in this topic and prepares students for today's lesson. Group counting by eights and nines anticipates multiplication using those units later in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Eights to 80
- Nines to 90

Decompose the Multiplication Sentence (4 minutes)

Materials: (S) Personal white board

Note: This activity reviews using the distributive property from Lesson 6.

- T: (Write $6 \times 6 = (5 + \underline{\quad}) \times 6$.) On your personal white board, copy and fill in the equation.
- S: (Write $(6 \times 6) = (5 + 1) \times 6$.)
- T: (Write $= (\underline{\quad} \times 6) + (\underline{\quad} \times 6)$.) Copy and fill in the equation.
- S: (Write $(5 \times 6) + (1 \times 6)$.)
- T: Write an addition equation. Below it, write your answer.
- S: (Write $30 + 6$ and 36 below it.)

Continue with the following suggested sequence: 8×6 , 7×6 , and 9×6 .



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Scaffold *Decompose the Multiplication Sentence* with pre-made sentence frames where students can simply fill in the blank, or solicit oral student responses only. Alternatively, model the equation while students draw number bonds or arrays.

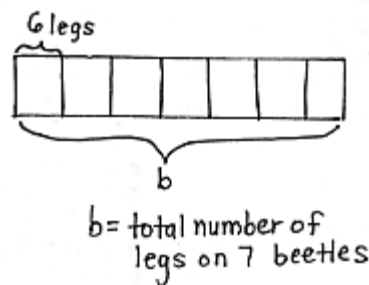
$$\begin{aligned}(6 \times 6) &= (5 + 1) \times 6 \\ &= (5 \times 6) + (1 \times 6) \\ &= 30 + 6 \\ &= 36\end{aligned}$$

Concept Development (10 minutes)

Materials: (S) Personal white board

Thad sees 7 beetles when he weeds his garden. Each beetle has 6 legs. How many legs are there on all 7 beetles?

- T: Talk to a partner. What kind of picture can we draw to model this problem?
- S: We can draw 7 beetles, each with 6 legs. → We can draw an array with 7 rows and 6 dots in each row. → We can draw a tape diagram with 7 parts and 6 in each part.
- T: On your personal white board, draw and label a tape diagram to model this problem. Use the letter b for the beetles' legs to represent the unknown. (Draw and label the tape diagram as shown.)



- T: What does the letter on your tape diagram represent?
- S: The unknown. → The total number of legs on 7 beetles.
- T: Next to your tape diagram, write an equation for the problem and solve it. At the signal, show your board. (Signal after students write equation.)
- S: (Show $7 \times 6 = b$, $b = 42$.)
- T: Talk to a partner. What strategy did you use to find b ?
- T: Write a sentence to answer the problem, and then read your sentence to a partner.

If time allows, repeat the process with $48 \div 6$, providing a context for the problem.

Application Problems (25 minutes)

Materials: (S) Problem Set (Page 2)

During this time, students work together in groups to complete the second page of the Problem Set. When the group work is done, they meet with members of other groups to discuss answers and problem solving strategies.

MP.1

Divide students into groups of four. Each group is assigned a word problem to model and solve from Page 2 of the Problem Set. Depending on the class size, there may be multiple groups solving the same problem. Circulate and clear up any misconceptions as students work.

After students have finished, bring the class back together and redistribute the students into new groups. New groups should be comprised of at least four students who each solved a different word problem. They then take turns discussing how their group solved their designated problem, paying special attention to the various strategies that were used. Allow 10 minutes for discussion.

Problem Set (5 minutes)

The second page of the Problem Set is designed to be completed in the group work activity, as described above. The first page of the Problem Set can be completed during this time.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Depending on the level of English proficiency of English language learners, make the problems and discussion available in their first language, if possible. Alternatively, provide extra time, reduce the amount of work, provide sentence frames for discussion, and/or read the words in Problem 1 as students match it to the correct equation.

Name Gina Date _____

1. Match the words to the correct equation.

a number times 6 equals 30	$n \times 7 = 21$
7 times a number equals 42	$n \times 6 = 30$
6 times 7 equals a number	$6 \times 7 = n$
63 divided by a number equals 9	$7 \times n = 42$
36 divided by a number equals 6	$36 \div n = 6$
a number times 7 equals 21	$63 \div n = 9$

2. Write an equation to represent the tape diagram below and solve for the unknown.

S	S	S	S	S	S	S	S
---	---	---	---	---	---	---	---

k

Equation: $6 \times 8 = k$ $k = 48$

Student Debrief (10 minutes)

Lesson Objective: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

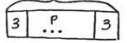
Any combination of the questions below may be used to lead the discussion.

- What is the value of n in each equation in Problem 1?
- What equation did you use to solve Problem 2?
- Explain to a partner what your drawing looks like for Problem 3(a).
- In Problems 3(a) through 3(d), what was unknown? Was it the group size or number of groups?
- What strategies did your group use to solve Problems 3(a) through 3(d)? Why?
- How did the Application Problems connect to today's lesson?

3. Model each problem with a drawing. Then, write an equation using a letter to represent the unknown and solve for the unknown.

a. Each student gets 3 pencils. There are a total of 21 pencils. How many students are there?

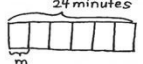
21 pencils



$21 \div 3 = P$
 $P = 7$
There are 7 students who get pencils.

b. Henry spends 24 minutes practicing 6 different basketball drills. He spends the same amount of time on each drill. How much time does Henry spend on each drill?

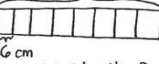
24 minutes



$24 \div 6 = m$
 $m = 4$
Henry spends 4 minutes on each drill.

c. Jessica has 8 pieces of yarn for a project. Each piece of yarn is 6 centimeters long. What is the total length of the yarn?

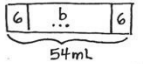
6 cm



$8 \times 6 = y$
 $y = 48$
The total length of the yarn is 48 cm.

d. Ginny measures 6 milliliters of water into each beaker. She pours a total of 54 milliliters. How many beakers does Ginny use?

6 mL



$6 \times 6 = 54$
 $b = 9$
Ginny uses 9 beakers.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Multiply.

$7 \times 1 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$ $7 \times 4 = \underline{\quad\quad}$

$7 \times 5 = \underline{\quad\quad}$ $7 \times 1 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 1 = \underline{\quad\quad}$

$7 \times 3 = \underline{\quad\quad}$ $7 \times 1 = \underline{\quad\quad}$ $7 \times 4 = \underline{\quad\quad}$ $7 \times 1 = \underline{\quad\quad}$

$7 \times 5 = \underline{\quad\quad}$ $7 \times 1 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$

$7 \times 2 = \underline{\quad\quad}$ $7 \times 4 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 5 = \underline{\quad\quad}$

$7 \times 2 = \underline{\quad\quad}$ $7 \times 1 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$

$7 \times 1 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$

$7 \times 4 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$ $7 \times 5 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$

$7 \times 4 = \underline{\quad\quad}$ $7 \times 1 = \underline{\quad\quad}$ $7 \times 4 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$

$7 \times 4 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$ $7 \times 4 = \underline{\quad\quad}$ $7 \times 5 = \underline{\quad\quad}$

$7 \times 4 = \underline{\quad\quad}$ $7 \times 5 = \underline{\quad\quad}$ $7 \times 1 = \underline{\quad\quad}$ $7 \times 5 = \underline{\quad\quad}$

$7 \times 2 = \underline{\quad\quad}$ $7 \times 5 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$ $7 \times 5 = \underline{\quad\quad}$

$7 \times 4 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 4 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$

$7 \times 5 = \underline{\quad\quad}$ $7 \times 3 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 4 = \underline{\quad\quad}$

$7 \times 3 = \underline{\quad\quad}$ $7 \times 5 = \underline{\quad\quad}$ $7 \times 2 = \underline{\quad\quad}$ $7 \times 4 = \underline{\quad\quad}$

multiply by 7 (1–5)

Name _____

Date _____

1. Match the words to the correct equation.

a number times 6 equals 30



7 times a number equals 42



6 times 7 equals a number



63 divided by a number equals 9



36 divided by a number equals 6



a number times 7 equals 21



$$n \times 7 = 21$$

$$n \times 6 = 30$$

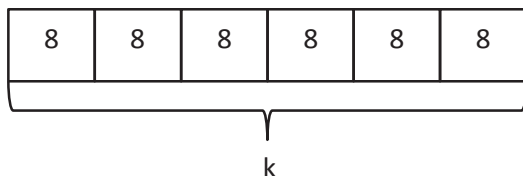
$$6 \times 7 = n$$

$$7 \times n = 42$$

$$36 \div n = 6$$

$$63 \div n = 9$$

2. Write an equation to represent the tape diagram below, and solve for the unknown.



Equation: _____

3. Model each problem with a drawing. Then, write an equation using a letter to represent the unknown, and solve for the unknown.
- Each student gets 3 pencils. There are a total of 21 pencils. How many students are there?
 - Henry spends 24 minutes practicing 6 different basketball drills. He spends the same amount of time on each drill. How much time does Henry spend on each drill?
 - Jessica has 8 pieces of yarn for a project. Each piece of yarn is 6 centimeters long. What is the total length of the yarn?
 - Ginny measures 6 milliliters of water into each beaker. She pours a total of 54 milliliters. How many beakers does Ginny use?

Name _____

Date _____

Model each problem with a drawing. Then, write an equation using a letter to represent the unknown, and solve for the unknown.

1. Three boys and three girls each buy 7 bookmarks. How many bookmarks do they buy all together?

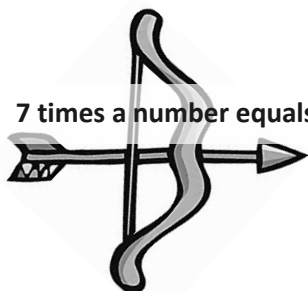
2. Seven friends equally share the cost of a \$56 meal. How much does each person pay?

Name _____

Date _____

1. Match the words on the arrow to the correct equation on the target.

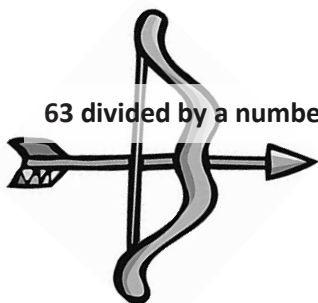
7 times a number equals 42



$$n \times 7 = 21$$



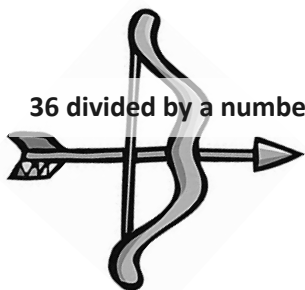
63 divided by a number equals 9



$$7 \times n = 42$$



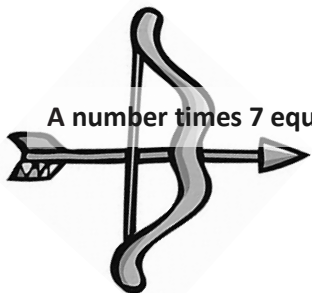
36 divided by a number equals 6



$$63 \div n = 9$$



A number times 7 equals 21



$$36 \div n = 6$$



2. Ari sells 6 boxes of pens at the school store.
- Each box of pens sells for \$7. Draw a tape diagram, and label the total amount of money he makes as m . Write an equation, and solve for m .
 - Each box contains 6 pens. Draw a tape diagram, and label the total number of pens as p . Write an equation, and solve for p .
-
3. Mr. Lucas divides 28 students into 7 equal groups for a project. Draw a tape diagram, and label the number of students in each group as n . Write an equation, and solve for n .



Topic C

Multiplication and Division Using Units up to 8

3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7, 3.OA.1, 3.OA.2, 3.OA.6, 3.OA.8

Focus Standards:	3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
	3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$</i>
	3.OA.5	Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i>
	3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
Instructional Days:	4	
Coherence -Links from:	G2–M3	Place Value, Counting, and Comparison of Numbers to 1,000
	G2–M6	Foundations of Multiplication and Division
	G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
-Links to:	G3–M4	Multiplication and Area
	G4–M3	Multi-Digit Multiplication and Division
	G4–M5	Fraction Equivalence, Ordering, and Operations
	G4–M7	Exploring Measurement with Multiplication

Students are informally familiar with parentheses from having seen them in distributive property lessons in Topic B and in Module 1. In Lesson 8, they understand parentheses as tools for grouping and learn the conventional order for performing Grade 3 operations. This practice anticipates applying parentheses in Lesson 9 as students formally study the associative property.

In Lesson 9, students model and demonstrate how to multiplicatively compose or decompose to make problems using units up to 8 easier to solve. For example, 8×5 may be thought of as:

$$\begin{aligned} 8 \times 5 &= (4 \times 2) \times 5 \\ &= 4 \times (2 \times 5) \\ &= 4 \times 10 \end{aligned}$$

Lessons 10 and 11 in this topic parallel Lessons 6 and 7 in Topic B. In Lesson 10, students use the $5 + n$ pattern as a strategy for solving multiplication and division problems using units of 8 with the distributive property. They learn that multiples of 8 can be thought of as $(5 + 3) \times n$. In division problems, students practice decomposing the dividend using multiples of 5. They recognize the efficacy of using this strategy when the quotient of a division equation is greater than 5 and also realize that the dividend must be decomposed into numbers that are divisible by the divisor. For example, to solve $64 \div 8$, 64 can be decomposed as 40 and 24 because both are divisible by 8.

In Lesson 11, students analyze, model, and solve multiplication and division word problems using units of 8. They understand division as both a quantity divided into equal groups, as well as an unknown factor problem. They draw models and write equations to interpret and solve problems, using a letter to represent the unknown in various positions.

A Teaching Sequence Toward Mastery of Multiplication and Division Using Units up to 8

Objective 1: Understand the function of parentheses and apply to solving problems.
(Lesson 8)

Objective 2: Model the associative property as a strategy to multiply.
(Lesson 9)

Objective 3: Use the distributive property as a strategy to multiply and divide.
(Lesson 10)

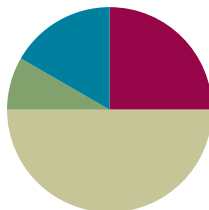
Objective 4: Interpret the unknown in multiplication and division to model and solve problems.
(Lesson 11)

Lesson 8

Objective: Understand the function of parentheses and apply to solving problems.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Multiply By 7 **3.OA.7** (6 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Add 6 and 7 Mentally **2.NBT.5** (5 minutes)

Multiply By 7 (6 minutes)

Materials: (S) Multiply By 7 (6–10) (Pattern Sheet)

Note: This activity builds fluency with multiplication facts using units of seven. It supports students knowing from memory all products of two one-digit numbers. See Lesson 5 for the directions for administering a Multiply By Pattern Sheet.

- T: (Write $6 \times 7 = \underline{\quad}$.) Let's skip-count up by sevens to solve. I'll raise a finger for each seven.
(Count with fingers to 6 as students count.)
- S: 7, 14, 21, 28, 35, 42.
- T: Let's skip-count down to find the answer, too.
Start at 70. (Count down with fingers as students count.)
- S: 70, 63, 56, 49, 42.

Continue with the following suggested sequence: 8×7 , 7×7 , and 9×7 .

- T: (Distribute the Multiply By 7 Pattern Sheet.) Let's practice multiplying by 7. Be sure to work left to right across the page.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Multiply By 7 is carefully scaffolded to support student success. Adjust the activity to suit students' diverse needs. For example, focus on one skill, such as skip-counting down to solve. Or, have students review and solidify their memorization of skip-counting up by seven by doing the Group Counting first.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes reviews multiplication using units of six from Topic B. Group counting eights prepares students for multiplication in this topic. Group counting nines anticipates multiplication using units of nine later in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Eights to 80
- Nines to 90

Add 6 and 7 Mentally (5 minutes)

Materials: (S) Personal white board

Note: This activity reviews the make ten strategy used for skip-counting by sixes and sevens in Lessons 4 and 5.

T: (Project $6 + 6 = \underline{\quad}$.) Say the expression.

S: $6 + 6$.

T: 6 and what make ten?

S: 4.

T: (Draw a number bond beneath the second 6.) On your personal white board, break apart the second 6, taking out the 4.

S: (Write the number bond.)

T: Say the addition sentence.

S: $6 + 6 = 12$.

$$\begin{array}{r} 6 + 6 = 12 \\ \quad \swarrow \searrow \\ \quad 4 \quad 2 \end{array}$$

Continue with the following possible sequence: $12 + 6$, $18 + 6$, $24 + 6$, $30 + 6$, $36 + 6$, $42 + 6$, $48 + 6$, $54 + 6$, $7 + 7$, $14 + 7$, $21 + 7$, $28 + 7$, $35 + 7$, $42 + 7$, $49 + 7$, $56 + 7$, and $63 + 7$.

Application Problem (5 minutes)

Richard has 2 cartons with 6 eggs in each. As he opens the cartons, he drops 2 eggs. How many unbroken eggs does Richard have left?

$$\begin{array}{c} \boxed{6 \text{ eggs}} \quad \boxed{6 \text{ eggs}} \\ \hline ? \\ 2 \times 6 \text{ eggs} = 12 \text{ eggs} \end{array}$$

$$12 \text{ eggs} - 2 \text{ eggs} = 10 \text{ eggs}$$

Richard has 10 unbroken eggs left.

Note: This problem provides a context for solving equations involving multiple operations which is central to the Concept Development.

Concept Development (30 minutes)

Materials: (S) Personal white board

Part 1: Solve equations containing parentheses.

- T: The two equations used to solve the Application Problem are $2 \times 6 = 12$ and $12 - 2 = 10$. (Show picture to the right.) This picture shows both. Talk to your partner: How could we include all of this information in one equation?
- S: We can rewrite them as one equation. Maybe $2 \times 6 - 2 = 10$?
- T: Let's check to make sure the new equation equals 10. Should we multiply first or subtract first? Does it matter?
- S: I don't think it matters. \rightarrow Before we multiplied first, so let's do that again.
- T: Let's find out. Solve the equation twice. The first time you solve it, multiply first. The second time you solve, subtract first. (Allow time for students to calculate.)
- S: When I multiplied first, I still got 10, but when I subtracted first, I got 8.
- T: For this problem, the order *does* matter. We can use parentheses in our equation to show what to do first. Remind me, which part of the equation do we need to do first and why?
- S: 2×6 . We have to find the total number of eggs Richard has in 2 cartons first.
- T: Watch how I use parentheses to show that. (Write $(2 \times 6) - 2 = 10$.)
- T: What is the product of 2×6 ?
- S: 12.
- T: Rewrite 2×6 as 12. What expression is left?
- S: $12 - 2$.
- T: What does $12 - 2$ equal?
- S: 10.
- T: In a complete sentence, how many eggs does Richard have left?
- S: Richard has 10 eggs left.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Alternatively, challenge students working above grade level to write equations in which multiple operations are used on both sides of the equation. For example, a student might extend $(2 \times 6) - 2 = 10$, as $(2 \times 6) - 2 = (7 - 2) \times 2$.

Continue with the following suggested sequence:

- $4 + 2 = 6$ and $6 \times 6 = 36 \rightarrow (4 + 2) \times 6 = 36$
- $12 \div 3 = 4$ and $15 - 4 = 11 \rightarrow 15 - (12 \div 3) = 11$

Note: Have students refer back to the original problem, as the situation dictates the placement of the parentheses.

Part 2: Explore how moving the parentheses can change the answer in an equation.

Write or project the following equation and the picture below, right: $(25 - 10) \div 5 = 3$.

T: Check my work. Is it correct?

S: Yes, because $25 - 10$ equals 15, and $15 \div 5$ equals 3.

T: Let's divide 10 by 5 first. What should we do with the parentheses to show that?

S: Move them over! → Make them go around $10 \div 5$.

T: Now the equation looks like this. (Write $25 - (10 \div 5) = n$.) Write the equation on your personal white board. Why is there a letter where the 3 was before?

S: We should write 3 because the numbers didn't change. → We don't know if it equals 3 anymore.

T: Really? Why not? The numbers are the same as before.

S: The parentheses moved.

T: Solve the problem with your partner. Does this equation still have an answer of 3?

S: (Work and discuss.) No, the answer is 23.

T: Why is the answer different?

S: We divided first. → One way, we divided 15 by 5. → The other way, we subtracted 2 from 25. → We divided and then subtracted. Before, we subtracted, and then divided.

T: What does this tell you about the way we use parentheses to group the math in equations? Is it important? Why or why not?

S: The parentheses tell us what math gets done first. → Yes, it's important because moving the parentheses can change the answer.

Continue with the following suggested sequence:

- $(2 + 3) \times 7$ and $2 + (3 \times 7)$
- $(3 \times 4) \div 2$ and $3 \times (4 \div 2)$

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

$$(25 - 10) \div 5 = 3$$

$$25 - (10 \div 5) = 23$$

Name Gina Date _____

1. Solve.

a. $(12 - 4) + 6 = \underline{14}$	i. $\underline{10} = (12 \div 2) + 4$
b. $12 - (4 + 6) = \underline{2}$	j. $\underline{2} = 12 \div (2 + 4)$
c. $\underline{5} = 15 - (7 + 3)$	k. $9 + (15 \div 3) = \underline{14}$
d. $\underline{11} = (15 - 7) + 3$	l. $(9 + 15) \div 3 = \underline{8}$
e. $\underline{30} = (3 + 2) \times 6$	m. $60 \div (10 - 4) = \underline{10}$
f. $\underline{15} = 3 + (2 \times 6)$	n. $(60 \div 10) - 4 = \underline{2}$
g. $4 \times (7 - 2) = \underline{20}$	o. $\underline{37} = 35 + (10 \div 5)$
h. $(4 \times 7) - 2 = \underline{26}$	p. $\underline{9} = (35 \div 10) + 5$

2. Use parentheses to make the equations true.

a. $(16 - 4) \div 7 = 19$	b. $16 - (4 + 7) = 5$
c. $2 \times 22 - (15 + 5)$	d. $12 - (22 - 15) = 5$
e. $(3 + 7) \times 6 = 60$	f. $3 \times (7 \times 6) = 45$
g. $5 - (10 \div 10) \times 5$	h. $50 - (100 \div 10) \times 5$
i. $(26 - 5) \div 7 = 3$	j. $36 \div 4 \times (25 - 16)$

Student Debrief (10 minutes)

Lesson Objective: Understand the function of parentheses and apply to solving problems.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Look at Problem 1(j). Would the answer be the same if I solved $(12 \div 2) + (12 \div 4)$? Why not? (Lead students to understand that they cannot distribute in this problem.)
- Look at Problem 1(i). Would the answer be the same if I solved $(9 \div 3) + (15 \div 3)$? Why?
- How did you discover where the parentheses belonged in Problem 2?
- Why does moving the parentheses in an equation only change the answer sometimes?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

3. The teacher writes $24 \div 4 + 2 = \underline{\hspace{2cm}}$ on the board. Chad says it equals 8. Samir says it equals 4. Explain how placing the parentheses in the equation can make both answers true.

The answer can equal 8 because $(24 \div 4) + 2 = 6 + 2 = 8$.
 The answer can equal 4 because $24 \div (4 + 2) = 24 \div 6 = 4$.
 Both students are correct!

4. Natasha solves the equation below by finding the sum of 5 and 12. Place the parentheses in the equation to show her thinking. Then, solve.

$$12 + 15 \div 3 = \underline{\hspace{2cm}}$$

$$12 + 5 = 17$$

5. Find two possible answers to the expression $7 + 3 \times 2$ by placing the parentheses in different places.

$$(7 + 3) \times 2 = 10 \times 2 = 20.$$

$$7 + (3 \times 2) = 7 + 6 = 13.$$

Multiply.

$7 \times 1 = \underline{\quad\quad\quad}$ $7 \times 2 = \underline{\quad\quad\quad}$ $7 \times 3 = \underline{\quad\quad\quad}$ $7 \times 4 = \underline{\quad\quad\quad}$

$7 \times 5 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$ $7 \times 8 = \underline{\quad\quad\quad}$

$7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 10 = \underline{\quad\quad\quad}$ $7 \times 5 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$

$7 \times 5 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$ $7 \times 5 = \underline{\quad\quad\quad}$ $7 \times 8 = \underline{\quad\quad\quad}$

$7 \times 5 = \underline{\quad\quad\quad}$ $7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 5 = \underline{\quad\quad\quad}$ $7 \times 10 = \underline{\quad\quad\quad}$

$7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 5 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$

$7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 8 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 9 = \underline{\quad\quad\quad}$

$7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$

$7 \times 8 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$ $7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$

$7 \times 8 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 8 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$

$7 \times 8 = \underline{\quad\quad\quad}$ $7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$

$7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$ $7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 8 = \underline{\quad\quad\quad}$

$7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 8 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 9 = \underline{\quad\quad\quad}$

$7 \times 7 = \underline{\quad\quad\quad}$ $7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 8 = \underline{\quad\quad\quad}$

$7 \times 9 = \underline{\quad\quad\quad}$ $7 \times 7 = \underline{\quad\quad\quad}$ $7 \times 6 = \underline{\quad\quad\quad}$ $7 \times 8 = \underline{\quad\quad\quad}$

multiply by 7 (6–10)

Name _____

Date _____

1. Solve.

a. $(12 - 4) + 6 = \underline{\hspace{2cm}}$

i. $\underline{\hspace{2cm}} = (12 \div 2) + 4$

b. $12 - (4 + 6) = \underline{\hspace{2cm}}$

j. $\underline{\hspace{2cm}} = 12 \div (2 + 4)$

c. $\underline{\hspace{2cm}} = 15 - (7 + 3)$

k. $9 + (15 \div 3) = \underline{\hspace{2cm}}$

d. $\underline{\hspace{2cm}} = (15 - 7) + 3$

l. $(9 + 15) \div 3 = \underline{\hspace{2cm}}$

e. $\underline{\hspace{2cm}} = (3 + 2) \times 6$

m. $60 \div (10 - 4) = \underline{\hspace{2cm}}$

f. $\underline{\hspace{2cm}} = 3 + (2 \times 6)$

n. $(60 \div 10) - 4 = \underline{\hspace{2cm}}$

g. $4 \times (7 - 2) = \underline{\hspace{2cm}}$

o. $\underline{\hspace{2cm}} = 35 + (10 \div 5)$

h. $(4 \times 7) - 2 = \underline{\hspace{2cm}}$

p. $\underline{\hspace{2cm}} = (35 + 10) \div 5$

2. Use parentheses to make the equations true.

a. $16 - 4 + 7 = 19$	b. $16 - 4 + 7 = 5$
c. $2 = 22 - 15 + 5$	d. $12 = 22 - 15 + 5$
e. $3 + 7 \times 6 = 60$	f. $3 + 7 \times 6 = 45$
g. $5 = 10 \div 10 \times 5$	h. $50 = 100 \div 10 \times 5$
i. $26 - 5 \div 7 = 3$	j. $36 = 4 \times 25 - 16$

3. The teacher writes $24 \div 4 + 2 = \underline{\hspace{2cm}}$ on the board. Chad says it equals 8. Samir says it equals 4. Explain how placing the parentheses in the equation can make both answers true.

4. Natasha solves the equation below by finding the sum of 5 and 12. Place the parentheses in the equation to show her thinking. Then, solve.

$12 + 15 \div 3 = \underline{\hspace{2cm}}$

5. Find two possible answers to the expression $7 + 3 \times 2$ by placing the parentheses in different places.

Name _____

Date _____

1. Use parentheses to make the equations true.

a. $24 = 32 - 14 + 6$

b. $12 = 32 - 14 + 6$

c. $2 + 8 \times 7 = 70$

d. $2 + 8 \times 7 = 58$

2. Marcos solves $24 \div 6 + 2 = \underline{\hspace{2cm}}$. He says it equals 6. Iris says it equals 3. Show how the position of parentheses in the equation can make both answers true.

Name _____

Date _____

1. Solve.

a. $9 - (6 + 3) = \underline{\hspace{2cm}}$

b. $(9 - 6) + 3 = \underline{\hspace{2cm}}$

c. $\underline{\hspace{2cm}} = 14 - (4 + 2)$

d. $\underline{\hspace{2cm}} = (14 - 4) + 2$

e. $\underline{\hspace{2cm}} = (4 + 3) \times 6$

f. $\underline{\hspace{2cm}} = 4 + (3 \times 6)$

g. $(18 \div 3) + 6 = \underline{\hspace{2cm}}$

h. $18 \div (3 + 6) = \underline{\hspace{2cm}}$

2. Use parentheses to make the equations true.

a. $14 - 8 + 2 = 4$

b. $14 - 8 + 2 = 8$

c. $2 + 4 \times 7 = 30$

d. $2 + 4 \times 7 = 42$

e. $12 = 18 \div 3 \times 2$

f. $3 = 18 \div 3 \times 2$

g. $5 = 50 \div 5 \times 2$

h. $20 = 50 \div 5 \times 2$

3. Determine if the equation is true or false.

a. $(15 - 3) \div 2 = 6$	<i>Example: True</i>
b. $(10 - 7) \times 6 = 18$	
c. $(35 - 7) \div 4 = 8$	
d. $28 = 4 \times (20 - 13)$	
e. $35 = (22 - 8) \div 5$	

4. Jerome finds that $(3 \times 6) \div 2$ and $18 \div 2$ are equal. Explain why this is true.

5. Place parentheses in the equation below so that you solve by finding the difference between 28 and 3. Write the answer.

$4 \times 7 - 3 = \underline{\hspace{2cm}}$

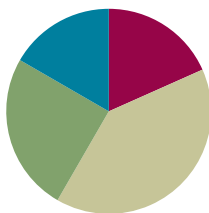
6. Johnny says that the answer to $2 \times 6 \div 3$ is 4 no matter where he puts the parentheses. Do you agree? Place parentheses around different numbers to help you explain his thinking.

Lesson 9

Objective: Model the associative property as a strategy to multiply.

Suggested Lesson Structure

■ Fluency Practice	(11 minutes)
■ Application Problems	(15 minutes)
■ Concept Development	(24 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (11 minutes)

- Divide by 6 and 7 **3.OA.7** (3 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Write In the Parentheses **3.OA.7** (4 minutes)

Divide by 6 and 7 (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews using a letter to represent the unknown taught in Lesson 3.

T: (Write $a \times 6 = 12$.) On your personal white board, write the value of a .

S: (Write $a = 2$.)

T: (Write $12 \div 6 = a$.) Say the division sentence.

S: $12 \div 6 = 2$.

Continue with the following suggested sequence: $a \times 6 = 30$, $b \times 6 = 24$, $c \times 6 = 36$, $d \times 6 = 60$, $e \times 6 = 54$, $f \times 7 = 35$, $g \times 7 = 28$, $h \times 7 = 42$, $j \times 7 = 70$, and $k \times 7 = 56$.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Group counting by eights prepares students for multiplication in this topic. Group counting nines anticipates multiplication using units of nine later in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Eights to 80
- Nines to 90

Write In the Parentheses (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the use of parentheses taught in Lesson 8.

T: (Write $10 - 5 + 3 = 8$.) On your board, copy the equation. Then, insert parentheses to make the statement true.

S: (Write $(10 - 5) + 3 = 8$.)

Continue with the following suggested sequence: $10 - 5 + 3 = 2$, $10 = 20 - 7 + 3$, $16 = 20 - 7 + 3$, $8 + 2 \times 4 = 40$, $8 + 2 \times 4 = 40$, $12 = 12 \div 2 \times 2$, $3 = 12 \div 2 \times 2$, $10 = 35 - 5 \times 5$, and $20 - 10 \div 5 = 2$.

Application Problems (15 minutes)

Materials: (S) Application Problems Sheet

Note: These problems give students practice solving equations with parentheses. This sequence of problems is specifically designed so that students recognize that the position of the parentheses does not change the answer in multiplication problems with more than two factors. (The same is true for addition. Problem 1 hints at this.) Debrief the Application Problems so that this is clear to students with respect to multiplication. This understanding is critical for the Concept Development. Begin the discussion by having them analyze the difference between the problems they circled and those they did not.

Name Gina Date _____

Solve the following pairs of problems. Circle the pairs where both problems have the same answer.

1. $\overset{10}{a. 7 + (6 + 4) = 17}$ $\underset{19}{b. (7 + 6) + 4 = 17}$	5. $\overset{9}{a. (3 + 2) \times 5 = 25}$ $\underset{10}{b. 3 + (2 \times 5) = 13}$
2. $\overset{6}{a. (3 \times 2) \times 4 = 24}$ $\underset{8}{b. 3 \times (2 \times 4) = 24}$	6. $\overset{4}{a. (8 + 2) \times 2 = 8}$ $\underset{4}{b. 8 + (2 \times 2) = 2}$
3. $\overset{2}{a. (2 \times 1) \times 5 = 10}$ $\underset{5}{b. 2 \times (1 \times 5) = 10}$	7. $\overset{4}{a. (9 - 5) + 3 = 7}$ $\underset{8}{b. 9 - (5 + 3) = 1}$
4. $\overset{8}{a. (4 \times 2) \times 2 = 16}$ $\underset{4}{b. 4 \times (2 \times 2) = 16}$	8. $\overset{40}{a. (8 \times 5) - 4 = 36}$ $\underset{1}{b. 8 \times (5 - 4) = 8}$

Concept Development (24 minutes)

Materials: (S) Personal white board

T: (Write 16×3 .) This is a difficult problem for a third grader to solve. Let's simplify it. Work with your partner to list factors that have a product of 16. Write them on your personal white board.

S: 4 times 4 makes 16. \rightarrow 8 and 2 also works.

T: 4, 8, and 2 are much friendlier factors than 16. Let's rewrite 16 as 8×2 . (Write $(8 \times 2) \times 3$.) Why do you think I put 8×2 in parentheses?

S: The parentheses show that when you group those numbers together and multiply, you get 16.



A NOTE ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

One way to scaffold listing factors of 16 is to give students 16 beans they can put into equal groups.

- T: Even with the 16 rewritten, this problem isn't too friendly because I still have to multiply 16×3 in the last step. Suppose I move the parentheses to change the way the numbers are grouped. Will it completely change my answer?
- S: No, we saw that it's okay to move the parentheses when it's all multiplication in our Application Problems.
- T: Write the equation on your board. Use the parentheses to group the numbers differently. Check your work with your partner's.
- S: (Write $8 \times (2 \times 3)$, and check work with a partner.)

- T: (Draw array.) My array shows how I regrouped the numbers to show 8 groups of (2×3) . Is this problem friendlier than 16×3 ?

- S: Oh, it's just 8×6 . That's the same as 48.
That was easy!

- T: So, what is the answer to 16×3 ?

- S: 48.

- T: Tell your partner the steps we took to simplify the problem and solve.

- S: First, we rewrote 16 as a multiplication problem with two easier factors. Then, we grouped the numbers with parentheses to make a multiplication problem that was easy to solve.

- T: (Do not erase the $16 \times 3 = 8 \times (2 \times 3)$ array.) When we brainstormed factors with a product of 16, some of you thought of 4×4 . Let's see if rewriting the 16 that way helps us simplify. Rewrite 16×3 using 4×4 .

- S: (Write $(4 \times 4) \times 3$.)

- T: Is it easy to solve yet?

- S: No!

- T: Try and simplify by using the parentheses to group the numbers differently.

- S: (Write $4 \times (4 \times 3)$.)

- T: (Draw the array.) Here is the array that shows our 4 groups of (4×3) . Did the problem get easier?

- S: Not really. It's still 4×12 , and that's hard.

- T: Let's compare the two arrays. What do you notice?

- S: They show 16×3 in different ways. → The first array shows 8 groups of 6, and the second array shows 4 groups of 12. → The second array has fewer groups but multiplies a larger number. → So, both arrays still show a total of 48, but the first array breaks it up into easier numbers.

- T: True. If we use repeated addition to find the answer to 4×12 , we'll find the answer is still 48. We didn't do anything wrong, but rewriting the 16 as 4×4 and moving the parentheses didn't do what we wanted it to. It didn't help us simplify. With your partner, compare the two arrays. What happened when we rewrote 16 as 4×4 and 8×2 ? What does the comparison tell you about this strategy?

- S: It doesn't always work. → It means you have to be careful about which numbers you choose. → Yeah, some are helpful and some aren't. → Sometimes you might have to try more than one pair of numbers before you find the pair that helps you simplify.

$$16 \times 3 = 8 \times (2 \times 3)$$

$$16 \times 3 = 4 \times (4 \times 3)$$

Continue with 15×3 . Point out that the order in which 15 is rewritten can make a difference. For example, ask students to notice which is easier:

a. $(3 \times 5) \times 3$	b. $(5 \times 3) \times 3$
$3 \times (5 \times 3)$	$5 \times (3 \times 3)$
3×15	5×9

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Learners who have not memorized sixes and sevens facts may not benefit from using the associative property to solve 14×3 on the Problem Set. Adjust the numbers, or encourage students to use a more personally efficient strategy, such as the distributive property.

Student Debrief (10 minutes)

Lesson Objective: Model the associative property as a strategy to multiply.


The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.


Any combination of the questions below may be used to lead the discussion.


- In Problem 1, how do the problems on the bottom simplify the problems on the top?
- Invite students to share how they knew where to draw parentheses for the equations in Problem 2.
- In Problem 3, how did Charlotte simplify?
- How are the commutative property and this new strategy helpful for finding unknown, larger facts?
- How did the Application Problems relate to the lesson today?


Name Gina Date _____

1. Use the array to complete the equation.

 a. $3 \times 12 = \underline{36}$

 b. $(3 \times 3) \times 4$
 $= \underline{9} \times 4$
 $= \underline{36}$

 c. $3 \times 14 = \underline{42}$

 d. $(3 \times 2) \times 7$
 $= \underline{6} \times 7$
 $= \underline{42}$

- In the Application Problems, we noticed that it is okay to move the parentheses when every operation is multiplication. Is that true for the other operations too? (Provide subtraction and division examples, where it is not okay to move parentheses and obtain the same answer. Provide addition examples that students can use in conjunction with Application Problem 1 to generalize that it is also true for addition.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

2. Place parentheses in the equations to simplify. Then, solve. The first one has been done for you.

a. $3 \times 16 = 3 \times (2 \times 8)$
 $= (3 \times 2) \times 8$
 $= 6 \times 8$
 48

b. $2 \times 14 = 2 \times (2 \times 7)$
 $= (2 \times 2) \times 7$
 $= 4 \times 7$
 28

c. $3 \times 12 = 3 \times (3 \times 4)$
 $= (3 \times 3) \times 4$
 $= 9 \times 4$
 36

d. $3 \times 14 = 3 \times (2 \times 7)$
 $= (3 \times 2) \times 7$
 $= 6 \times 7$
 42

e. $15 \times 3 = (5 \times 3) \times 3$
 $= 5 \times (3 \times 3)$
 $= 5 \times 9$
 45

f. $15 \times 2 = (5 \times 3) \times 2$
 $= 5 \times (3 \times 2)$
 $= 5 \times 6$
 30

3. Charlotte finds the answer to 16×2 by thinking about 8×4 . Explain her strategy.

$16 \times 2 = (8 \times 2) \times 2$
 $= 8 \times (2 \times 2)$
 $= 8 \times 4$
 $= 32$

Charlotte rewrites 16 as 8×2 . Then she moves the parentheses over to 2×2 . That's how she got the related fact, 8×4 .

