Problem of the Month
Part and Whole

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:
In the Problem of the Month *Part and Whole,* students explore rational numbers and solve problems involving symmetry, congruence, determining equal area, sub-dividing area models, reasoning about measurements, and generalizing about fractions. The mathematical topics that underlie this POM are understanding rational numbers through different representations. Students explore fractions through area models using symmetry, congruence, measurement, and mathematical notation.

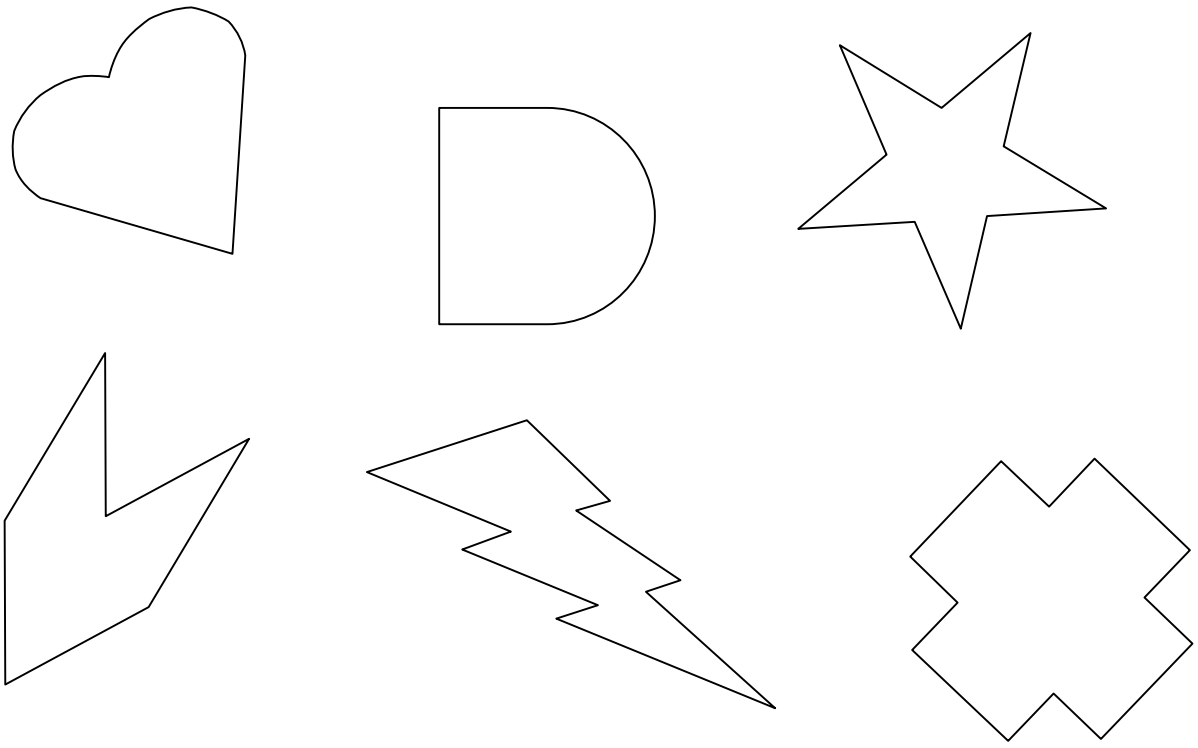
In the first level of the POM, students view different geometric figures and determine whether they can divide the figure into two identical pieces. Their task is to use symmetry to answer question of same shape and equal area parts. In level B, students are given a picture of flat geometric shapes made out of clay. Then students are asked to find a way to make a straight line cut in order to divide the figure into two parts of equal amounts of clay. In level C, the students are presented with a rectangular map. The map is divided into six different regions of various sizes. The task for the students is to determine the fractional part of each region in terms of the whole rectangular area. In level D, students analyze a triangular region to once again find the fractional parts of the whole. The students are then asked to design their own map with sub-divided regions. In the final level of the POM, students are presented with an investigation to find five different unit fractions with a sum of 1. Students must determine whether there is more than one set of five unit fractions that sum to 1, and if so they must determine a general method for finding other sets. If not, they prove why not. They also explore other size sets of unit fractions that can be found to sum to 1.

