Problem of the Month: *Cut It Out*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month, *Cut It Out,* students use properties of two-dimensional geometry to solve problems involving spatial visualization. The mathematical topics that underlie this POM are the attributes of polygons, symmetry, spatial visualization, transformations, patterns, functions, and fractal geometry.

The problem asks students to use spatial reasoning to make sense of part of a visual image. In the first level of the POM, students are asked to fold a square piece of paper into fourths such that the result is a smaller square. At the most folded corner the students are to visualize cutting a tiny square out of the paper. Their task is to draw a picture of the unfolded square noting where the hole(s) are located and the

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approximate size of the square. In Level B, students are presented with a similar situation that involves determining what the unfolded square looks like when different types of cuts are made. The students need to visualize the results of multiple cuts. In Level C, students work with the inverse challenge. Students are given visual images of the unfolded square piece of paper that has been folded a number of times and in which holes have been cut. The students must determine how many folds were required and how the cut was made. In Level D, the students explore a geometric fractal image. The fractal image is generated from folding a square paper and cutting out a small corner. The process is repeated and the students draw the image, find the area of the paper, and the perimeter of all edges at each stage. Using knowledge of patterns and functions the students predict what the fractal looks like and its size in terms of area and perimeter at any stage of the fractal's development. In Level E, students design their own fractal, following some geometric pattern, and they compute formulas for generating the area and perimeter of a fractal.

Mathematical Concepts

Spatial visualization plays an important part in real-world experiences. Whether designing the most complex structures created by designers, architects, and construction workers or arranging the furniture in a room, spatial awareness and visualization are essential. In this POM, students explore various aspects of spatial visualization. This involves examining symmetrical patterns and understanding the relationship between multiple lines of reflection. Students will use their spatial sense and develop understandings of attributes in plane geometry and the use of basic transformations. In addition to the geometric aspects of this POM, students are seeking to find patterns, develop functional relationships, and examine the measurement aspect of perimeter and area in fractals. The mathematics involved in the higher level is fractal geometry and chaos theory.

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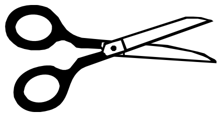
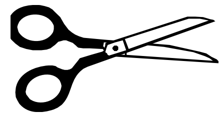
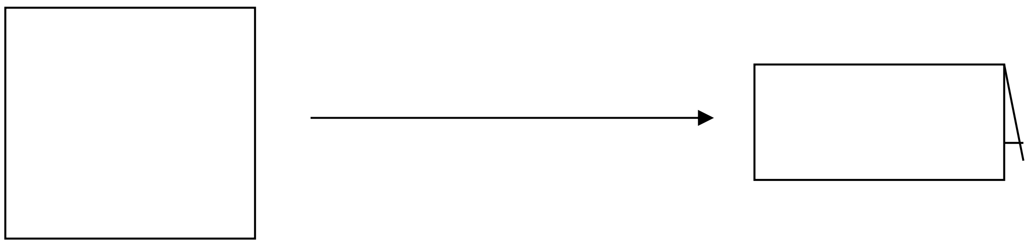


Problem of the Month
Cut It Out

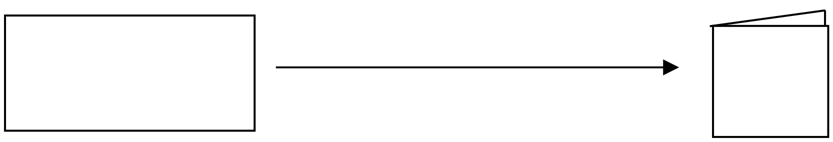


Level A

Start with a square piece of paper that is 8 inches in length. Fold the piece of paper in half, bending the top edge down to meet the bottom edge of the paper.

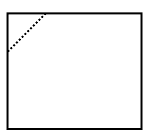


Now fold the sheet again in half by bending the left side over to meet the right side.



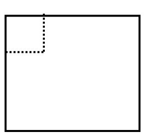
How do the shape and size of the folded paper compare to the original sheet of paper? Describe its dimensions and area.

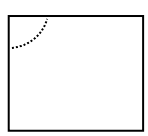
Now make the cut (dotted line) at the top left-hand corner of the folded paper.



Without unfolding the paper, draw and describe what the paper will look like when unfolded. Explain how you know.