Name _____ Date _____

Solve the following pairs of problems. Circle the pairs where both problems have the same answer.

1. a. $7 + (6 + 4)$

b. $(7 + 6) + 4$

5. a. $(3 + 2) \times 5$

b. $3 + (2 \times 5)$

2. a. $(3 \times 2) \times 4$

b. $3 \times (2 \times 4)$

6. a. $(8 \div 2) \times 2$

b. $8 \div (2 \times 2)$

3. a. $(2 \times 1) \times 5$

b. $2 \times (1 \times 5)$

7. a. $(9 - 5) + 3$

b. $9 - (5 + 3)$

4. a. $(4 \times 2) \times 2$

b. $4 \times (2 \times 2)$

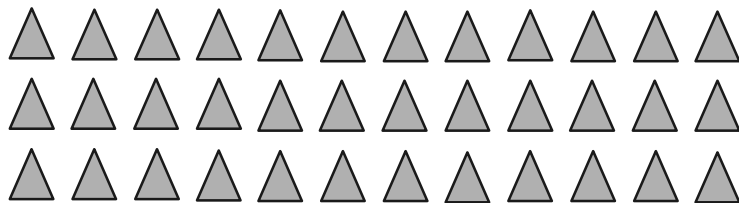
8. a. $(8 \times 5) - 4$

b. $8 \times (5 - 4)$

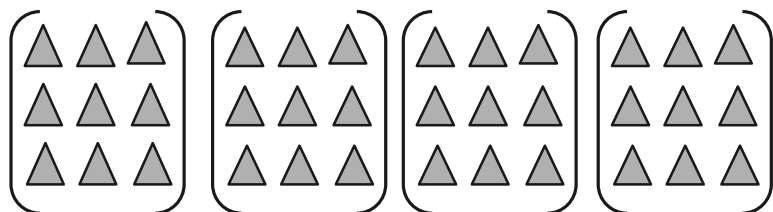
Name _____

Date _____

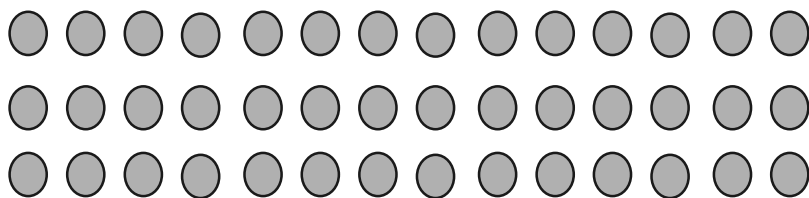
1. Use the array to complete the equation.



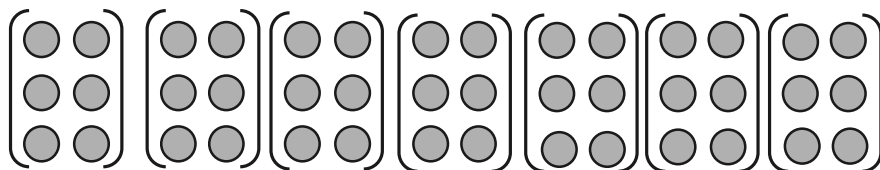
a. $3 \times 12 = \underline{\hspace{2cm}}$



b. $(3 \times 3) \times 4$
 $= \underline{\hspace{2cm}} \times 4$
 $= \underline{\hspace{2cm}}$



c. $3 \times 14 = \underline{\hspace{2cm}}$



d. $(\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}) \times 7$
 $= \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

2. Place parentheses in the equations to simplify. Then, solve. The first one has been done for you.

a. $3 \times 16 = 3 \times (2 \times 8)$
 $= (3 \times 2) \times 8$
 $= \underline{6} \times 8$

48

b. $2 \times 14 = 2 \times (2 \times 7)$
 $= (2 \times 2) \times 7$
 $= \underline{\quad} \times 7$

c. $3 \times 12 = 3 \times (3 \times 4)$
 $= 3 \times 3 \times 4$
 $= \underline{\quad} \times \underline{\quad}$

d. $3 \times 14 = 3 \times 2 \times 7$
 $= 3 \times 2 \times 7$
 $= \underline{\quad} \times \underline{\quad}$

e. $15 \times 3 = 5 \times 3 \times 3$
 $= 5 \times 3 \times 3$
 $= \underline{\quad} \times \underline{\quad}$

f. $15 \times 2 = 5 \times 3 \times 2$
 $= 5 \times 3 \times 2$
 $= \underline{\quad} \times \underline{\quad}$

3. Charlotte finds the answer to 16×2 by thinking about 8×4 . Explain her strategy.

Name _____

Date _____

Simplify to find the answer to 18×3 . Show your work, and explain your strategy.

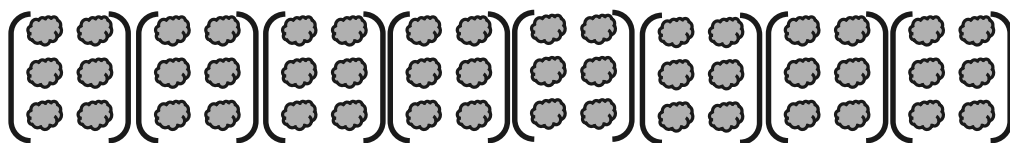
Name _____

Date _____

1. Use the array to complete the equation.



a. $3 \times 16 = \underline{\hspace{2cm}}$



b. $(3 \times \underline{\hspace{1cm}}) \times 8$
 $= \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$



c. $4 \times 18 = \underline{\hspace{2cm}}$



d. $(4 \times \underline{\hspace{1cm}}) \times 9$
 $= \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

2. Place parentheses in the equations to simplify and solve.

$$\left. \begin{array}{l} 12 \times 4 = (6 \times 2) \times 4 \\ = 6 \times (2 \times 4) \\ = 6 \times \underline{8} \end{array} \right\} = \underline{48}$$

$$\text{a. } \left. \begin{array}{l} 3 \times 14 = 3 \times (2 \times 7) \\ = 3 \times 2 \times 7 \\ = \underline{\quad} \times 7 \end{array} \right\} = \underline{\quad}$$

$$\text{b. } \left. \begin{array}{l} 3 \times 12 = 3 \times (3 \times 4) \\ = 3 \times 3 \times 4 \\ = \underline{\quad} \times 4 \end{array} \right\} = \underline{\quad}$$

3. Solve. Then, match the related facts.

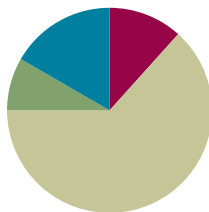
a. $20 \times 2 = \underline{40} =$	$6 \times (5 \times 2)$
b. $30 \times 2 = \underline{\quad} =$	$8 \times (5 \times 2)$
c. $35 \times 2 = \underline{\quad} =$	$4 \times (5 \times 2)$
d. $40 \times 2 = \underline{\quad} =$	$7 \times (5 \times 2)$

Lesson 10

Objective: Use the distributive property as a strategy to multiply and divide.

Suggested Lesson Structure

■ Fluency Practice	(7 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(38 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (7 minutes)

- Group Counting **3.OA.1** (4 minutes)
- Decompose Multiples of 8 **3.OA.5** (3 minutes)

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes and sevens reviews multiplication using those units in Topic B. Group counting by eights prepares students for multiplication in this topic. Group counting nines anticipates multiplication using units of nine later in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

Decompose Multiples of 8 (3 minutes)

Materials: (S) Personal white board

Note: This activity prepares students to use the distributive property in today's lesson.

- T: (Project a number bond with a whole of 48 and 16 as a part.) On your personal white board, fill in the unknown part in the number bond.

Continue with the following suggested sequence: whole of 56 and 24 as a part, whole of 64 and 40 as a part, whole of 40 and 16 as a part, and whole of 72 and 24 as a part.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Students working below grade level may benefit from clear directions as to how to find the unknown part of the number bond. Model and instruct students to subtract to find the unknown part. Begin with smaller numbers as applicable to student needs.

Application Problem (5 minutes)

Use the 5 plus something break apart and distribute strategy to solve 6×8 . Model with a tape diagram.

Note: This problem reviews modeling the break apart and distribute strategy using a tape diagram from Lesson 6. Up until today's lesson, students learned to break apart the first factor and distribute the second factor. Today's Concept Development reverses the order using the fact in this Application Problem.

$$(5 \times 8) + (1 \times 8)$$

$$40 + 8 = 48, n = 48$$

$$6 \times 8 = 48$$

Concept Development (38 minutes)

Materials: (S) Personal white board

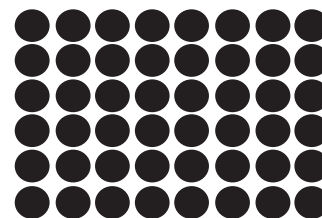
Problem 1: Multiply.

- T: When we use the break apart and distribute strategy, which factor do we break apart?
- S: We break apart the number of groups.
- T: Do you think our strategy would work if we broke apart the size of the groups and distributed the factor representing the number of groups instead? Think about the commutative property. Talk to your partner.
- S: I'm not sure. I don't think so. → The commutative property says that you can switch factors around in multiplication, so maybe it would work.
- T: Let's try using the break apart and distribute strategy that way to solve 6×8 . Then, we can compare what happens with our work on the Application Problem.
- T: Take a look at my array. (Project the 6 by 8 array, shown at right.) Which factor will we break apart?
- S: The 8. → The size of the groups.
- T: Breaking it into 5 plus something helps us make two smaller facts. We can break 8 into 5 and what?
- S: 5 and 3.
- T: (Write $6 \times (5 + 3)$ under the array.) Is 8 represented by the number of columns or the number of rows in the array?
- S: The columns.
- T: How should I draw my line to show that we broke apart the columns?
- S: Maybe an up and down line? → You could make a vertical line after 5 columns. Then, one part would show 5 columns, and the other would show 3.



**NOTES ON
MULTIPLE MEANS
OF ACTION AND
EXPRESSION:**

Support English language learners as they engage in today's discussion. Offer extra time for them to formulate their thoughts and discuss with their partners. If appropriate, preview words such as *factor*. Conduct subtle and frequent checks for understanding. Elaborate, expand, or paraphrase the dialogue as needed.



$$6 \times (5 + 3)$$

MP.7

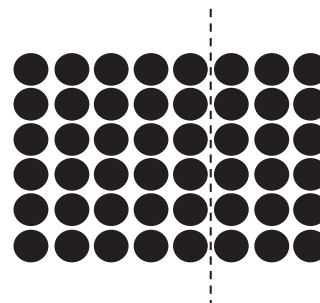
T: (Draw a dotted line after the fifth column.) On your personal white board, write the multiplication facts you would use to label each part of the array.

S: (Write 6×5 and 6×3 .)

T: What did we break our 6 eights into?

S: 6 fives and 6 threes.

T: Talk to your partner about how $6 \times (5 + 3)$ shows 6 fives and 6 threes. Use the array to help you explain.



$$6 \times (5 + 3)$$

MP.7

T: Solve the problem.

S: (May use $6 \times (5 + 3)$ or $(6 \times 5) + (6 \times 3)$ to solve.)

T: What does it equal?

S: 48.

T: Look back at your work on the Application Problem. Compare it with this way of solving. Notice what is the same or different. Talk to your partner about what you see.

S: We switched around the factors that we broke apart and distributed. → In the Application Problem, the units never changed. They were always eights. The one we just did had two different units, fives and threes, but what stayed the same was the number of fives and the number of threes.

T: Does the break apart and distribute strategy work both ways?

S: Yes!

Continue with the following suggested problem: 7×8 .

Problem 2: Divide.

T: Let's use the break apart and distribute strategy to solve $64 \div 8$. Draw a number bond with $64 \div 8$ as the whole. Leave the parts empty. (Allow time for students to draw.)

T: Let's think about how to break apart 64 into two numbers that are easier for us to divide. Make a list with your partner. Remember that when we break apart 64, both numbers need to be divisible by 8.

S: How about 60 and 4? → No, you can't divide those by 8. → Maybe 32 and 32. → Or 40 and 24.

T: Using 32 and 32 works nicely because it's a double. Forty and 24 also works well; $40 \div 8$ makes 5. Five is easy to add to, so let's try 40 and 24. Write $40 \div 8$ as one of the parts on your number bond.

T: What division fact goes inside the other part?

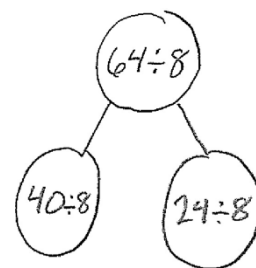
S: $24 \div 8$.

T: How do you know?

S: 40 plus 24 equals 64. → We started with 64, used 40, and need 24 more.

T: Write that division fact as the other part.

Our number bond shows us that $64 \div 8$ has the same value as combining $40 \div 8$ and $24 \div 8$. Work with your partner to write that as an addition sentence on your board.



$$64 \div 8 = (40 \div 8) + (24 \div 8)$$

$$64 \div 8 = 5 + 3$$

$$64 \div 8 = 8$$

S: (Write $64 \div 8 = (40 \div 8) + (24 \div 8)$.)

T: Work with your partner to solve.

S: (Write $5 + 3 = 8$.)

T: What is $64 \div 8$?

S: 8.

Continue with the following suggested sequence:

- $96 \div 8$
- $54 \div 6$

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use the distributive property as a strategy to multiply and divide.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Describe the steps you took to solve for the unknown numbers in Problem 1(a).
- How did you know what division fact to write for the unknown part in Problem 3?
- What multiplication sentence is used to solve Problem 4? How do you know?

Name: Cirra Date: _____

1. Label the arrays. Then fill in the blanks below to make the statements true.

a) $8 \times 8 = 64$

$(8 \times 5) = 40$; $(8 \times 3) = 24$

$8 \times 8 = 8 \times (5 + 3)$
 $= (8 \times 5) + (8 \times 3)$
 $= 40 + 24$
 $= 64$

b) $8 \times 9 = 72$

$(8 \times 5) = 40$; $(8 \times 4) = 32$

$8 \times 9 = 8 \times (5 + 4)$
 $= (8 \times 5) + (8 \times 4)$
 $= 40 + 32$
 $= 72$

2. Break apart and distribute to solve $56 \div 8$.

$56 \div 8$

$40 \div 8$ $16 \div 8$

$56 \div 8 = (40 \div 8) + (16 \div 8)$
 $= 5 + 2$
 $= 7$

3. Break apart and distribute to solve $72 \div 8$.

$72 \div 8$

$40 \div 8$ $32 \div 8$

$72 \div 8 = (40 \div 8) + (32 \div 8)$
 $= 5 + 4$
 $= 9$

4. An octagon has 8 sides. Skip-count each octagon to find the total number of sides on 9 octagons.

8 16 24 32 40 48 56 64 72

Nine octagons have a total of 72 sides.

5. Multiply.

$4 \times 8 = 32$

$6 \times 8 = 48$

$3 \times 8 = 24$

$8 \times 10 = 80$

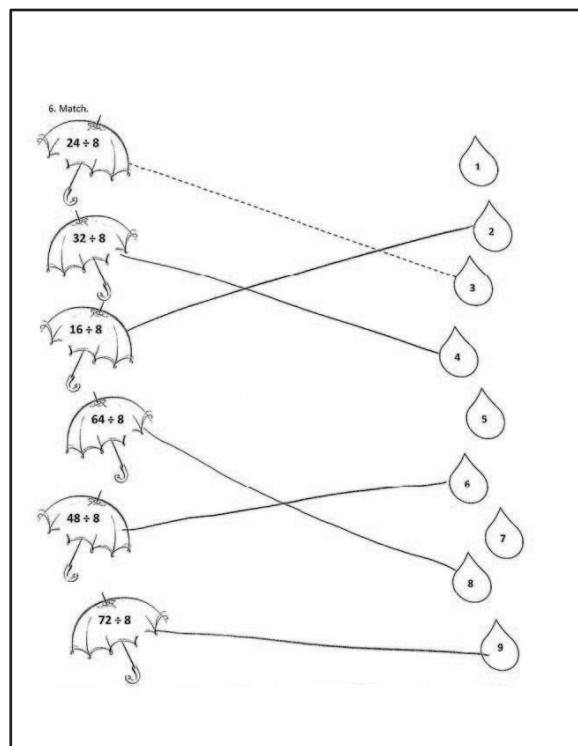
$8 \times 8 = 64$

$7 \times 8 = 56$

- Invite students to share how to apply the break apart strategy to any of the expressions in Problem 5.
- In what ways does the break apart and distribute strategy remind you of the simplifying strategy we learned yesterday?
- How did our math work today help make multiplication and division with larger numbers simpler?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.



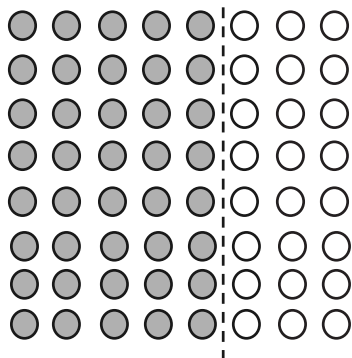
Name _____

Date _____

1. Label the arrays. Then, fill in the blanks below to make the statements true.

a. $8 \times 8 = \underline{\hspace{2cm}}$

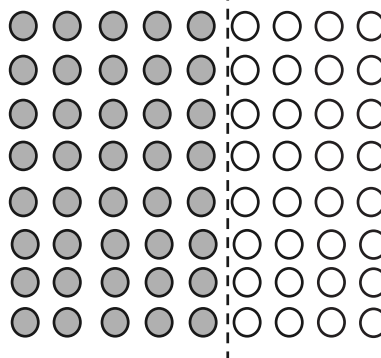
$(8 \times 5) = \underline{\hspace{2cm}} \quad (8 \times \underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$



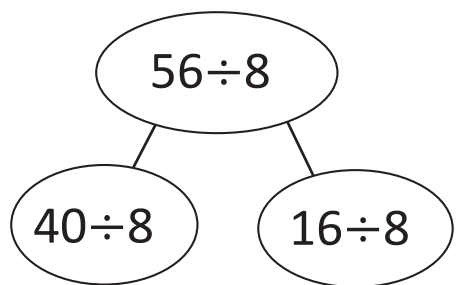
$$\begin{aligned}
 8 \times 8 &= 8 \times (5 + \underline{\hspace{1cm}}) \\
 &= (8 \times 5) + (8 \times \underline{\hspace{1cm}}) \\
 &= \underline{40} + \underline{\hspace{1cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

b. $8 \times 9 = 9 \times 8 = \underline{\hspace{2cm}}$

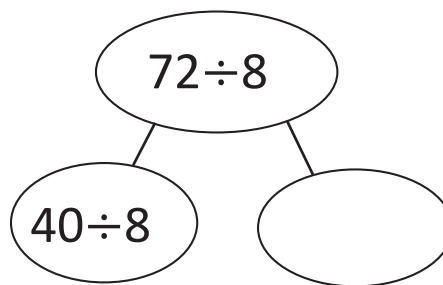
$(8 \times 5) = \underline{\hspace{2cm}} \quad (8 \times \underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$



$$\begin{aligned}
 9 \times 8 &= 8 \times (5 + \underline{\hspace{1cm}}) \\
 &= (8 \times 5) + (8 \times \underline{\hspace{1cm}}) \\
 &= \underline{40} + \underline{\hspace{1cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

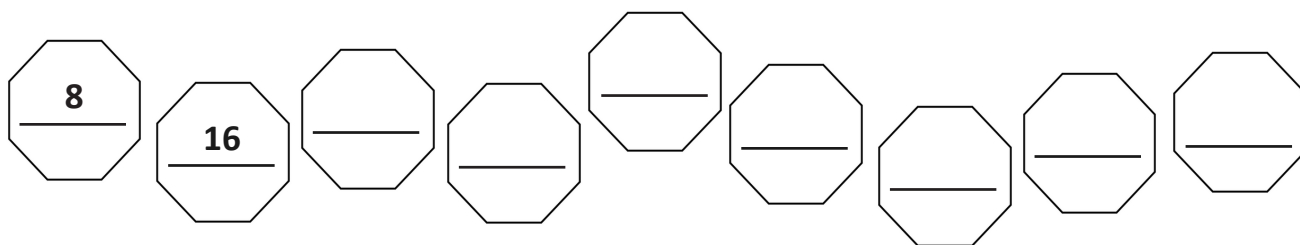
2. Break apart and distribute to solve $56 \div 8$.

$$\begin{aligned}
 56 \div 8 &= (40 \div 8) + (\underline{\hspace{2cm}} \div 8) \\
 &= 5 + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

3. Break apart and distribute to solve $72 \div 8$.

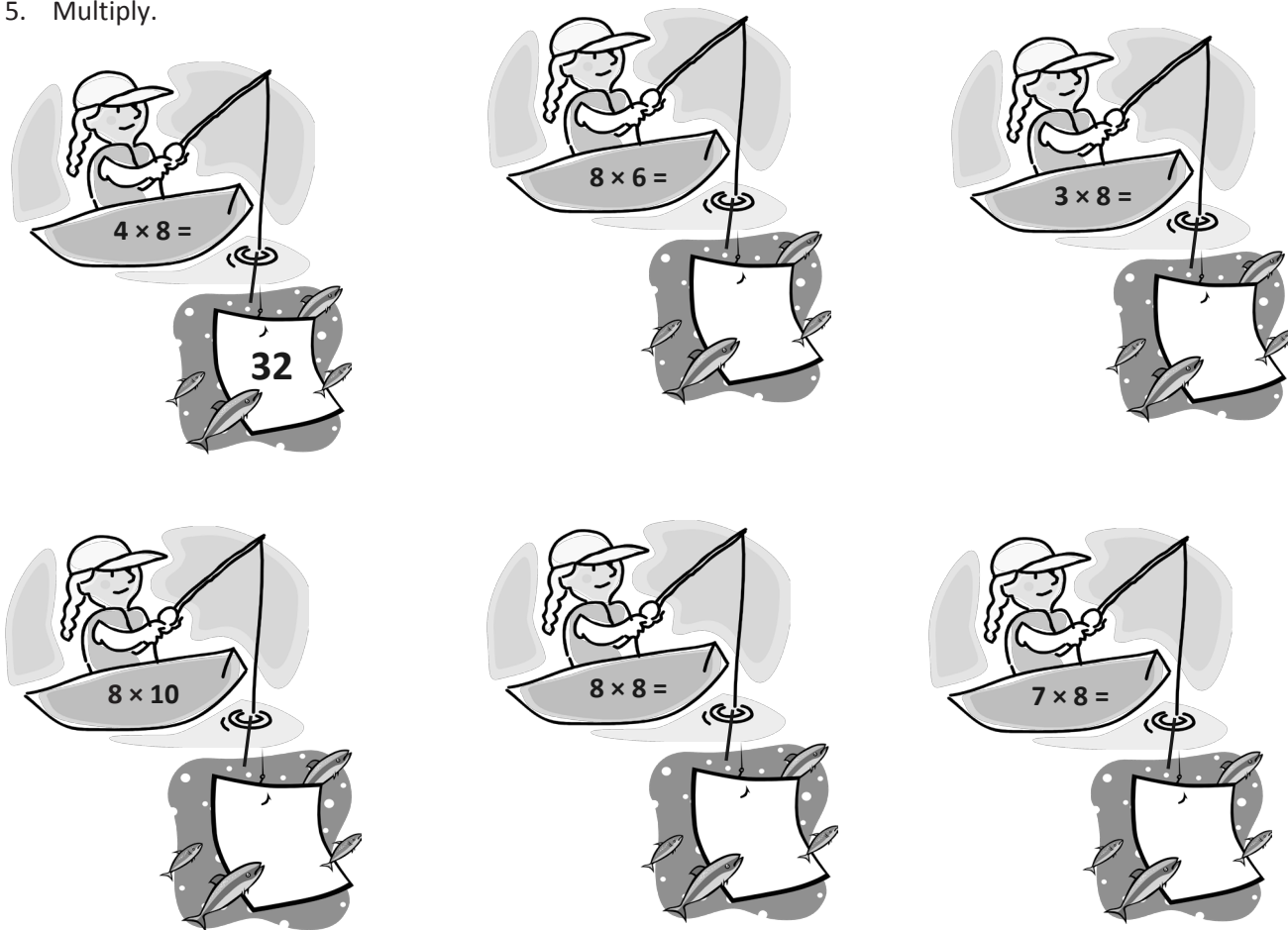
$$\begin{aligned}
 72 \div 8 &= (40 \div 8) + (\underline{\hspace{2cm}} \div 8) \\
 &= 5 + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

4. An octagon has 8 sides. Skip-count to find the total number of sides on 9 octagons.



Nine octagons have a total of _____ sides.

5. Multiply.



6. Match.

The image shows a matching exercise. On the left, there are six umbrellas, each with a division problem written on it. On the right, there are nine water droplets, each with a number written inside. A dashed line connects the first umbrella to the third droplet.

Umbrellas (left to right):

- $24 \div 8$
- $32 \div 8$
- $16 \div 8$
- $64 \div 8$
- $48 \div 8$
- $72 \div 8$

Water Droplets (top to bottom):

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9

Name _____

Date _____

Use the break apart and distribute strategy to solve the following problem. You may choose whether or not to draw an array.

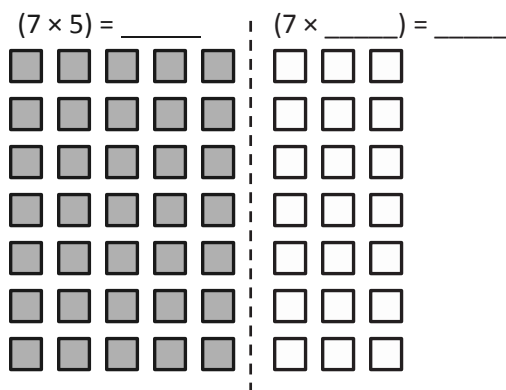
$7 \times 8 = \underline{\quad}$

Name _____

Date _____

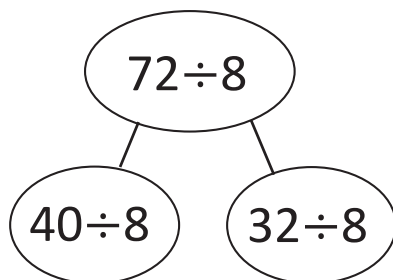
1. Label the array. Then, fill in the blanks to make the statements true.

$$8 \times 7 = 7 \times 8 = \underline{\hspace{2cm}}$$



$$\begin{aligned}
 8 \times 7 &= 7 \times (5 + \underline{\hspace{1cm}}) \\
 &= (7 \times 5) + (7 \times \underline{\hspace{1cm}}) \\
 &= \underline{35} + \underline{\hspace{1cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

2. Break apart and distribute to solve $72 \div 8$.



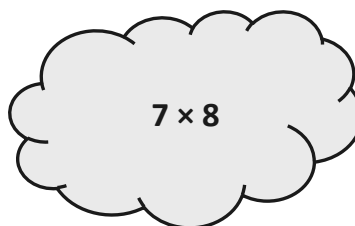
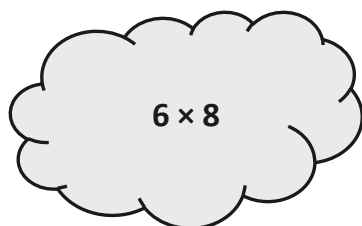
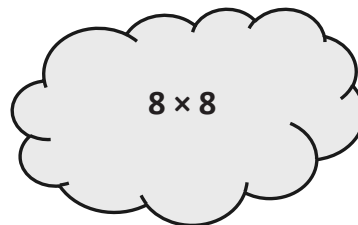
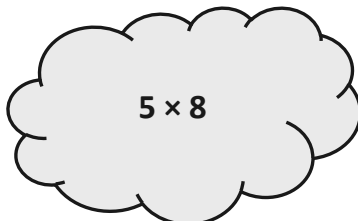
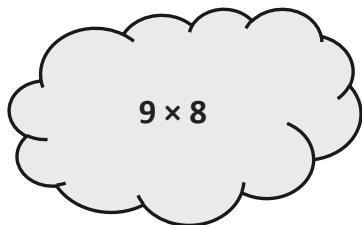
$$72 \div 8 = (40 \div 8) + (\underline{\hspace{2cm}} \div 8)$$

$$= 5 + \underline{\hspace{2cm}}$$

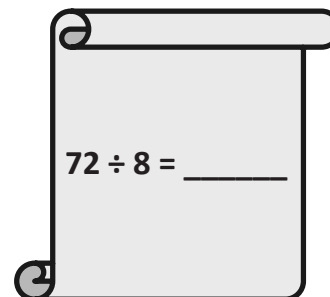
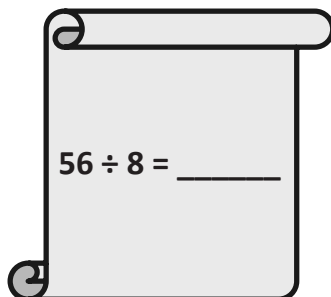
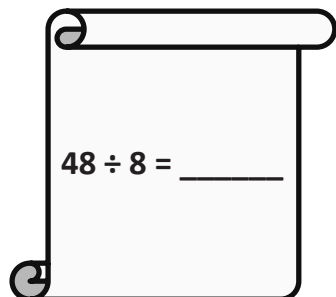
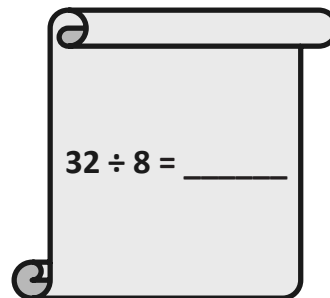
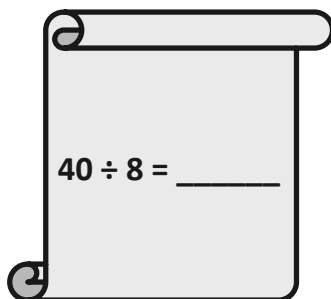
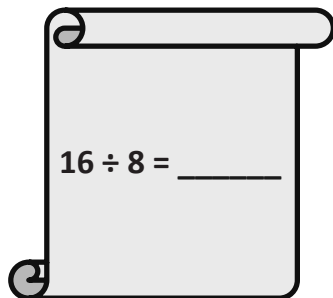
$$= \underline{\hspace{2cm}}$$

3. Count by 8. Then, match each multiplication problem with its value.

8



4. Divide.

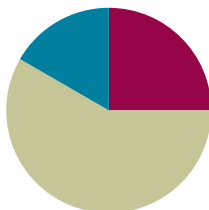


Lesson 11

Objective: Interpret the unknown in multiplication and division to model and solve problems.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Concept Development	(35 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Multiply By 8 **3.OA.7** (7 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Decompose the Multiplication Sentence **3.OA.5** (4 minutes)

Multiply By 8 (7 minutes)

Materials: (S) Multiply By 8 (1–5) (Pattern Sheet)

Note: This activity builds fluency with multiplication facts using units of 8. It supports students knowing from memory all products of two one-digit numbers. See Lesson 5 for the directions for administering a Multiply By Pattern Sheet.

T: (Write $5 \times 8 = \underline{\quad}$.) Let's skip-count by eights to find the answer. I'll raise a finger for each eight. (Count with fingers to 5 as students count, and record the count-by sequence on the board.)

S: 8, 16, 24, 32, 40.

T: (Circle 40 and write $5 \times 8 = 40$ above it. Write $3 \times 8 = \underline{\quad}$.) Let's skip-count up by eights again. (Count with fingers to 3 as students count.)

S: 8, 16, 24.

T: Let's see how we can skip-count down to find the answer, too. Start at 40 with 5 fingers, 1 for each eight. (Count down with fingers as students say numbers.)

S: 40 (5 fingers), 32 (4 fingers), 24 (3 fingers).

Repeat the process for 4×8 .

T: (Distribute the Multiply By 8 Pattern Sheet.) Let's practice multiplying by 8. Be sure to work left to right across the page.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes and sevens reviews multiplication using those units in Topic B. Group counting nines anticipates multiplication in the next topic. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Nines to 90

Decompose the Multiplication Sentence (4 minutes)

Materials: (S) Personal white board

Note: This activity reviews multiplying using the distributive property from Lesson 10.

T: (Write $8 \times 8 = (5 + \underline{\quad}) \times 8$.) On your personal white board, copy and fill in the equation.

S: (Write $8 \times 8 = (5 + 3) \times 8$.)

T: (Write $= (\underline{\quad} \times 8) + (\underline{\quad} \times 8)$.) Copy and fill in the equation.

S: (Write $(5 \times 8) + (3 \times 8)$.)

T: Find the products, and write an addition sentence. Below it, write your answer.

S: (Write $40 + 24$ and 64 below it.)

$$\begin{aligned} 8 \times 8 &= (5 + 3) \times 8 \\ &= (5 \times 8) + (3 \times 8) \\ &= 40 + 24 \\ &= 64 \end{aligned}$$

Continue with the following suggested sequence: 7×8 , 6×8 , and 9×8 .

Concept Development (35 minutes)

Materials: (S) Personal white board

Problem 1: Interpret the unknown in multiplication.

Write the following problem: Asmir buys 8 boxes of 9 candles for his dad's birthday. After putting some candles on the cake, there are 28 candles left. How many candles does Asmir use?

T: Model the problem. Then, tell your partner the steps you'll need to take to solve.

S: (Model.) First, you have to find out how many candles Asmir has. After that, you could subtract 28 from the total to see how many he used.

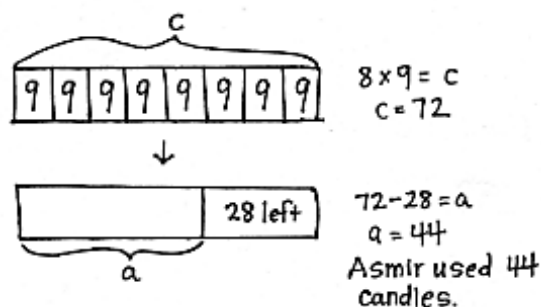
T: Write an equation to find the total number of candles. Instead of using a question mark, use the letter c to represent the unknown.

T: Read your equation out loud.

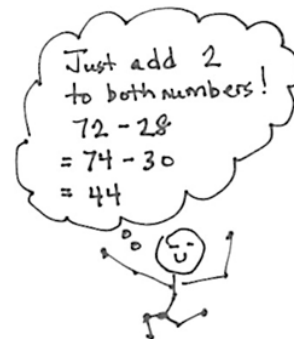
S: 8 times 9 equals c .

T: What does c represent?

S: The product. \rightarrow The total number of candles.



- T: Choose a strategy, and find the value of c . (Possible strategies: known from memory, skip-count, distributive property, associative property.)
- T: Use a complete sentence to tell what c equals.
- S: He bought 72 candles, so c equals 72.
- T: Did we solve the problem?
- S: No, we have to find how many candles Asmir uses.
- T: Write an equation to represent the second step of the problem; this time, use the letter a to represent the unknown.
- S: (Write $72 - 28 = a$.)
- T: Find the value of a . This is a good opportunity to practice your mental math strategies. (Allow time for solving.) What is the value of a ?
- S: 44.
- T: Answer the question in a complete sentence.
- S: Asmir uses 44 candles.



Problem 2: Interpret the unknown in division.

Write the following problem: The fabric store sells one meter of cloth for \$8. Maria buys some cloth that costs a total of \$56. She then uses 3 meters to sew a dress. How many meters of cloth does she have left?

- T: Draw a model to represent the problem. Choose letters to represent the unknowns.
- T: What is unknown in this problem?
- S: The total meters of cloth Maria buys. → There's something else, too. We don't know how many meters of cloth Maria has left.
- T: Tell your partner why you need to know how many meters of cloth Maria buys.
- S: First, you have to find out how many meters of cloth Maria buys. After that, you could subtract 3 meters from the total to see how many meters she has left.
- T: What will be your first step to solving this problem?
- S: Finding the total meters of cloth Maria buys.
- T: Whisper to your partner how you'll do that, and then write an equation using a letter for the unknown.
- S: I'm going to do the total cost divided by the cost of one meter of fabric. So, $\$56 \div \$8 = t$.
- T: Tell your partner why you picked the letter you used to represent the unknown. How does it relate to the problem?
- S: (Possible response: I chose letter t to stand for the *total* meters of cloth Maria buys.)

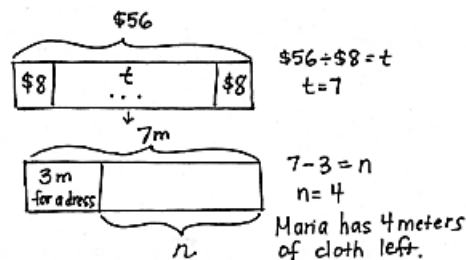


NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Many learners benefit from this step-by-step guidance from planning a strategy to finding a solution.

Students above grade level and others may be motivated by more choice and autonomy. In addition, welcome various strategies, plans for solving, and modeling.

- T: Whisper what the unknown in your equation equals.
- S: (Possible response: t equals 7 meters.)
- T: Tell your partner your next step for solving. Then, write an equation using a letter for the unknown.
- S: Now that I know that Maria bought a total of 7 meters, I'll do $7 - 3 = n$. Letter n stands for the *number* of meters she has left.
- T: Is your letter the same as the one you used for the first step? Why or why not?
- S: It's different because it represents something different. → Oh yeah! I need to change mine.
- T: Finish solving, and then answer the question using words.
- S: (Solve to find n is 4 meters. Write Maria has 4 meters of cloth left.)
- T: Does Maria have enough cloth to sew another dress? Why or why not?
- S: Yes, she has 4 meters left, and she only needs 3 meters. → So, even after making a second dress, she will still have 1 meter of cloth left.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Students working above grade level and others may want to omit the tape diagram as they mentally solve the first few problems of the Problem Set. Redirect their focus to the accurate labeling of the unknown. After accurately labeling their tape diagrams, ask students to erase the values and experiment with numbers they choose.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Interpret the unknown in multiplication and division to model and solve problems.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Name Gina Date _____

1. Ms. Santor divides 32 students into 8 equal groups for a field trip. Draw a tape diagram and label the number of students in each group as n . Write an equation and solve for n .

$32 \div 8 = n$
 $n = 4$
There are 4 students in each group.

2. Tara buys 6 packs of printer paper. Each pack of paper costs \$8. Draw a tape diagram and label the total amount she spends as m . Write an equation and solve for m .

$m = 6 \times \$8$
 $m = \$48$
Tara spends \$48 on printer paper.

3. Mr. Reed spends \$24 on coffee beans. How many kilograms of coffee beans does he buy? Draw a tape diagram and label the total amount of coffee beans he buys as c . Write an equation and solve for c .