Problem of the Month
Part and Whole

Level A:

You and your friend have made a batch of cookies that have different shapes. You want to share each cookie between you and your friend so that you can taste each one. You decided you want to make sure to share the cookie so both pieces are the same. How should you cut your cookies to make sure each of you have the same shape and size of the cookies?

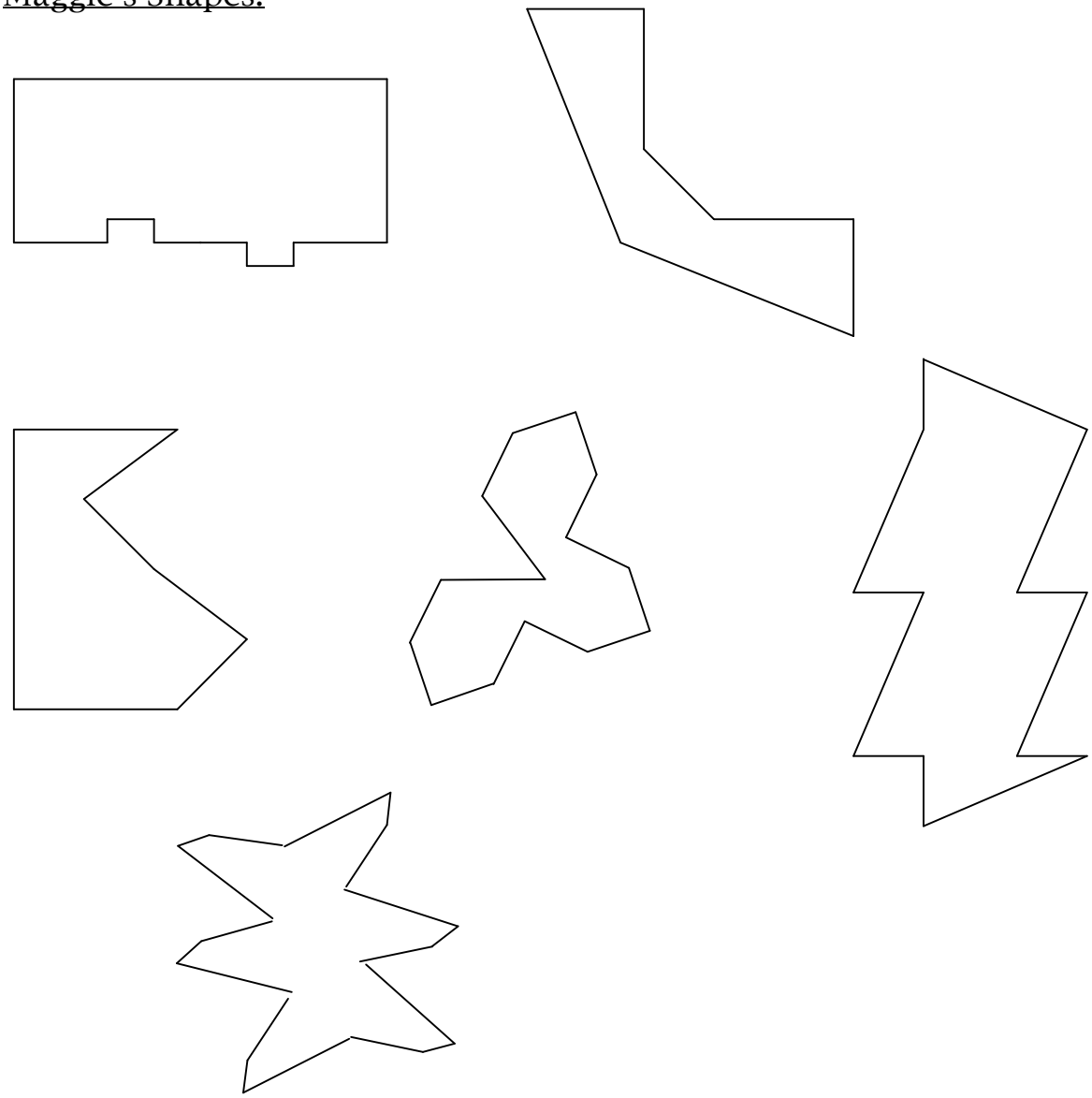
Draw a line through the cookies where you would make a cut and explain why the two pieces are the same.