Repeat the folding process with new sheets of paper. Make the following cuts.





Before unfolding the paper predict what the original paper will look like after the cut. Draw an illustration and explain your reasoning.

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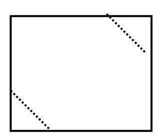
Level B

Once again, start with a square piece of paper that is 8 inches in length. Fold the piece of paper in half, bending the top edge down to meet the bottom edge of the paper.

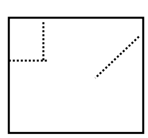
Now fold the sheet in half again by bending the left side over to meet the right side.

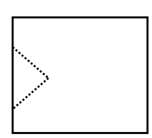
Now make the cuts (dotted lines) at the top right-hand corner and bottom left-hand corner of the folded paper.



Without unfolding the paper, draw and describe what the paper will look like when unfolded. Explain how you know.

Repeat the folding process with new sheets of paper. Make the following cuts.





Before unfolding the paper predict what the original paper will look like after the cut. Draw an illustration and explain your reasoning.

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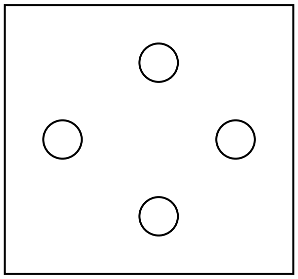
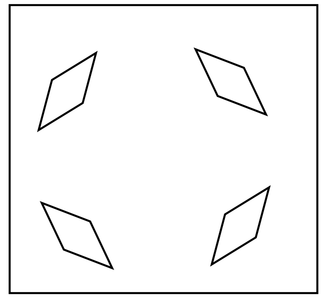
Level C

Examine each of the following images. Each image is a blank sheet of paper that contains a set of holes. The paper was folded a **number of times** and then **one continuous cut** was made to produce the image. Determine how the paper was folded and how the cut was made to produce each of the six images below.

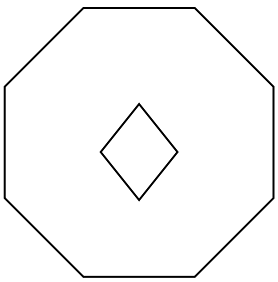
A.



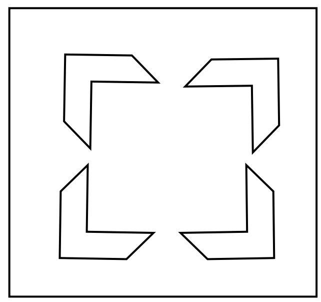
B.



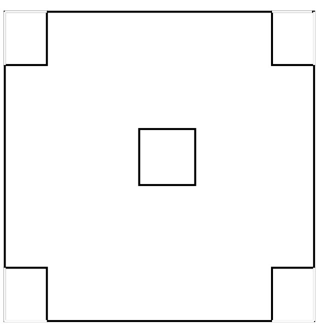
C.



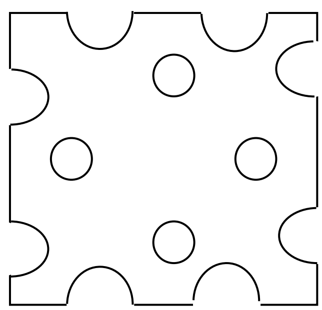
D.



E.



F.



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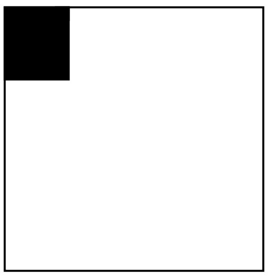
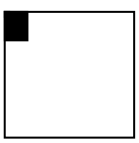
Level D

A fractal is an image that has self-similarity. In this activity, create a fractal. Start with a square sheet of tissue paper 8 inches on each side. List the area and perimeter of the paper.



1. Fold the paper by bringing the left side over to the right. Then bring the top down to the bottom. If you were to open the paper up, it would be divided into 4 congruent squares. Now imagine cutting a square measuring 1 inch on a side out of the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the hole). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance bordering each hole). How do the area and perimeter compare to the original paper?



2. Take the folded sheet and fold it again by bringing the left side over to the right and the top down to the bottom. If you were to open the paper up, it would be divided into 16 squares. Now imagine cutting out a square measuring 1/4 inch on a side from the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the holes). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance around each hole). How do the area and perimeter compare to the original paper?