$c = 24 \div 8$
 $c = 3$
Mr. Reed buys 3 kg of coffee.

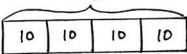
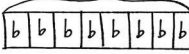
$\$8$ for 1 kg

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, did you solve to find the number of groups or the number of items in each group?
- What equations can be used to solve Problem 1?
- In Problem 4, how many parts did each pack need to be split into in order for each boy to get 1 part? (Two equal parts.) Could we use that fact to solve the problem without first finding the total number of cards? Why or why not?
- Problems 4–6 are multiple-step problems. Why is it useful to use different letters to represent two unknowns in the same problem?

4. Eight boys equally share 4 packs of baseball cards. Each pack contains 10 cards. How many cards does each boy get?

$$4 \times 10 = c$$

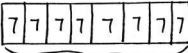
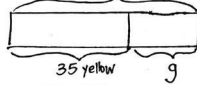
$$c = 40$$

$$40 \div 8 = b$$

$$b = 5$$

Each boy gets 5 cards.

5. There are 8 bags of yellow and green balloons. Each bag contains 7 balloons. If there are 35 yellow balloons, how many green balloons are there?

$$8 \times 7 = b$$

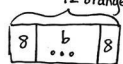
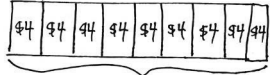
$$b = 56$$

$$56 - 35 = g$$

$$g = 21$$

There are 21 green balloons.

6. The fruit seller packs 72 oranges into bags of 8 each. He sells all the oranges at \$4 a bag. How much money did he receive?

$$72 \div 8 = b$$

$$b = 9$$

$$9 \times \$4 = m$$

$$m = \$36$$

The fruit seller receives \$36.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Multiply.

$8 \times 1 = \underline{\quad\quad\quad}$ $8 \times 2 = \underline{\quad\quad\quad}$ $8 \times 3 = \underline{\quad\quad\quad}$ $8 \times 4 = \underline{\quad\quad\quad}$

$8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 1 = \underline{\quad\quad\quad}$ $8 \times 2 = \underline{\quad\quad\quad}$ $8 \times 1 = \underline{\quad\quad\quad}$

$8 \times 3 = \underline{\quad\quad\quad}$ $8 \times 1 = \underline{\quad\quad\quad}$ $8 \times 4 = \underline{\quad\quad\quad}$ $8 \times 1 = \underline{\quad\quad\quad}$

$8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 1 = \underline{\quad\quad\quad}$ $8 \times 2 = \underline{\quad\quad\quad}$ $8 \times 3 = \underline{\quad\quad\quad}$

$8 \times 2 = \underline{\quad\quad\quad}$ $8 \times 4 = \underline{\quad\quad\quad}$ $8 \times 2 = \underline{\quad\quad\quad}$ $8 \times 5 = \underline{\quad\quad\quad}$

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$8 \times 4 = \underline{\quad\quad\quad}$ $8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 1 = \underline{\quad\quad\quad}$ $8 \times 5 = \underline{\quad\quad\quad}$

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$8 \times 3 = \underline{\quad\quad\quad}$ $8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 2 = \underline{\quad\quad\quad}$ $8 \times 4 = \underline{\quad\quad\quad}$

multiply by 8 (1–5)

Name _____

Date _____

1. Ms. Santor divides 32 students into 8 equal groups for a field trip. Draw a tape diagram, and label the number of students in each group as n . Write an equation, and solve for n .

-
2. Tara buys 6 packs of printer paper. Each pack of paper costs \$8. Draw a tape diagram, and label the total amount she spends as m . Write an equation, and solve for m .

-
3. Mr. Reed spends \$24 on coffee beans. How many kilograms of coffee beans does he buy? Draw a tape diagram, and label the total amount of coffee beans he buys as c . Write an equation, and solve for c .



4. Eight boys equally share 4 packs of baseball cards. Each pack contains 10 cards. How many cards does each boy get?

-
5. There are 8 bags of yellow and green balloons. Each bag contains 7 balloons. If there are 35 yellow balloons, how many green balloons are there?

-
6. The fruit seller packs 72 oranges into bags of 8 each. He sells all the oranges at \$4 a bag. How much money did he receive?

Name _____

Date _____

Erica buys some packs of rubber bracelets. There are 8 bracelets in each pack.

- a. How many packs of rubber bracelets does she buy if she has a total of 56 bracelets? Draw a tape diagram, and label the total number of packages as p . Write an equation, and solve for p .
- b. After giving some bracelets away, Erica has 18 left. How many bracelets did she give away?

Name _____

Date _____

1. Jenny bakes 10 cookies. She puts 7 chocolate chips on each cookie. Draw a tape diagram, and label the total amount of chocolate chips as c . Write an equation, and solve for c .

-
2. Mr. Lopez arranges 48 dry erase markers into 8 equal groups for his math stations. Draw a tape diagram, and label the number of dry erase markers in each group as v . Write an equation, and solve for v .

-
3. There are 35 computers in the lab. Five students each turn off an equal number of computers. How many computers does each student turn off? Label the unknown as m , and then solve.

4. There are 9 bins of books. Each bin has 6 comic books. How many comic books are there altogether?

-
5. There are 8 trail mix bags in one box. Clarissa buys 5 boxes. She gives an equal number of bags of trail mix to 4 friends. How many bags of trail mix does each friend receive?

-
6. Leo earns \$8 each week for doing chores. After 7 weeks, he buys a gift and has \$38 left. How much money does he spend on the gift?

Name _____

Date _____

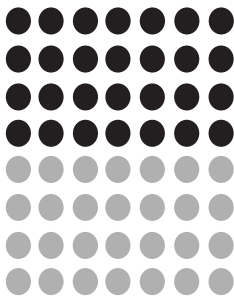
1. The carnival is in town for 21 days. How many weeks is the carnival in town? (There are 7 days in 1 week.) Write an equation, and solve.
2. There are 48 liters of water needed to finish filling the dunk tank at the carnival. Each container holds 8 liters of water. How many containers are needed to finish filling the dunk tank? Represent the problem using multiplication and division sentences and a letter for the unknown. Solve.

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} \div \underline{\quad} = \underline{\quad}$$

3. There are 4 rows of 7 chairs setup for the Magic Show. A worker sees the large number of people lined up and doubles the number of rows of chairs. They are shown below.

Explain and label to show how the array represents both 8×7 and $2 \times (4 \times 7)$.



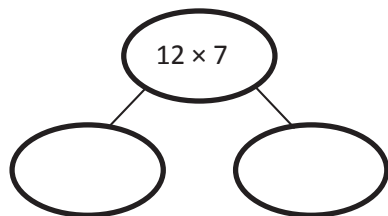
4. a. Fabrizio wins a bumblebee doll with 6 stripes. He notices that 5 other children in line for the Magic Show won the same doll. How many stripes are on 6 bumblebee dolls? Write an equation using a letter to represent the unknown. Solve.

The magician uses a magic box. Every time he puts an object in, it gets multiplied. Fabrizio writes down what happens each time and tries to find a pattern. Look at his notes to the right.

- b. Use the pattern to fill in the number of bean bags.
- c. What does the magic box do? Explain how you know.

In	Out
2 Feathers	14 Feathers
3 Marbles	21 Marbles
4 Dice	28 Dice
5 Wands	35 Wands
6 Bean bags	___ Bean bags

- d. The magician puts 12 rings into the magic box. Fabrizio draws a number bond to find the total number of rings after they are multiplied in the magic box. Use the number bond to show how Fabrizio might have solved the problem.



- e. After the show, Fabrizio and 5 friends equally share the cost of a \$54 magic set. They use the equation $6 \times n = \$54$ to figure out how much each person pays. How much does Fabrizio pay?

Mid-Module Assessment Task Standards Addressed

Topics A–C

Represent and solve problems involving multiplication and division.

- 3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$.*

Understand properties of multiplication and the relationship between multiplication and division.

- 3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

Multiply and divide within 100.

- 3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

- 3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop on their way to proficiency. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery

Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 3.OA.3 3.OA.4	Student is unable to write an equation for the problem. The attempt shows student may not understand the meaning of the question.	Student mixes up the order of numbers in the division equation (e.g., $21 \div 3 = ?$).	Student writes the correct equation but divides incorrectly (e.g., $21 \div 7 =$ wrong answer).	Student correctly: <ul style="list-style-type: none"> Writes $21 \div 7 = 3$. Identifies that the answer represents the number of weeks.
2 3.OA.3 3.OA.4	Student is unable to write both equations and does not correctly solve the problem. The attempt shows student may not understand the meaning of the questions.	Student gives an incorrect answer with reasonable attempt that includes: <ul style="list-style-type: none"> Attempt to represent the problem with multiplication and division equations. Use of a letter to represent the unknown. 	Student provides a partially correct answer. Student correctly: <ul style="list-style-type: none"> Writes $n \times 8 = 48$. Writes $48 \div 8 = n$. 	Student correctly: <ul style="list-style-type: none"> Writes $n \times 8 = 48$. Writes $48 \div 8 = n$. Solves to find 6 containers.
3 3.OA.5	Student is unable to explain and label how the array represents both expressions.	Student attempts to explain and label how the array represents one of the expressions.	Student accurately labels how the array represents both expressions, but the explanation lacks clarity.	Student accurately explains and labels how the array represents both expressions, showing understanding of the associative property of multiplication.



A Progression Toward Mastery

Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
4 3.OA.3 3.OA.4 3.OA.5 3.OA.9	Student answers one question correctly.	Student answers two questions correctly.	Student answers three questions correctly. Mistakes may include: <ul style="list-style-type: none"> Completing the equation in Part (a) incorrectly (e.g., $6 \times 6 = n$; $n =$ wrong answer). Providing inaccurate explanation in Part (c). Providing incorrect total in Part (d) (e.g., $12 \times 7 =$ wrong total). 	Student correctly: <ul style="list-style-type: none"> Writes and solves an equation using a letter to represent the total number of stripes in Part (a) ($6 \times 6 = b$; $b = 36$). Fills in 42 bean bags in the chart in Part (b). Accurately explains how the magic box multiplies objects by 7 in Part (c). Uses a number bond to break apart the 12×7 and distributes to find the total number of rings, 84, in Part (d). Writes $n = \\$9$ in Part (e).

Name

Gina

Date

1. The carnival is in town for 21 days. How many weeks is the carnival in town? (There are 7 days in 1 week.) Write an equation, and solve.

7, 14, 21

① ② ③

$$21 \div 7 = 3$$

The carnival is in town
for 3 weeks

2. There are 48 liters of water needed to finish filling the dunk tank at the carnival. Each container holds 8 liters of water. How many containers are needed to finish filling the dunk tank? Represent the problem using multiplication and division sentences and a letter for the unknown. Solve.

$$n \times 8 = 48$$

$$48 \div 8 = n$$

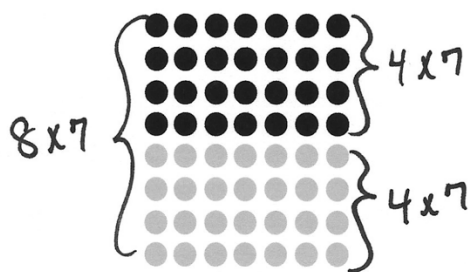
n = the number of containers

$$n = 6$$

6 containers are needed to
finish filling the dunk tank.

3. There are 4 rows of 7 chairs setup for the magic show. A worker sees the large number of people lined up and doubles the number of rows of chairs. They are shown below.

Explain and label to show how the array represents both 8×7 and $2 \times (4 \times 7)$.



You can see the array 2 ways.
You can see the total array as
8 rows of 7, or you can see
4 rows of 7 two times (the
black rows and gray rows).
They both have the same
total of 56 chairs.

4. a. Fabrizio wins a bumblebee doll with 6 stripes. He notices that 5 other children in line for the magic show won the same doll. How many stripes are on 6 bumblebee dolls? Write an equation using a letter to represent the unknown. Solve.

$$6 \times 6 = S$$

$$S = 36$$

$S =$ total number of stripes
There are 36 stripes on 6 bumblebee dolls.

The magician uses a magic box. Every time he puts an object in, it gets multiplied. Fabrizio writes down what happens each time and tries to find a pattern. Look at his notes to the right.

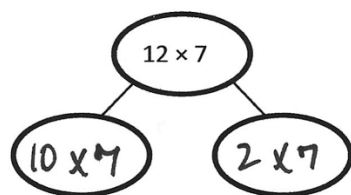
- b. Use the pattern to fill in the number of bean bags.

- c. What does the magic box do? Explain how you know.

The magic box multiplies objects by 7. When you put an object in, it grows 7 times. That's why when you put 6 bean bags in, 42 come out.

In	Out
2 Feathers $\times 7 \rightarrow$	14 Feathers
3 Marbles $\times 7 \rightarrow$	21 Marbles
4 Dice $\times 7 \rightarrow$	28 Dice
5 Wands $\times 7 \rightarrow$	35 Wands
6 Bean bags $\times 7 \rightarrow$	<u>42</u> Bean bags

- d. The magician puts 12 rings into the magic box. Fabrizio draws a number bond to find the total number of rings after they are multiplied in the magic box. Use the number bond to show how Fabrizio might have solved the problem.



$$(10 \times 7) + (2 \times 7) = 12 \times 7$$

$$70 + 14 = 84$$

When the magician puts 12 rings into the box, 84 rings come out.

- e. After the show, Fabrizio and 5 friends equally share the cost of a \$54 magic set. They use the equation $6 \times n = \$54$ to figure out how much each person pays. How much does Fabrizio pay?

$6 \times n = \$54$ is the same as $\$54 \div 6 = n$, where $n =$ the amount each person pays.

$$n = \$9$$

Fabrizio pays \$9.



Topic D

Multiplication and Division Using Units of 9

3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7, 3.OA.9, 3.OA.1, 3.OA.2, 3.OA.6

Focus Standards:	3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
	3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$</i>
	3.OA.5	Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i>
	3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
	3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i>
Instructional Days:	4	
Coherence -Links from:	G2–M3	Place Value, Counting, and Comparison of Numbers to 1,000
	G2–M6	Foundations of Multiplication and Division
	G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
	-Links to:	
	G3–M4	Multiplication and Area
	G4–M3	Multi-Digit Multiplication and Division
	G4–M5	Fraction Equivalence, Ordering, and Operations
	G4–M7	Exploring Measurement with Multiplication

In Lesson 12, students use the distributive property to establish the $9 = 10 - 1$ pattern for multiplication. Conceptual understanding of the pattern enables students to see this method of multiplication as a tool rather than a trick. This lesson lays the foundation for exploring other patterns that emerge with multiplication using units of 9 in the subsequent lessons.

Lessons 13 and 14 focus on the study of patterns as they relate to the fact $9 = 10 - 1$. Students discover that the tens digit in the product of a nines fact is 1 less than the multiplier and that the ones digit in the product is 10 minus the multiplier. For example, $9 \times 3 = 27$, $2 = 3 - 1$, and $7 = 10 - 3$. They also see that the digits of the nines facts' products produce a sum of 9, as in the example above ($2 + 7 = 9$).

Lesson 15 parallels the final lessons of Topics B and C. Students analyze multiplication and division problems using units of 9, drawing models, and writing equations using a letter to represent the unknown. These lessons are intended to provide students with continuous experience relating three numbers to find the unknown, as well as to deepen their understanding of the relationship between multiplication and division.

A Teaching Sequence Toward Mastery of Multiplication and Division Using Units of 9

Objective 1: Apply the distributive property and the fact $9 = 10 - 1$ as a strategy to multiply.
(Lesson 12)

Objective 2: Identify and use arithmetic patterns to multiply.
(Lessons 13–14)

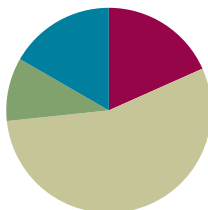
Objective 3: Interpret the unknown in multiplication and division to model and solve problems.
(Lesson 15)

Lesson 12

Objective: Apply the distributive property and the fact $9 = 10 - 1$ as a strategy to multiply.

Suggested Lesson Structure

■ Fluency Practice	(11 minutes)
■ Application Problem	(6 minutes)
■ Concept Development	(33 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (11 minutes)

- Multiply By 8 **3.OA.7** (7 minutes)
- Take from the Ten **3.OA.5** (4 minutes)

Multiply by 8 (7 minutes)

Materials: (S) Multiply By 8 (6–10) (Pattern Sheet)

Note: This activity builds fluency with respect to multiplication facts using units of 8. It supports students knowing from memory all products of two one-digit numbers. See Lesson 5 for the directions regarding administration of a Multiply By Pattern Sheet.

- T: (Write $6 \times 8 = \underline{\quad}$.) Let's skip-count up by eights to solve. (Count with fingers to 6 as students count.)
- S: 8, 16, 24, 32, 40, 48.
- T: Let's skip-count down to find the answer, too. Start at 80. (Count down from 10 fingers as students count.)
- S: 80, 72, 64, 56, 48.
- T: Let's skip-count up again to find the answer, but this time start at 40. (Count up from 5 fingers as students count.)
- S: 40, 48.

Continue with the following possible sequence: 8×8 , 7×8 , and 9×8 .

- T: (Distribute the Multiply By 8 Pattern Sheet.) Let's practice multiplying by 8. Be sure to work left to right across the page.

Take from the Ten (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for today's Concept Development.

T: (Write $20 - 2 = \underline{\quad}$.) Say the subtraction sentence in unit form.

S: 2 tens – 2 ones.

T: (Point to the 20.) Let's break apart the 20, taking out 10 ones. How many tens are left?

S: 1 ten.

T: What's 10 ones – 2 ones?

S: 8 ones.

T: (Write 8.)

T: What's $20 - 2$?

S: 18.

T: (Write $20 - 2 = 18$.)

T: (Write $30 - 3 = \underline{\quad}$.) After writing the equation, break apart the 30, taking out 10 ones.

S: (Break apart the 30 into 20 and 10.)

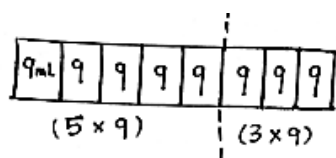
T: Take 3 ones from 10 ones and complete the equation.

S: (Take 3 from 10 to get 7; $30 - 3 = 27$.)

Continue with the following possible sequence: $40 - 4$, $50 - 5$, $60 - 6$, $70 - 7$, $80 - 8$, and $90 - 9$.

Application Problem (6 minutes)

A scientist fills 5 test tubes with 9 milliliters of fresh water in each. She fills another 3 test tubes with 9 milliliters of salt water in each. How many milliliters of water does she use in all? Use the break apart and distribute strategy to solve.



$$\begin{aligned} 8 \times 9 &= (5+3) \times 9 \\ &= (5 \times 9) + (3 \times 9) \\ &= 45 + 27 \\ &\quad \begin{array}{r} 45 \\ +27 \\ \hline 72 \end{array} \\ &= 72 \end{aligned}$$

She used 72 mL of water in all.

Note: The Application Problem is meant to reinforce the $5 + n$ break apart and distribute strategy to support Problem 1 in the Problem Set and also provide a point of comparison between the $5 + n$ strategy and $9 = 10 - 1$ strategy for multiplying with a factor of 9. Notice that, to add 45 and 27, the student has taken 3 from 45 to make 30 from 27.

Concept Development (33 minutes)

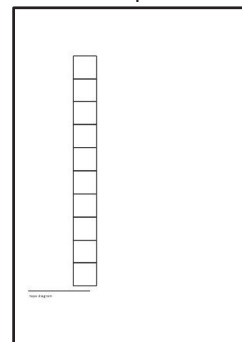
Materials: (S) Personal white board, tape diagram (Template)

Use the $9 = 10 - 1$ strategy to solve $9 \times n$ facts.

Have students insert templates into their personal white boards.

- T: We solved 8×9 in the Application Problem. Does 8×9 show 8 units of 9 or 9 units of 8?
- S: 8 units of 9.
- T: What multiplication fact represents 9 units of 8?
- S: 9×8 .
- T: How can our work solving 8×9 help us solve 9×8 ?
- S: We can use the commutative property to know that, if $8 \times 9 = 72$, then so does 9×8 .
- T: Sometimes we can't use the commutative property because we don't know the product of either fact. Let's look at how we can use a tens fact to help solve a nines fact when that happens. What's easier to solve, 9×8 or 10×8 ?
- S: 10×8 because we already know tens facts.
- T: How many eights are in 10×8 ?
- S: 10 eights!
- T: Label them on your tape diagram.
- T: How many eights in 9×8 ?
- S: 9 eights!
- T: Change your tape diagram so it shows 9 eights. (Allow students time to finish their work.)
- T: What change did you make?
- S: I crossed off an eight. \rightarrow I took away 1 eight. \rightarrow I subtracted one unit.
- T: 9 eights (point to the tape diagram) equals 10 eights minus...?
- S: 1 eight!
- T: Work with your partner to write a number sentence showing that.
- S: (Write $9 \times 8 = (10 \times 8) - (1 \times 8)$.)
- T: Rewrite your equation using the products of 10×8 and 1×8 .
- S: (Write $9 \times 8 = 80 - 8$.)
- T: What is $80 - 8$?
- S: 72.
- T: Tell your partner how we used a tens fact to solve a nines fact.
- S: We just took the product of 10×8 and subtracted 1 eight. \rightarrow That made the math simple. I can do $80 - 8$ in my head!

Template



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Teachers should adjust their rate of speech for English language learners and others as students write equations in response to oral prompts. Label the equations. For example, write *9 eights* under 9×8 .

MP.3

- T: (Write $9 \times 8 = (5 + 4) \times 8$.) One way we've learned to solve 9×8 is by breaking 9 eights up into 5 eights plus 4 eights. Why did it work well to subtract this time instead?
- S: Because we only had to subtract 1 eight. → Yeah, 9 is really close to 10, and tens are easy to use. We already know 10×8 , and besides, it's easy to subtract from a tens fact.
- T: Work with your partner to change the equation I just wrote for 9×8 . Make sure it shows how we used subtraction to solve.
- S: (Change the equation to $9 \times 8 = (10 - 1) \times 8$.)
- T: What part of the equation did you change?
- S: We changed $5 + 4$ to $10 - 1$.
- T: Why?
- S: Because we didn't add; we subtracted. We started with 10 eights and then took away 1 eight.

Continue with the following suggested sequence: 9×7 and 9×6 .



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

As students solve the Problem Set, some learners may solve Problem 1 more efficiently using the $9 = 10 - 1$ strategy.

Students working above grade level can be encouraged to write equations using parentheses for Problem 2. Challenge students to offer multiple equations. Ask, "How many equations can you write for Problem 2(a)?"

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.


Student Debrief (10 minutes)

Lesson Objective: Apply the distributive property and the fact $9 = 10 - 1$ as a strategy to multiply.


The Student Debrief is intended to invite reflection and active processing of the total lesson experience.


Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name Gina Date _____

1. Each  has a value of 9. Find the value of each row. Then, add the rows to find the total.


a. $5 \times 9 = 45$


 $5 \times 9 = 45$

 $1 \times 9 = 9$

$8 \times 9 = (5 + 3) \times 9$
 $= (5 \times 9) + (3 \times 9)$
 $= 45 + 27$
 $= 72$


b. $7 \times 9 = 63$


 $7 \times 9 = 63$

 $2 \times 9 = 18$

$7 \times 9 = (5 + 2) \times 9$
 $= (5 \times 9) + (2 \times 9)$
 $= 45 + 18$
 $= 63$


c. $8 \times 9 = 72$


 $8 \times 9 = 72$

 $3 \times 9 = 27$

$8 \times 9 = (5 + 3) \times 9$
 $= (5 \times 9) + (3 \times 9)$
 $= 45 + 27$
 $= 72$

d. $9 \times 9 = 81$

 $9 \times 9 = 81$

 $4 \times 9 = 36$

$9 \times 9 = (5 + 4) \times 9$
 $= (5 \times 9) + (4 \times 9)$
 $= 45 + 36$
 $= 81$

Any combination of the questions below may be used to lead the discussion.

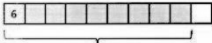
- What does the nine represent in Problem 1? (It represents the value of each unit.)
What does the nine represent in Problem 2? (It represents the number of units.)
- How can multiplication be used to solve the division facts in Problem 4?
- Think about the strategy used to solve Problem 2(a). How could a similar strategy be used to solve 8×6 instead of 9×6 ?
- Today, we solved 9×8 in different ways. How are the strategies we used in the Application Problem and Concept Development similar? How are they different?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

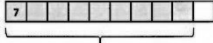
2. Find the total value of the shaded blocks.

a. $9 \times 6 = 54$



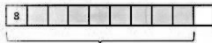
$9 \text{ sixes} = 10 \text{ sixes} - 1 \text{ six}$
 $= 60 - 6$
 $= 54$

b. $9 \times 7 = 63$



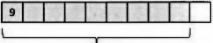
$9 \text{ sevens} = 10 \text{ sevens} - 1 \text{ seven}$
 $= 70 - 7$
 $= 63$

c. $9 \times 8 = 72$




$9 \text{ eights} = 10 \text{ eights} - 1 \text{ eight}$
 $= 80 - 8$
 $= 72$

d. $9 \times 9 = 81$



$9 \text{ nines} = 10 \text{ nines} - 1 \text{ nine}$
 $= 90 - 9$
 $= 81$

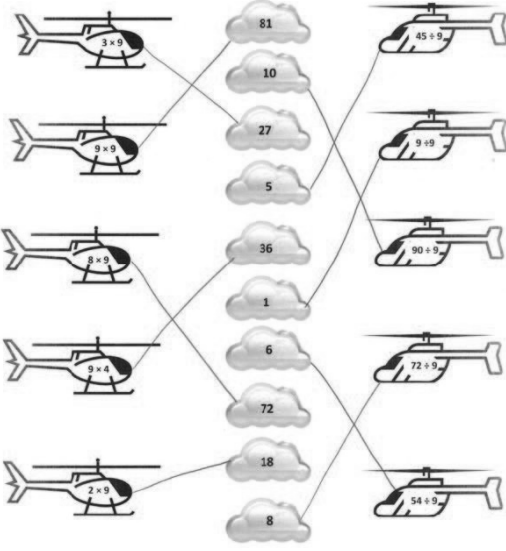
3. Matt buys a pack of postage stamps. He counts 9 rows of 4 stamps. He thinks of 10 fours to find the total number of stamps. Show the strategy that Matt might have used to find the total number of stamps.



9×4

$9 \text{ fours} = 10 \text{ fours} - 1 \text{ four}$
 $40 - 4 = 36$
 Matt bought 36 postage stamps.

4. Match.



Helicopters (left): 3×9 , 8×9 , 3×9 , 9×4 , 2×9

Helicopters (right): $45 \div 9$, $9 \div 9$, $90 \div 9$, $72 \div 9$, $54 \div 9$

Clouds (center): 81, 10, 27, 5, 36, 1, 6, 72, 18, 8

Multiply.

$8 \times 1 = \underline{\quad\quad\quad}$ $8 \times 2 = \underline{\quad\quad\quad}$ $8 \times 3 = \underline{\quad\quad\quad}$ $8 \times 4 = \underline{\quad\quad\quad}$

$8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$ $8 \times 8 = \underline{\quad\quad\quad}$

$8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 10 = \underline{\quad\quad\quad}$ $8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$

$8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$ $8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 8 = \underline{\quad\quad\quad}$

$8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 10 = \underline{\quad\quad\quad}$

$8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 5 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$

$8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 8 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 9 = \underline{\quad\quad\quad}$

$8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$

$8 \times 8 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$ $8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$

$8 \times 8 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 8 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$

$8 \times 8 = \underline{\quad\quad\quad}$ $8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$

$8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$ $8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 8 = \underline{\quad\quad\quad}$

$8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 8 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 9 = \underline{\quad\quad\quad}$


$8 \times 7 = \underline{\quad\quad\quad}$ $8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 8 = \underline{\quad\quad\quad}$

$8 \times 9 = \underline{\quad\quad\quad}$ $8 \times 7 = \underline{\quad\quad\quad}$ $8 \times 6 = \underline{\quad\quad\quad}$ $8 \times 8 = \underline{\quad\quad\quad}$

multiply by 8 (6–10)

Name _____

Date _____

1. Each  has a value of 9. Find the value of each row. Then, add the rows to find the total.

a. $6 \times 9 = \underline{\hspace{2cm}}$



$5 \times 9 = 45$



$1 \times 9 = \underline{\hspace{2cm}}$

$$\begin{aligned} 6 \times 9 &= (5 + 1) \times 9 \\ &= (5 \times 9) + (1 \times 9) \\ &= 45 + \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

b. $7 \times 9 = \underline{\hspace{2cm}}$



$5 \times 9 = 45$



$\underline{\hspace{2cm}} \times 9 = \underline{\hspace{2cm}}$

$$\begin{aligned} 7 \times 9 &= (5 + \underline{\hspace{2cm}}) \times 9 \\ &= (5 \times 9) + (\underline{\hspace{2cm}} \times 9) \\ &= 45 + \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

c. $8 \times 9 = \underline{\hspace{2cm}}$



$5 \times 9 = \underline{\hspace{2cm}}$



$\underline{\hspace{2cm}} \times 9 = \underline{\hspace{2cm}}$

$$\begin{aligned} 8 \times 9 &= (5 + \underline{\hspace{2cm}}) \times 9 \\ &= (5 \times 9) + (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}}) \\ &= 45 + \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

d. $9 \times 9 = \underline{\hspace{2cm}}$



$5 \times 9 = \underline{\hspace{2cm}}$

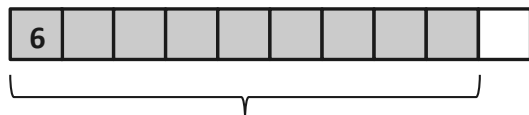


$\underline{\hspace{2cm}} \times 9 = \underline{\hspace{2cm}}$

$$\begin{aligned} 9 \times 9 &= (5 + \underline{\hspace{2cm}}) \times 9 \\ &= (5 \times 9) + (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}}) \\ &= 45 + \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

2. Find the total value of the shaded blocks.

a. $9 \times 6 =$

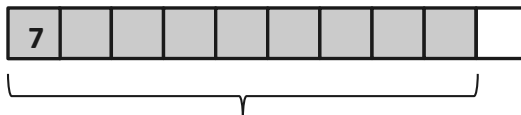


9 sixes = 10 sixes – 1 six

= _____ – 6

= _____

b. $9 \times 7 =$

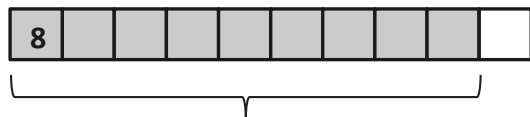


9 sevens = 10 sevens – 1 seven

= _____ – 7

= _____

c. $9 \times 8 =$

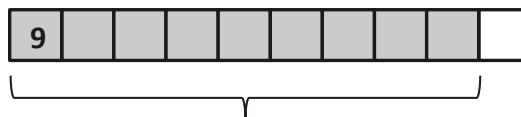


9 eights = 10 eights – 1 eight

= _____ – 8

= _____

d. $9 \times 9 =$



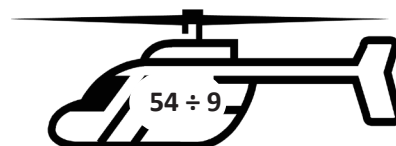
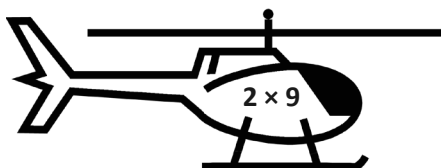
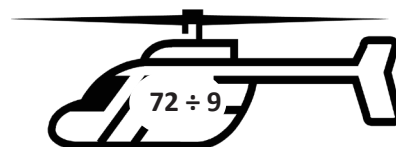
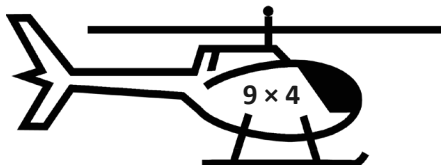
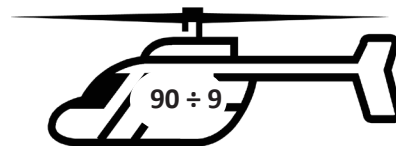
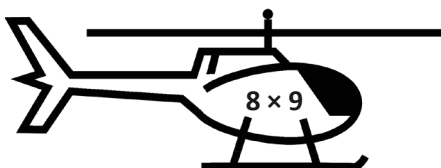
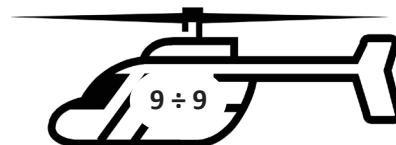
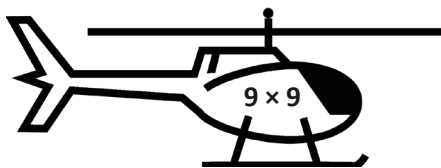
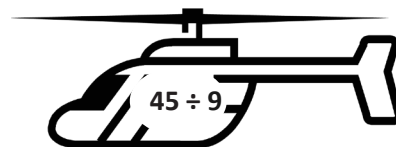
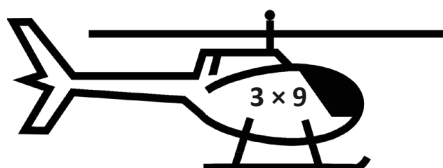
9 nines = 10 nines – 1 nine

= _____ – _____

= _____


3. Matt buys a pack of postage stamps. He counts 9 rows of 4 stamps. He thinks of 10 fours to find the total number of stamps. Show the strategy that Matt might have used to find the total number of stamps.

4. Match.



Name _____

Date _____

1. Each  has a value of 9. Complete the equations to find the total value of the tower of blocks.



$$\begin{aligned} \underline{\hspace{1cm}} \times 9 &= (5 + \underline{\hspace{1cm}}) \times 9 \\ &= (5 \times \underline{\hspace{1cm}}) + (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}) \\ &= 45 + \underline{\hspace{1cm}} \\ &= \underline{\hspace{1cm}} \end{aligned}$$

2. Hector solves 9×8 by subtracting 1 eight from 10 eights. Draw a model, and explain Hector's strategy.

Name _____

Date _____

1. Find the value of each row. Then, add the rows to find the total.

a. Each  has a value of 6.

$9 \times 6 = \underline{\hspace{2cm}}$



$5 \times 6 = 30$



$4 \times 6 = \underline{\hspace{2cm}}$

$$\begin{aligned}
 9 \times 6 &= (5 + 4) \times 6 \\
 &= (5 \times 6) + (4 \times 6) \\
 &= 30 + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

b. Each  has a value of 7.

$9 \times 7 = \underline{\hspace{2cm}}$



$5 \times 7 = \underline{\hspace{2cm}}$



$\underline{\hspace{2cm}} \times 7 = \underline{\hspace{2cm}}$

$$\begin{aligned}
 9 \times 7 &= (5 + \underline{\hspace{2cm}}) \times 7 \\
 &= (5 \times 7) + (\underline{\hspace{2cm}} \times 7) \\
 &= 35 + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

c. Each  has a value of 8.

$9 \times 8 = \underline{\hspace{2cm}}$



$5 \times 8 = \underline{\hspace{2cm}}$



$\underline{\hspace{2cm}} \times 8 = \underline{\hspace{2cm}}$

$$\begin{aligned}
 9 \times 8 &= (5 + \underline{\hspace{2cm}}) \times 8 \\
 &= (5 \times 8) + (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}}) \\
 &= 40 + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

d. Each  has a value of 9.

$9 \times 9 = \underline{\hspace{2cm}}$



$5 \times 9 = \underline{\hspace{2cm}}$

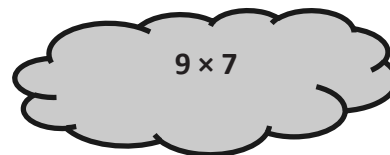
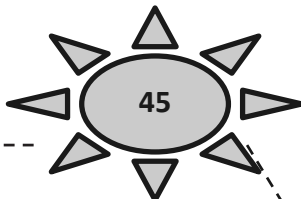


$\underline{\hspace{2cm}} \times 9 = \underline{\hspace{2cm}}$

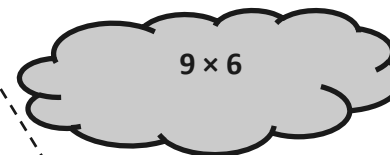
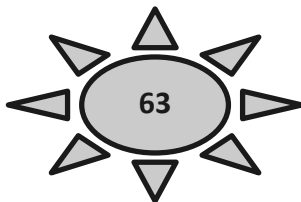
$$\begin{aligned}
 9 \times 9 &= (5 + \underline{\hspace{2cm}}) \times 9 \\
 &= (5 \times 9) + (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}}) \\
 &= 45 + \underline{\hspace{2cm}} \\
 &= \underline{\hspace{2cm}}
 \end{aligned}$$

2. Match.

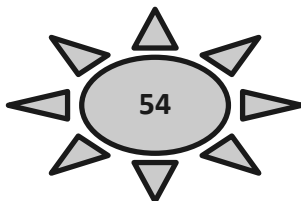
a. **9 fives** = 10 fives – 1 five
= $50 - 5$



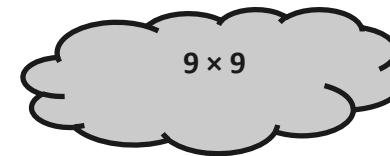
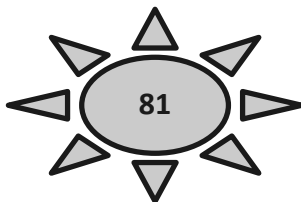
b. **9 sixes** = 10 sixes – 1 six
= $\underline{\quad} - 6$



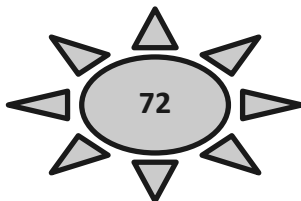
c. **9 sevens** = 10 sevens – 1 seven
= $\underline{\quad} - 7$



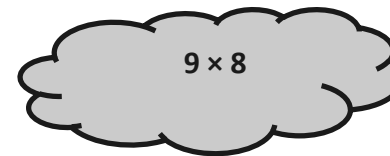
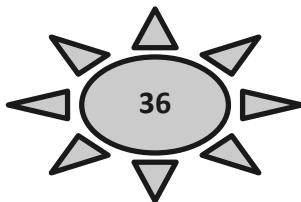
d. **9 eights** = 10 eights – 1 eight
= $\underline{\quad} - 8$



e. **9 nines** = 10 nines – 1 nine
= $\underline{\quad} - \underline{\quad}$



f. **9 fours** = 10 fours – 1 four
= $\underline{\quad} - \underline{\quad}$



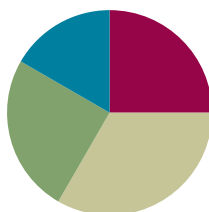
tape diagram

Lesson 13

Objective: Identify and use arithmetic patterns to multiply.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Concept Development	(20 minutes)
■ Application Problem	(15 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



NOTES ON LESSON STRUCTURE:

The Problem Set follows immediately after the Application Problem in this lesson. The 10 minutes for the Problem Set are included in the time allotted for the Application Problem rather than the Concept Development.