Level B:

Maggie and Lexie were making funny shapes out of flat clay. They decided to play a game. Maggie would make a clay shape and Lexie would have to divide the clay shape using one cut-line. The two pieces would not have to look the same, but they would have to be the same size (same amount of clay). Below are Maggie's clay shapes. Show where Lexie should make a cut-line to make two pieces so both would be the same amount of clay.

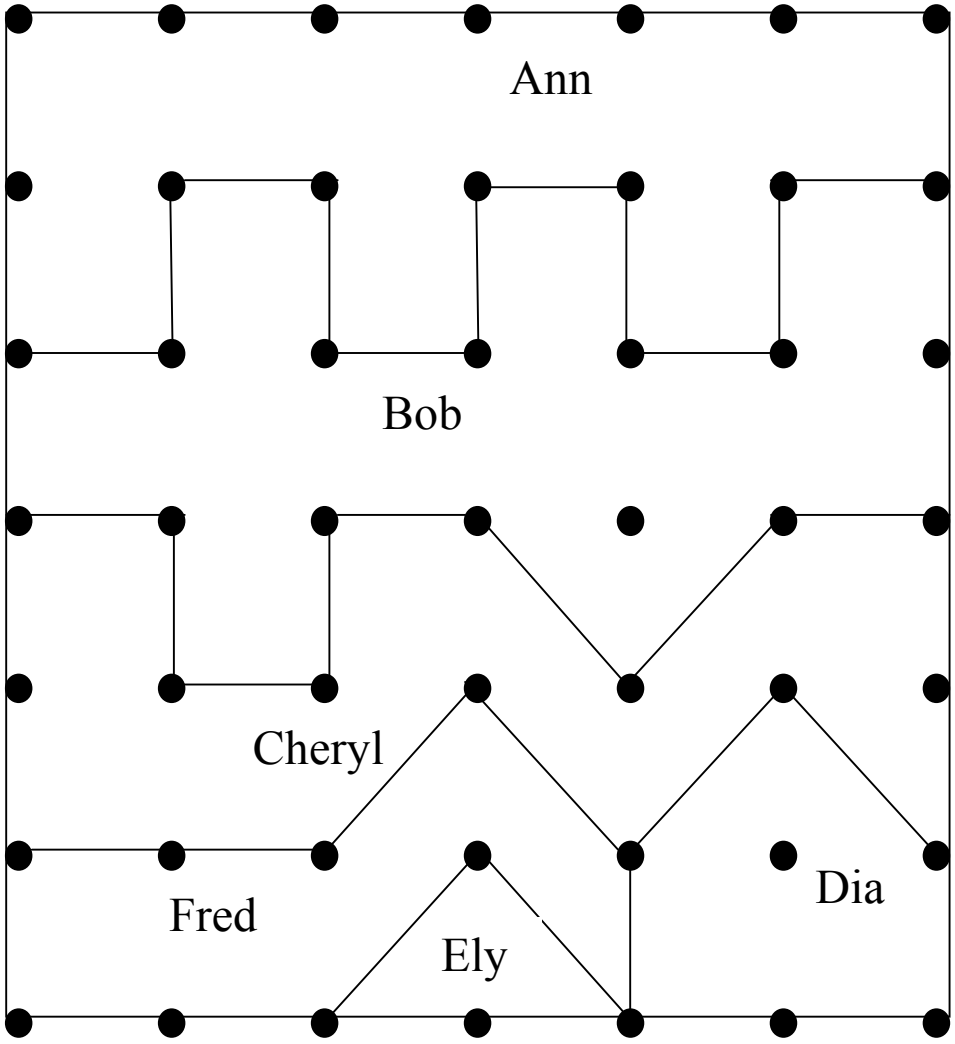
Maggie's Shapes:



Explain to Lexie why you know your methods are right.