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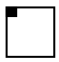
3. Take the folded sheet and fold it again by bringing the left side over to the right and the top down to the bottom. If you were to open the paper up, it would be divided into 64 squares. Now imagine cutting a square measuring 1/16 inch on a side out of the most folded corner (top left corner) of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Cut the square out of the upper left corner of the folded paper and check to determine how your prediction compared to the actual result. Determine the new area of the paper (excluding the holes). If we define perimeter to be the boundary around the area of the remaining paper, then calculate the new perimeter (the distance around the outside of the paper, plus the distance around each hole). How do the area and perimeter compare to the original paper?



4. Imagine taking the folded sheet and folding it again for the fourth time using the same process. How many sub-squares would the folded paper contain? Now imagine cutting a square measuring 1/64 inch on a side out of the upper left corner of the folded paper. What do you think the paper will look like when it is opened up? Draw a picture to represent the paper after the square has been cut out. Determine the new area and perimeter of the paper. How do the area and perimeter compare to the original paper?

5. Examine the process you followed in the previous steps. A fractal contains an infinite number of iterations (steps). Explain what the fractal would ultimately look like. Draw a diagram of the fractal. Explain the size of the fractal at the first five iterations (steps). Determine the actual size of the fractal in terms of area and perimeter. Discuss how you found your answers and explain your mathematical reasoning.

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Level E

A fractal is a geometric figure that has self-similarity, that is created using a recursive process and that is infinite in structure. There are two categories of fractals – geometric and random. A geometric fractal is an endlessly generating pattern of self-similarity. The pattern continually replicates itself in smaller versions. Thus, when a small portion of a geometric fractal is magnified, it looks exactly like the original version. A random fractal also contains self-similar images of itself, only in a disorderly, non-predictable pattern. Beautiful computer-generated images such as the Mandelbrot Set are examples of these fractals.

Design a poster/object that contains a fractal. The fractal could be a self-similar collage, a series of pictures inside a picture, a self-similar geometric design, or another self-similar unique creation. It must be an original drawing or design. Your poster may contain photographs, pictures from periodicals, enlargements and reductions from copiers, and/or computer-generated designs. The fractal may be created using a random (chaos) technique or a self-similar drawing. You may produce a 3-dimensional model of a fractal. Your design must contain at least four iterations of a process that produces some self-similar shapes.

Write a report that describes the fractal and the process that you used to create the design. Be sure to describe the relationship between similar objects in your design. Identify the self-similar shapes or pictures that you used in the fractal. Demonstrate a procedure for finding the size (length, area, volume, angular distance, etc.) of the self-similar objects at any given level of the fractal.

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Problem of the Month

Cut It Out

Primary Version Level A

Materials: Square tissue paper and scissors for the teacher to use during discussion, square tissue paper for every student to hold and examine

Discussion on the rug: Teacher holds up the square tissue paper. **"We are going to think hard and make a picture in our brains. Follow me in folding our pieces of paper. First fold the paper in half like a hotdog bun. Next, fold this same paper in half again, but this time like a hamburger. If we unfolded the paper, what will it look like?"** Students think about the folded parts that make four squares. After soliciting some guesses, students unfold the paper to examine the results. **"Why did that happen?"**

In small groups: Students have re-folded their papers. Teacher demonstrates cutting the folded corner with a diagonal cut. **"If I unfold the paper, what will it look like now?"** Students make a drawing of what they visualized. The teacher unfolds the paper and asks students to explain how they pictured the altered paper. The process is repeated with a new paper; the teacher cuts out a small square in the folded corner. The third time, the teacher cuts a quarter of a circle out of the folded corner.