Fluency Practice (15 minutes)

- Sprint: Multiply or Divide by 8 **3.OA.7** (8 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Decompose Multiples of 9 **3.OA.5** (3 minutes)

Sprint: Multiply or Divide by 8 (8 minutes)

Materials: (S) Multiply or Divide by 8 Sprint

Note: This Sprint reviews Lessons 10 and 11, focusing on the relationship between multiplying and dividing using units of 8.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes, sevens, and eights reviews multiplication taught previously in the module. Group counting nines prepares students for multiplication in this lesson. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90



A NOTE ON STANDARDS ALIGNMENT:

Some problems in the Sprint and Part 2 of the Concept Development extend beyond Grade 3 multiplication and division standards.

By extending to products above 90 and quotients above 10 in the Sprint, students working above grade level are provided the stimulus to stretch their conceptual understanding, which may keep them engaged and invigorated to improve on Sprint B. Students who have mastered their times tables are likely to otherwise go unchallenged.

Decompose Multiples of 9 (3 minutes)

Materials: (S) Personal white board

Note: This activity prepares students to use the distributive property using units of 9.

T: (Project a number bond with a whole of 45 and 18 as a part.) On your personal white board, complete the unknown part in the number bond.

S: (Write 27.)

Continue with the following possible sequence: whole of 90 and 27 as a part, whole of 54 and 36 as a part, whole of 72 and 27 as a part, and whole of 63 and 18 as a part.

9

$$9 + 10 = 19$$

18 9 is 1 less than 10 so:
 $9 + 9 = 18$

$$18 + 10 = 28$$

27 9 is 1 less than 10 so:
 $18 + 9 = 27$

$$27 + 10 = 37$$

36 9 is 1 less than 10 so:
 $27 + 9 = 36$

$$36 + 10 = 46$$

45 9 is 1 less than 10 so:
 $36 + 9 = 45$

$$45 + 10 = 55$$

54 9 is 1 less than 10 so:
 $45 + 9 = 54$

Concept Development (20 minutes)

Materials: (S) Personal white board, Problem Set

Part 1: Identify patterns in multiples of 9.

T: During the fluency activity, we group counted nines to say the **multiples** of 9. When we skip-count by nines, what are we adding each time?

S: 9.

T: Adding nines can be tricky. What's a simplifying strategy for adding 9?

S: I can break apart 9 to make the next ten and then add what's left of the 9 to it. → I can add 10 and then subtract 1.

T: (Lead students through applying the add 10, subtract 1 strategy in Problem 2 on the Problem Set. Model the first example. Students can then work in pairs to find the rest. Allow time for students to finish their work.)

T: Compare the digits in the ones and tens places of the multiples. What pattern do you notice?

S: The digit in the tens place increases by 1. → The digit in the ones place decreases by 1.

MP.7

T: Now, with your partner, analyze the sum of the digits for each multiple of 9. What pattern do you notice?

S: The sum of the digits in every multiple of 9 is equal to 9.

T: How does knowing the sum of the digits in every multiple of 9 is equal to 9 help you with nines facts?

S: To check my answer, I can add up the digits. If the sum isn't equal to 9, I made a mistake.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Simplify and clarify *sum of the digits* for English language learners and others. Distinguish *some* from *sum*. Perhaps express the request in another manner, such as, "Look at the product. (Point.) Add the digit in the ones place (point) to the digit in the tens place (point). What's the sum? Now, look at the next product..."

Part 2: Apply strategies to solve nines facts.

Have students write and solve all facts from 1×9 to 10×9 in a column on their personal white boards.

- T: Let's examine $1 \times 9 = 9$. Here, what is 9 multiplied by?
 S: 9 is multiplied by 1.
 T: What number is in the tens place of the product for 1×9 ?
 S: Zero.
 T: How is the number in the tens place related to 1?
 S: It is 1 less. \rightarrow Zero is one less than 1.
 T: Say the product of 2×9 at my signal. (Signal.)
 S: 18.
 T: Which digit is in the tens place of the product?
 S: 1.
 T: How is the digit in the tens place related to the 2?
 S: It's one less again. \rightarrow 1 is one less than 2.

$$\begin{array}{l} 1 \times 9 = 9 \\ 2 \times 9 = 18 \\ 3 \times 9 = 27 \\ 4 \times 9 = 36 \\ 5 \times 9 = 45 \\ 6 \times 9 = 54 \\ 7 \times 9 = 63 \\ 8 \times 9 = 72 \\ 9 \times 9 = 81 \\ 10 \times 9 = 90 \end{array}$$

Repeat the process with 3×9 and 4×9 .

- T: What pattern do you notice with the digit in the tens place for each of those products?
 S: The number in the tens place is 1 less than the number of groups.
 T: With your partner, see if that pattern fits for the rest of the nines facts to ten.
 S: It does! The pattern keeps going!
 T: Let's see if we can find a pattern involving the ones place. We know that 2×9 equals 18. The 2 and 8 are related in some way. We also know that 3×9 equals 27. The 3 and 7 are related in the *same* way. Discuss with your partner how they are related.
 S: $2 + 8 = 10$ and $3 + 7 = 10$. $\rightarrow 10 - 2 = 8$ and $10 - 3 = 7$.
 T: When you take the number of groups and subtract it from 10, what do you get?
 S: The ones place in the product!
 T: With your partner, see if that pattern fits for the rest of the nines facts. (Allow students time to finish their work.)
 T: Did the pattern work for every fact, 1×9 through 10×9 ?
 S: Yes!
 T: Let's try 11×9 . What is the product?
 S: 99.
 T: What is the number of groups?
 S: 11.



**A NOTE ON
STANDARDS
ALIGNMENT:**

The fact 11×9 extends beyond the Grade 3 multiplication standards. Its use here allows students to construct boundaries for the pattern that they discovered with the nines facts (the number in the tens place in the product is one less than the number of groups, and the number in the ones place in the product is 10 minus the number of groups). By examining the product of 11×9 , students learn that the pattern only applies to 1×9 through 10×9 .

- T: Talk to your partner: Does the pattern work for 11×9 ? Why or why not?
- S: No, the pattern doesn't make sense. You can't have 10 in the tens place, and we don't know how to solve $10 - 11$ to find what digit is in the ones place.
- T: The pattern can give you the answer to any nines fact from 1×9 to 10×9 , but it doesn't work for nines facts bigger than 10×9 .

Application Problem (15 minutes)

Michaela and Gilda read the same book. It takes Michaela about 8 minutes to read a chapter and Gilda about 10 minutes. There are 9 chapters in the book. How many fewer minutes does Michaela spend reading than Gilda?

$$\begin{array}{l} \text{Michaela } 8 \times 9 = 72 \\ \text{Gilda } 10 \times 9 = 90 \\ 90 - 72 = 18 \\ \text{Michaela spends 18 fewer minutes reading.} \end{array}$$

Note: This problem comes after the Concept Development, so students have the opportunity to apply some of the strategies they learned in the context of problem solving. Encourage them to check their answers to the nines facts using new learning.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Consider adjusting the Problem Set to assist learners with perceptual disabilities. Provide a table for students to record the nine skip-count, and/or highlight the tens or ones.

As students solve Problem 2, it may be helpful to write the nines fact for each given product (e.g., in Problem 2(f), have students write 5×9 above 45).

Student Debrief (10 minutes)

Lesson Objective: Identify and use arithmetic patterns to multiply.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- What patterns did you use to solve Problem 1?
- The add 10, subtract 1 strategy can be used to quickly find **multiples** of 9. How could you change it to quickly find multiples of 8?
- How is the add 10, subtract 1 strategy related to the $9 = 10 - 1$ break apart and distribute strategy we learned recently?
- In Problem 3(d) how did you figure out where Kent's strategy stops working? Why doesn't this strategy work past 10×9 ?
- How can the number of groups in a nines fact help you find the product?
- How did group counting during the fluency activity help prepare us for today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Gina Date _____

1. a. Skip-count by nine.
9, 18, 27, 36, 45, 54, 63, 72, 81, 90

b. Look at the tens place in the count-by. What is the pattern?
The tens place increases by 1.

c. Look at the ones place in the count-by. What is the pattern?
The ones place decreases by 1.

2. Complete to make true statements.

a. 10 more than 0 is <u>10</u> 1 less is <u>9</u> $1 \times 9 =$ <u>9</u>	f. 10 more than 45 is <u>55</u> 1 less is <u>54</u> $6 \times 9 =$ <u>54</u>
b. 10 more than 9 is <u>19</u> 1 less is <u>18</u> $2 \times 9 =$ <u>18</u>	g. 10 more than 54 is <u>64</u> 1 less is <u>63</u> $7 \times 9 =$ <u>63</u>
c. 10 more than 18 is <u>28</u> 1 less is <u>27</u> $3 \times 9 =$ <u>27</u>	h. 10 more than 63 is <u>73</u> 1 less is <u>72</u> $8 \times 9 =$ <u>72</u>
d. 10 more than 27 is <u>37</u> 1 less is <u>36</u> $4 \times 9 =$ <u>36</u>	i. 10 more than 72 is <u>82</u> 1 less is <u>81</u> $5 \times 9 =$ <u>81</u>
e. 10 more than 36 is <u>46</u> 1 less is <u>45</u> $5 \times 9 =$ <u>45</u>	j. 10 more than 81 is <u>91</u> 1 less is <u>90</u> $10 \times 9 =$ <u>90</u>

4. Each equation contains a letter representing the unknown. Find the value of each unknown. Then write the letters that match the answers to solve the riddle.

$a \times 9 = 54$ <u>a = 6</u>	$81 \div 9 = g$ <u>g = 9</u>	$9 \times d = 72$ <u>d = 8</u>
$o \div 9 = 10$ <u>o = 90</u>	$e \times 9 = 63$ <u>e = 7</u>	$9 \times n = 27$ <u>n = 3</u>
$9 \times s = 36$ <u>s = 4</u>	$t \times 9 = 18$ <u>t = 2</u>	$l \div 9 = 5$ <u>l = 45</u>

How do you make one vanish?

A d d a g and i t s g o n e .

6 8 8 6 3 6 3 8 45 2 4 90 3 7

3. a. Analyze the equations in problem 2. What is the pattern?
The pattern is add 10 then subtract 1. $10 - 1$.
To get a nines fact you add 10 then subtract 1.

b. Use the pattern to find the next 4 facts. Show your work.

$11 \times 9 =$ $90 + 10 = 100$ $100 - 1 = 99$ $11 \times 9 = 99$	$12 \times 9 =$ $99 + 10 = 109$ $109 - 1 = 108$ $12 \times 9 = 108$	$13 \times 9 =$ $108 + 10 = 118$ $118 - 1 = 117$ $13 \times 9 = 117$	$14 \times 9 =$ $117 + 10 = 127$ $127 - 1 = 126$ $14 \times 9 = 126$
--	--	---	---

c. Kent notices another pattern in problem 2. His work is shown below. He sees that:

- the tens digit in the product is 1 less than the number of groups
- the ones digit in the product is 10 minus the number of groups

	tens digit	ones digit
$2 \times 9 = 18$	$1 = 2 - 1$	$8 = 10 - 2$
$3 \times 9 = 27$	$2 = 3 - 1$	$7 = 10 - 3$
$4 \times 9 = 36$	$3 = 4 - 1$	$6 = 10 - 4$
$5 \times 9 = 45$	$4 = 5 - 1$	$5 = 10 - 5$

Use Kent's strategy to solve 6×9 and 7×9 .

$6 \times 9 = 54 \rightarrow 5 = 6 - 1$ and $4 = 10 - 6$
 $7 \times 9 = 63 \rightarrow 6 = 7 - 1$ and $3 = 10 - 7$

d. Show an example of when Kent's pattern doesn't work.

$12 \times 9 = 108$. 0 does not equal $12 - 1$. And 8 does not equal $10 - 12$.

A

Number Correct: _____

Multiply or divide by 8

1.	$2 \times 8 =$	
2.	$3 \times 8 =$	
3.	$4 \times 8 =$	
4.	$5 \times 8 =$	
5.	$1 \times 8 =$	
6.	$16 \div 8 =$	
7.	$24 \div 8 =$	
8.	$40 \div 8 =$	
9.	$8 \div 1 =$	
10.	$32 \div 8 =$	
11.	$6 \times 8 =$	
12.	$7 \times 8 =$	
13.	$8 \times 8 =$	
14.	$9 \times 8 =$	
15.	$10 \times 8 =$	
16.	$64 \div 8 =$	
17.	$56 \div 8 =$	
18.	$72 \div 8 =$	
19.	$48 \div 8 =$	
20.	$80 \div 8 =$	
21.	$\underline{\hspace{1cm}} \times 8 = 40$	
22.	$\underline{\hspace{1cm}} \times 8 = 16$	

23.	$\underline{\hspace{1cm}} \times 8 = 80$	
24.	$\underline{\hspace{1cm}} \times 8 = 32$	
25.	$\underline{\hspace{1cm}} \times 8 = 24$	
26.	$80 \div 8 =$	
27.	$40 \div 8 =$	
28.	$8 \div 1 =$	
29.	$16 \div 8 =$	
30.	$24 \div 8 =$	
31.	$\underline{\hspace{1cm}} \times 8 = 48$	
32.	$\underline{\hspace{1cm}} \times 8 = 56$	
33.	$\underline{\hspace{1cm}} \times 8 = 72$	
34.	$\underline{\hspace{1cm}} \times 8 = 64$	
35.	$56 \div 8 =$	
36.	$72 \div 8 =$	
37.	$48 \div 8 =$	
38.	$64 \div 8 =$	
39.	$11 \times 8 =$	
40.	$88 \div 8 =$	
41.	$12 \times 8 =$	
42.	$96 \div 8 =$	
43.	$14 \times 8 =$	
44.	$112 \div 8 =$	

B

Number Correct: _____

Improvement: _____

Multiply or divide by 8

1.	$1 \times 8 =$	
2.	$2 \times 8 =$	
3.	$3 \times 8 =$	
4.	$4 \times 8 =$	
5.	$5 \times 8 =$	
6.	$24 \div 8 =$	
7.	$16 \div 8 =$	
8.	$32 \div 8 =$	
9.	$8 \div 1 =$	
10.	$40 \div 8 =$	
11.	$10 \times 8 =$	
12.	$6 \times 8 =$	
13.	$7 \times 8 =$	
14.	$8 \times 8 =$	
15.	$9 \times 8 =$	
16.	$56 \div 8 =$	
17.	$48 \div 8 =$	
18.	$64 \div 8 =$	
19.	$80 \div 8 =$	
20.	$72 \div 8 =$	
21.	_____ $\times 8 = 16$	
22.	_____ $\times 8 = 40$	

23.	_____ $\times 8 = 48$	
24.	_____ $\times 8 = 80$	
25.	_____ $\times 8 = 24$	
26.	$16 \div 8 =$	
27.	$8 \div 1 =$	
28.	$80 \div 8 =$	
29.	$40 \div 8 =$	
30.	$24 \div 8 =$	
31.	_____ $\times 8 = 64$	
32.	_____ $\times 8 = 32$	
33.	_____ $\times 8 = 72$	
34.	_____ $\times 8 = 56$	
35.	$64 \div 8 =$	
36.	$72 \div 8 =$	
37.	$48 \div 8 =$	
38.	$56 \div 8 =$	
39.	$11 \times 8 =$	
40.	$88 \div 8 =$	
41.	$12 \times 8 =$	
42.	$96 \div 8 =$	
43.	$13 \times 8 =$	
44.	$104 \div 8 =$	

Name _____

Date _____

1. a. Skip-count by nine.

9 36 72

_____, _____, _____, _____, _____, _____, _____, _____

- b. Look at the
- tens*
- place in the count-by. What is the pattern?

- c. Look at the
- ones*
- place in the count-by. What is the pattern?

2. Complete to make true statements.

- a. 10 more than 0 is
- 10
- ,

1 less is 9.

$1 \times 9 = \underline{9}$

- f. 10 more than 45 is _____,

1 less is _____.

$6 \times 9 = \underline{\hspace{2cm}}$

- b. 10 more than 9 is
- 19
- ,

1 less is 18.

$2 \times 9 = \underline{\hspace{2cm}}$

- g. 10 more than 54 is _____,

1 less is _____.

$7 \times 9 = \underline{\hspace{2cm}}$

- c. 10 more than 18 is _____,

1 less is _____.

$3 \times 9 = \underline{\hspace{2cm}}$

- h. 10 more than 63 is _____,

1 less is _____.

$8 \times 9 = \underline{\hspace{2cm}}$

- d. 10 more than 27 is _____,

1 less is _____.

$4 \times 9 = \underline{\hspace{2cm}}$

- i. 10 more than 72 is _____,

1 less is _____.

$9 \times 9 = \underline{\hspace{2cm}}$

- e. 10 more than 36 is _____,

1 less is _____.

$5 \times 9 = \underline{\hspace{2cm}}$

- j. 10 more than 81 is _____,

1 less is _____.

$10 \times 9 = \underline{\hspace{2cm}}$

3. a. Analyze the equations in Problem 2. What is the pattern?

b. Use the pattern to find the next 4 facts. Show your work.

$11 \times 9 =$

$12 \times 9 =$

$13 \times 9 =$

$14 \times 9 =$

c. Kent notices another pattern in Problem 2. His work is shown below. He sees the following:

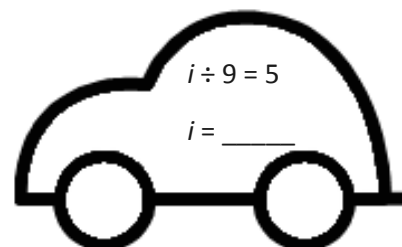
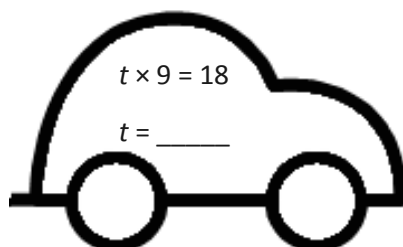
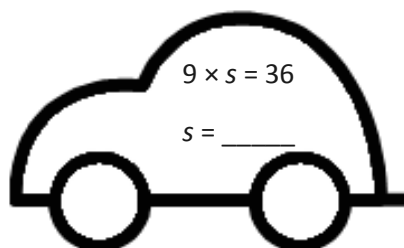
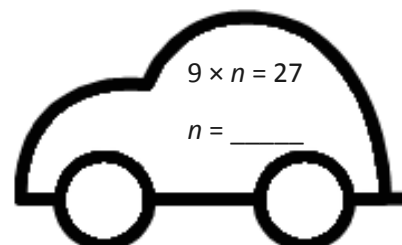
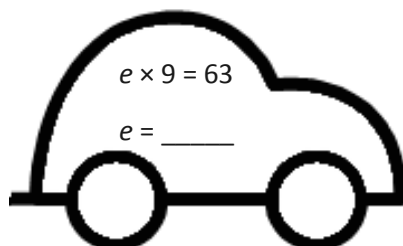
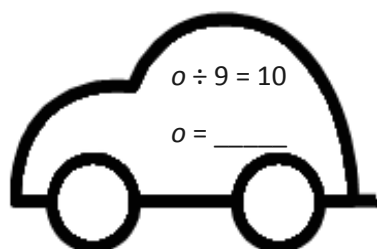
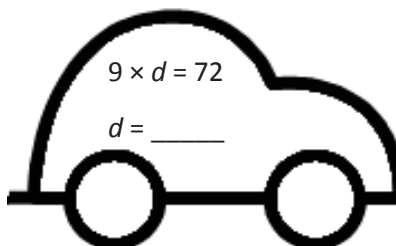
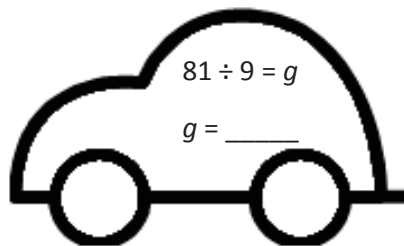
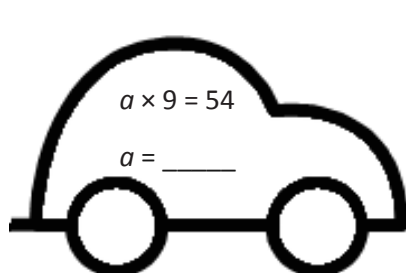
- The tens digit in the product is 1 less than the number of groups.
- The ones digit in the product is 10 minus the number of groups.

		Tens digit	Ones digit
$2 \times 9 = \underline{18}$	\rightarrow	$\underline{1} = 2 - 1$	$\underline{8} = 10 - 2$
$3 \times 9 = \underline{27}$	\rightarrow	$\underline{2} = 3 - 1$	$\underline{7} = 10 - 3$
$4 \times 9 = \underline{36}$	\rightarrow	$\underline{3} = 4 - 1$	$\underline{6} = 10 - 4$
$5 \times 9 = \underline{45}$	\rightarrow	$\underline{4} = 5 - 1$	$\underline{5} = 10 - 5$

Use Kent's strategy to solve 6×9 and 7×9 .

d. Show an example of when Kent's pattern doesn't work.

4. Each equation contains a letter representing the unknown. Find the value of each unknown. Then, write the letters that match the answers to solve the riddle.



How do you make one vanish?

$\frac{6}{6} \frac{8}{8} \frac{8}{8} \frac{6}{6} \frac{9}{9} \frac{6}{6} \frac{3}{3} \frac{8}{8} \frac{45}{45} \frac{2}{2} \frac{4}{4} \frac{9}{9} \frac{90}{90} \frac{3}{3} \frac{7}{7} !$

Name _____

Date _____

1. $6 \times 9 = 54$

$8 \times 9 = 72$

What is 10 more than 54? _____

What is 10 more than 72? _____

What is 1 less? _____

What is 1 less? _____

$7 \times 9 =$ _____

$9 \times 9 =$ _____

2. Explain the pattern used in Problem 1.

Name _____

Date _____

1. a. Skip-count by nines down from 90.

90, _____, 72, _____, _____, _____, 36, _____, _____, _____

- b. Look at the *tens* place in the count-by. What is the pattern?

- c. Look at the *ones* place in the count-by. What is the pattern?

2. Each equation contains a letter representing the unknown. Find the value of each unknown.

$$a \times 9 = 18$$

$$a = \underline{\hspace{2cm}}$$

$$m \div 9 = 3$$

$$m = \underline{\hspace{2cm}}$$

$$e \times 9 = 45$$

$$e = \underline{\hspace{2cm}}$$

$$f \div 9 = 4$$

$$f = \underline{\hspace{2cm}}$$

$$9 \times d = 81$$

$$d = \underline{\hspace{2cm}}$$

$$w \div 9 = 6$$

$$w = \underline{\hspace{2cm}}$$

$$9 \times s = 90$$

$$s = \underline{\hspace{2cm}}$$

$$k \div 9 = 8$$

$$k = \underline{\hspace{2cm}}$$

3. Solve.

a. What is 10 more than 0? ____ b. What is 10 more than 9? ____ c. What is 10 more than 18? ____

What is 1 less? ____

What is 1 less? ____

What is 1 less? ____

$1 \times 9 = \underline{\quad}$

$2 \times 9 = \underline{\quad}$

$3 \times 9 = \underline{\quad}$

d. What is 10 more than 27? ____ e. What is 10 more than 36? ____ f. What is 10 more than 45? ____

What is 1 less? ____

What is 1 less? ____

What is 1 less? ____

$4 \times 9 = \underline{\quad}$

$5 \times 9 = \underline{\quad}$

$6 \times 9 = \underline{\quad}$

g. What is 10 more than 54? ____ h. What is 10 more than 63? ____ i. What is 10 more than 72? ____

What is 1 less? ____

What is 1 less? ____

What is 1 less? ____

$7 \times 9 = \underline{\quad}$

$8 \times 9 = \underline{\quad}$

$9 \times 9 = \underline{\quad}$

j. What is 10 more than 81? ____

What is 1 less? ____

$10 \times 9 = \underline{\quad}$

4. Explain the pattern in Problem 3, and use the pattern to solve the next 3 facts.

$11 \times 9 = \underline{\quad}$

$12 \times 9 = \underline{\quad}$

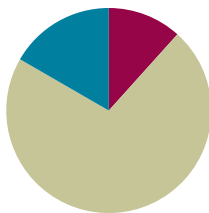
$13 \times 9 = \underline{\quad}$

Lesson 14

Objective: Identify and use arithmetic patterns to multiply.

Suggested Lesson Structure

■ Fluency Practice	(7 minutes)
■ Concept Development	(43 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (7 minutes)

- Multiply By 9 **3.OA.7** (7 minutes)

Multiply By 9 (7 minutes)

Materials: (S) Multiply By 9 (1–5) (Pattern Sheet)

Note: This activity builds fluency with respect to multiplication facts using units of 9. It supports students knowing from memory all products of two one-digit numbers. See Lesson 5 for the directions regarding administering a Multiply By Pattern Sheet.

T: (Write $5 \times 9 = \underline{\quad}$.) Let's skip-count by nines to find the answer. (Count with fingers to 5 as students count and record the count-by sequence on the board.)

S: 9, 18, 27, 36, 45.

T: (Circle 45 and write $5 \times 9 = 45$ above it. Write $3 \times 9 = \underline{\quad}$.) Let's skip-count up by nines again. (Count with fingers to 3 as students count.)

S: 9, 18, 27.

T: (Circle 27 and write $3 \times 9 = 27$ above it.) Let's see how we can skip-count down to find the answer, too. Start at 45 with 5 fingers, 1 for each nine. (Count down with your fingers as students say numbers.)

S: 45 (5 fingers), 36 (4 fingers), 27 (3 fingers).

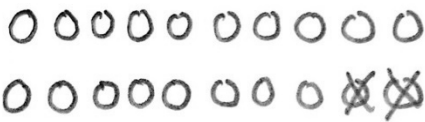
Repeat the process for 4×9 .

T: (Distribute the Multiply By 9 Pattern Sheet.) Let's practice multiplying by 9. Be sure to work left to right across the page.

Concept Development (43 minutes)

Materials: (S) Personal white board

Part 1: Extend the $9 = 10 - 1$ strategy of multiplying with units of 9.

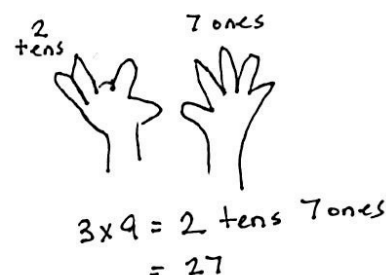
- T: How is the $9 = 10 - 1$ strategy, or add ten, subtract 1, from the last lesson used to solve 2×9 ?
- S: You can do $1 \times 9 = 9$, then add ten and subtract one like this: $(9 + 10) - 1 = 18$.
- T: Let's use this strategy to find 2×9 another way. (Draw a 2×10 array.) When we start with 2×10 , how many tens do we have?
- S: 2 tens.
- T: In unit form, what is the fact we are finding?
- S: 2 nines.
- T: To get 1 nine, we subtract 1 from a ten. In our problem, there are 2 nines, so we need to subtract 2 from our 2 tens. (Cross off 2 from the array, as shown.) When we subtract 2, how many tens are left?
- 
- S: 1 ten.
- T: What happened to the other ten?
- S: We subtracted 2, so now there are 8 left, not 10. → It's not a full ten anymore after we took off 2 ones. → There are just 1 ten and 8 ones.
- T: $2 \times 9 = 18$. Tell your partner how we used the $9 = 10 - 1$ strategy with 2×10 to find 2×9 .
- S: (Explain.)
- T: Let's use the $9 = 10 - 1$ strategy to solve 3×9 . Draw an array for 3×10 . (Allow time for students to draw.) To solve, how many should we subtract?
- S: 3.
- T: Tell your partner why 3.
- S: Because we are trying to find 3 nines. The teacher made 3 tens, and you have to take 1 away from each ten to make it 3 nines. So, you subtract 3.
- T: Cross off 3, and then talk to your partner: How many tens and ones are left in the array?
- S: (Cross off 3.) There are still 2 complete tens but only 7 ones in the third row.
- T: What does our array show is the product of 3×9 ?
- S: 27.
- T: How is the array related to the strategy of using the number of groups, 3, to help you solve 3×9 ?
- S: There are only 2 tens in 27, and $3 - 1 = 2$. There are 7 ones in 27, and $10 - 3 = 7$.
- T: You can use your fingers to quickly solve a nines fact using this strategy. Put your hands out in front of you with all 10 fingers up, like this. (Model, palms facing away.)
- T: Imagine your fingers are numbered 1 through 10 with your pinky on the left being number 1 and your pinky on the right being number 10. Let's count from 1 to 10 together, lowering the finger that matches each number. (Count from 1 to 10 with the class.)

- T: To solve a nines fact, lower the finger that matches the number of nines. Let's try together with 3×9 . Hands out, fingers up!
- T: For 3×9 , which finger matches the number of nines?
- S: My third finger from the left!
- T: Lower that finger. (Model.) How many fingers are to the left of the lowered finger?
- S: 2 fingers!
- T: 2 is the digit in the tens place. How many fingers are to the right of the lowered finger?
- S: 7 fingers!
- T: 7 is the digit in the ones place.
- T: What is the product of 3×9 shown by our fingers?
- S: 27.
- T: Does it match the product we found using our array?
- S: Yes!



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Display a picture of 10 fingers (numbered) on two hands for English language learners and others. Present a demonstration of the finger strategy, or make an animated video. Make it fun for students with rhythm or a song, which the teacher or students compose.



Continue with the following possible sequence: 7×9 , 10×9 , and 11×9 . Use the previous example to discuss with students that the finger strategy is limited to facts where the number of groups is between 1 and 10.

- T: Discuss with your partner. How is the finger strategy we just learned related to the strategy of using the number of groups to help solve a nines fact?
- S: (Discuss.)

Part 2: Apply strategies for solving nines facts and reason about their effectiveness.

Part 2 is intended to be a station-based activity where small groups of students rotate through five stations. At each station, they use a different strategy to solve nines facts. The suggestions below indicate which recently learned strategy students might use to solve nines facts at each station.

Station 1: Use the add 10, subtract 1 strategy to list facts from 1×9 to 10×9 .

Station 2: Use $9 \times n = (10 \times n) - (1 \times n)$, a distributive strategy, to solve facts from 1×9 to 10×9 .

Station 3: Use the finger strategy to solve facts from 1×9 to 10×9 .

Station 4: Use the number of groups to find the digits in the tens and ones places of the product to solve facts from 6×9 to 9×9 .

Station 5: Use $9 \times n = (5 \times n) + (4 \times n)$, a distributive strategy, to solve facts from 6×9 to 9×9 .

MP.5

After finishing, discuss the effectiveness of the strategies used to solve nines facts. Use the following suggested discussion questions:

- Is there a strategy that is easiest for you? What makes it easier than the others?
- What strategy is quickest in helping you solve a nines fact with a large number of groups, such as $12 \times 9 = n$? Which strategies would not work for such a large fact?
- Which strategies could easily be used to solve a division fact?



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

If appropriate for the class, consider having students complete station work in math journals or reflecting in writing so that they have their work as a reference.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Identify and use arithmetic patterns to multiply.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Invite students to explain the strategy used in each problem.
- Encourage students to explain a different strategy that could be used to solve Problem 3.
- Why is it important to know several strategies for solving larger multiplication facts? Which strategies for solving nines facts can be modified to apply to a different set of facts (sixes, sevens, eights, etc.)?

Name: Gina Date: _____

1. Multiply. Then add the tens digit and ones digit of each product.

$9 \times 1 = 9$	$0 + 9 = 9$
$9 \times 2 = 18$	$1 + 8 = 9$
$9 \times 3 = 27$	$2 + 7 = 9$
$9 \times 4 = 36$	$3 + 6 = 9$
$9 \times 5 = 45$	$4 + 5 = 9$
$9 \times 6 = 54$	$5 + 4 = 9$
$9 \times 7 = 63$	$6 + 3 = 9$
$9 \times 8 = 72$	$7 + 2 = 9$
$9 \times 9 = 81$	$8 + 1 = 9$
$9 \times 10 = 90$	$9 + 0 = 9$

b. What was the sum of the digits in each product? How can this strategy help you check your work with the nines facts?

The sum of the digits in each product is 9. So if I add up the digits in my answer and it equals 9, my answer is correct.

c. Araceli continues to count by nines. She writes, "90, 99, 108, 117, 126, 135, 144, 153, 162, 171, 180, 189, 198." Wow! The sum of the digits is still 9! Is she correct? Why or why not?

Araceli is incorrect. She counts by nine correctly, but 189 and 198 do not follow the rule. If you add up the digits, it does not equal 9. So the strategy of the sum of the digits only happens up to 180.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

2. Araceli uses the number of groups in 8×9 to help her find the product. She uses $8 - 1 = 7$ to get the digit in the tens place, and $10 - 8 = 2$ to get the digit in the ones place. Use her strategy to find 4 more facts.

① $5 \times 9 = 45$ $5 - 1 = 4$ ↑ $10 - 5 = 5$	② $6 \times 9 = 54$ $6 - 1 = 5$ ↑ $10 - 6 = 4$	③ $7 \times 9 = 63$ $7 - 1 = 6$ ↑ $10 - 7 = 3$	④ $8 \times 9 = 72$ $8 - 1 = 7$ ↑ $10 - 8 = 2$
--	--	--	--

3. Dennis calculates 9×8 by thinking about it as $80 - 8 = 72$. Explain Dennis' strategy.


Dennis uses $9 = 10 - 1$.

Ten eights = 80. Minus 1 eight is 72.

$80 - 8 = 72$.

$72 = 9 \times 8$

4. Sonya figures out the answer to 7×9 by putting down her right index finger, shown below. What is the answer? Explain how to use Sonya's finger strategy.



Sonya is thinking each finger matches a number from 1 to 10. 1 on the left, 10 on the right. She puts down her 7th finger to match the 7 in 7×9 . Then she sees there are 6 fingers to the left (tens place) and 3 to the right (ones place).

The product = 63.

Multiply.

$9 \times 1 = \underline{\quad\quad}$ $9 \times 2 = \underline{\quad\quad}$ $9 \times 3 = \underline{\quad\quad}$ $9 \times 4 = \underline{\quad\quad}$

$9 \times 5 = \underline{\quad\quad}$ $9 \times 1 = \underline{\quad\quad}$ $9 \times 2 = \underline{\quad\quad}$ $9 \times 1 = \underline{\quad\quad}$

$9 \times 3 = \underline{\quad\quad}$ $9 \times 1 = \underline{\quad\quad}$ $9 \times 4 = \underline{\quad\quad}$ $9 \times 1 = \underline{\quad\quad}$

$9 \times 5 = \underline{\quad\quad}$ $9 \times 1 = \underline{\quad\quad}$ $9 \times 2 = \underline{\quad\quad}$ $9 \times 3 = \underline{\quad\quad}$

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multiply by 9 (1–5)

Name _____

Date _____

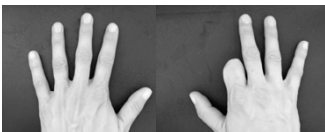
1. a. Multiply. Then, add the tens digit and ones digit of each product.

$1 \times 9 = 9$	$\underline{0} + \underline{9} = \underline{9}$
$2 \times 9 = 18$	$\underline{1} + \underline{8} = \underline{\quad}$
$3 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$4 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$5 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$6 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$7 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$8 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$9 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$10 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$

- b. What is the sum of the digits in each product? How can this strategy help you check your work with the nines facts?

- c. Araceli continues to count by nines. She writes, “90, 99, 108, 117, 126, 135, 144, 153, 162, 171, 180, 189, 198. Wow! The sum of the digits is still 9.” Is she correct? Why or why not?

2. Araceli uses the number of groups in 8×9 to help her find the product. She uses $8 - 1 = 7$ to get the digit in the tens place and $10 - 8 = 2$ to get the digit in the ones place. Use her strategy to find 4 more facts.
3. Dennis calculates 9×8 by thinking about it as $80 - 8 = 72$. Explain Dennis' strategy.
4. Sonya figures out the answer to 7×9 by putting down her right index finger (shown). What is the answer? Explain how to use Sonya's finger strategy.



Name _____ Date _____

Donald writes $6 \times 9 = 54$. Explain two strategies you could use to check his work.

Name _____

Date _____

1. a. Multiply. Then, add the digits in each product.

$10 \times 9 = 90$	$\underline{9} + \underline{0} = \underline{9}$
$9 \times 9 = 81$	$\underline{8} + \underline{1} = \underline{9}$
$8 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$7 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$6 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$5 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$4 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$3 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$2 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$
$1 \times 9 =$	$\underline{\quad} + \underline{\quad} = \underline{\quad}$

- b. What pattern did you notice in Problem 1(a)? How can this strategy help you check your work with nines facts?

2. Thomas calculates 9×7 by thinking about it as $70 - 7 = 63$. Explain Thomas' strategy.

-
3. Alexia figures out the answer to 6×9 by lowering the thumb on her right hand (shown). What is the answer? Explain Alexia's strategy.



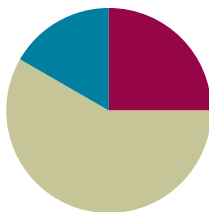
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4. Travis writes $72 = 9 \times 8$. Is he correct? Explain at least 2 strategies Travis can use to check his work.

Lesson 15

Objective: Interpret the unknown in multiplication and division to model and solve problems.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Concept Development	(35 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Multiply By 9 **3.OA.7** (7 minutes)
- Group Counting **3.OA.1** (4 minutes)
- Divide by 9 **3.OA.7** (4 minutes)

Multiply By 9 (7 minutes)

Materials: (S) Multiply By 9 (6–10) (Pattern Sheet)

Note: This activity builds fluency with respect to multiplication facts using units of 9. It supports students knowing from memory all products of two one-digit numbers. See Lesson 5 for the directions regarding administering a Multiply By Pattern Sheet.

T: (Write $6 \times 9 = \underline{\quad}$.) Let's skip-count up by nine to solve. (Count with fingers to 6 as students count.)

S: 9, 18, 27, 36, 45, 54.

T: Let's skip-count down to find the answer, too. Start at 90. (Starting with 10 fingers, count down as students count.)

S: 90, 81, 72, 63, 54.

T: Let's skip-count up again to find the answer, but this time start at 45. (Starting with 5 fingers, count up as students count.)

S: 45, 54.

Continue with the following possible suggestions: 8×9 , 7×9 , and 9×9 .

T: (Distribute the Multiply By 9 Pattern Sheet.) Let's practice multiplying by 9. Be sure to work left to right across the page.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes, sevens, and eights reviews multiplication taught previously in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80

Divide by 9 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews using a letter to represent the unknown, which was first taught in Lesson 3.