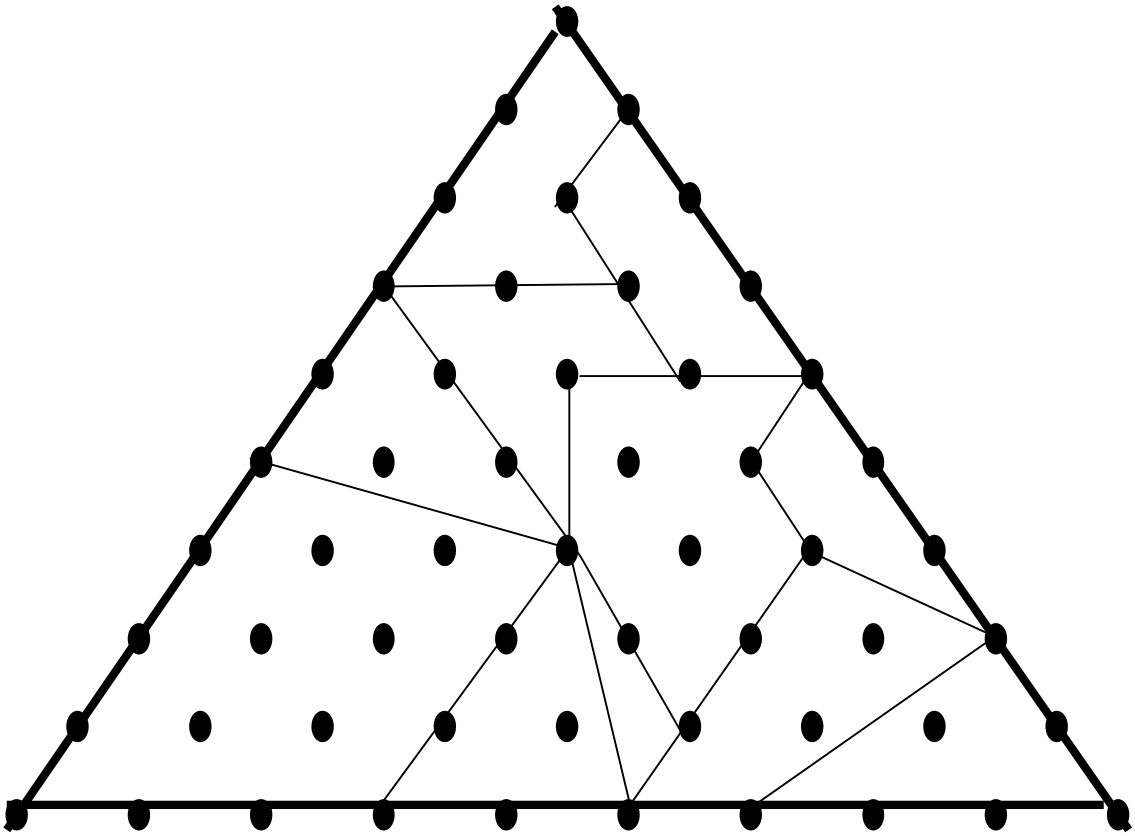
Level C:

Great Uncle Landowner has a parcel of land he owned. In his will, he left a map of the land that is divided into different regions. He wrote the names of each of his nephews and nieces on different regions of the map. He wrote the name in each region to indicate who will inherit that section of land. The regions range in size. Your job is to determine the fractional part of each region as it relates to the whole parcel. Examine the map below and determine the fraction piece of each region of land. Explain how you determined that fractional part awarded to each niece and nephew.