At the end of the investigation: Students either discuss or dictate a response to this summary question. **"Explain how you know what the paper will look like after we fold it and the paper is cut."**

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Problem of the Month
Cut It Out
Task Description – Level A
The problem of the month asks the students to use spatial reasoning to make sense of part of a visual image. Students are asked to fold a square piece of paper into fourths such that the result is a smaller square. At the most folded corner the students are to visualize cutting a tiny square out of the paper. Their task is to draw a picture of the unfolded square noting where the hole(s) are located and the approximate size of the square.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2 Correctly name shapes regardless of their orientations or overall size. **Analyze, compare, create, and compose shapes.** K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **Reason with shapes and attributes.** 1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **Reason with shapes and their attributes.** 2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
Common Core State Standards Math – Standards of Mathematical Practice
MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

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Problem of the Month
Cut It Out
Task Description – Level B
The problem of the month asks students to use spatial reasoning to make sense of part of a visual image. In Level B students are presented with a situation similar to that in Level A which involves determining what the unfolded square looks like after different types of cuts are made. Students need to visualize the results of multiple cuts.
Common Core State Standards Math - Content Standards
<u>**Geometry**</u> **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2 Correctly name shapes regardless of their orientations or overall size. **Analyze, compare, create, and compose shapes.** K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **Reason with shapes and attributes.** 1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Common Core State Standards Math – Standards of Mathematical Practice
MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

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Problem of the Month
Cut It Out
Task Description – Level C
The problem of the month asks students to use spatial reasoning to make sense of part of a visual image. In Level C, students work with the inverse challenge. The student is given visual images of the unfolded square piece of paper that has been folded a number of times and in which holes have been cut. The students must determine how many folds were required and how the cut was made.
Common Core State Standards Math - Content Standards
<u>**Geometry**</u> **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2. Correctly name shapes regardless of their orientations or overall size. **Analyze, compare, create, and compose shapes.** K.G.5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **Reason with shapes and attributes.** 1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** 4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.1. Verify experimentally the properties of rotations, reflections, and translations…
Common Core State Standards Math – Standards of Mathematical Practice
MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

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Problem of the Month:
Cut It Out
Task Description – Level D
The problem of the month asks students to use spatial reasoning to make sense of part of a visual image. In Level D, students explore a geometric fractal image. The fractal image is generated by folding a square paper and cutting out a small corner. The process is repeated and the students draw the image, find the area of the paper and the perimeter of all edges at each stage. Using knowledge of patterns and functions, the students predict what the fractal looks like and its size in terms of area and perimeter at any stage of the fractal's development.
Common Core State Standards Math - Content Standards
<u>**Measurement and Data**</u> **Geometric measurement: understand concepts of area and relate area to multiplication and addition.** 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. 3.MD.5.a A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. 3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** 3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. <u>**Geometry**</u> **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.1 Verify experimentally the properties of rotations, reflections, and translations… <u>**Functions**</u> **Define, evaluate, and compare functions.** 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. … <u>**High School – Functions - Interpreting Functions**</u> **Understand the concept of a function and use function notation.** F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. … <u>**High School – Functions - Building Functions**</u> **Build a function that models a relationship between two quantities.** F-BF.1 Write a function that describes a relationship between two quantities.
Common Core State Standards Math – Standards of Mathematical Practice
MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

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Problem of the Month
Cut It Out
Task Description – Level E
The problem of the month asks students to use spatial reasoning to make sense of part of a visual image. In Level E, students design their own fractal following some geometric pattern, and they compute formulas for generating the area and perimeter of a fractal.
Common Core State Standards Math - Content Standards
<u>**Measurement and Data**</u> **Geometric measurement: understand concepts of area and relate area to multiplication and addition.** 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. 3.MD.5.a A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. 3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** 3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. <u>**Geometry**</u> **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **Understand congruence and similarity using physical models, transparencies, or geometry software.** 8.G.1 Verify experimentally the properties of rotations, reflections, and translations… <u>**Functions**</u> **Define, evaluate, and compare functions.** 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. … <u>**High School – Functions - Interpreting Functions**</u> **Understand the concept of a function and use function notation.** F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. … <u>**High School – Functions - Building Functions**</u> **Build a function that models a relationship between two quantities.** F-BF.1 Write a function that describes a relationship between two quantities.
Common Core State Standards Math – Standards of Mathematical Practice
MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

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Problem of the Month
Cut It Out
Task Description – Primary Level
This task challenges students to explore various aspects of circular geometry. This level asks students to describe and identify circular shapes of real world objects such as coins, counters, containers, etc. Then students are asked to ring and identify all shapes in a given picture that are shaped like a circle.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. K.G.2 Correctly name shapes regardless of their orientations or overall size. **Analyze, compare, create, and compose shapes.** K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **Reason with shapes and attributes.** 1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **Reason with shapes and their attributes.** 2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. 2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
Common Core State Standards Math – Standards of Mathematical Practice
MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

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