T: (Write $a \times 9 = 18$.) On your personal white board, write the value of a .

S: (Write $a = 2$.)

T: (Write $18 \div 9 = \underline{\hspace{1cm}}$.) Say the division sentence.

S: $18 \div 9 = 2$.

Continue with the following possible sequence: $b \times 9 = 45$, $c \times 9 = 36$, $d \times 9 = 54$, $e \times 9 = 27$, $f \times 9 = 90$, $g \times 9 = 81$, and $h \times 9 = 72$.

Concept Development (35 minutes)

Materials: (S) Personal white board

Problem 1: Interpret the unknown in multiplication.

Write or project the following problem: Ada buys 9 packs of highlighters with 4 in each pack. After giving 1 highlighter to each classmate, she has 17 left. How many highlighters does Ada give away?

T: Model the problem. Then, tell your partner the steps you'll follow to solve it.

S: (Model.) First, you have to find out how many highlighters Ada has. → After that, subtract 17 from the total to see how many she gives away.

T: Write and solve an equation to find the total number of highlighters. Use h to represent the unknown. (Allow students time to finish their work.)

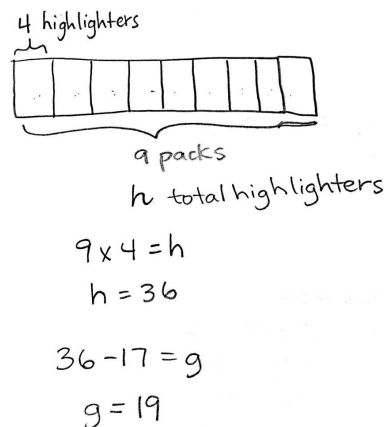
T: What equation did you use?

S: $9 \times 4 = h$.

**NOTES ON
MULTIPLE MEANS
OF ENGAGEMENT:**

Students working above grade level and others may not identify a strategy for solving $9 \times 4 = 36$, instead saying, "It's easy! I just knew it." Challenge students to articulate strategies that work so well they seem automatic. Ask, "Did you count by fours? Did you switch the factors and calculate 4×9 instead? Did you use a pattern to solve? If so, what pattern?"

- T: Talk to your partner. What strategy for solving nines facts did you use?
- S: (Discuss.)
- T: What is the value of h ?
- S: The value of h is 36.
- T: How many highlighters does Ada have at the start?
- S: 36 highlighters.
- T: Is the problem complete yet?
- S: No, we have to find how many highlighters Ada gives away.
- T: Solve the second step of the problem using the letter g to represent the unknown.
- S: (Write $36 - 17 = g$ and solve.)
- T: What is the value of g ?
- S: The value of g is 19.
- T: How many highlighters does Ada give away?
- S: Ada gives away 19 highlighters.
- T: Can we tell how many classmates Ada has? How do you know?
- S: Ada has 19 classmates! I know because she gave 1 highlighter to each classmate, and she gave away 19 highlighters.



Problem 2: Interpret the unknown in division.

Write the following problem: Eliza finds a bag of 72 marbles and runs to share them with 8 of her friends. She's so excited that she drops the bag and loses 18 marbles. How many marbles will Eliza and each of her friends get?

- T: What should we do first, subtract or divide? Why?
- S: We should subtract. → Subtract because we need to find out how many marbles Eliza has left after she loses some.
- T: Write an equation to solve for the first unknown. Use m to represent the number of marbles Eliza has.
- S: (Solve $72 - 18 = m$.)
- T: What is the value of m ?
- S: 54.
- T: So, how many marbles does Eliza have to give away?
- S: 54 marbles!
- T: Is our work on this problem finished?
- S: No! Now we have to find out how many marbles each friend gets!
- T: Draw a model that represents how many marbles each friend gets.



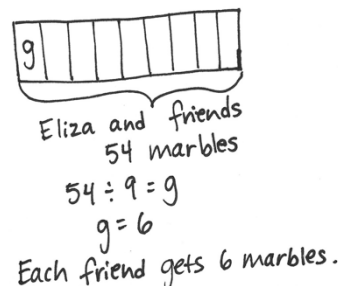
NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Some students may mistakenly interpret the divisor to be 8 rather than 9. Use this example to highlight the advantages of carefully reading word problems and drawing a picture to solve. Guide students to underline *Eliza and each of her friends* in the final question before attempting an equation.

- T: (After finishing.) How many units are in your model?
- S: 9 units.
- T: What part of the problem tells you that your model needs 9 units?
- S: Where it says, "Eliza and each of her friends." She has 8 friends, and $8 + 1 = 9$.
- T: Write an equation to solve for the second unknown. Use g to represent the number of marbles each friend gets.
- S: (Solve $54 \div 9 = g$.)
- T: How many marbles does each friend get? How do you know?
- S: Each friend gets 6 marbles. \rightarrow 6 marbles because the value of g is 6, and g represents the number of marbles each friend gets.

$$72 - 18 = m$$

$$m = 54$$



Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Interpret the unknown in multiplication and division to model and solve problems.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In your model for Problem 1, is the unknown the number of units or the size of each unit?
- In Problem 3, how did you show what letter you used to represent the unknown and what it stood for?
- How did you solve the large division fact in Problem 4?
- What longer equation, including parentheses, can be used to solve Problem 6?

Name: Gina Date: _____

Write an equation, and use a letter to represent the unknown for Problems 1–6.

1. Mrs. Parson gave each of her grandchildren \$9. She gave a total of \$36. How many grandchildren does Mrs. Parson have?

$$36 \div 9 = g$$

$$g = 4$$

Mrs. Parson has 4 grandchildren.

2. Shiva pours 27 liters of water equally into 9 containers. How many liters of water are in each container?

$$27 \div 9 = c$$

$$c = 3$$

There are 3 liters in each container.

3. Derek cuts 7 pieces of wire. Each piece is 9 meters long. What is the total length of the 7 pieces?

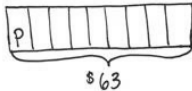
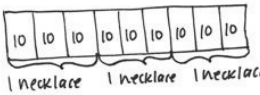
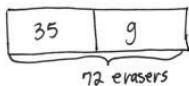
$$7 \times 9 = w$$

$$w = 63$$

The total length of 7 pieces of wire is 63 meters.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

4. Aunt Deena and Uncle Chris share the cost of a limousine ride with their 7 friends. The ride cost a total of \$63. If everyone shares the cost equally, how much does each person pay?
- Aunt Deena + Uncle Chris + 7 friends is 9
- 
- $$63 \div 9 = p$$
- $$p = 7$$
- Each person pays \$7.
5. Cara bought 9 packs of beads. There are 10 beads in each pack. She always uses 30 beads to make each necklace. How many necklaces can she make if she uses all the beads?
- $$9 \times 10 = 90$$
- She buys 90 beads.
- 
- Cara can make 3 necklaces.
6. There are 8 erasers in a set. Damon buys 9 sets. After giving some erasers away, Damon has 35 erasers left. How many erasers did he give away?
- $$9 \times 8 = 72$$
- Damon buys 72 erasers.
- 
- $$72 - 35 = g$$
- $$g = 37$$
- Damon gave away 37 erasers.

Multiply.

$9 \times 1 = \underline{\quad\quad}$ $9 \times 2 = \underline{\quad\quad}$ $9 \times 3 = \underline{\quad\quad}$ $9 \times 4 = \underline{\quad\quad}$

$9 \times 5 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$ $9 \times 8 = \underline{\quad\quad}$

$9 \times 9 = \underline{\quad\quad}$ $9 \times 10 = \underline{\quad\quad}$ $9 \times 5 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$

$9 \times 5 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$ $9 \times 5 = \underline{\quad\quad}$ $9 \times 8 = \underline{\quad\quad}$

$9 \times 5 = \underline{\quad\quad}$ $9 \times 9 = \underline{\quad\quad}$ $9 \times 5 = \underline{\quad\quad}$ $9 \times 10 = \underline{\quad\quad}$

$9 \times 6 = \underline{\quad\quad}$ $9 \times 5 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$

$9 \times 6 = \underline{\quad\quad}$ $9 \times 8 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$ $9 \times 9 = \underline{\quad\quad}$

$9 \times 6 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$

$9 \times 8 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$ $9 \times 9 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$

$9 \times 8 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$ $9 \times 8 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$

$9 \times 8 = \underline{\quad\quad}$ $9 \times 9 = \underline{\quad\quad}$ $9 \times 9 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$

$9 \times 9 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$ $9 \times 9 = \underline{\quad\quad}$ $9 \times 8 = \underline{\quad\quad}$

$9 \times 9 = \underline{\quad\quad}$ $9 \times 8 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$ $9 \times 9 = \underline{\quad\quad}$

$9 \times 7 = \underline{\quad\quad}$ $9 \times 9 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$ $9 \times 8 = \underline{\quad\quad}$

$9 \times 9 = \underline{\quad\quad}$ $9 \times 7 = \underline{\quad\quad}$ $9 \times 6 = \underline{\quad\quad}$ $9 \times 8 = \underline{\quad\quad}$

multiply by 9 (6–10)

Name _____

Date _____

Write an equation, and use a letter to represent the unknown for Problems 1–6.

1. Mrs. Parson gave each of her grandchildren \$9. She gave a total of \$36. How many grandchildren does Mrs. Parson have?
2. Shiva pours 27 liters of water equally into 9 containers. How many liters of water are in each container?
3. Derek cuts 7 pieces of wire. Each piece is 9 meters long. What is the total length of the 7 pieces?

Name _____

Date _____

Use a letter to represent the unknown.

1. Mrs. Aquino pours 36 liters of water equally into 9 containers. How much water is in each container?
2. Marlon buys 9 packs of hot dogs. There are 6 hot dogs in each pack. After the barbeque, 35 hot dogs are left over. How many hot dogs were eaten?

Name _____

Date _____

1. The store clerk equally divides 36 apples among 9 baskets. Draw a tape diagram, and label the number of apples in each basket as a . Write an equation, and solve for a .
2. Elijah gives each of his friends a pack of 9 almonds. He gives away a total of 45 almonds. How many packs of almonds did he give away? Model using a letter to represent the unknown, and then solve.
3. Denice buys 7 movies. Each movie costs \$9. What is the total cost of 7 movies? Use a letter to represent the unknown. Solve.



Topic E

Analysis of Patterns and Problem Solving Including Units of 0 and 1

3.OA.3, 3.OA.7, 3.OA.8, 3.OA.9, 3.OA.1, 3.OA.2, 3.OA.4, 3.OA.6

Focus Standards:	3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
	3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
	3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)
	3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i>
Instructional Days:	3	
Coherence -Links from:	G2–M6	Foundations of Multiplication and Division
	G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
-Links to:	G3–M4	Multiplication and Area
	G4–M3	Multi-Digit Multiplication and Division
	G4–M7	Exploring Measurement with Multiplication

In Lesson 16, students multiply and divide by 0 and 1. They use patterns to understand that $n \times 0 = 0$ and show why the result of dividing a number by 0 is undefined but that dividing 0 by another number results in 0. Lesson 17 synthesizes students' knowledge of factors from 0 to 10 in an exploration of patterns using the multiplication table. Students recognize the patterns of particular factors and make connections between multiplication and division.

In Lesson 18, students apply the tools, representations, and concepts they have learned in order to solve two-step word problems using all four operations. They call on rounding skills learned in Module 2 to estimate solutions and use their estimations to assess the reasonableness of answers.

A Teaching Sequence Toward Mastery of Analysis of Patterns and Problem Solving Including Units of 0 and 1

Objective 1: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division.
(Lesson 16)

Objective 2: Identify patterns in multiplication and division facts using the multiplication table.
(Lesson 17)

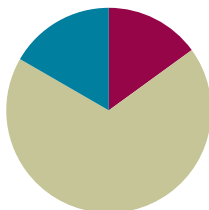
Objective 3: Solve two-step word problems involving all four operations and assess the reasonableness of solutions.
(Lesson 18)

Lesson 16

Objective: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division.

Suggested Lesson Structure

■ Fluency Practice	(9 minutes)
■ Concept Development	(41 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (9 minutes)

- Sprint: Multiply or Divide By 9 **3.OA.7** (9 minutes)

Sprint: Multiply or Divide by 9 (9 minutes)

Materials: (S) Multiply or Divide By 9 Sprint

Note: This Sprint reviews Lessons 12–15, focusing on the relationship between multiplication and division using units of 9.

Concept Development (41 minutes)

Materials: (S) Personal white board

Problem 1: Multiply and divide using units of 1.

- T: Draw three large circles on your personal white board. Draw an equal number of dots in each circle. You can draw between 2 and 10 dots in each circle. You choose! How many groups are there?
- S: 3.
- T: Write a multiplication equation to represent your picture. Read your equation to a partner.
- T: Erase one of the circles. How many groups now?
- S: 2.



NOTES ON STANDARDS ALIGNMENT:

The last six problems on the Sprint extend beyond Grade 3 multiplication and division standards because students who have mastered their times tables are likely to otherwise go unchallenged during this activity. By extending to products above 90 and quotients above 10, strong students are provided the stimulus to stretch their conceptual understanding, which may keep them engaged and invigorated to improve on Sprint B.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

One group is often challenging for students to represent because they tend to interpret it as the number in the set. The opening activity starts with 3 groups and works back to 1 group to avoid this misconception. Stay alert to it while circulating, and support students with their work in Problem 1.

- T: Write a multiplication equation to represent your picture. Read your equation to a partner.
- T: Erase one of the circles. How many groups now?
- S: 1.
- T: Write a multiplication equation to represent your picture. Read your equation to me. (Call on students to read equations and record.)
- T: Rewrite your equation. Let n equal the number of dots in each group. What is 1 times n dots?
- S: It's n because the number of dots in each group is the same as the total number of dots.
- T: What is 1 times a number equal to?
- S: That number!
- T: Write the related division fact for your multiplication equation. Read it at my signal. (Signal.)
- S: $n \div 1 = n$.
- T: Use your picture to discuss with a partner. Why is our division equation true?
- S: It shows the total number of dots, n , divided into 1 group. That equals n counters in each group.
- T: What is a number divided by 1?
- S: That number!

Repeat this process, drawing n circles with 3 dots in each circle. Students erase 1 dot from each circle and write multiplication equations to represent their pictures until they are left with n circles and 1 dot in each circle. This demonstrates $n \times 1 = n$ and $n \div n = 1$.

- T: What patterns did we discover for multiplying and dividing by units of 1?
- S: Any number times 1 equals that number, any number divided by 1 equals that number, and any number divided by itself equals 1.

Problem 2: Multiply and divide using units of 0.

- T: (Write $4 \times 0 = b$ on the board.) What does this equation represent?
- S: Four groups of 0.
- T: Draw a picture of the equation using circles to show the groups and dots to show the number in each group. (Allow students time to draw.) How many dots did you draw in each group?
- S: Zero!



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

English language learners and others may benefit from a scaffolded review of unknowns represented as letters. Start with a frame using blanks ($_\times 1 = _\$), then question marks ($? \times 1 = ?$), then *a number* $\times 1 = a$ number, and finally, $n \times 1 = n$.



$$3 \times 4 = 12$$



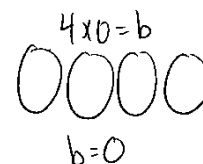
$$2 \times 4 = 8$$



$$1 \times 4 = 4$$

$$1 \times n = n$$

$$n \div 1 = n$$



T: There are a total of 0 dots, so 4 times 0 equals?

S: Zero!

T: What is the value of b in the equation?

S: b equals 0.

Continue with the following possible suggestions: 7×0 , 6×0 , and 0×0 .

T: What pattern did you notice?

S: Any number times 0 equals 0.

T: Write that equation using n to represent a number.

S: $n \times 0 = 0$.

T: Write the related division equation on your board.

S: $0 \div n = 0$.

T: What does this equation represent?

S: Zero divided by a number equals 0.

T: Let's choose a value for n and see if we get a true equation. Rewrite the equation, letting n equal 7. (After students write $0 \div 7 = 0$.) What does this equation represent?

S: Zero things divided into 7 groups equals 0.

T: Draw a picture of the equation using circles to show the groups and dots to show the number in each group. (Allow students time to draw.) How many dots did you draw in each group?

S: Zero!

T: Zero divided by 7 equals?

S: Zero!

T: Rewrite the equation to show 7 things divided into 0 groups equals n .

S: (Write $7 \div 0 = n$.)

T: What is the related multiplication fact?

S: $0 \times n = 7$.

T: What does this equation represent?

S: Zero times a number equals 7.

T: Talk with your partner; is this possible?

S: No, because any number times 0 equals 0, not 7.

T: There's no value for n that would make a true multiplication sentence, and the same is true for the division equation.

T: Let's look at a special case of dividing by 0. Write $0 \div 0 = n$ on your board. What is the related multiplication fact?

S: $0 \times n = 0$.

T: What does this equation represent?

S: Zero times a number equals 0.

T: Talk with a partner; what is the value of n ?

S: Any number! $\rightarrow n$ can be any number because when you multiply any number times 0, it equals 0.

$$n \times 0 = 0$$

$$0 \div n = 0$$

$$0 \div 7 = 0$$



$$7 \div 0 = n$$

$$0 \times n = 7$$

$$0 \div 0 = n$$

$$0 \times n = 0$$

$$n = \text{any number}$$

MP.7

MP.7

- T: n could be 3, 2, 5, 6, or any other number. n can be any number in the multiplication equation, and the same is true for the division equation. Work with your partner to try a few different numbers in the multiplication and division equations.
- S: (Plug in a variety of values.)
- T: What do you notice?
- S: Lots of numbers work!
- T: Right, there isn't one single value for n in this case. Talk with a partner about what patterns you discovered for dividing by 0.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

MP.7

- Discuss with a partner, what patterns for multiplying and dividing by 0 and 1 helped you solve Problem 1?
- What pattern for multiplying by 1 does Problem 3 represent?
- Which problems show that we can't define a single specific value when we divide by 0? Explain your answer to a partner.
- How are multiplying by 1 and multiplying units of 1 similar to adding 0 to a number?
- How can the patterns for multiplying and dividing by 1 or multiplying and dividing 0 by a number help you solve equations with larger factors (e.g., $346 \times 1 = b$)?

Name Gina Date _____

1. Complete.

a. 6 $\times 1 = 6$ b. 0 $\div 7 = 0$ c. $8 \times$ 1 $= 8$ d. $9 \div$ 1 $= 9$

e. $0 \div 5 =$ 0 f. 5 $\times 0 = 0$ g. $4 \div$ 4 $= 1$ h. 3 $\times 1 = 3$

2. Match each equation with its solution.

3. Let n be a number. Complete the blanks below with the products.

1	2	3	4	5	6	7	8	9	...	n
$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 1$		$\times 1$
1	2	3	4	5	6	7	8	9		n

What pattern do you notice?
The products grow by 1 each time. Also, the product is the same as the factor that's not 1. Any number $\times 1 =$ that number.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

4. Josie says that any number divided by 1 equals that number.

a. Write a division equation using n to represent Josie's statement.

$$n \div 1 = n$$

b. Use your equation from Part (a). Let $n = 6$. Write a new equation and draw a picture to show that your equation is true.

$$6 \div 1 = 6$$



c. Write the related multiplication equation that you can use to check your division equation.

$$1 \times 6 = 6$$

5. Matt explains what he learned about dividing with zero to his little sister.

a. What might Matt tell his sister about solving $0 \div 9$? Explain your answer.

He will tell her that it equals 0 because 0 divided by any number equals 0.

b. What might Matt tell his sister about solving $8 \div 0$? Explain your answer.

If you think about the related multiplication fact, there is no number that makes it true, and that's the same for the division fact.

c. What might Matt tell his sister about solving $0 \div 0$? Explain your answer.

The answer could be any number! So you can't really know what the answer is.

A

Number Correct: _____

Multiply or divide by 9

1.	$2 \times 9 =$	
2.	$3 \times 9 =$	
3.	$4 \times 9 =$	
4.	$5 \times 9 =$	
5.	$1 \times 9 =$	
6.	$18 \div 9 =$	
7.	$27 \div 9 =$	
8.	$45 \div 9 =$	
9.	$9 \div 9 =$	
10.	$36 \div 9 =$	
11.	$6 \times 9 =$	
12.	$7 \times 9 =$	
13.	$8 \times 9 =$	
14.	$9 \times 9 =$	
15.	$10 \times 9 =$	
16.	$72 \div 9 =$	
17.	$63 \div 9 =$	
18.	$81 \div 9 =$	
19.	$54 \div 9 =$	
20.	$90 \div 9 =$	
21.	$\underline{\hspace{1cm}} \times 9 = 45$	
22.	$\underline{\hspace{1cm}} \times 9 = 9$	

23.	$\underline{\hspace{1cm}} \times 9 = 90$	
24.	$\underline{\hspace{1cm}} \times 9 = 18$	
25.	$\underline{\hspace{1cm}} \times 9 = 27$	
26.	$90 \div 9 =$	
27.	$45 \div 9 =$	
28.	$9 \div 9 =$	
29.	$18 \div 9 =$	
30.	$27 \div 9 =$	
31.	$\underline{\hspace{1cm}} \times 9 = 54$	
32.	$\underline{\hspace{1cm}} \times 9 = 63$	
33.	$\underline{\hspace{1cm}} \times 9 = 81$	
34.	$\underline{\hspace{1cm}} \times 9 = 72$	
35.	$63 \div 9 =$	
36.	$81 \div 9 =$	
37.	$54 \div 9 =$	
38.	$72 \div 9 =$	
39.	$11 \times 9 =$	
40.	$99 \div 9 =$	
41.	$12 \times 9 =$	
42.	$108 \div 9 =$	
43.	$14 \times 9 =$	
44.	$126 \div 9 =$	

B

Number Correct: _____

Improvement: _____

Multiply or divide by 9

1.	$1 \times 9 =$	
2.	$2 \times 9 =$	
3.	$3 \times 9 =$	
4.	$4 \times 9 =$	
5.	$5 \times 9 =$	
6.	$27 \div 9 =$	
7.	$18 \div 9 =$	
8.	$36 \div 9 =$	
9.	$9 \div 9 =$	
10.	$45 \div 9 =$	
11.	$10 \times 9 =$	
12.	$6 \times 9 =$	
13.	$7 \times 9 =$	
14.	$8 \times 9 =$	
15.	$9 \times 9 =$	
16.	$63 \div 9 =$	
17.	$54 \div 9 =$	
18.	$72 \div 9 =$	
19.	$90 \div 9 =$	
20.	$81 \div 9 =$	
21.	$\underline{\hspace{1cm}} \times 9 = 9$	
22.	$\underline{\hspace{1cm}} \times 9 = 45$	

23.	$\underline{\hspace{1cm}} \times 9 = 18$	
24.	$\underline{\hspace{1cm}} \times 9 = 90$	
25.	$\underline{\hspace{1cm}} \times 9 = 27$	
26.	$18 \div 9 =$	
27.	$9 \div 9 =$	
28.	$90 \div 9 =$	
29.	$45 \div 9 =$	
30.	$27 \div 9 =$	
31.	$\underline{\hspace{1cm}} \times 9 = 27$	
32.	$\underline{\hspace{1cm}} \times 9 = 36$	
33.	$\underline{\hspace{1cm}} \times 9 = 81$	
34.	$\underline{\hspace{1cm}} \times 9 = 63$	
35.	$72 \div 9 =$	
36.	$81 \div 9 =$	
37.	$54 \div 9 =$	
38.	$63 \div 9 =$	
39.	$11 \times 9 =$	
40.	$99 \div 9 =$	
41.	$12 \times 9 =$	
42.	$108 \div 9 =$	
43.	$13 \times 9 =$	
44.	$117 \div 9 =$	

Name _____

Date _____

1. Complete.

a. $\underline{\hspace{1cm}} \times 1 = 6$

b. $\underline{\hspace{1cm}} \div 7 = 0$

c. $8 \times \underline{\hspace{1cm}} = 8$

d. $9 \div \underline{\hspace{1cm}} = 9$

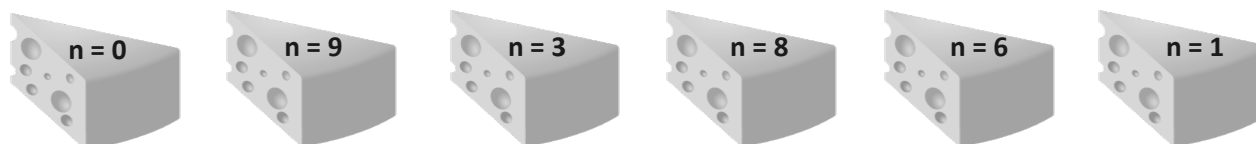
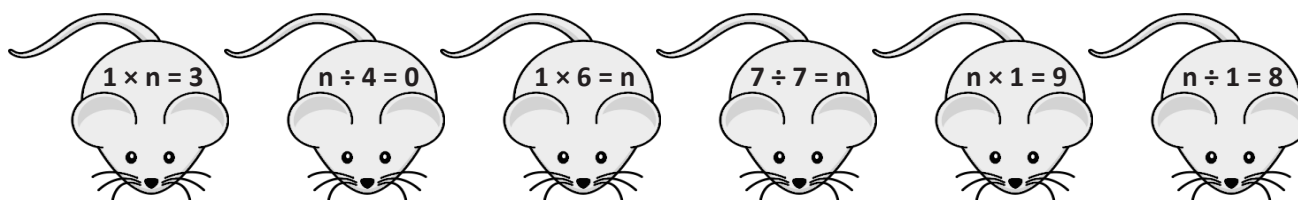
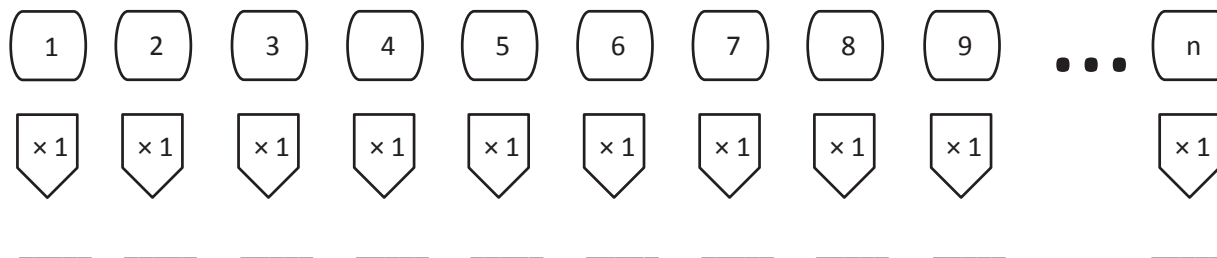
e. $0 \div 5 = \underline{\hspace{1cm}}$

f. $\underline{\hspace{1cm}} \times 0 = 0$

g. $4 \div \underline{\hspace{1cm}} = 1$

h. $\underline{\hspace{1cm}} \times 1 = 3$

2. Match each equation with its solution.

3. Let n be a number. Complete the blanks below with the products.

What pattern do you notice?

4. Josie says that any number divided by 1 equals that number.
- Write a division equation using n to represent Josie's statement.
 - Use your equation from Part (a). Let $n = 6$. Write a new equation, and draw a picture to show that your equation is true.
 - Write the related multiplication equation that you can use to check your division equation.
5. Matt explains what he learned about dividing with zero to his little sister.
- What might Matt tell his sister about solving $0 \div 9$? Explain your answer.
 - What might Matt tell his sister about solving $8 \div 0$? Explain your answer.
 - What might Matt tell his sister about solving $0 \div 0$? Explain your answer.

Name _____

Date _____

1. Complete.

a. _____ \times 1 = 5

b. 6 \times _____ = 6

c. _____ \div 7 = 0

d. 5 \times _____ = 0

e. 1 = 9 \div _____

f. 8 = 1 \times _____

2. Luis divides 8 by 0 and says it equals 0. Is he correct? Explain why or why not.

Name _____

Date _____

1. Complete.

a. $4 \times 1 = \underline{\quad}$

b. $4 \times 0 = \underline{\quad}$

c. $\underline{\quad} \times 1 = 5$

d. $\underline{\quad} \div 5 = 0$

e. $6 \times \underline{\quad} = 6$

f. $\underline{\quad} \div 6 = 0$

g. $0 \div 7 = \underline{\quad}$

h. $7 \times \underline{\quad} = 0$

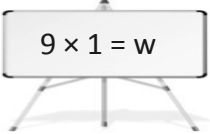
i. $8 \div \underline{\quad} = 8$

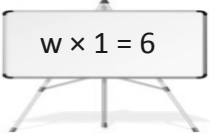
j. $\underline{\quad} \times 8 = 8$

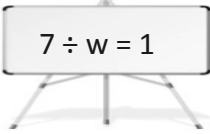
k. $9 \times \underline{\quad} = 9$

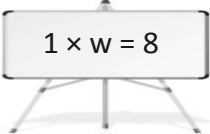
l. $9 \div \underline{\quad} = 1$

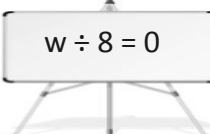
2. Match each equation with its solution.

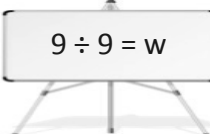
 $9 \times 1 = w$

 $w \times 1 = 6$

 $7 \div w = 1$

 $1 \times w = 8$

 $w \div 8 = 0$

 $9 \div 9 = w$

 $w = 6$

 $w = 7$

 $w = 8$

 $w = 9$

 $w = 1$

 $w = 0$

3. Let $c = 8$. Determine whether the equations are true or false. The first one has been done for you.

a. $c \times 0 = 8$	<i>False</i>
b. $0 \times c = 0$	
c. $c \times 1 = 8$	
d. $1 \times c = 8$	
e. $0 \div c = 8$	
f. $8 \div c = 1$	
g. $0 \div c = 0$	
h. $c \div 0 = 8$	

4. Rajan says that any number multiplied by 1 equals that number.

a. Write a multiplication equation using n to represent Rajan's statement.

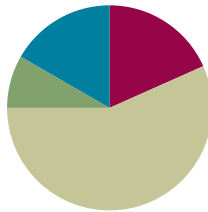
b. Using your equation from Part (a), let $n = 5$, and draw a picture to show that the new equation is true.

Lesson 17

Objective: Identify patterns in multiplication and division facts using the multiplication table.

Suggested Lesson Structure

■ Fluency Practice	(11 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(34 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (11 minutes)

- Multiply By 10 **3.NBT.3** (3 minutes)
- Multiply or Divide **3.OA.7** (4 minutes)
- Complete the Number Sentence **3.OA.5** (4 minutes)

Multiply with 10 (3 minutes)

Note: This fluency activity anticipates Lesson 19, which involves multiplying by multiples of 10 using the place value chart.

T: I'll say a fact. You say the whole equation. 10×1 .

S: $10 \times 1 = 10$.

Continue with the following possible sequence: 10×2 , 10×3 , 10×8 , and 10×5 .

T: I'll say a product that is a multiple of 10. You say the multiplication fact starting with 10. 20.

S: $10 \times 2 = 20$.

Continue with the following possible sequence: 30, 40, 90, 50, and 10.

Multiply or Divide (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews multiplication and division facts within 100.

T: (Write $6 \times 1 = \underline{\quad}$.) Say the multiplication sentence.

S: $6 \times 1 = 6$.

Continue with the following possible sequence: 6×2 and 6×3 .

T: On your personal white board, show the answer to 6×7 . If you need to, skip-count.

S: (Write 42.)

Continue with the following possible sequence, asking students to write answers to the harder problems on their personal white boards, while asking them to orally answer the easier problems: $30 \div 6$, $24 \div 6$, $60 \div 6$, $54 \div 6$, 7×1 , 7×2 , 7×3 , 7×8 , $35 \div 7$, $28 \div 7$, $70 \div 7$, $63 \div 7$, $49 \div 7$, 8×1 , 8×2 , 8×3 , 8×9 , $40 \div 8$, $48 \div 8$, $32 \div 8$, $80 \div 8$, $64 \div 8$, 9×1 , 9×2 , 9×3 , 9×8 , $45 \div 9$, $36 \div 9$, $54 \div 9$, $90 \div 9$, $81 \div 9$, and $63 \div 9$.

Complete the Number Sentence (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews multiplication and division using units of 0 and 1.

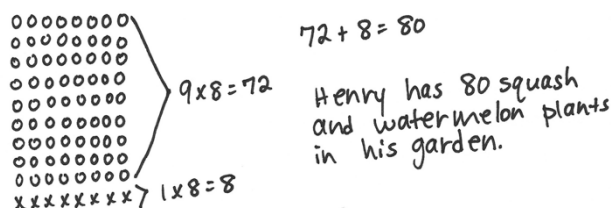
T: (Write $\underline{\quad} \times 1 = 6$.) On your personal white board, complete the equation.

S: (Write $6 \times 1 = 6$.)

Continue with the following possible sequence: $\underline{\quad} \times 1 = 7$, $9 \times \underline{\quad} = 9$, $8 \times \underline{\quad} = 8$, $7 \div \underline{\quad} = 7$, $9 \div \underline{\quad} = 9$, $7 \div \underline{\quad} = 1$, $9 \div \underline{\quad} = 1$, $8 \times \underline{\quad} = 0$, $6 \times \underline{\quad} = 0$, $0 \div 7 = \underline{\quad}$, $0 \div 9 = \underline{\quad}$, $\underline{\quad} \div 8 = 0$, $\underline{\quad} \div 6 = 0$, $\underline{\quad} \times 1 = 8$, $7 \times \underline{\quad} = 7$, $6 \div \underline{\quad} = 6$, $9 \times \underline{\quad} = 0$, $6 \div \underline{\quad} = 1$, $0 \div 6 = \underline{\quad}$, $\underline{\quad} \div 9 = 0$, and $9 \div \underline{\quad} = 1$.

Application Problem (5 minutes)

Henry's garden has 9 rows of squash plants. Each row has 8 squash plants. There is also 1 row with 8 watermelon plants. How many squash and watermelon plants does Henry have in all?



Note: This problem reviews multiplying by units of 9 and 1. Depending on how students choose to solve the problem, it can be used to review multiplying with units of 10 (e.g., 10×8). Be sure to discuss the various strategies that can be used to solve this problem.

Concept Development (34 minutes)

Materials: (S) Personal white board, Problem Set, orange crayon

Problems 1(a) and 1(b)

- T: Write the products to complete the table in Problem 1. Then, color all the squares that have even products orange.
- T: Let's look at the first orange square in the table. Write the multiplication equation on your board for the product in this square. (Students write.) Are the factors 2 and 1 odd or even?
- S: 2 is even and 1 is odd.
- T: Look at the orange square below this one. Write the multiplication equation on your board for the product in this square. (Students write.) Are the factors 2 and 2 odd or even?
- S: They're both even!
- T: Work with a partner to continue to look at the orange squares, and tell if the factors are odd or even. (Students finish working.) What did you notice about the factors of even products?
- S: The factors are either both even or one is odd and one is even.
- T: (Write the following.) Even times even equals even. Odd times even equals even.
- T: Work with a partner to find out what kinds of factors are required to produce an odd product. (Students finish working.) What did you notice?
- S: Odd times odd equals odd!
- T: Answer Problems 1(a) and 1(b) on the Problem Set.

Completed Table from Problem 1:

1×1	2×1	3×1	4×1	5×1	6×1	7×1	8×1
1	2	3	4	5	6	7	8
1×2	2×2	3×2	4×2	5×2	6×2	7×2	8×2
2	4	6	8	10	12	14	16
1×3	2×3	3×3	4×3	5×3	6×3	7×3	8×3
3	6	9	12	15	18	21	24
1×4	2×4	3×4	4×4	5×4	6×4	7×4	8×4
4	8	12	16	20	24	28	32
1×5	2×5	3×5	4×5	5×5	6×5	7×5	8×5
5	10	15	20	25	30	35	40
1×6	2×6	3×6	4×6	5×6	6×6	7×6	8×6
6	12	18	24	30	36	42	48
1×7	2×7	3×7	4×7	5×7	6×7	7×7	8×7
7	14	21	28	35	42	49	56
1×8	2×8	3×8	4×8	5×8	6×8	7×8	8×8
8	16	24	32	40	48	56	64



NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If possible and necessary, transcribe the rapid oral responses of learners who may otherwise work at a slower pace. Use color to outline rows and columns to help learners better discern the content. If highlighting the column of threes facts, for example, ask students to circle even products (instead of coloring them orange).

Problems 1(c) and 1(d)

- T: Compare the shaded columns and shaded rows. Which factors do they have in common?
- S: 2, 4, 6, 8.
- T: What is 5×4 ?
- S: 20.
- T: What is 2×4 ?
- S: 8.

- T: How do these 2 facts help you find 7×4 ? Talk to your partner and answer Problem 1(c) on the Problem Set.
- S: 20 and 8 is 28. \rightarrow 2 fours + 5 fours is 7 fours. \rightarrow 2 plus 5 is 7, so the products of these 2 facts can be added together to get the product of 7×4 . (Answer Problem 1(c) on the Problem Set.)
- T: Is the product of 7 and 16 on this table?
- S: No!
- T: Talk to a partner. How can we use this table and what we know to solve 7×16 ?
- S: 10 sevens and 6 sevens is 16 sevens, $70 + 42$. \rightarrow Doubling 8 sevens equals $56 + 56$. \rightarrow 9 sevens and 7 sevens, $63 + 49$. \rightarrow We can think of 16 as $8 + 8$ and then the problem is $(7 \times 8) + (7 \times 8)$. \rightarrow We could also add 4 sevens four times! $28 + 28 + 28 + 28$.
- T: Answer Problem 1(d).

Problem 2

- T: Complete the chart in Problem 2 by writing the products for each equation. (Students finish working.) Read the products to me.
- S: 1, 4, 9, 16, 25, 36.
- T: If this chart continued, what would the next equation be?
- S: $7 \times 7 = 49$.
- T: And the next equation?
- S: $8 \times 8 = 64$.
- T: Draw arrays to match each of these equations in Problem 2. (Students finish working.) Now, record the change in the number of squares from one array to the next.
- T: (Allow students time to finish.) Discuss with a partner. What is the pattern in the number of squares being added?
- S: It's 1, 3, 5, 7, like that! \rightarrow The increase in squares is the same as counting by the odd numbers, 1, 3, 5, 7, 9, 11, 13, 15.
- T: Answer Problem 2(b).
- T: What are the first 2 odd numbers when you start counting at 0?
- S: 1 and 3.
- T: What is their sum?
- S: 4.
- T: Look at Problem 2. Four is the product of what?
- MP.7** S: 2×2 .
- T: The sum of the first 2 odd numbers is the same as the product of 2×2 .
- T: What is the sum of the first 3 odd numbers?
- S: 9.
- T: Look at Problem 2. Nine is the product of what?
- S: 3×3 .

MP.7

- T: Use the table in Problem 2: What do you think is the sum of the first 5 odd numbers?
- S: 25.
- T: Check your work: What is $1 + 3 + 5 + 7 + 9$?
- S: 25.
- T: Answer Problems 2(c) and 2(d).