Level D:

You work for a puzzle company. You need to determine the fractional size of each piece so that company will know the materials needed for the different size pieces. They sent you the following puzzle. Determine the fractional size of each piece and explain your reasoning.



You have been assigned to create a more complicated puzzle. Create a design and provide a key to the fractional size of each shape, explaining how you determined its size.

Level E:
A unit fraction has a numerator of one and a natural number denominator. Find five different unit fractions with a sum of 1.

Determine if there are more sets of five unit fractions, if so determine a general method for finding other sets. If not, prove why not.

What other *n* number of unit fractions can be found that sum to 1. Explain your reasoning and justify your conclusions.

Problem of the Month
Part and Whole

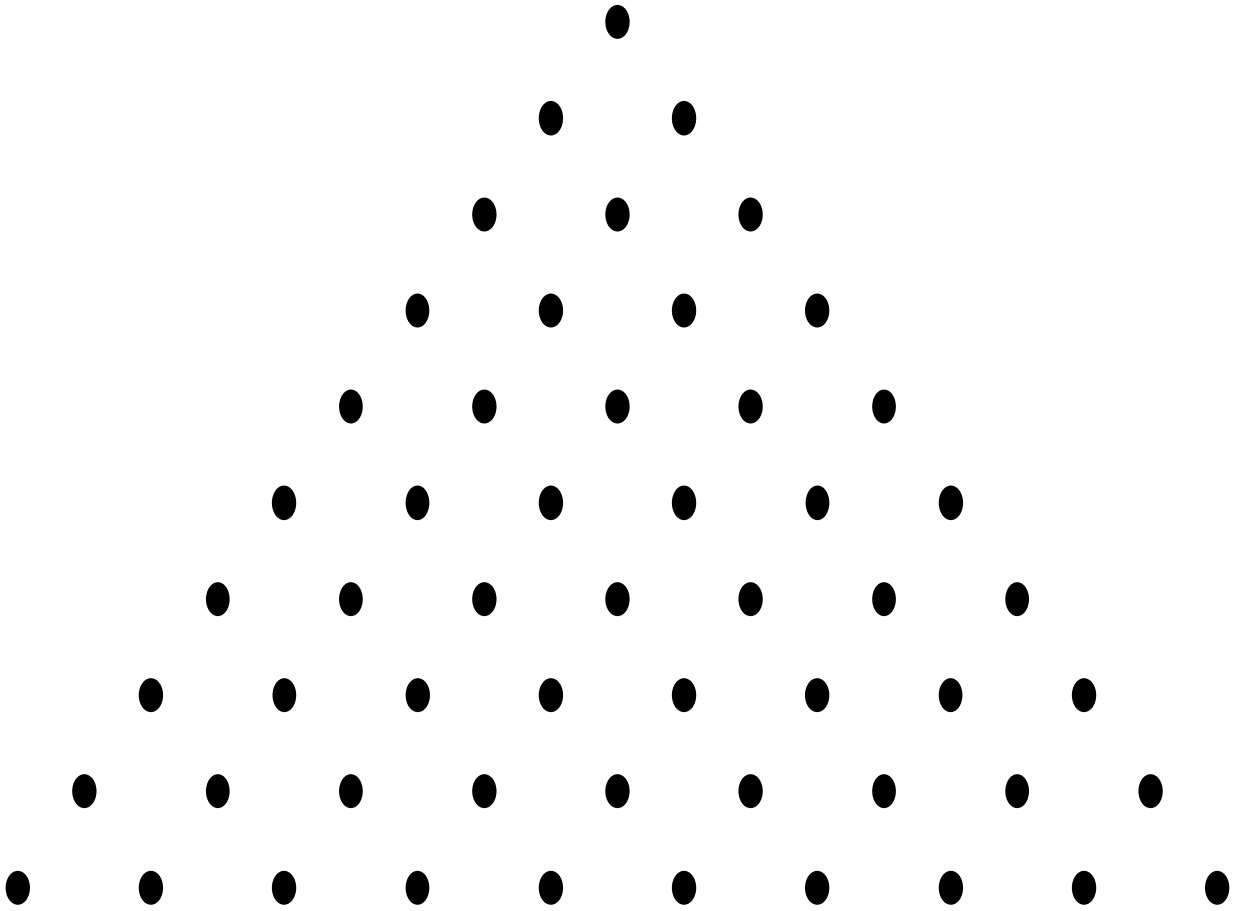
Primary Version Level A

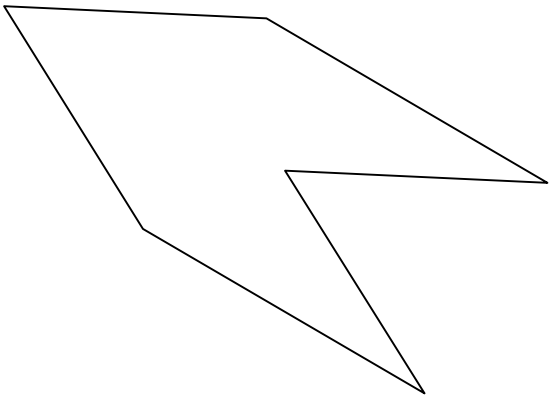
Materials: The enlarged paper with Cookies all different shapes, scissors, pencils and rulers.

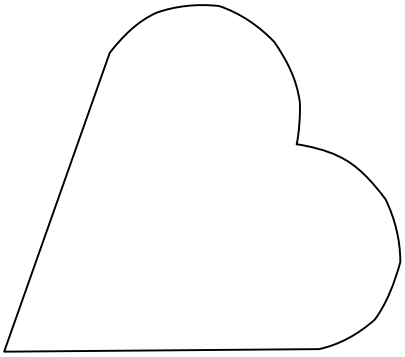
Discussion on the rug: (Teacher holds up the heart that was cut from shape.) "We want to share this cookie between two friends. How can we make one cut so each friend gets the same size piece?" (Students think about how to share the cookie. After soliciting some ideas from students (folding, drawing different lines, measuring, etc.), the teacher asks, "How will we know for sure?"

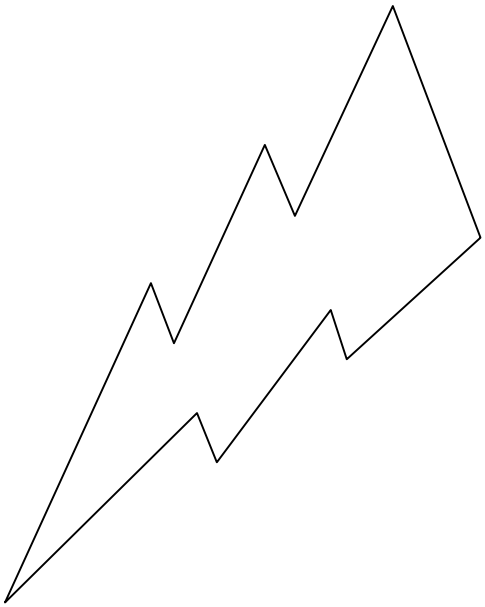
In small groups: (Students have enlarged cookie paper, rulers, pencils, tissue paper and scissors available)
Teacher says, "Here are different shaped cookies. You want to share each cookie, so you need to cut each one in half so the pieces are the same. Where should you cut it to make sure each one of you has the same size cookie? Draw a line to show where to cut." (Students draw a line to show where to make the cut. After the students are done, the teacher asks how we can show that both friends get the same amount. The class may actually cut out some of the shapes and cut them to show whether or not they are cut in half.)

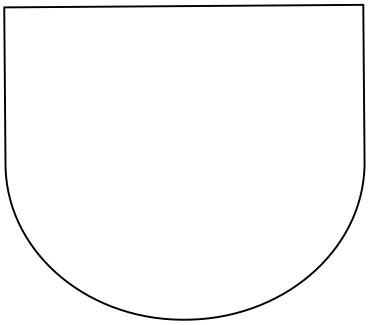
At the end of the investigation: (Students either discuss or dictate a response to this summary question.) "Explain how you know that both friends have the same amount after you cut the cookie?"