Student Debrief (10 minutes)

Lesson Objective: Identify patterns in multiplication and division facts using the multiplication table.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Talk to a partner: How do the patterns you discovered in Problem 1 for odd and even products help you when multiplying?
- What is the name of the strategy that you used to solve Problem 1(c)? Explain to a partner how this strategy could be used to solve another fact that isn't on the chart, like 6×18 .
- Look at the arrays you drew for Problem 2. If you drew an array for 7×7 , how many little squares would you add to the array that you drew for 6×6 ? How do you know?
- In Problem 2(c), you proved that 9×9 is the sum of the first 9 odd numbers. Is 10×10 the sum of the first 10 odd numbers? Where can you see the odd numbers on the two-colored multiplication table? Can you state a rule that this pattern shows using n to represent a number? (Guide students to see that $n \times n$ is the sum of the first n odd numbers. These types of problems are included in the homework.)

Name Gina Date _____

1. Write the products into the squares as fast as you can.

1×1	2×1	3×1	4×1	5×1	6×1	7×1	8×1
1	2	3	4	5	6	7	8
1×2	2×2	3×2	4×2	5×2	6×2	7×2	8×2
2	4	6	8	10	12	14	16
1×3	2×3	3×3	4×3	5×3	6×3	7×3	8×3
3	6	9	12	15	18	21	24
1×4	2×4	3×4	4×4	5×4	6×4	7×4	8×4
4	8	12	16	20	24	28	32
1×5	2×5	3×5	4×5	5×5	6×5	7×5	8×5
5	10	15	20	25	30	35	40
1×6	2×6	3×6	4×6	5×6	6×6	7×6	8×6
6	12	18	24	30	36	42	48
1×7	2×7	3×7	4×7	5×7	6×7	7×7	8×7
7	14	21	28	35	42	49	56
1×8	2×8	3×8	4×8	5×8	6×8	7×8	8×8
8	16	24	32	40	48	56	64

a. Color all the squares with even products orange. Can an even product ever have an odd factor?
Yes, because odd times even equals even.

b. Can an odd product ever have an even factor?
No, because odd products only happen with 2 odd factors.

c. Everyone knows that $7 \times 4 = (5 \times 4) + (2 \times 4)$. Explain how this is shown in the table.
The table shows $5 \times 4 = 20$ and $2 \times 4 = 8$. So, $20 + 8 = 28$, which is the product of 7×4 .

d. Use what you know to find the product of 7×16 or $8 \text{ sevens} + 8 \text{ sevens}$.

$$7 \times 16 = (7 \times 8) + (7 \times 8)$$

$$= 56 + 56$$

$$= 112$$

2. In the table, only the products on the diagonal are shown.

a. Label each product on the diagonal.

b. Draw an array to match each expression in the table below. Then label the number of squares you added to make each new array. The first two arrays have been done for you.

1×1	2×2	3×3	4×4	5×5	6×6
1	4	9	16	25	36

3 5 7 9 11

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

- c. What pattern do you notice in the number of squares that are added to each new array?
The pattern is skip-counting by odd numbers.

- d. Use the pattern you discovered in part b to prove this: 9×9 is the sum of the first 9 odd numbers.

First 9 odd numbers: 1, 3, 5, 7, 9, 11, 13, 15, 17

Sum: $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$

$10 + 11 = 21$

Name _____

Date _____

1. Write the products into the squares as fast as you can.

1×1	2×1	3×1	4×1	5×1	6×1	7×1	8×1
1×2	2×2	3×2	4×2	5×2	6×2	7×2	8×2
1×3	2×3	3×3	4×3	5×3	6×3	7×3	8×3
1×4	2×4	3×4	4×4	5×4	6×4	7×4	8×4
1×5	2×5	3×5	4×5	5×5	6×5	7×5	8×5
1×6	2×6	3×6	4×6	5×6	6×6	7×6	8×6
1×7	2×7	3×7	4×7	5×7	6×7	7×7	8×7
1×8	2×8	3×8	4×8	5×8	6×8	7×8	8×8








- a. Color all the squares with even products orange. Can an even product ever have an odd factor?
- b. Can an odd product ever have an even factor?
- c. Everyone knows that $7 \times 4 = (5 \times 4) + (2 \times 4)$. Explain how this is shown in the table.
- d. Use what you know to find the product of 7×16 or 8 sevens + 8 sevens.

2. In the table, only the products on the diagonal are shown.

a. Label each product on the diagonal.

1×1					
	2×2				
		3×3			
			4×4		
				5×5	
					6×6

b. Draw an array to match each expression in the table below. Then, label the number of squares you added to make each new array. The first two arrays have been done for you.

1×1	2×2	3×3	4×4	5×5	6×6
					
					
3					

- c. What pattern do you notice in the number of squares that are added to each new array?
- d. Use the pattern you discovered in Part (b) to prove this: 9×9 is the sum of the first 9 odd numbers.

Name _____

Date _____

1. Use what you know to find the product of 8×12 or 6 eights + 6 eights.
2. Luis says $3 \times 233 = 626$. Use what you learned about odd times odd to explain why Luis is wrong.

Name _____

Date _____

1. a. Write the products into the chart as fast as you can.

×	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

- b. Color the rows and columns with even factors yellow.
- c. What do you notice about the factors and products that are left unshaded?

- d. Complete the chart by filling in each blank and writing an example for each rule.

Rule	Example
odd times odd equals _____	
even times even equals _____	
even times odd equals _____	

- e. Explain how $7 \times 6 = (5 \times 6) + (2 \times 6)$ is shown in the table.

- f. Use what you know to find the product of 4×16 or 8 fours + 8 fours.

2. Today in class, we found that $n \times n$ is the sum of the first n odd numbers. Use this pattern to find the value of n for each equation below. The first is done for you.

a. $1 + 3 + 5 = n \times n$

$$9 = 3 \times 3$$

b. $1 + 3 + 5 + 7 = n \times n$

c. $1 + 3 + 5 + 7 + 9 + 11 = n \times n$

d. $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = n \times n$

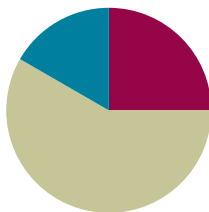
e. $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = n \times n$

Lesson 18

Objective: Solve two-step word problems involving all four operations and assess the reasonableness of solutions.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Concept Development	(35 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Sprint: Multiply and Divide with 1 and 0 **3.OA.5** (8 minutes)
- Multiply with 10 **3.NBT.3** (3 minutes)
- Group Counting **3.OA.1** (4 minutes)

Sprint: Multiply and Divide with 1 and 0 (8 minutes)

Materials: (S) Multiply and divide with 1 and 0 Sprint

Note: This Sprint reviews Lesson 16, which involves rules and properties when multiplying and dividing with 1 and 0.

Multiply with 10 (3 minutes)

Note: This fluency activity anticipates Lesson 19, which involves multiplying by multiples of 10 using the place value chart.

T: I'll say a fact. You say the whole equation. 10×1 .

S: $10 \times 1 = 10$.

Continue with the following possible sequence: 10×2 , 10×3 , 10×9 , and 10×7 .

T: I'll say a product that is a multiple of 10. You say the multiplication fact starting with 10. 20.

S: $10 \times 2 = 20$.

Continue with the following possible sequence: 30, 40, 80, and 60.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. These counts review multiplication taught previously in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

Concept Development (35 minutes)

Materials: (S) Personal white board

Project the following word problem: Joe has \$173 in the bank. He earns the same amount of money each week for 7 weeks and puts this money in the bank. Now, Joe has \$208 in the bank. How much money does Joe earn each week?

T: Draw a model to show the total amount of money Joe has in the bank at the end of the 7 weeks. At my signal, show me your personal white board. (Signal.)

T: Do we know the amount of money Joe puts in the bank?

S: No.

T: Label this unknown on your model using the letter m for money. Then, write what m represents. (Students write.) Write an equation to show how to solve for m .

S: (Write $\$208 - \$173 = m$.)

T: Solve for m , and write its value on your model.

S: (Write $m = \$35$.)

T: Is this answer reasonable?

S: Yes, because $\$173 + \35 equals $\$208$, which is the total amount Joe has in the bank.

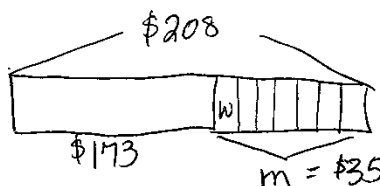
T: Did we answer the question in the problem?

S: No, we're trying to figure out how much money he earns each week.

T: Adjust your model to show what you know about the amount of money Joe earns in 7 weeks.

S: (Split $\$35$ into 7 equal pieces.)

T: Label the unknown with the letter w to represent how much money Joe earns each week. Then, write what w represents.



m = number of dollars Joe put in bank

$$\$208 - \$173 = m$$

$$m = \$35$$

w = number of dollars earned each week

$$\$35 \div 7 = w$$

$$w = \$5$$

- S: (Label. Then, write what w represents.)
- T: Write an equation on your board and solve for w .
- S: (Write $\$35 \div 7 = w$; $w = \$5$.)
- T: Discuss with a partner. Is it reasonable that Joe earns \$5 a week?
- S: Yes, it makes sense because that's how much I earn each week for my allowance! → It's reasonable because \$5 a week for 7 weeks is \$35 and that's about \$40. \$173 is about \$170 and $\$40 + \170 is \$210, which is close to \$208.
- T: It's important to make sure the answer makes sense for every part of the problem!

Problem Set (20 minutes)

Students should do their personal best to complete the Problem Set within the allotted 20 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

When completing the Problem Set, students working above grade level may enjoy an open-ended extension. Offer students an option to choose one of the models and equations from the Problem Set to write their own word problem.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Take advantage of the opportunity in the Student Debrief to review personal and class goals regarding problem solving. Guide students to identify their strengths and weaknesses as problem solvers. Construct new goals for future work.

Student Debrief (10 minutes)

Lesson Objective: Solve two-step word problems involving all four operations and assess the reasonableness of solutions.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, you found that Sasha gives Rose a piece of yarn that is 27 centimeters long. Into how many 9-centimeter pieces can Rose cut this piece?
- In Problem 2, did Julio spend more time on his spelling homework or his math homework? How do you know?
- How are Problems 3 and 4 similar? Discuss with a partner.
- In Problem 5, if Cora weighs 5 pencils, what is the total weight for the pencils and the ruler? How do you know?
- Discuss with a partner the importance of checking the reasonableness of your answer.

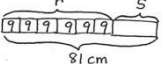
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name Grina Date _____

Directions: Use the RDW process for each problem. Explain why your answer is reasonable.

1. Rose has 6 pieces of yarn that are each 9 centimeters long. Sasha gives Rose a piece of yarn. Now Rose has a total of 81 centimeters of yarn. What is the length of the yarn that Sasha gives Rose?




r = number of cm Rose has
 $6 \times 9 = r$
 $r = 54$

s = number of cm of yarn Sasha gives Rose
 $81 - 54 = s$
 $s = 27$
 Sasha gives Rose 27 cm of yarn

$54 + 27 \approx 50 + 30 = 80$
 My answer is reasonable because my estimate, 80 cm, is close to 81 cm.

2. Julio spends 29 minutes doing his spelling homework. He then completes each math problem in 4 minutes. There are 7 math problems. How many minutes does Julio spend on his homework in all?



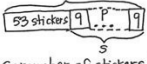
m = number of minutes spent on math homework
 $7 \times 4 = m$
 $m = 28$

h = number of minutes spent on homework
 $29 + 28 = h$
 $h = 57$

My answer is reasonable because 57 minutes is almost an hour and 29 and 28 are each about half an hour. Two half hours equal an hour.

Julio spends 57 minutes on his homework.

3. Pearl buys 125 stickers. She gives 53 stickers to her little sister. Pearl then puts 9 stickers on each page of her album. If she uses all of her remaining stickers, how many pages does Pearl put stickers on?

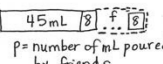


s = number of stickers Pearl has left.
 $125 - 53 = s$
 $s = 72$

P = number of pages Pearl puts stickers on.
 $72 \div 9 = P$
 $P = 8$
 Pearl puts stickers on 8 pages.

My answer is reasonable because $9 \times 8 = 72$ and $72 + 53 \approx 72 + 50 = 122$ and that is close to 125.

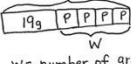
4. Tanner's beaker had 45 milliliters of water in it at first. After each of his friends poured in 8 milliliters, the beaker contained 93 milliliters. How many friends poured water into Tanner's beaker?



f = number of friends
 $48 \div 8 = f$
 $f = 6$
 6 friends poured water into Tanner's beaker.

My answer is reasonable because I know $8 \times 6 = 48$ and $48 + 45 \approx 50 + 45 = 95$ and that is close to 93.

5. Cora weighs 4 new, identical pencils and a ruler. The total weight of these items is 55 grams. She weighs the ruler by itself and it weighs 19 grams. How much does each pencil weigh?



w = number of grams 4 pencils weigh.
 $55 - 19 = w$
 $w = 36$

P = number of grams each pencil weighs
 $36 \div 4 = P$
 $P = 9$
 Each pencil weighs 9 grams.

$4 \times 9 = 36$ and $36 + 19 = 35 + 20 = 55$
 My answer is reasonable because I checked my work with multiplication and addition and got 55.

A

Number Correct: _____

Multiply and Divide with 1 and 0

1.	_____ \times 1 = 2	
2.	_____ \times 1 = 3	
3.	_____ \times 1 = 4	
4.	_____ \times 1 = 9	
5.	8 \times _____ = 0	
6.	9 \times _____ = 0	
7.	4 \times _____ = 0	
8.	5 \times _____ = 5	
9.	6 \times _____ = 6	
10.	7 \times _____ = 7	
11.	3 \times _____ = 3	
12.	0 \div 1 = _____	
13.	0 \div 2 = _____	
14.	0 \div 3 = _____	
15.	0 \div 6 = _____	
16.	1 \times _____ = 1	
17.	4 \div _____ = 4	
18.	5 \div _____ = 5	
19.	6 \div _____ = 6	
20.	8 \div _____ = 8	
21.	_____ \times 1 = 5	
22.	3 \times _____ = 0	

23.	9 \div _____ = 9	
24.	8 \times _____ = 8	
25.	_____ \times 1 = 1	
26.	0 \div 3 = _____	
27.	_____ \times 1 = 7	
28.	6 \times _____ = 0	
29.	4 \times _____ = 4	
30.	0 \div 8 = _____	
31.	0 \times _____ = 0	
32.	1 \div 1 = _____	
33.	_____ \times 1 = 24	
34.	17 \times _____ = 0	
35.	32 \times _____ = 32	
36.	0 \div 19 = _____	
37.	46 \times _____ = 0	
38.	0 \div 51 = _____	
39.	64 \times _____ = 64	
40.	_____ \times 1 = 79	
41.	0 \div 82 = _____	
42.	_____ \times 1 = 96	
43.	27 \times _____ = 27	
44.	43 \times _____ = 0	

B

Number Correct: _____

Improvement: _____

Multiply and Divide with 1 and 0

1.	_____ \times 1 = 3	
2.	_____ \times 1 = 4	
3.	_____ \times 1 = 5	
4.	_____ \times 1 = 8	
5.	7 \times _____ = 0	
6.	8 \times _____ = 0	
7.	3 \times _____ = 0	
8.	4 \times _____ = 4	
9.	5 \times _____ = 5	
10.	6 \times _____ = 6	
11.	2 \times _____ = 2	
12.	0 \div 2 = _____	
13.	0 \div 3 = _____	
14.	0 \div 4 = _____	
15.	0 \div 7 = _____	
16.	1 \times _____ = 1	
17.	3 \div _____ = 3	
18.	4 \div _____ = 4	
19.	5 \div _____ = 5	
20.	7 \div _____ = 7	
21.	_____ \times 1 = 6	
22.	4 \times _____ = 0	

23.	8 \div _____ = 8	
24.	7 \times _____ = 7	
25.	_____ \times 1 = 1	
26.	0 \div 5 = _____	
27.	_____ \times 1 = 9	
28.	5 \times _____ = 0	
29.	9 \times _____ = 9	
30.	0 \div 6 = _____	
31.	1 \div 1 = _____	
32.	0 \times _____ = 0	
33.	_____ \times 1 = 34	
34.	16 \times _____ = 0	
35.	31 \times _____ = 31	
36.	0 \div 18 = _____	
37.	45 \times _____ = 0	
38.	0 \div 52 = _____	
39.	63 \times _____ = 63	
40.	_____ \times 1 = 78	
41.	0 \div 81 = _____	
42.	_____ \times 1 = 97	
43.	26 \times _____ = 26	
44.	42 \times _____ = 0	

Name _____

Date _____

Use the RDW process for each problem. Explain why your answer is reasonable.

1. Rose has 6 pieces of yarn that are each 9 centimeters long. Sasha gives Rose a piece of yarn. Now, Rose has a total of 81 centimeters of yarn. What is the length of the yarn that Sasha gives Rose?
2. Julio spends 29 minutes doing his spelling homework. He then completes each math problem in 4 minutes. There are 7 math problems. How many minutes does Julio spend on his homework in all?

Name _____

Date _____

Use the RDW process to solve. Explain why your answer is reasonable.

On Saturday, Warren swims laps in the pool for 45 minutes. On Sunday, he runs 8 miles. It takes him 9 minutes to run each mile. How long does Warren spend exercising over the weekend?

Name _____

Date _____

Use the RDW process for each problem. Explain why your answer is reasonable.

1. Mrs. Portillo's cat weighs 6 kilograms. Her dog weighs 22 kilograms more than her cat. What is the total weight of her cat and dog?
2. Darren spends 39 minutes studying for his science test. He then does 6 chores. Each chore takes him 3 minutes. How many minutes does Darren spend studying and doing chores?
3. Mr. Abbot buys 8 boxes of granola bars for a party. Each box has 9 granola bars. After the party, there are 39 bars left. How many bars were eaten during the party?

4. Leslie weighs her marbles in a jar, and the scale reads 474 grams. The empty jar weighs 439 grams. Each marble weighs 5 grams. How many marbles are in the jar?
5. Sharon uses 72 centimeters of ribbon to wrap gifts. She uses 24 centimeters of her total ribbon to wrap a big gift. She uses the remaining ribbon for 6 small gifts. How much ribbon will she use for each small gift if she uses the same amount on each?
6. Six friends equally share the cost of a gift. They pay \$90 and receive \$42 in change. How much does each friend pay?



Topic F

Multiplication of Single-Digit Factors and Multiples of 10

3.OA.5, 3.OA.8, 3.OA.9, 3.NBT.3, 3.OA.1

Focus Standards:	3.OA.5	Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) <i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i>
	3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)
	3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i>
	3.NBT.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.
Instructional Days:	3	
Coherence -Links from:	G2–M3	Place Value, Counting, and Comparison of Numbers to 1,000
	G2–M6	Foundations of Multiplication and Division
	G3–M1	Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
-Links to:	G3–M4	Multiplication and Area
	G4–M3	Multi-Digit Multiplication and Division
	G4–M7	Exploring Measurement with Multiplication

In Lesson 19, students initially use the place value chart to multiply by multiples of 10. To solve 2×40 , for example, they begin by modeling 2×4 in the ones place. Students relate this to multiplying 2×4 tens, locating the same basic fact in the tens column. They see that when multiplied by 10, the digits in the product shift one place value to the left. Complexities are addressed as regrouping becomes involved with problems like 4×6 , where the product has mixed units of tens and ones. However, the same principle applies—the digits shift once to the left.

Lesson 20 carries students' understanding from Lesson 19 to more abstract situations using a wider range of multiples of 10. Students learn to model place value strategies using the associative property, for example, $2 \times 30 = 2 \times (3 \times 10) = (2 \times 3) \times 10$, and $4 \times 60 = 4 \times (6 \times 10) = (4 \times 6) \times 10$. In Lesson 21, students apply learning from Topic F to solving two-step word problems and multiplying single-digit factors and multiples of 10. They use the rounding skills learned in Module 2 to estimate and assess the reasonableness of their solutions.

A Teaching Sequence Toward Mastery of Multiplication of Single-Digit Factors and Multiples of 10

Objective 1: Multiply by multiples of 10 using the place value chart.
(Lesson 19)

Objective 2: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where n and m are less than 10) to multiply by multiples of 10.
(Lesson 20)

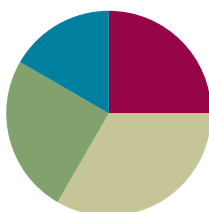
Objective 3: Solve two-step word problems involving multiplying single-digit factors and multiples of 10.
(Lesson 21)

Lesson 19

Objective: Multiply by multiples of 10 using the place value chart.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Concept Development	(20 minutes)
■ Application Problem	(15 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



NOTES ON TIMING:

The Application Problem comes after the Concept Development in this lesson. The 15 minutes allotted for the Application Problem includes 5 minutes for the Application Problem and 10 minutes for the Problem Set.

Fluency Practice (15 minutes)

- Group Counting **3.OA.1** (4 minutes)
- Multiply with 10 **3.NBT.3** (3 minutes)
- Multiply by Different Units **3.NBT.3** (4 minutes)
- Exchange Place Value Disks **3.NBT.3** (4 minutes)

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. These counts review the multiplication taught previously in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

Multiply with 10 (3 minutes)

Note: This fluency activity prepares students for this lesson.

T: I'll say a multiplication problem. You say the whole equation. 10×1 .

S: $10 \times 1 = 10$.

Continue with the following possible sequence: 10×2 , 10×3 , 10×8 , and 10×6 .

T: I'll say a multiple of 10. You say the multiplication fact starting with 10. 20.

S: $10 \times 2 = 20$.

Continue with the following possible sequence: 30, 40, 90, 70, and 50.

Multiply by Different Units (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for this lesson.

T: (Write $2 \times 3 = \underline{\quad}$.) Say the multiplication equation in unit form.

S: 2×3 ones = 6 ones.

T: (Write 2×3 cats = $\underline{\quad}$.) On your personal white board, write the multiplication equation.

Continue with the following possible sequence: 3×4 , 3×4 dogs; 4×5 , 4×5 pencils; 5×6 , 5×6 books; 6×7 , 6×7 cars; 7×8 , 7×8 turtles; 8×9 , 8×9 chairs; and 9×7 , 9×7 flowers.

Exchange Place Value Disks (4 minutes)

Materials: (S) Place value disks

Note: This fluency activity prepares students for this lesson.

T: Make an array showing 3 by 2 ones. As a multiplication equation, say how many ones you have.

S: 3×2 ones = 6 ones.

Continue with the following possible sequence: 3 by 3 ones, 4 by 2 ones, and 5 by 2 ones.

T: 10 ones can be exchanged for 1 of what unit?

S: 1 ten.

T: Exchange 10 ones for 1 ten.

T: Make an array showing 4 by 5 ones.

T: Say how many ones you have as a multiplication equation.

S: 4×5 ones = 20 ones.

T: Say the multiplication equation again; this time, say the answer in units of 10.

S: 4×5 ones = 2 tens.

T: Exchange 20 ones for 2 tens.

Concept Development (20 minutes)

Materials: (T/S) Place value disks (S) Personal white board

Problem 1: Multiply by multiples of 10 using place value disks.

T: Use your disks to show 2 rows of 3 ones.

S: (Model 2×3 ones array.)

T: (Write 2×3 ones = $\underline{\quad}$ ones.) Our array shows this equation, true?

S: True.

**NOTES ON
MULTIPLE MEANS
OF ACTION AND
EXPRESSION:**

During the Concept Development, check for understanding as students use concrete place value disks. Make sure students are distinguishing between ones disks and tens disks. Ask students to count out, “1 ten, 2 tens, 3 tens, etc.,” as they make their array. Alternatively, students may draw the disks.

T: How many ones do we have in total?

S: 6 ones.

T: Say the multiplication equation in standard form.

S: $2 \times 3 = 6$.

T: Use your disks to show 2 rows of 3 tens.

S: (Model 2×3 tens array.)

T: (Write 2×3 tens = _____ tens.) How many tens do we have in total?

S: 6 tens.

T: What is the value of 6 tens?

S: 60.

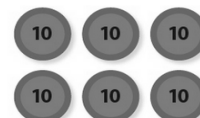
T: Say the multiplication equation in standard form.

S: $2 \times 30 = 60$.



$$2 \times 3 \text{ ones} = 6 \text{ ones}$$

$$2 \times 3 = 6$$



$$2 \times 3 \text{ tens} = 6 \text{ tens}$$

$$2 \times 30 = 60$$

Repeat the process with 3×4 ones and 3×4 tens; 2×6 ones and 2×6 tens.

Problem 2: Multiply by multiples of 10 using a place value chart.

T: (Project or draw the place value chart shown at right.) Use the chart to write an equation in both unit form and standard form.

S: (Write 2×5 ones = 10 ones and $2 \times 5 = 10$.)

T: How many ones do I have in total?

S: 10 ones.

T: (Project or draw the place value chart shown at bottom right.) Compare the two charts. What do you notice about the number of dots?

S: The number of dots is exactly the same in both charts.
→ The only thing that changes is where they are placed.
The dots moved over to the tens place.

T: Because we still have a total of ten dots, what change do you think we will make in our equations?

S: The units will change from ones to tens.

T: Write your equations now.

S: (Write equations.)

T: Say the full equation in standard form.

S: 2 times 50 equals 100.

tens	ones
	• • • • •
	• • • • •

$$2 \times 5 \text{ ones} = \underline{\hspace{2cm}} \text{ ones}$$

$$2 \times 5 = \underline{\hspace{2cm}}$$

tens	ones
• • • • •	
• • • • •	

$$2 \times 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$$

$$2 \times 50 = \underline{\hspace{2cm}}$$

Repeat the process with 3×6 ones and 3×6 tens.

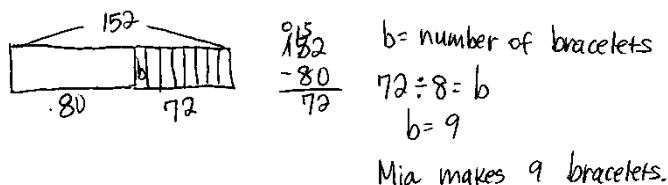
T: (Write $80 \times 6 = \underline{\hspace{2cm}}$.) How would you use this strategy to solve a more complicated problem like the one on the board?

MP.7 S: We can first think of the problem as $8 \text{ ones} \times 6$, which is 48. We know that fact since we've been practicing our sixes. → Then, all we have to do is move the answer over to the tens place, so it becomes 48 tens. → So, the answer is 480.

Repeat the process with 7×90 and 60×4 to give the students an opportunity to discuss the unit form strategy with more complex problems.

Application Problem (15 minutes)

Mia has 152 beads. She uses some to make bracelets. Now there are 80 beads. If she uses 8 beads for each bracelet, how many bracelets does she make?



$$\begin{array}{r} 152 \\ - 80 \\ \hline 72 \end{array}$$

$$b = \text{number of bracelets}$$

$$72 \div 8 = b$$

$$b = 9$$
 Mia makes 9 bracelets.



NOTES ON TIMING:

The 15 minutes allotted for the Application Problem includes 5 minutes for the word problem and 10 minutes for the Problem Set.



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Instead of having them complete the Problem Set, give English language learners the option of writing a response to either of the first two Debrief questions. This chance to reflect and prepare their response in English may increase their confidence and participation in the Student Debrief.

Note: This problem reviews Lesson 18, which consisted of solving two-step word problems involving more than one operation.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Multiply by multiples of 10 using the place value chart.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How do the disks in Problem 1 show the strategy we learned today?
- What is the relationship between the charts in the left column and the charts in the right column in Problem 2? How did the left column help you solve the problems in the right column?
- How does knowing your multiplication facts help you easily multiply by multiples of 10?


- Now that we know a strategy for multiplying with multiples of 10, how would we use the same process for multiplying with multiples of 100? What would be the same? (The multiplication facts.) What would change? (The units.)

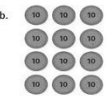
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

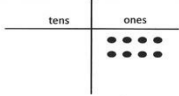
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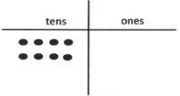
1. Use the disks to fill in the blanks in the equations.

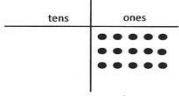
a.  $4 \times 3 \text{ ones} = \underline{12} \text{ ones}$
 $4 \times 3 = \underline{12}$

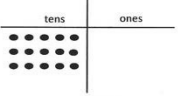
b.  $4 \times 3 \text{ tens} = \underline{12} \text{ tens}$
 $4 \times 30 = \underline{120}$

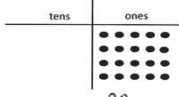
2. Use the chart to complete the blanks in the equations.

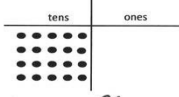
a.  $2 \times 4 \text{ ones} = \underline{8} \text{ ones}$
 $2 \times 4 = \underline{8}$

b.  $2 \times 4 \text{ tens} = \underline{8} \text{ tens}$
 $2 \times 40 = \underline{80}$

c.  $3 \times 5 \text{ ones} = \underline{15} \text{ ones}$
 $3 \times 5 = \underline{15}$

d.  $3 \times 5 \text{ tens} = \underline{15} \text{ tens}$
 $3 \times 50 = \underline{150}$

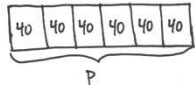
e.  $4 \times 5 \text{ ones} = \underline{20} \text{ ones}$
 $4 \times 5 = \underline{20}$

f.  $4 \times 5 \text{ tens} = \underline{20} \text{ tens}$
 $4 \times 50 = \underline{200}$

3. Fill in the blank to make the equation true.

a. <u>14</u> = 7×2	b. <u>14</u> tens = $7 \text{ tens} \times 2$
c. <u>24</u> = 8×3	d. <u>24</u> tens = $8 \text{ tens} \times 3$
e. <u>300</u> = 60×5	f. <u>320</u> = 4×80
g. $7 \times 40 = \underline{280}$	h. $50 \times 8 = \underline{400}$

4. A bus can carry 40 passengers. How many passengers can 6 buses carry? Model with a tape diagram.

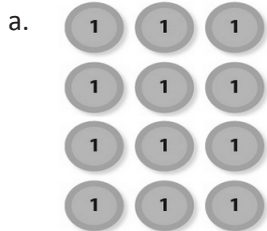


$6 \times 40 = P$
 $P = 240$
 6 buses can carry 240 passengers.

Name _____

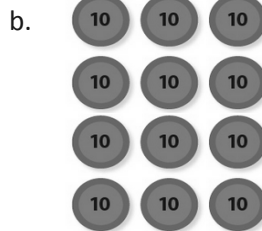
Date _____

1. Use the disks to fill in the blanks in the equations.



$$4 \times 3 \text{ ones} = \underline{\hspace{2cm}} \text{ ones}$$

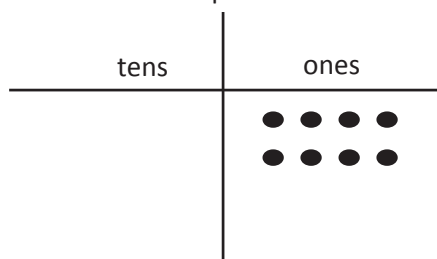
$$4 \times 3 = \underline{\hspace{2cm}}$$



$$4 \times 3 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$$

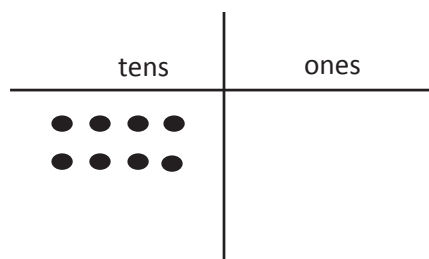
$$4 \times 30 = \underline{\hspace{2cm}}$$

2. Use the chart to complete the blanks in the equations.



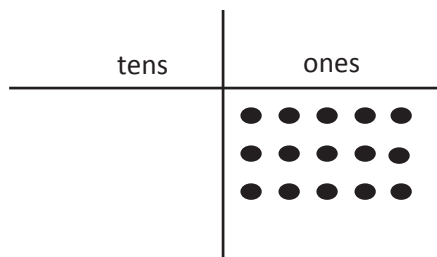
a. $2 \times 4 \text{ ones} = \underline{\hspace{2cm}} \text{ ones}$

$$2 \times 4 = \underline{\hspace{2cm}}$$



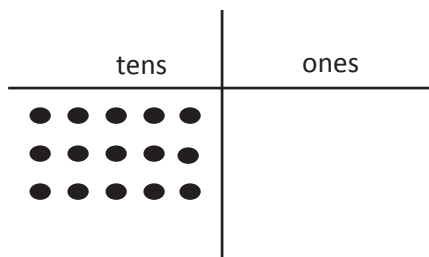
b. $2 \times 4 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$

$$2 \times 40 = \underline{\hspace{2cm}}$$



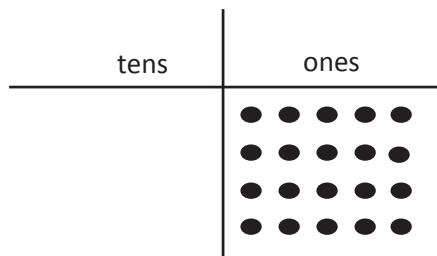
c. $3 \times 5 \text{ ones} = \underline{\hspace{2cm}} \text{ ones}$

$$3 \times 5 = \underline{\hspace{2cm}}$$



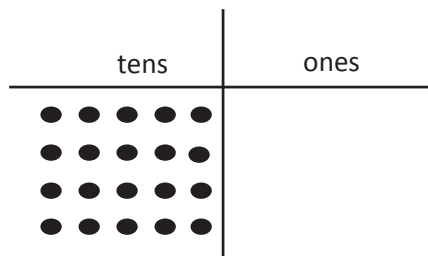
d. $3 \times 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$

$$3 \times 50 = \underline{\hspace{2cm}}$$



e. 4×5 ones = _____ ones

$4 \times 5 =$ _____



f. 4×5 tens = _____ tens

$4 \times 50 =$ _____

3. Fill in the blank to make the equation true.


a. _____ = 7×2	b. _____ tens = $7 \text{ tens} \times 2$
c. _____ = 8×3	d. _____ tens = $8 \text{ tens} \times 3$
e. _____ = 60×5	f. _____ = 4×80
g. $7 \times 40 =$ _____	h. $50 \times 8 =$ _____


4. A bus can carry 40 passengers. How many passengers can 6 buses carry? Model with a tape diagram.

Name _____

Date _____

1. Use the chart to complete the blanks in the equations.

tens	ones
	

tens	ones
	

$6 \times 5 \text{ ones} = \underline{\hspace{2cm}} \text{ ones}$

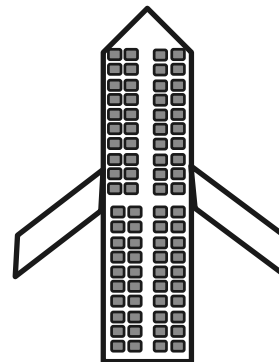
$6 \times 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$

$6 \times 5 = \underline{\hspace{2cm}}$

$6 \times 50 = \underline{\hspace{2cm}}$

2. A small plane has 20 rows of seats. Each row has 4 seats.

- a. Find the total number of seats on the plane.



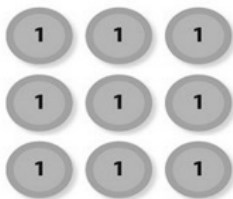
- b. How many seats are on 3 small planes?

Name _____

Date _____

1. Use the disks to complete the blanks in the equations.

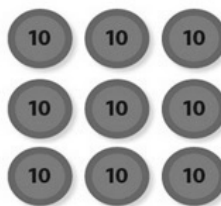
a.



$$3 \times 3 \text{ ones} = \underline{\hspace{2cm}} \text{ ones}$$

$$3 \times 3 = \underline{\hspace{2cm}}$$

b.



$$3 \times 3 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$$

$$30 \times 3 = \underline{\hspace{2cm}}$$

2. Use the chart to complete the blanks in the equations.

tens	ones

$$\text{a. } 2 \times 5 \text{ ones} = \underline{\hspace{2cm}} \text{ ones}$$

$$2 \times 5 = \underline{\hspace{2cm}}$$

tens	ones

$$\text{b. } 2 \times 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$$

$$2 \times 50 = \underline{\hspace{2cm}}$$

tens	ones

$$\text{c. } 5 \times 5 \text{ ones} = \underline{\hspace{2cm}} \text{ ones}$$

$$5 \times 5 = \underline{\hspace{2cm}}$$

tens	ones

$$\text{d. } 5 \times 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$$

$$5 \times 50 = \underline{\hspace{2cm}}$$

3. Match.

6×2

120

6 tens \times 2

21

7×3

12

7 tens \times 3

270

70×5

210

3×90

350

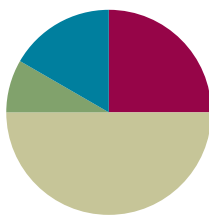
4. Each classroom has 30 desks. What is the total number of desks in 8 classrooms? Model with a tape diagram.

Lesson 20

Objective: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where n and m are less than 10) to multiply by multiples of 10.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Application Problem	(5 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



Fluency Practice (15 minutes)

- Group Counting **3.OA.1** (3 minutes)
- Multiply by Different Units **3.NBT.3** (6 minutes)
- Write in the Parentheses **3.OA.7** (6 minutes)

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. The counts in these lessons review the multiplication taught previously in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

Multiply by Different Units (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 19.

- T: (Write $2 \times 3 = \underline{\quad}$.) Say the multiplication equation in unit form.
 S: 2 ones \times 3 = 6 ones.
 T: Say it in standard form.
 S: $2 \times 3 = 6$.

T: (Write $2 \text{ tens} \times 3 = \underline{\hspace{1cm}}$.) On your personal white board, write the multiplication equation in unit form.

S: (Write $2 \text{ tens} \times 3 = 6 \text{ tens}$.)

T: Below your equation, write a second multiplication equation in standard form.

S: (Write $20 \times 3 = 60$.)

Continue with the following possible sequence: 4×2 , $4 \text{ tens} \times 2$, 5×3 , $5 \times 3 \text{ tens}$, 6×4 , and $6 \times 4 \text{ tens}$.

T: (Write $7 \times 6 = \underline{\hspace{1cm}}$.) Say the multiplication equation.

S: $7 \times 6 = 42$.

T: (Write $70 \times 6 = \underline{\hspace{1cm}}$.) Write the multiplication equation.

S: (Write $70 \times 6 = 420$.)

Continue with the following possible sequence: 8×8 , 8×80 ; 9×8 , 90×8 ; 6×6 , 60×6 ; 8×7 , 8×70 ; 4×9 , 40×9 ; and 9×6 , 90×6 .

Write in the Parentheses (6 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews the use of parentheses and prepares students for today's lesson.

T: (Write $4 \times 5 = 2 \times 2 \times 5$.) What's 4×5 ?