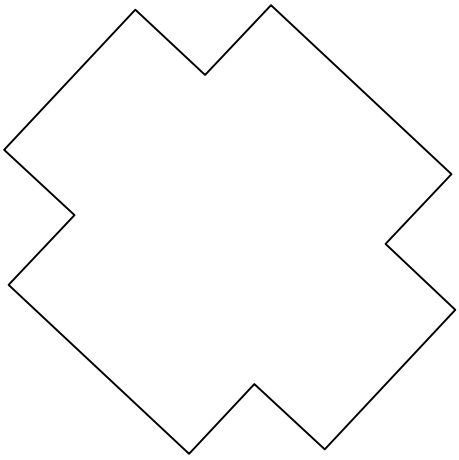


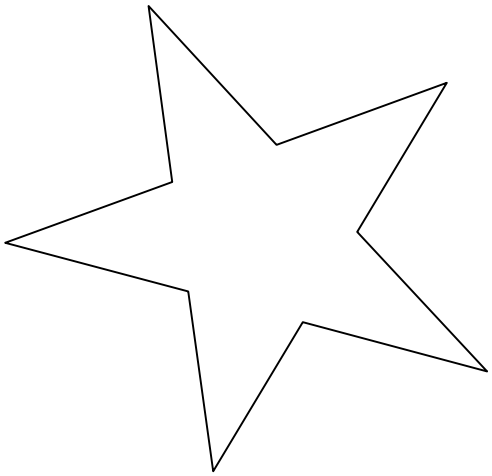












Problem of the Month

Part and Whole

Task Description – Level A

This task challenges a student to use symmetry to divide shapes into two identical pieces, equal area and equal shape.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

Geometry

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.3 Recognize a line of symmetry for two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month

Part and Whole

Task Description – Level B

This task challenges a student to reason about subdividing shapes into two parts with equal areas, but not necessarily equal shapes using one line. Students need to develop arguments to show that the areas are the same without calculating the areas.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

Geometry

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month

Part and Whole

Task Description – Level C

This task challenges a student to determine fractional parts of a rectangle divided into 6 different-size regions.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.6 Measure areas by counting unit squares.

Geometry

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas.

Number and Operations - Fractions

Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is portioned into b equal parts, understand a fraction a/b as the quantity formed by a parts of size 1/b.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month

Part and Whole

Task Description – Level D

This task challenges a student to analyze a triangular region to find the fractional parts of the whole. The students are then asked to design their own map with sub-divided regions.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.6 Measure areas by counting unit squares.

Geometry

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes, apply techniques in the context of solve real-world and mathematical problems.

Number and Operations - Fractions

Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is portioned into b equal parts, understand a fraction a/b as the quantity formed by a parts of size 1/b.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are

comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.7 Look for and make use of structure.
Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Problem of the Month

Part and Whole

Task Description – Level E

This task challenges a student to investigate to find five different unit fractions with a sum of 1. Students determine whether there is more than one set of five unit fractions that sum to 1, and if so they determine a general method for finding other sets. If not, they prove why not. They also explore other size sets of unit fractions that can be found to sum to 1.

Common Core State Standards Math - Content Standards

Number and Operations - Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1 Add and subtract with unlike denominators by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Common Core State Standards Math – Standards of Mathematical Practice

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y -2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month

Part and Whole

Task Description – Primary Level

This task challenges a student to use tools such as scissors, rulers, and tracing paper to reason about symmetry and dividing shapes into equal pieces, same size and same shape.

Common Core State Standards Math - Content Standards

Measurement and Data

Describe and compare measureable attributes.

K.MD.2 Directly compare two objects with a measureable attribute in common, to see which object has "more of/less of" the attribute, and describe the difference.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

Geometry

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.