S: 20.

T: On your personal white board, copy the equation. Then, underneath the equation, write in parentheses and solve.

S: (Write $4 \times 5 = 2 \times 2 \times 5$. Beneath it, write $20 = (2 \times 2) \times 5$.)

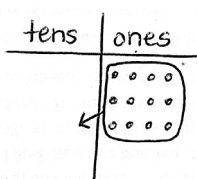
$$4 \times 5 = 2 \times 2 \times 5$$

$$20 = (2 \times 2) \times 5$$

Continue with the following possible sequence: $6 \times 4 = 6 \times 2 \times 2$, $6 \times 6 = 6 \times 2 \times 3$, $4 \times 7 = 2 \times 2 \times 7$, $7 \times 8 = 7 \times 4 \times 2$, $8 \times 4 = 8 \times 2 \times 2$, $8 \times 6 = 8 \times 3 \times 2$, $9 \times 6 = 9 \times 3 \times 2$, and $9 \times 8 = 9 \times 4 \times 2$.

Application Problem (5 minutes)

Model 3×4 on a place value chart. Then, explain how the array can help you solve 30×4 .



This array shows $3 \text{ ones} \times 4 = 12 \text{ ones}$. 30×4 is just $3 \text{ tens} \times 4$ which is equal to 12 tens, or 120. We can move the dots over to the tens place to show this, because the only thing that changes is the unit.



NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

Allow English language learners more time to compose their explanation, access to a math picture dictionary, an example of a well-written response, and an opportunity to share their response (perhaps during the Debrief).

Note: This problem reviews multiplying by multiples of 10 from Lesson 19. In today's Concept Development, students build on their understanding from Lesson 19 to multiply by multiples of 10 using the associative property.

MP.3

Concept Development (30 minutes)

Materials: (S) Personal white board

- T: (Write 40×2 .) Ten times what number gives us a product of 40?
- S: 4.
- T: Let's rewrite our equation. (Write $(10 \times 4) \times 2$.) Why do you think I put 10×4 in parentheses?
- S: The parentheses show that, when you group those numbers together and multiply, you get 40.
→ The parentheses remind us that we put 10×4 where 40 used to be.
- T: Let's move the parentheses to change the way the numbers are grouped.
- T: On your personal white board, use the parentheses to group the numbers differently.
- S: (Write $10 \times (4 \times 2)$.)
- T: Is this problem friendlier than 40×2 ?
- S: Oh, it's just 10×8 . That's the same as 80. That was a little easier than multiplying by 40.

Repeat the process with 20×3 , 30×3 , and 50×2 .

- T: (Project or draw Image A shown.) Use the chart to write a multiplication equation in unit form.
- S: (Write $3 \times 6 \text{ ones} = 18 \text{ ones}$.)
- T: Now, I want to multiply 18 ones by ten. Watch as I show this on the chart. I redraw dots into the tens place and draw an arrow (draw arrow) to remind myself that they shift to the next unit. Let's multiply our 3 groups of 6 ones by 10.
- T: (Write $(3 \times 6 \text{ ones}) \times 10 = \underline{\hspace{2cm}}$.) What is the answer to $18 \text{ ones} \times 10$ in unit form?
- S: 18 tens!
- T: What is the value of 18 tens?
- S: 180.
- T: (Project or draw Image B, shown on the next page.) This time, I already moved 6 ones to make them 6 tens. Use the chart to write a multiplication equation in unit form.
- S: (Write $6 \text{ ones} \times 10 = 6 \text{ tens}$.)
- T: Now, I want to multiply 6 tens by 3. How many rows do I need to add to show 3 rows of 6 tens?
- S: 2 rows.

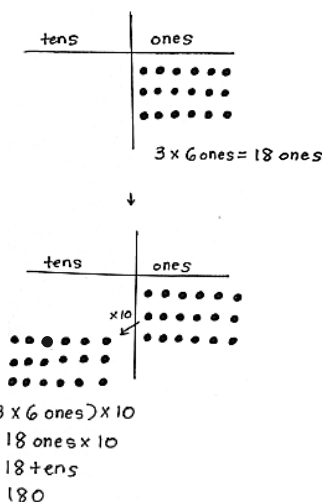


NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

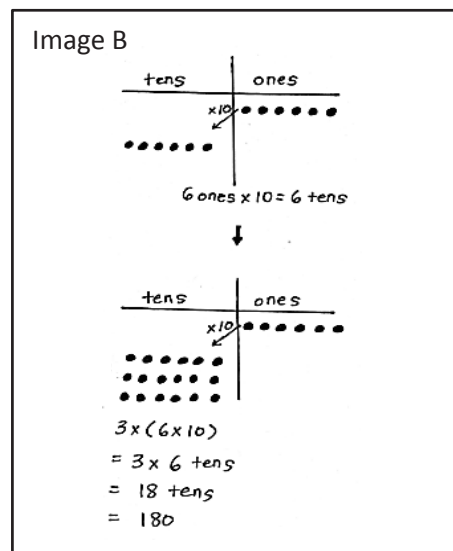
Allow students who are working above grade level more autonomy to experiment with the manner they use to solve, as well as with the numbers they choose. Example prompts are given below:

- Write a multiplication fact that you think is best solved using the associative property.
- Write another three-factor multiplication equation with a product of 40. Compare the two equations. What do you notice?
- In the equation $10 \times (4 \times 2)$, what would happen if you changed the factors inside the parentheses to numbers greater than 10?

Image A



- T: (Add 2 rows of 6 tens and write $3 \times (6 \times 10)$.) How does my array show this expression? Tell your partner.
- S: There are 3 rows of 6 tens. \rightarrow Six tens is the same as 6×10 . It has the parentheses around it because we did that first on the chart. \rightarrow Then, we multiplied the 6×10 by 3.
- T: What is the answer to 3×6 tens in unit form?
- S: 18 tens!
- T: What is the value of 18 tens?
- S: 180.
- T: Compare the equations $(3 \times 6 \text{ ones}) \times 10$ and $3 \times (6 \times 10)$. What do you notice about the factors we used?
- S: The factors are the same! 3, 6, and 10. The units are different and so is the order of what you multiply first.
- T: In both charts, we saw how multiplying the ten, even at different times, made it easier to solve.



Repeat the process with $(4 \times 5) \times 10$ and $4 \times (5 \times 10)$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where n and m are less than 10) to multiply by multiples of 10.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Name Gina Date _____

1. Use the chart to complete the equations. Then solve. The first one has been done for you.

a) $(2 \times 4) \times 10$
 $= (8 \text{ ones}) \times 10$
 $= \underline{80}$

b) $2 \times (4 \times 10)$
 $= 2 \times (4 \text{ tens})$
 $= \underline{80}$

c) $(3 \times 5) \times 10$
 $= (15 \text{ ones}) \times 10$
 $= \underline{150}$

d) $3 \times (5 \times 10)$
 $= 3 \times (5 \text{ tens})$
 $= \underline{150}$

Any combination of the questions below may be used to lead the discussion

- In Problem 1, which grouping is easier for you to solve? Why?
- How do you see the movement of the parentheses in the place value charts in Problem 1?
- Share with a partner how you knew where to draw parentheses for the equations in Problem 2.
- In Problem 3, how did Gabriella simplify the problem?
- Why didn't we need to have a hundreds column in our place value charts?
- How is this new strategy helpful for finding unknown, larger facts?

2. Place parentheses in the equations to find the related fact. Then solve. The first one has been done for you.

$$\begin{aligned}
 2 \times 20 &= 2 \times (2 \times 10) \\
 &= (2 \times 2) \times 10 \\
 &= \underline{4} \times 10 \\
 &= \underline{40}
 \end{aligned}$$

$$\begin{aligned}
 2 \times 30 &= 2 \times (3 \times 10) \\
 &= (2 \times 3) \times 10 \\
 &= \underline{6} \times 10 \\
 &= \underline{60}
 \end{aligned}$$

$$\begin{aligned}
 3 \times 30 &= 3 \times (3 \times 10) \\
 &= (3 \times 3) \times 10 \\
 &= \underline{9} \times 10 \\
 &= \underline{90}
 \end{aligned}$$

$$\begin{aligned}
 2 \times 50 &= 2 \times (2 \times 5 \times 10) \\
 &= (2 \times 2 \times 5) \times 10 \\
 &= \underline{10} \times 10 \\
 &= \underline{100}
 \end{aligned}$$

3. Gabriella solves 20×4 by thinking about 10×8 . Explain her strategy.

$$\begin{aligned}
 20 \times 4 &= (10 \times 2) \times 4 \\
 &= 10 \times (2 \times 4) \\
 &= 10 \times 8 \\
 &= 80
 \end{aligned}$$

Gabriella breaks the 20×4 into $10 \times 2 \times 4$. Then she moves the $()$ over to 2×4 . This makes the problem easier to solve. Instead of thinking of the problem as 20×4 , she can solve by thinking of an easier fact, 10×8 .

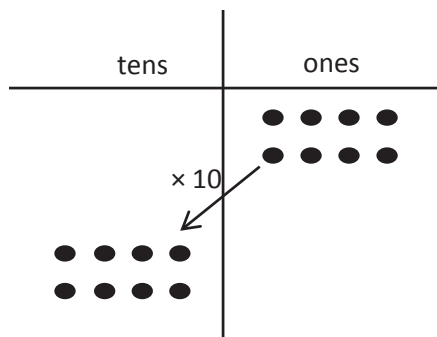
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____

Date _____

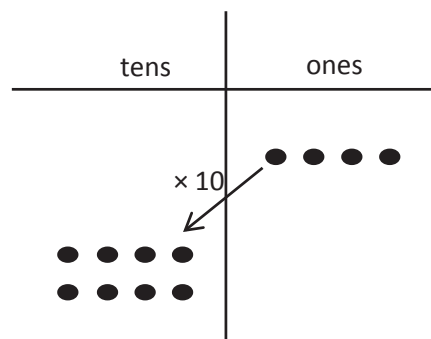
1. Use the chart to complete the equations. Then, solve. The first one has been done for you.



a. $(2 \times 4) \times 10$

$= (8 \text{ ones}) \times 10$

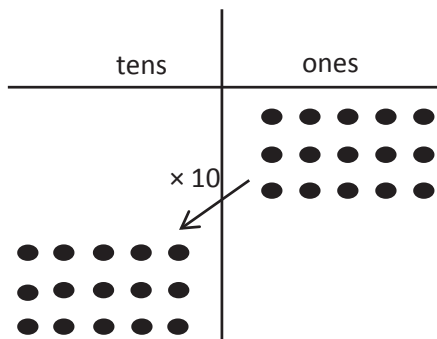
$= \underline{80}$



b. $2 \times (4 \times 10)$

$= 2 \times (4 \text{ tens})$

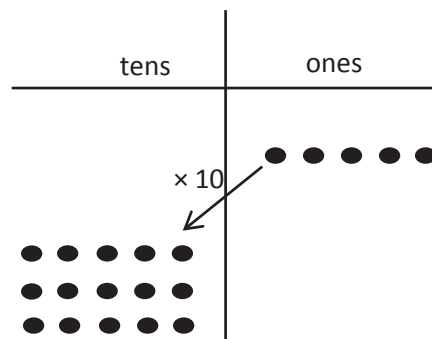
$= \underline{\hspace{2cm}}$



c. $(3 \times 5) \times 10$

$= (\underline{\hspace{1cm}} \text{ ones}) \times 10$

$= \underline{\hspace{2cm}}$

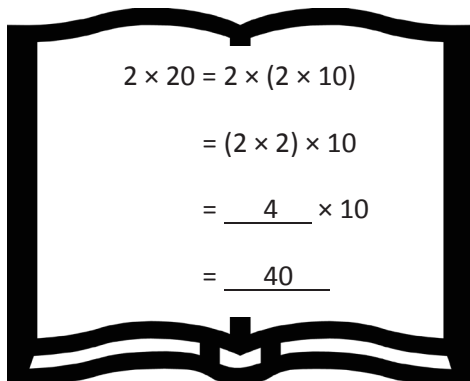


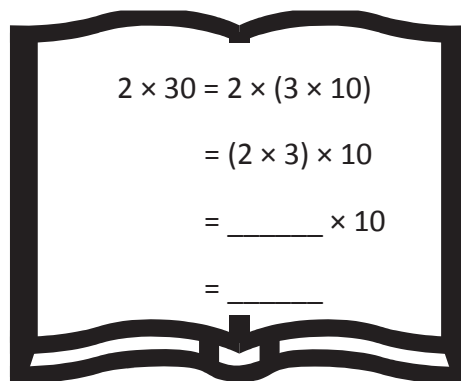
d. $3 \times (5 \times 10)$

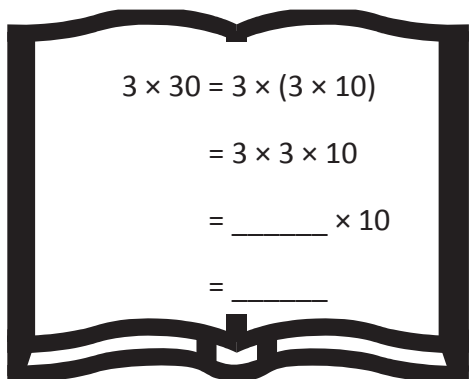
$= 3 \times (\underline{\hspace{1cm}} \text{ tens})$

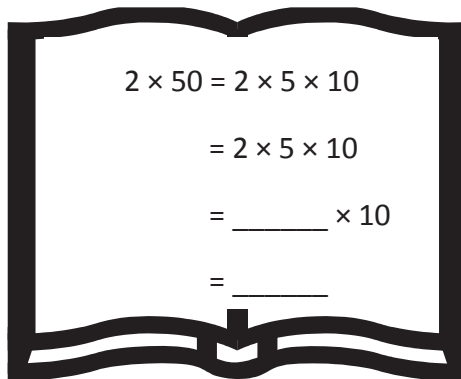
$= \underline{\hspace{2cm}}$

2. Place parentheses in the equations to find the related fact. Then, solve. The first one has been done for you.


$$\begin{aligned}2 \times 20 &= 2 \times (2 \times 10) \\&= (2 \times 2) \times 10 \\&= \underline{4} \times 10 \\&= \underline{40}\end{aligned}$$


$$\begin{aligned}2 \times 30 &= 2 \times (3 \times 10) \\&= (2 \times 3) \times 10 \\&= \underline{\quad} \times 10 \\&= \underline{\quad}\end{aligned}$$


$$\begin{aligned}3 \times 30 &= 3 \times (3 \times 10) \\&= 3 \times 3 \times 10 \\&= \underline{\quad} \times 10 \\&= \underline{\quad}\end{aligned}$$


$$\begin{aligned}2 \times 50 &= 2 \times 5 \times 10 \\&= 2 \times 5 \times 10 \\&= \underline{\quad} \times 10 \\&= \underline{\quad}\end{aligned}$$

3. Gabriella solves 20×4 by thinking about 10×8 . Explain her strategy.

Name _____ Date _____

1. Place parentheses in the equations to find the related fact. Then, solve.

a. $4 \times 20 = 4 \times 2 \times 10$

$= 4 \times 2 \times 10$

$= \underline{\hspace{2cm}} \times 10$

$= \underline{\hspace{2cm}}$

b. $3 \times 30 = 3 \times 3 \times 10$

$= 3 \times 3 \times 10$

$= \underline{\hspace{2cm}} \times 10$

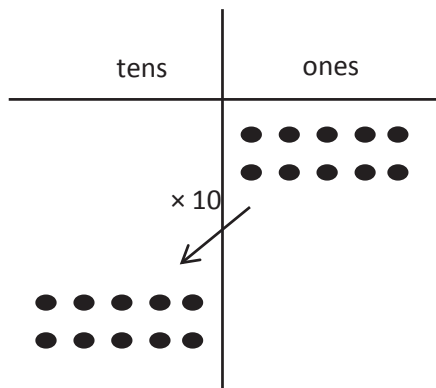
$= \underline{\hspace{2cm}}$

2. Jamila solves 20×5 by thinking about 10 tens. Explain her strategy.

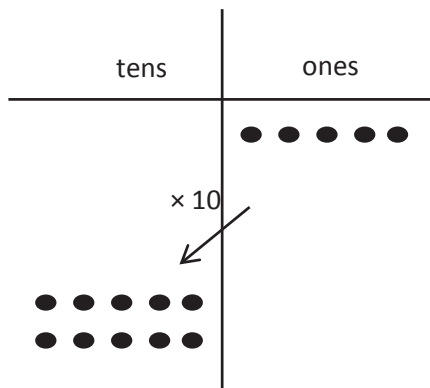
Name _____

Date _____

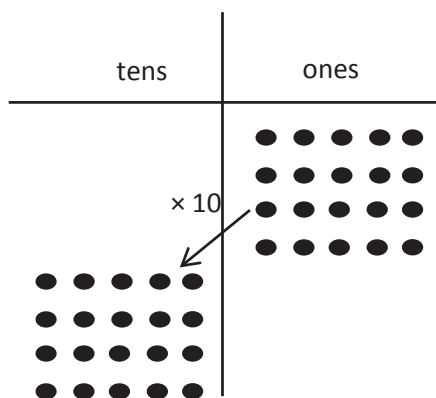
1. Use the chart to complete the equations. Then, solve.



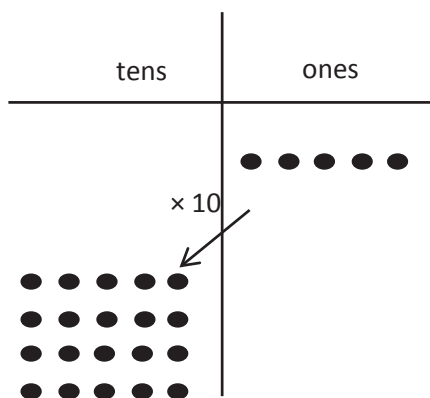
a. $(2 \times 5) \times 10$
 $= (10 \text{ ones}) \times 10$
 $=$ _____



b. $2 \times (5 \times 10)$
 $= 2 \times (5 \text{ tens})$
 $=$ _____



c. $(4 \times 5) \times 10$
 $= (\text{_____ ones}) \times 10$
 $=$ _____



d. $4 \times (5 \times 10)$
 $= 4 \times (\text{_____ tens})$
 $=$ _____

2. Solve. Place parentheses in (c) and (d) as needed to find the related fact.

a. $3 \times 20 = 3 \times (2 \times 10)$

$$= (3 \times 2) \times 10$$

$$= \underline{6} \times 10$$

$$= \underline{\quad}$$

b. $3 \times 30 = 3 \times (3 \times 10)$

$$= (3 \times 3) \times 10$$

$$= \underline{\quad} \times 10$$

$$= \underline{\quad}$$

c. $3 \times 40 = 3 \times (4 \times 10)$

$$= 3 \times 4 \times 10$$

$$= \underline{\quad} \times 10$$

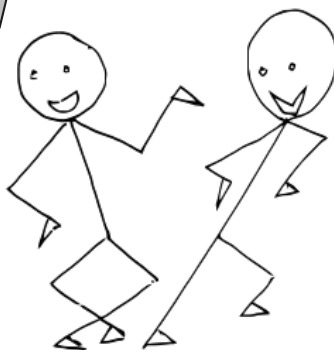
$$= \underline{\quad}$$

d. $3 \times 50 = 3 \times 5 \times 10$

$$= 3 \times 5 \times 10$$

$$= \underline{\quad} \times 10$$

$$= \underline{\quad}$$



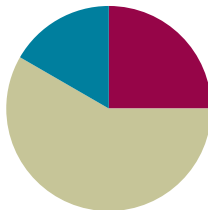
3. Danny solves 5×20 by thinking about 10×10 . Explain his strategy.

Lesson 21

Objective: Solve two-step word problems involving multiplying single-digit factors and multiples of 10.

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Concept Development	(35 minutes)
■ Student Debrief	(10 minutes)
Total Time	(60 minutes)



NOTES ON STANDARDS ALIGNMENT:

This lesson incorporates the use of seconds in both the Concept Development and Problem Set. Although seconds are included in the Grade 4 standard **4.MD.1**, students are familiar with them from Grade 3 Module 2 Lesson 1. In this lesson, the conversion between minutes and seconds is always provided when needed.

Fluency Practice (15 minutes)

- Sprint: Multiply by Multiples of 10 **3.NBT.3** (9 minutes)
- Group Counting **3.OA.1** (3 minutes)
- Write in the Parentheses **3.OA.7** (3 minutes)

Sprint: Multiply by Multiples of 10 (9 minutes)

Materials: (S) Multiply by Multiples of 10 Sprint

Note: This Sprint reviews Lesson 19, which involved multiplying single-digit numbers by multiples of 10.

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. These counts review multiplication taught previously in the module. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

Write in the Parentheses (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 20.

- T: (Write $2 \times 40 = 2 \times 4 \times 10$.) What's 2×40 ?
- S: 80.

$$2 \times 40 = 2 \times 4 \times 10$$

$$2 \times 40 = (2 \times 4) \times 10$$

$$2 \times 40 = 8 \times 10$$

$$2 \times 40 = 80$$

T: On your personal white board, copy the number sentence. Then, write in parentheses and solve.

S: (Write as shown in the box.)

Continue with the following possible sequence: $3 \times 30 = 3 \times 3 \times 10$ and $2 \times 50 = 2 \times 5 \times 10$.

Concept Development (35 minutes)

Materials: (T) Stopwatch, multiples of 10 multiplication cards (Template) (S) Personal white board

Place one card face down on each student's desk. At the prompt of "Go!," each student solves his or her problem. Students then line up as a class, ordering their products from least to greatest. Instruct students to complete these tasks silently and quickly. Let them know that they will be timed and that extra time will be added as a penalty if they are too noisy.

Template

$10 \times 6 =$	$9 \times 60 =$	$40 \times 2 =$	$10 \times 6 =$
$70 \times 3 =$	$50 \times 6 =$	$80 \times 9 =$	$20 \times 3 =$
$8 \times 30 =$	$3 \times 30 =$	$5 \times 30 =$	$4 \times 60 =$
$6 \times 80 =$	$70 \times 7 =$	$20 \times 7 =$	$10 \times 7 =$
$90 \times 7 =$	$2 \times 60 =$	$50 \times 7 =$	$80 \times 3 =$
$60 \times 6 =$	$9 \times 50 =$	$30 \times 9 =$	$4 \times 80 =$

T: It took you 4 minutes and 13 seconds to find the products and order them from least to greatest. How do we find the total number of seconds it took to complete this activity?

S: Add the total seconds in 4 minutes to 13 seconds. → We need to know how many seconds are in 1 minute first.

T: There are 60 seconds in 1 minute. Draw and label a tape diagram to show the total number of seconds in 4 minutes. Label the unknown as n . Then, check with a partner.

S: (Draw and label. Then, check with a partner.)

T: Write an equation. Then, solve.

S: (Write $4 \times 60 = n$, $n = 240$.)

T: Discuss with a partner the strategy you used to solve 4×60 .

T: (After discussion, call on some students to share.)

S: I thought of it as 4×6 tens, which equals 24 tens. And, 24 tens is 240. → I thought of it as $(4 \times 6) \times 10$, which is 24×10 , which equals 240. → It's like 24 tens is 10 tens + 10 tens + 4 tens or $100 + 100 + 40 = 240$.

T: Four minutes is equal to how many seconds?

S: 240 seconds.

T: Whisper the next step to your partner.

S: Add 13 seconds to 240 seconds.

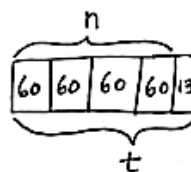
T: Add a unit of 13 to your diagram and label the total number of seconds using t for the unknown. Then, solve for t . How many seconds did it take you to complete the activity?

S: 253 seconds!



NOTES ON STANDARDS ALIGNMENT:

The time used in this problem, 4 minutes and 13 seconds, is an arbitrary time used to demonstrate how to solve this problem. Be sure to use the actual time it takes the class to complete the activity. Seconds are part of the Grade 4 standard **4.MD.1**.



$$\begin{aligned} 4 \times 60 &= n \\ n &= 240 \\ 240 + 13 &= t \\ t &= 253 \end{aligned}$$

There are 253 seconds in 4 minutes and 13 seconds.

MP.1

Project the following problems on the board and invite students to problem solve independently or in pairs using the RDW process:

- Each day Andrea does 25 squats to warm up for gymnastics practice and 15 squats to cool down after practice. How many squats does she do in all when she practices Monday through Friday?
- Benny gets \$5 a week for allowance. After saving his money for 20 weeks, how much more does Benny need to buy a bike that costs \$108?
- Genevieve makes 43 bracelets. She gives 13 bracelets away as gifts and sells the rest for \$4 each. How much money does Genevieve make in all?



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Give English language learners and others practice reading aloud the word problems on the Problem Set. To improve understanding, have students read the problems to their partners and paraphrase what the question asks them to find.

The above problems represent a variety of two-step word problem types and provide varied practice for the students.

Problem Set (15 minutes)

Students should do their personal best to complete the Problem Set within the allotted 15 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Solve two-step word problems involving multiplying single-digit factors and multiples of 10.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion

- In Problem 2, how many more months will Lupe need to save so she has enough to buy the art supplies? How do you know?

Name Gina Date _____

Use the RDW process to solve each problem. Use a letter to represent the unknown.

- There are 60 seconds in 1 minute. Use a tape diagram to find the total number of seconds in 5 minutes and 45 seconds.

$$5 \times 60 = n$$

$$n = 300$$

$$t = 300 + 45$$

$$t = 345$$

There are 345 seconds in 5 minutes and 45 seconds.
- Lupe saves \$30 each month for 4 months. Does she have enough money to buy the art supplies below? Explain why or why not.

$$4 \times \$30 = m$$

$$m = \$120$$

Lupe does not have enough money. She saved \$120, but she needs \$142. She needs \$22 more.
- Brad receives 5 cents for each can or bottle he recycles. How many cents does Brad earn if he recycles 48 cans and 32 bottles?

$$t = 48 \times 5$$

$$t = 240$$

$$m = 80 \times 5$$

$$m = 400$$

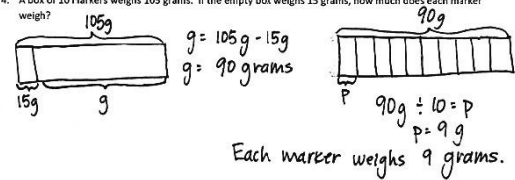
Brad earns 400 cents from recycling.

- In Problem 3, how many dollars does Brad earn? (Consider prompting students by asking how many cents are in 1 dollar.)
- Discuss the second step of Problem 4 with a partner. How was this different than the other problems? Explain how you could solve it with multiplication.
- Explain the three unknowns you needed to find to solve Problem 5.
- Explain to a partner how you solved Problem 6. Explain how you could have used the multiplying by 10 strategy to help solve this problem.

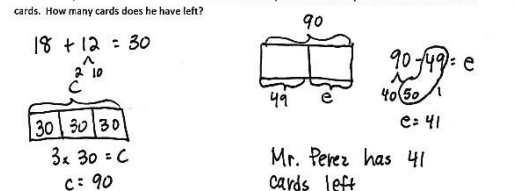
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

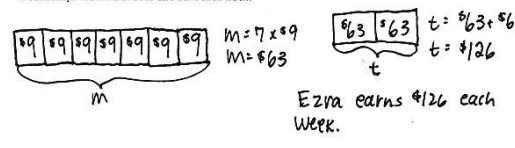
4. A box of 10 markers weighs 105 grams. If the empty box weighs 15 grams, how much does each marker weigh?



5. Mr. Perez buys 3 sets of cards. Each set comes with 18 striped cards and 12 polka dot cards. He uses 49 cards. How many cards does he have left?



6. Ezra earns \$9 an hour working at a book store. She works for 7 hours each day on Mondays and Wednesdays. How much does Ezra earn each week?



A

Number Correct: _____

Multiply by Multiples of 10

1.	$2 \times 3 =$	
2.	$2 \times 30 =$	
3.	$20 \times 3 =$	
4.	$2 \times 2 =$	
5.	$2 \times 20 =$	
6.	$20 \times 2 =$	
7.	$4 \times 2 =$	
8.	$4 \times 20 =$	
9.	$40 \times 2 =$	
10.	$5 \times 3 =$	
11.	$50 \times 3 =$	
12.	$3 \times 50 =$	
13.	$4 \times 4 =$	
14.	$40 \times 4 =$	
15.	$4 \times 40 =$	
16.	$6 \times 3 =$	
17.	$6 \times 30 =$	
18.	$60 \times 3 =$	
19.	$7 \times 5 =$	
20.	$70 \times 5 =$	
21.	$7 \times 50 =$	
22.	$8 \times 4 =$	

23.	$8 \times 40 =$	
24.	$80 \times 4 =$	
25.	$9 \times 6 =$	
26.	$90 \times 6 =$	
27.	$2 \times 5 =$	
28.	$2 \times 50 =$	
29.	$3 \times 90 =$	
30.	$40 \times 7 =$	
31.	$5 \times 40 =$	
32.	$6 \times 60 =$	
33.	$70 \times 6 =$	
34.	$8 \times 70 =$	
35.	$80 \times 6 =$	
36.	$9 \times 70 =$	
37.	$50 \times 6 =$	
38.	$8 \times 80 =$	
39.	$9 \times 80 =$	
40.	$60 \times 8 =$	
41.	$70 \times 7 =$	
42.	$5 \times 80 =$	
43.	$60 \times 9 =$	
44.	$9 \times 90 =$	

B

Number Correct: _____

Improvement: _____

Multiply by Multiples of 10

1.	$4 \times 2 =$	
2.	$4 \times 20 =$	
3.	$40 \times 2 =$	
4.	$3 \times 3 =$	
5.	$3 \times 30 =$	
6.	$30 \times 3 =$	
7.	$3 \times 2 =$	
8.	$3 \times 20 =$	
9.	$30 \times 2 =$	
10.	$5 \times 5 =$	
11.	$50 \times 5 =$	
12.	$5 \times 50 =$	
13.	$4 \times 3 =$	
14.	$40 \times 3 =$	
15.	$4 \times 30 =$	
16.	$7 \times 3 =$	
17.	$7 \times 30 =$	
18.	$70 \times 3 =$	
19.	$6 \times 4 =$	
20.	$60 \times 4 =$	
21.	$6 \times 40 =$	
22.	$9 \times 4 =$	

23.	$9 \times 40 =$	
24.	$90 \times 4 =$	
25.	$8 \times 6 =$	
26.	$80 \times 6 =$	
27.	$5 \times 2 =$	
28.	$5 \times 20 =$	
29.	$3 \times 80 =$	
30.	$40 \times 8 =$	
31.	$4 \times 50 =$	
32.	$8 \times 80 =$	
33.	$90 \times 6 =$	
34.	$6 \times 70 =$	
35.	$60 \times 6 =$	
36.	$7 \times 70 =$	
37.	$60 \times 5 =$	
38.	$6 \times 80 =$	
39.	$7 \times 80 =$	
40.	$80 \times 6 =$	
41.	$90 \times 7 =$	
42.	$8 \times 50 =$	
43.	$80 \times 9 =$	
44.	$7 \times 90 =$	

Name _____

Date _____

Use the RDW process to solve each problem. Use a letter to represent the unknown.

1. There are 60 seconds in 1 minute. Use a tape diagram to find the total number of seconds in 5 minutes and 45 seconds.

2. Lupe saves \$30 each month for 4 months. Does she have enough money to buy the art supplies below? Explain why or why not.



3. Brad receives 5 cents for each can or bottle he recycles. How many cents does Brad earn if he recycles 48 cans and 32 bottles?

4. A box of 10 markers weighs 105 grams. If the empty box weighs 15 grams, how much does each marker weigh?
5. Mr. Perez buys 3 sets of cards. Each set comes with 18 striped cards and 12 polka dot cards. He uses 49 cards. How many cards does he have left?
6. Ezra earns \$9 an hour working at a book store. She works for 7 hours each day on Mondays and Wednesdays. How much does Ezra earn each week?

Name _____

Date _____

Use the RDW process to solve. Use a letter to represent the unknown.

Frederick buys a can of 3 tennis balls. The empty can weighs 20 grams, and each tennis ball weighs 60 grams. What is the total weight of the can with 3 tennis balls?

Name _____

Date _____

Use the RDW process for each problem. Use a letter to represent the unknown.

1. There are 60 minutes in 1 hour. Use a tape diagram to find the total number of minutes in 6 hours and 15 minutes.
2. Ms. Lemus buys 7 boxes of snacks. Each box has 12 packets of fruit snacks and 18 packets of cashews. How many snack packets does she buy altogether?
3. Tamara wants to buy a tablet that costs \$437. She saves \$50 a month for 9 months. Does she have enough money to buy the tablet? Explain why or why not.

4. Mr. Ramirez receives 4 sets of books. Each set has 16 fiction books and 14 nonfiction books. He puts 97 books in his library and donates the rest. How many books does he donate?
5. Celia sells calendars for a fundraiser. Each calendar costs \$9. She sells 16 calendars to her family members and 14 calendars to the people in her neighborhood. Her goal is to earn \$300. Does Celia reach her goal? Explain your answer.
6. The video store sells science and history movies for \$5 each. How much money does the video store make if it sells 33 science movies and 57 history movies?

$30 \times 6 =$	$9 \times 60 =$	$40 \times 2 =$	$10 \times 6 =$
$70 \times 3 =$	$50 \times 6 =$	$80 \times 9 =$	$20 \times 5 =$
$8 \times 30 =$	$3 \times 30 =$	$5 \times 50 =$	$4 \times 40 =$
$6 \times 80 =$	$70 \times 7 =$	$20 \times 7 =$	$10 \times 7 =$
$90 \times 7 =$	$2 \times 60 =$	$50 \times 7 =$	$80 \times 5 =$
$60 \times 6 =$	$9 \times 50 =$	$30 \times 9 =$	$4 \times 80 =$

multiples of 10 multiplication cards

Name _____

Date _____

1. Aunt Korina and her 3 friends decide to share a cab to go to the mall. If they each spent \$6, how much did the cab ride cost altogether? Write an equation using a letter to represent the unknown. Solve.

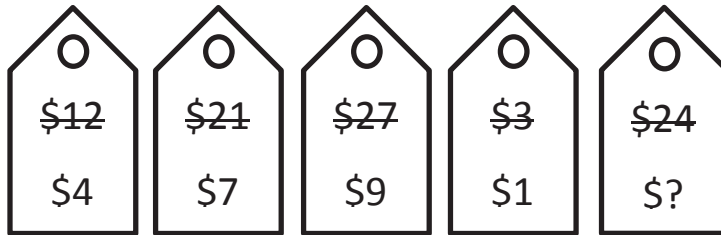
2. Aunt Korina's 3 friends each order pasta and a lemonade for lunch. Aunt Korina orders only chicken salad.

- a. Use the menu to find how much they spend altogether.
Write equations using letters to represent the unknown. Solve.

Lunch Menu	
Pasta	\$7
Chicken Salad	\$9
Lemonade	\$2

- b. Aunt Korina mentally checks the total using $4 \times \$9$. Explain her strategy.

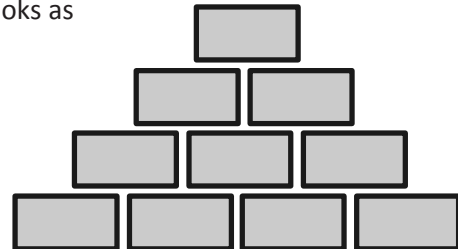
3. After lunch, the friends notice a sale. Compare the crossed out prices to the new sale prices. If all sale prices are calculated in the same way, what would the sale price be on an item that originally cost \$24? Use words and equations to explain how you know.



4. a. A shopkeeper in the bookstore arranges the boxed sets of books as shown to the right.

If each box contains 9 books, how many books are there?

- Write an equation using a letter to represent the unknown, and then solve.
- Explain how you know your answer is reasonable.

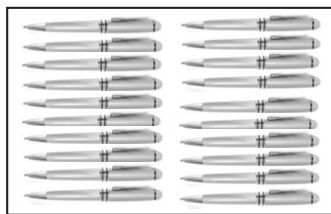


- b. Aunt Korina figures out how many books are in the arrangement. Her work is shown below. Explain Aunt Korina's strategy.

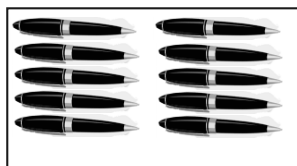
$$10 \times 10 - 10 = 90$$

- c. In the book store, Aunt Korina buys 3 boxes of pens. Each box contains 2 bundles of 10 gray pens. Her friend buys 6 packs of pens. Each pack contains 10 black pens. Explain how the equation below shows how Aunt Korina and her friend buy the same number of pens.

$$6 \times 10 = 3 \times 2 \times 10$$



Box of Gray Pens



Pack of Black Pens

5. Complete as many problems as you can in 100 seconds. The teacher will time you and tell you when to stop.

$2 \times 1 = \underline{\quad}$ $4 \div 2 = \underline{\quad}$ $\underline{\quad} = 10 \div 5$ $3 \times 3 = \underline{\quad}$ $2 \times \underline{\quad} = 4$

$\underline{\quad} \times 6 = 12$ $21 \div 7 = \underline{\quad}$ $8 \times \underline{\quad} = 24$ $\underline{\quad} = 9 \times 3$ $\underline{\quad} = 30 \div 10$

$5 \times 3 = \underline{\quad}$ $8 \div 2 = \underline{\quad}$ $\underline{\quad} \times 3 = 12$ $\underline{\quad} = 16 \div 4$ $6 \times 4 = \underline{\quad}$

$9 \times 4 = \underline{\quad}$ $7 \times \underline{\quad} = 35$ $40 \div 8 = \underline{\quad}$ $\underline{\quad} = 3 \times 5$ $\underline{\quad} \times 4 = 20$

$7 \times \underline{\quad} = 35$ $\underline{\quad} = 54 \div 9$ $\underline{\quad} \times 6 = 36$ $8 \times 6 = \underline{\quad}$ $24 \div 4 = \underline{\quad}$

$9 \times 6 = \underline{\quad}$ $\underline{\quad} = 49 \div 7$ $8 \times \underline{\quad} = 56$ $\underline{\quad} = 6 \times 7$ $21 \div 3 = \underline{\quad}$

$7 \times 7 = \underline{\quad}$ $\underline{\quad} \times 9 = 63$ $\underline{\quad} = 64 \div 8$ $6 \times \underline{\quad} = 48$ $\underline{\quad} = 4 \times 8$

$24 \div 3 = \underline{\quad}$ $81 \div 9 = \underline{\quad}$ $63 \div 7 = \underline{\quad}$ $8 \times 9 = \underline{\quad}$ $9 \times \underline{\quad} = 81$

End-of-Module Assessment Task Standards Addressed

Topics A–F

Represent and solve problems involving multiplication and division.

- 3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$*

Understand properties of multiplication and the relationship between multiplication and division.

- 3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

Multiply and divide within 100.

- 3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

- 3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)
- 3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)

- 3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop on their way to proficiency. In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next. Problem 5 is scored differently since it is a timed assessment of fluency. Students complete as many problems as they can in two minutes. Although this page of the assessment contains 40 questions, answering 30 correct within the time limit is considered passing.

A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 3.OA.3 3.OA.4	Student is unable to write and solve an equation.	Student writes a multiplication equation using the incorrect factors and without a letter to represent the unknown.	Student writes a multiplication equation using a letter to represent the unknown but calculates an incorrect answer (e.g., $4 \times \$6 = n$; $n = \text{wrong answer}$).	Student correctly writes and solves $4 \times \$6 = n$; $n = \$24$.
2 3.OA.3 3.OA.4 3.OA.8	Student is unable to answer either question correctly.	Student attempts to solve Part (a) but does not use letters to represent the unknown.	Student writes correct equations and solves for the unknown in Part (a) (e.g., $F = \text{the amount in dollars spent by Aunt Korina's friends}$, $K = \text{the amount in dollars spent by Aunt Korina}$, $F + K = \$27 + \$9 = \$36$), but provides inaccurate explanation in Part (b).	Student correctly: <ul style="list-style-type: none"> Writes equations using letters to represent the unknown (e.g., $F = \text{the amount in dollars spent by Aunt Korina's friends}$, $K = \text{the amount in dollars spent by Aunt Korina}$, $F + K = \\$27 + \\$9 = \\$36$). Finds the total, \$36, in Part (a). Provides accurate explanation of strategy in Part (b).



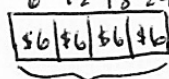
A Progression Toward Mastery

3 3.OA.9	<p>Student is unable to find and explain the pattern.</p>	<p>Student attempts to find and explain the pattern.</p>	<p>Student understands how the sale prices are calculated but incorrectly finds the sale price of \$24 (e.g., $\\$24 \div 3 =$ wrong answer).</p>	<p>Student clearly:</p> <ul style="list-style-type: none"> Explains the sale prices are calculated by dividing the original price by 3. Writes $\\$24 \div 3 = \\8.
4 3.OA.3 3.OA.4 3.OA.5 3.OA.8 3.OA.9 3.NBT.3	<p>Student is unable to answer any question correctly.</p>	<p>Student answers one question correctly.</p>	<p>Student answers two questions correctly. Mistakes may include:</p> <ul style="list-style-type: none"> Providing inaccurate explanations in Parts (a) or (b). Inability to explain why the equations are equal in Part (c). 	<p>Student clearly:</p> <ul style="list-style-type: none"> Writes $10 \times 9 = n$; $n = 90$ books. Provides accurate explanation of the equation in Part (a). Provides accurate explanation of the $9 = 10 - 1$ pattern in Part (b). Provides accurate explanation of the associative property of multiplication in Part (c) to find 60 pens altogether.
5 3.OA.7	<p>Use the attached sample work to correct students' answers on the fluency page of the assessment.</p> <p>Students who answer 30 or more questions correctly within the allotted time pass this portion of the assessment. For students who do not pass, consider re-administering this fluency page with each subsequent End-of-Module Assessment until they are successful.</p> <p>Analyze the mistakes students make on this assessment to further guide fluency instruction. Below are possible questions to ask as part of this analysis:</p> <ul style="list-style-type: none"> Did this student struggle with multiplication, division, or both? Did this student struggle with a particular factor? Did the student consistently miss problems with the unknown in a particular position? 			

Name Gina

Date _____

1. Aunt Korina and her 3 friends decide to share a cab to go to the mall. If they each spent \$6, how much did the cab ride cost altogether? Write an equation using a letter to represent the unknown. Solve.

$6 \quad 12 \quad 18 \quad 24$

 $4 \times \$6 = n$
 $n = \$24$
 $n = \text{total cost of the cab ride.}$
 The cab ride costs \$24.

2. Aunt Korina's 3 friends each order pasta and a lemonade for lunch. Aunt Korina orders only chicken salad.

- a. Use the menu to find how much they spend altogether. Write equations using letters to represent the unknown. Solve.

$F = \text{the amount spent by Aunt Korina's friends (in \$)}$
 $F = \underbrace{\$9 \quad \$9 \quad \$9}_{? \quad F = \$27}$

$K = \text{the amount spent by Aunt Korina (in \$)}$
 $K = \$9$

$F + K = \$27 + \9
 $= \$36$

They spend \$36 altogether.

Lunch Menu	
Pasta	\$7
Chicken Salad	\$9
Lemonade	\$2

$\$7 + \$2 = \$9$

- b. Aunt Korina mentally checks the total using $4 \times \$9$. Explain her strategy.

3 friends each spent \$9. Aunt Korina also spent \$9. So, they spend $4 \times \$9$ altogether. The tape diagrams above also show 4 units of \$9.

3. After lunch, the friends notice a sale. Compare the crossed out prices to the new sale prices. If all sale prices are calculated in the same way, what would the sale price be on an item that originally cost \$24? Use words and equations to explain how you know.

\$12 \$3 ↓ \$4	\$21 \$3 ↓ \$7	\$27 \$3 ↓ \$9	\$3 \$3 ↓ \$1	\$24 \$3 ↓ \$?
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$\$24 \div 3 = \8

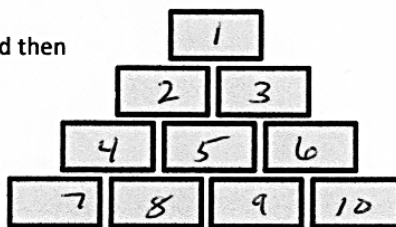
The sale price is \$8. The sale price is found by dividing the original price by 3.

4. a. A shopkeeper in the bookstore arranges the boxed sets of books as shown to the right. If each box contains 9 books, how many books are there?

- Write an equation using a letter to represent the unknown, and then solve.
- Explain how you know your answer is reasonable.

$$10 \times 9 = n \quad n = \text{total number of books.}$$

$$n = 90$$



Since there are 10 boxed sets of books and 9 books in each set, I had to multiply 10×9 to find the total number of books, 90.

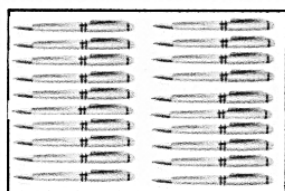
- b. Aunt Korina figures out how many books are in the arrangement. Her work is shown below. Explain Aunt Korina's strategy.

$$10 \times 10 - 10 = 90$$

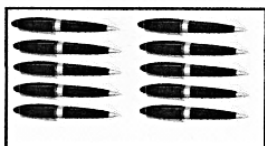
Aunt Korina found 10 tens, and then subtracted 1 ten to get 9 tens. 9 tens is 9×10 , which is the same as 10×9 .

- c. In the book store, Aunt Korina buys 3 boxes of pens. Each box contains 2 bundles of 10 gray pens. Her friend buys 6 packs of pens. Each pack contains 10 black pens. Explain how the equation below shows how Aunt Korina and her friend buy the same number of pens.

$$\text{Black pens} \rightarrow 6 \times 10 = (3 \times 2) \times 10 \leftarrow \text{Gray pens}$$



Box of gray pens



Pack of black pens

$3 \times 2 \times 10$ represents Korina's total gray pens. 6×10 represents her friend's total pens. We know they buy the same number of pens because $3 \times 2 = 6$, so $3 \times 2 \times 10$ is the same as 6×10 . They both have 60 pens.

5. Complete as many problems as you can in 100 seconds. The teacher will time you and tell you when to stop.

$2 \times 1 = 2 \quad 4 \div 2 = 2 \quad 2 = 10 \div 5 \quad 3 \times 3 = 9 \quad 2 \times 2 = 4$

$2 \times 6 = 12 \quad 21 \div 7 = 3 \quad 8 \times 3 = 24 \quad 27 = 9 \times 3 \quad 3 = 30 \div 10$

$5 \times 3 = 15 \quad 8 \div 2 = 4 \quad 4 \times 3 = 12 \quad 4 = 16 \div 4 \quad 6 \times 4 = 24$

$9 \times 4 = 36 \quad 7 \times 5 = 35 \quad 40 \div 8 = 5 \quad 15 = 3 \times 5 \quad 5 \times 4 = 20$

$7 \times 5 = 35 \quad 6 = 54 \div 9 \quad 6 \times 6 = 36 \quad 8 \times 6 = 48 \quad 24 \div 4 = 6$

$9 \times 6 = 54 \quad 7 = 49 \div 7 \quad 8 \times 7 = 56 \quad 42 = 6 \times 7 \quad 21 \div 3 = 7$

$7 \times 7 = 49 \quad 7 \times 9 = 63 \quad 8 = 64 \div 8 \quad 6 \times 8 = 48 \quad 32 = 4 \times 8$

$24 \div 3 = 8 \quad 81 \div 9 = 9 \quad 63 \div 7 = 9 \quad 8 \times 9 = 72 \quad 9 \times 9 = 81$



Answer Key

GRADE 3 • MODULE 3

Multiplication and Division with Units of 0, 1, 6–9, and
Multiples of 10

Lesson 1

Sprint

Side A

1. 2	12. 6	23. 14	34. 16
2. 4	13. 9	24. 25	35. 18
3. 6	14. 5	25. 30	36. 35
4. 4	15. 10	26. 35	37. 40
5. 8	16. 15	27. 20	38. 45
6. 12	17. 7	28. 24	39. 28
7. 6	18. 14	29. 28	40. 32
8. 12	19. 9	30. 15	41. 36
9. 8	20. 18	31. 18	42. 21
10. 16	21. 10	32. 21	43. 24
11. 3	22. 12	33. 14	44. 27

Side B

1. 5	12. 4	23. 35	34. 40
2. 10	13. 6	24. 10	35. 45
3. 15	14. 4	25. 12	36. 14
4. 3	15. 8	26. 14	37. 16
5. 6	16. 12	27. 15	38. 18
6. 9	17. 6	28. 18	39. 21
7. 7	18. 12	29. 21	40. 24
8. 14	19. 8	30. 20	41. 27
9. 9	20. 16	31. 24	42. 28
10. 18	21. 25	32. 28	43. 32
11. 2	22. 30	33. 35	44. 36

Problem Set

1.
 - a. Answers will vary.
 - b. 14; 3; 28; 35; 6
2. 24, 4, 6; 24, 6, 4
3.
 - a. 7
 - b. sixes; 18
 - c. tens; 80
 - d. 6; 24
 - e. 5; 40
 - f. 4; 4
 - g. 1; 27
 - h. 9; 36
 - i. 3; 32
 - j. 5; 30
 - k. 7; 3; 21
 - l. 5; 5; 20

Exit Ticket

1. 28, 4, 7; 28, 7, 4
2. Explanations will vary.

Homework

1.
 - a. 9; 4; 15; 6; 21
 - b. 5; 6; 28; 32; 36
 - c. 30; 7; 8; 9; 50
2. 24, 4, 6; 24, 6, 4
24, 3, 8; 24, 8, 3
3. Expressions accurately matched
4.
 - a. 6
 - b. 3; 18
 - c. 8; 32
 - d. 7; 7
 - e. 7, 2; 14
 - f. 5; 30

Lesson 2

Sprint

Side A

1. 4	12. 16	23. 30	34. 12
2. 6	13. 16	24. 30	35. 12
3. 6	14. 18	25. 35	36. 18
4. 8	15. 18	26. 35	37. 18
5. 8	16. 20	27. 40	38. 21
6. 10	17. 20	28. 40	39. 21
7. 10	18. 15	29. 45	40. 24
8. 12	19. 15	30. 45	41. 24
9. 12	20. 20	31. 50	42. 27
10. 14	21. 20	32. 50	43. 27
11. 14	22. 25	33. 9	44. 16

Side B

1. 10	12. 40	23. 12	34. 12
2. 10	13. 40	24. 12	35. 12
3. 15	14. 45	25. 14	36. 18
4. 15	15. 45	26. 14	37. 18
5. 20	16. 50	27. 16	38. 21
6. 20	17. 50	28. 16	39. 21
7. 25	18. 4	29. 18	40. 24
8. 30	19. 6	30. 18	41. 24
9. 30	20. 6	31. 20	42. 27
10. 35	21. 8	32. 20	43. 27
11. 35	22. 8	33. 9	44. 16

Problem Set

1. Sevens; 7, 7; 35
5, 1; 7; 42; 6, 7, 42; 7, 6, 42
2. a. Eights; 8, 8; 40
b. 48; answers will vary.
3. 63
4. 4
5. No; explanations will vary.

Exit Ticket

42; answers will vary.

Homework

1. 5 nines; 9, 9; 45
5, 1; 9; 54; 6, 9, 54; 9, 6, 54
2. 42
3. 6
4. 3

Lesson 3

Problem Set

- $e = 20$; $\ell = 7$; $i = 6$; $c = 3$; $s = 4$; $n = 10$; $t = 70$; $k = 9$; $b = 2$; $a = 24$; $h = 5$; kitchen tables
- $m = \$24$
 - $c = \$6$
- 4, n , 28; 28, 4, n ; $n = 7$; 7 loaves of bread
- Shorter game: 10 minutes; longer game: 22 minutes

Exit Ticket

- 45
- 5
- 3
- 28
- 3, n , 15; 15, 3, n ; $n = 5$; 5 rose bushes

Homework

- 40, 50, 70, 80, 100
 - $e = 30$; $f = 40$; $p = 50$; $w = 60$; $n = 70$; $g = 80$
- $n = 4$; $a = 4$; $p = 5$; $c = 3$; $d = 6$; $h = 35$; $f = 18$; $y = 8$
- $b = \$28$
 - $c = \$2$; answers will vary.
- 50 m; answers will vary.

Lesson 4

Problem Set

- 12, 24, 42, 54; each number matched to its corresponding multiplication fact
- 12, 18, 24; 4, 24; 24, 4
- 12, 18, 24, 30, 36, 42; 7, 42; 42, 7
- 12, 24, 18, 18, 36, 18, 30, 42
 - 8; 8
- No; explanations will vary.

Exit Ticket

- 54; explanations will vary.
- 48
 - 9

Homework

- 12
 - 18
 - 20, 4, 24
 - 20, 10, 30
 - 36
 - 40, 2, 42
 - Answers will vary; 48
 - Answers will vary; 54
 - Answers will vary; 60
- 12, 18, 24, 30; 5, 30; 30, 5
- 12, 18, 24, 30, 36; 6, 36; 36, 6
- 8; answers will vary.

Lesson 5

Pattern Sheet

6	12	18	24
30	6	12	6
18	6	24	6
30	6	12	18
12	24	12	30
12	6	12	18
6	18	12	18
24	18	30	18
24	6	24	12
24	18	24	30
24	30	6	30
12	30	18	30
24	12	24	18
30	18	12	24
18	30	12	24

Problem Set

- 14, 28, 35, 56, 63, 70
42, 6; 21, 3; 56, 8; 49, 7; 7, 1; 70, 10; 63, 9; 28, 4; 14, 2; 35, 5
- 21, 35, 49, 56, 70
 - 3, 21; 21, 3
 - 5, 35; 35, 5
 - 7, 49; 49, 7
 - 8, 56; 56, 8
 - 10, 70; 70, 10
- Explanations will vary.
- Both are correct; explanations will vary.

Exit Ticket

21, 35, 49, 56, 70

- a. 1, 7; 7, 1
- b. 2, 14; 14, 2
- c. 3, 21; 21, 3
- d. 4, 28; 28, 4
- e. 5, 35; 35, 5
- f. 6, 42; 42, 6
- g. 7, 49; 49, 7
- h. 8, 56; 56, 8
- i. 9, 63; 63, 9
- j. 10, 70; 70, 10

Homework

- 1.
 - a. 14
 - b. 20, 1, 21
 - c. 20, 8, 28
 - d. 30, 5, 35
 - e. 40, 2, 42
 - f. 40, 9, 49
 - g. 50, 6, 56; answers may vary.
 - h. 60, 3, 63; answers may vary.
- 2. 70, 63, 56, 42, 35, 21, 14
70, 63, 56, 49, 42, 35, 28, 21, 14, 7
70, 10; 63, 9; 56, 8; 49, 7; 42, 6; 35, 5; 28, 4; 21, 3; 14, 2; 7, 1

Lesson 6

Pattern Sheet

6	12	18	24
30	36	42	48
54	60	30	36
30	42	30	48
30	54	30	60
36	30	36	42
36	48	36	54
36	42	36	42
48	42	54	42
48	36	48	42
48	54	54	36
54	42	54	48
54	48	36	54
42	54	36	48
54	42	36	48

Problem Set

- 36; 30; 1, 6; 6; 36
 - 42; 30; 2, 12; 12; 42
 - 48; 30; 3, 18; 3; 3; 18; 48
 - 54; 30; 4, 24; 4; 4; 24; 54
- 24; 4; 9
- $14 \div 7$; 14; 2; 7
- Yes; explanations will vary.
- Answers will vary.

Exit Ticket

- 8; answers will vary.
- Both are correct; explanations will vary.

Homework

1.
 - a. Tape diagrams accurately labeled; 42; 35; 1, 7; 7, 42
 - b. Tape diagrams accurately labeled; 49; 35; 2, 14; 14, 49
 - c. Tape diagrams accurately labeled; 56; 35; 3, 21; 3; 3; 21; 56
 - d. Tape diagrams accurately labeled; 63; 35; 4, 28; 4; 4; 28; 63
2. 24; 24; 4; 9
3. 21; 35, 7; 21, 7; 3; 8
4. 7; explanations will vary.
5. Yes; explanations will vary.

Lesson 7

Pattern Sheet

7	14	21	28
35	7	14	7
21	7	28	7
35	7	14	21
14	28	14	35
14	7	14	21
7	21	14	21
28	21	35	21
28	7	28	14
28	21	28	35
28	35	7	35
14	35	21	35
28	14	28	21
35	21	14	28
21	35	14	28

Problem Set

- Words matched to corresponding equations
- $6 \times 8 = k$ or $8 \times 6 = k$; $k = 48$
- Picture models equation; 7
 - Picture models equation; 4 minutes
 - Picture models equation; 48 cm
 - Picture models equation; 9

Exit Ticket

- 42; equations may vary.
- \$8; equations may vary.

Homework

1. Words matched to corresponding equations
2.
 - a. $m = \$42$; tape diagram drawn and labeled; equations may vary.
 - b. $p = 36$; tape diagram drawn and labeled; equations may vary.
3. $n = 4$; tape diagram drawn and labeled; equations may vary.

Lesson 8

Pattern Sheet

7	14	21	28
35	42	49	56
63	70	35	42
35	49	35	56
35	63	35	70
42	35	42	49
42	56	42	63
42	49	42	49
56	49	63	49
56	42	56	49
56	63	63	42
63	49	63	56
63	56	42	63
49	63	42	56
63	49	42	56

Problem Set

1.
 - a. 14
 - b. 2
 - c. 5
 - d. 11
 - e. 30
 - f. 15
 - g. 20
 - h. 26
 - i. 10
 - j. 2
 - k. 14
 - l. 8
 - m. 10
 - n. 2
 - o. 37
 - p. 9
2.
 - a. $(16 - 4) + 7 = 19$
 - b. $16 - (4 + 7) = 5$
 - c. $2 = 22 - (15 + 5)$
 - d. $12 = (22 - 15) + 5$
 - e. $(3 + 7) \times 6 = 60$
 - f. $3 + (7 \times 6) = 45$
 - g. $5 = (10 \div 10) \times 5$
 - h. $50 = (100 \div 10) \times 5$
 - i. $(26 - 5) \div 7 = 3$
 - j. $36 = 4 \times (25 - 16)$
3. Chad used $(24 \div 4) + 2 = 8$;
Samir used $24 \div (4 + 2) = 4$.
4. $12 + (15 \div 3) = 17$
5. 13; 20

Exit Ticket

1.
 - a. $24 = (32 - 14) + 6$
 - b. $12 = 32 - (14 + 6)$
 - c. $(2 + 8) \times 7 = 70$
 - d. $2 + (8 \times 7) = 58$
2. Marcos used $(24 \div 6) + 2 = 6$; Iris used $24 \div (6 + 2) = 3$.

Homework

1.
 - a. 0
 - b. 6
 - c. 8
 - d. 12
 - e. 42
 - f. 22
 - g. 12
 - h. 2
2.
 - a. $14 - (8 + 2) = 4$
 - b. $(14 - 8) + 2 = 8$
 - c. $2 + (4 \times 7) = 30$
 - d. $(2 + 4) \times 7 = 42$
 - e. $12 = (18 \div 3) \times 2$
 - f. $3 = 18 \div (3 \times 2)$
 - g. $5 = 50 \div (5 \times 2)$
 - h. $20 = (50 \div 5) \times 2$
3.
 - a. Answer provided
 - b. True
 - c. False
 - d. True
 - e. False
4. Explanations may vary.
5. $(4 \times 7) - 3 = 25$
6. Agree; answers will vary.

Lesson 9

Application Problems

- | | | | |
|----|---------|----|-------|
| 1. | a. 17 | 5. | a. 25 |
| | b. 17 | | b. 13 |
| | Circled | 6. | a. 8 |
| 2. | a. 24 | | b. 2 |
| | b. 24 | 7. | a. 7 |
| | Circled | | b. 1 |
| 3. | a. 10 | 8. | a. 36 |
| | b. 10 | | b. 8 |
| | Circled | | |
| 4. | a. 16 | | |
| | b. 16 | | |
| | Circled | | |

Problem Set

- | | | | |
|----|-------------------|----|-------------------------|
| 1. | a. 36 | 2. | a. Answer provided. |
| | b. 9; 36 | | b. 4; 28 |
| | c. 42 | | c. 9, 4; 36 |
| | d. 3, 2; 6, 7; 42 | | d. 6, 7; 42 |
| | | | e. 5, 9; 45 |
| | | | f. 5, 6; 30 |
| | | 3. | Explanations will vary. |

Exit Ticket

54; explanations will vary.

Homework

1.
 - a. 48
 - b. 2; 6, 8; 48
 - c. 72
 - d. 2; 8, 9; 72
2.
 - a. 6, 42
 - b. 9, 36
3.
 - a. Answer provided.
 - b. $60; 6 \times (5 \times 2)$
 - c. $70; 7 \times (5 \times 2)$
 - d. $80; 8 \times (5 \times 2)$

Lesson 10

Problem Set

1. a. Array accurately labeled; 64; 40; 3, 24; 3; 3; 24; 64
b. Array accurately labeled; 72; 40; 4, 32; 4; 4; 32; 72
2. 16; 2; 7
3. $32 \div 8$; 32; 4; 9
4. 24, 32, 40, 48, 56, 64, 72; 72
5. Answer provided; 48; 24; 80; 64; 56
6. Answer provided; 4; 2; 8; 6; 9

Exit Ticket

56; strategy accurately used to solve

Homework

1. Array accurately labeled; 56; 35; 3, 21; 3; 3; 21; 56
2. 32; 4; 9
3. 16, 24, 32, 40, 48, 56, 64, 72, 80; 72, 40, 64, 48, 56
4. 2; 5; 4; 6; 7; 9

Lesson 11

Pattern Sheet

8	16	24	32
40	8	16	8
24	8	32	8
40	8	16	24
16	32	16	40
16	8	16	24
8	24	16	24
32	24	40	24
32	8	32	16
32	24	32	40
32	40	8	40
16	40	24	40
32	16	32	24
40	24	16	32
24	40	16	32

Problem Set

1. Tape diagram drawn and labeled; $n = 4$
2. Tape diagram drawn and labeled; $m = \$48$
3. Tape diagram drawn and labeled; $c = 3$
4. 5
5. 21
6. \$36

Exit Ticket

- a. Tape diagram drawn and labeled; $p = 7$
- b. 38

Homework

1. Tape diagram drawn and labeled; $c = 70$
2. Tape diagram drawn and labeled; $v = 6$
3. $m = 7$
4. 54
5. 10
6. \$18

Lesson 12

Pattern Sheet

8	16	24	32
40	48	56	64
72	80	40	48
40	56	40	64
40	72	40	80
48	40	48	56
48	64	48	72
48	56	48	56
64	56	72	56
64	48	64	56
64	72	72	48
72	56	72	64
72	64	48	72
56	72	48	64
72	56	48	64

Problem Set

- 54; 9; 9; 54
 - 63; 2, 18; 2; 2; 18; 63
 - 72; 45; 3, 27; 3; 3, 9; 27; 72
 - 81; 45; 4, 36; 4; 4, 9; 36; 81
- 54; 60; 54
 - 63; 70; 63
 - 72; 80; 72
 - 81; 90, 9; 81
- 36; answers will vary.
- Products and quotients matched

Exit Ticket

1. 6, 1; 9, 1, 9; 9; 54
2. Picture models equation; explanations may vary.

Homework

1.
 - a. 54; 24; 24; 54
 - b. 63; 35; 4, 28; 4; 4; 28; 63
 - c. 72; 40; 4, 32; 4; 4, 8; 32; 72
 - d. 81; 45; 4, 36; 4; 4, 9; 36; 81
2.
 - a. Answer provided
 - b. 60; 54; 9×6
 - c. 70; 63; 9×7
 - d. 80; 72; 9×8
 - e. 90, 9; 81; 9×9
 - f. 40, 4; 36; 9×4

Lesson 13

Sprint

Side A

- | | | | |
|--------|--------|--------|---------|
| 1. 16 | 12. 56 | 23. 10 | 34. 8 |
| 2. 24 | 13. 64 | 24. 4 | 35. 7 |
| 3. 32 | 14. 72 | 25. 3 | 36. 9 |
| 4. 40 | 15. 80 | 26. 10 | 37. 6 |
| 5. 8 | 16. 8 | 27. 5 | 38. 8 |
| 6. 2 | 17. 7 | 28. 8 | 39. 88 |
| 7. 3 | 18. 9 | 29. 2 | 40. 11 |
| 8. 5 | 19. 6 | 30. 3 | 41. 96 |
| 9. 8 | 20. 10 | 31. 6 | 42. 12 |
| 10. 4 | 21. 5 | 32. 7 | 43. 112 |
| 11. 48 | 22. 2 | 33. 9 | 44. 14 |

Side B

- | | | | |
|--------|--------|--------|---------|
| 1. 8 | 12. 48 | 23. 6 | 34. 7 |
| 2. 16 | 13. 56 | 24. 10 | 35. 8 |
| 3. 24 | 14. 64 | 25. 3 | 36. 9 |
| 4. 32 | 15. 72 | 26. 2 | 37. 6 |
| 5. 40 | 16. 7 | 27. 8 | 38. 7 |
| 6. 3 | 17. 6 | 28. 10 | 39. 88 |
| 7. 2 | 18. 8 | 29. 5 | 40. 11 |
| 8. 4 | 19. 10 | 30. 3 | 41. 96 |
| 9. 8 | 20. 9 | 31. 8 | 42. 12 |
| 10. 5 | 21. 2 | 32. 4 | 43. 104 |
| 11. 80 | 22. 5 | 33. 9 | 44. 13 |

Problem Set

1.
 - a. 18, 27, 45, 54, 63, 81, 90
 - b. +1
 - c. -1
2.
 - a. Answer provided
 - b. 18
 - c. 28; 27; 27
 - d. 37; 36; 36
 - e. 46; 45; 45
 - f. 55; 54; 54
 - g. 64; 63; 63
 - h. 73; 72; 72
 - i. 82; 81; 81
 - j. 91; 90; 90
3.
 - a. +10, -1
 - b. 99; 108; 117; 126
 - c. 54; 63; strategy accurately used to solve
 - d. Answers will vary.
4. $a = 6; g = 9; d = 8; o = 90; e = 7; n = 3; s = 4;$
 $t = 2; i = 45$
Add a "g" and it's gone!

Exit Ticket

1. 64; 63; 63
82; 81; 81
2. Answers will vary.

Homework

1.
 - a. 81, 63, 54, 45, 27, 18, 9
 - b. -1
 - c. +1
2. $a = 2; m = 27; e = 5; f = 36; d = 9; w = 54; s = 10;$
 $k = 72$
3.
 - a. 10; 9; 9
 - b. 19; 18; 18
 - c. 28; 27; 27
 - d. 37; 36; 36
 - e. 46; 45; 45
 - f. 55; 54; 54
 - g. 64; 63; 63
 - h. 73; 72; 72
 - i. 82; 81; 81
 - j. 91; 90; 90
4. Answers will vary; 99; 108; 117

Lesson 14

Pattern Sheet

9	18	27	36
45	9	18	9
27	9	36	9
45	9	18	27
18	36	18	45
18	9	18	27
9	27	18	27
36	27	45	27
36	9	36	18
36	27	36	45
36	45	9	45
18	45	27	45
36	18	36	27
45	27	18	36
27	45	18	36

Problem Set

- Answer provided
9
27, 2, 7, 9
36, 3, 6, 9
45, 4, 5, 9
54, 5, 4, 9
63, 6, 3, 9
72, 7, 2, 9
81, 8, 1, 9
90, 9, 0, 9
 - 9; answers will vary.
 - Incorrect; answers will vary.
- Answers will vary.
- Explanations will vary.
- 63; explanations will vary.

Exit Ticket

Answers will vary.

Homework

1. a. Answer provided
Answer provided
72, 7, 2, 9
63, 6, 3, 9
54, 5, 4, 9
45, 4, 5, 9
36, 3, 6, 9
27, 2, 7, 9
18, 1, 8, 9
9, 0, 9, 9
- b. Answers will vary.
2. Answers will vary.
3. 54; explanations will vary.
4. Correct; answers will vary.

Lesson 15

Pattern Sheet

9	18	27	36
45	54	63	72
81	90	45	54
45	63	45	72
45	81	45	90
54	45	54	63
54	72	54	81
54	63	54	63
72	63	81	63
72	54	72	63
72	81	81	54
81	63	81	72
81	72	54	81
63	81	54	72
81	63	54	72

Problem Set

- 4; solution includes equation and an unknown
- 3 L; solution includes equation and an unknown
- 63 m; solution includes equation and an unknown
- \$7; solution includes equation and an unknown
- 3; solution includes equation and an unknown
- 37; solution includes equation and an unknown

Exit Ticket

- 4 L; solution includes an unknown
- 19; solution includes an unknown

Homework

1. Tape diagram drawn and labeled; $36 \div 9 = a$; $a = 4$
2. 5; solution includes an unknown
3. \$63; solution includes an unknown
4. 9 m
5. 54
6. 3

Lesson 16

Sprint

Side A

1. 18	12. 63	23. 10	34. 8
2. 27	13. 72	24. 2	35. 7
3. 36	14. 81	25. 3	36. 9
4. 45	15. 90	26. 10	37. 6
5. 9	16. 8	27. 5	38. 8
6. 2	17. 7	28. 1	39. 99
7. 3	18. 9	29. 2	40. 11
8. 5	19. 6	30. 3	41. 108
9. 1	20. 10	31. 6	42. 12
10. 4	21. 5	32. 7	43. 126
11. 54	22. 1	33. 9	44. 14

Side B

1. 9	12. 54	23. 2	34. 7
2. 18	13. 63	24. 10	35. 8
3. 27	14. 72	25. 3	36. 9
4. 36	15. 81	26. 2	37. 6
5. 45	16. 7	27. 1	38. 7
6. 3	17. 6	28. 10	39. 99
7. 2	18. 8	29. 5	40. 11
8. 4	19. 10	30. 3	41. 108
9. 1	20. 9	31. 3	42. 12
10. 5	21. 1	32. 4	43. 117
11. 90	22. 5	33. 9	44. 13

Problem Set

1.
 - a. 6
 - b. 0
 - c. 1
 - d. 1
 - e. 0
 - f. Any number
 - g. 4
 - h. 3
2. Equations matched to solutions
3. 1, 2, 3, 4, 5, 6, 7, 8, 9, n
Answers will vary.
4.
 - a. $n \div 1 = n$
 - b. $6 \div 1 = 6$; picture drawn
 - c. $6 \times 1 = 6$
5.
 - a. Explanations may vary.
 - b. Explanations may vary.
 - c. Explanations may vary.

Exit Ticket

1.
 - a. 5
 - b. 1
 - c. 0
 - d. 0
 - e. 9
 - f. 8
2. No; explanations may vary.

Homework

1.
 - a. 4
 - b. 0
 - c. 5
 - d. 0
 - e. 1
 - f. 0
 - g. 0
 - h. 0
 - i. 1
 - j. 1
 - k. 1
 - l. 9
2. Equations matched to solutions
3.
 - a. Answer provided
 - b. True
 - c. True
 - d. True
 - e. False
 - f. True
 - g. True
 - h. False
4.
 - a. $n \times 1 = n$
 - b. Answers will vary.

Lesson 17

Problem Set

1. Products accurately recorded
 - a. Even-product squares colored; Yes
 - b. No
 - c. Explanations may vary.
 - d. 112
2.
 - a. Products accurately labeled
 - b. Arrays accurately drawn; 5, 7, 9, 11
 - c. Answers may vary.
 - d. Explanations may vary.

Exit Ticket

1. 96
2. Explanations will vary.

Homework

1.
 - a. Products accurately recorded
 - b. Even factors accurately identified
 - c. Explanations may vary.
 - d. Odd; even; even; examples will vary.
 - e. Explanations may vary.
 - f. 64
2.
 - a. Answer provided
 - b. $16 = 4 \times 4$
 - c. $36 = 6 \times 6$
 - d. $64 = 8 \times 8$
 - e. $100 = 10 \times 10$

Lesson 18

Sprint

Side A

1. 2	12. 0	23. 1	34. 0
2. 3	13. 0	24. 1	35. 1
3. 4	14. 0	25. 1	36. 0
4. 9	15. 0	26. 0	37. 0
5. 0	16. 1	27. 7	38. 0
6. 0	17. 1	28. 0	39. 1
7. 0	18. 1	29. 1	40. 79
8. 1	19. 1	30. 0	41. 0
9. 1	20. 1	31. Any number	42. 96
10. 1	21. 5	32. 1	43. 1
11. 1	22. 0	33. 24	44. 0

Side B

1. 3	12. 0	23. 1	34. 0
2. 4	13. 0	24. 1	35. 1
3. 5	14. 0	25. 1	36. 0
4. 8	15. 0	26. 0	37. 0
5. 0	16. 1	27. 9	38. 0
6. 0	17. 1	28. 0	39. 1
7. 0	18. 1	29. 1	40. 78
8. 1	19. 1	30. 0	41. 0
9. 1	20. 1	31. 1	42. 97
10. 1	21. 6	32. Any number	43. 1
11. 1	22. 0	33. 34	44. 0

Problem Set

1. 27 cm; solution includes model, equation, and explanation.
2. 57 min; solution includes model, equation, and explanation.
3. 8; solution includes model, equation, and explanation.
4. 6; solution includes model, equation, and explanation.
5. 9 g; solution includes model, equation, and explanation.

Exit Ticket

117 minutes; solution includes model, equation, and explanation.

Homework

1. 34 kg; solution includes model, equation, and explanation.
2. 57 min; solution includes model, equation, and explanation.
3. 33; solution includes model, equation, and explanation.
4. 7; solution includes model, equation, and explanation.
5. 8 cm; solution includes model, equation, and explanation.
6. \$8; solution includes model, equation, and explanation.

Lesson 19

Problem Set

1.
 - a. 12; 12
 - b. 12; 120
2.
 - a. 8; 8
 - b. 8; 80
 - c. 15; 15
 - d. 15; 150
 - e. 20; 20
 - f. 20; 200
3.
 - a. 14
 - b. 14
 - c. 24
 - d. 24
 - e. 300
 - f. 320
 - g. 280
 - h. 400
4. 240; tape diagram models equation.

Exit Ticket

1. 30, 30; 30, 300
2.
 - a. 80
 - b. 240

Homework

1.
 - a. 9; 9
 - b. 9; 90
2.
 - a. 10; 10
 - b. 10; 100
 - c. 25; 25
 - d. 25; 250
3. Products matched to corresponding solutions
4. 240; tape diagram models equation.

Lesson 20

Problem Set

- Answer provided
 - 80
 - 15; 150
 - 5; 150
- Answer provided
9; 90
6; 60
10; 100
- Explanations will vary.

Exit Ticket

- $(4 \times 2) \times 10$; 8; 80
 - $(3 \times 3) \times 10$; 9; 90
- Explanations will vary.

Homework

- | | | | |
|----|--|----|--|
| 1. | <ol style="list-style-type: none">10010020; 2005; 200 | 2. | <ol style="list-style-type: none">609; 9012; 12015; 150 |
| | | 3. | Explanations will vary. |

Lesson 21

Sprint

Side A

- | | | | |
|---------|---------|---------|---------|
| 1. 6 | 12. 150 | 23. 320 | 34. 560 |
| 2. 60 | 13. 16 | 24. 320 | 35. 480 |
| 3. 60 | 14. 160 | 25. 54 | 36. 630 |
| 4. 4 | 15. 160 | 26. 540 | 37. 300 |
| 5. 40 | 16. 18 | 27. 10 | 38. 640 |
| 6. 40 | 17. 180 | 28. 100 | 39. 720 |
| 7. 8 | 18. 180 | 29. 270 | 40. 480 |
| 8. 80 | 19. 35 | 30. 280 | 41. 490 |
| 9. 80 | 20. 350 | 31. 200 | 42. 400 |
| 10. 15 | 21. 350 | 32. 360 | 43. 540 |
| 11. 150 | 22. 32 | 33. 420 | 44. 810 |

Side B

- | | | | |
|---------|---------|---------|---------|
| 1. 8 | 12. 250 | 23. 360 | 34. 420 |
| 2. 80 | 13. 12 | 24. 360 | 35. 360 |
| 3. 80 | 14. 120 | 25. 48 | 36. 490 |
| 4. 9 | 15. 120 | 26. 480 | 37. 300 |
| 5. 90 | 16. 21 | 27. 10 | 38. 480 |
| 6. 90 | 17. 210 | 28. 100 | 39. 560 |
| 7. 6 | 18. 210 | 29. 240 | 40. 480 |
| 8. 60 | 19. 24 | 30. 320 | 41. 630 |
| 9. 60 | 20. 240 | 31. 200 | 42. 400 |
| 10. 25 | 21. 240 | 32. 640 | 43. 720 |
| 11. 250 | 22. 36 | 33. 540 | 44. 630 |

Problem Set

1. 345 seconds; tape diagram models equation
2. No; explanations will vary; solution includes model and equation with unknown.
3. 400¢; solution includes model and equation with unknown.
4. 9 g; solution includes model and equation with unknown.
5. 41; solution includes model and equation with unknown.
6. \$126; solution includes model and equation with unknown.

Exit Ticket

200 g; solution includes model and equation with unknown.

Homework

1. 375 minutes; solution includes model and equation with unknown.
2. 210; solution includes model and equation with unknown.
3. Yes; explanations will vary; solution includes model and equation with unknown.
4. 23; solution includes model and equation with unknown.
5. No; explanations will vary; solution includes model and equation with unknown.
6. \$450; solution includes model and equation with unknown.