GRADE 5

Unit

3

Teacher Adaptation Pack

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K5_Beta

Directions for Use

1. Read the current grade level unit standards and prior-grade connections.
2. Ask prior grade level teachers if students were taught the topics last year or show students a problem on the prior-grade level topic and anonymously ask students if they know how to solve the problem.
   a. If yes, start the current grade level section without the add-in lessons.
   b. If not, teach the prior grade level add-in lessons.
3. In the last add-in lesson, give the mini-assessment as the cool-down.
   a. If students get the questions correct, start the current grade level section.
   b. If students get some things correct and some not, still start the current grade level section, and use the ongoing practice materials to support students.

Recommended Implementation

Section A	Section B	Section C
Grade 5 Unit 3	*Grade 5 Unit 3*	*Grade 5 Unit 3*

Grade 5 Unit 3: Fraction Multiplication and Division	
Standards	• 5.NF.B.4.A, 5.NF.B.4.B, 5.NF.B.6, 5.NF.B.7.A, 5.NF.B.7.B, 5.NF.B.7.C
Prior-Grade Connections	• none
Rationale	The lessons in 5.3 build from sections B and C of the previous unit, where students represented and solved problems involving the multiplication of a whole number by a fraction or mixed number. They applied these skills to the concept of area and recognized that they can multiply the side-lengths of a rectangle to find its area when one of its sides has a fractional length. In this unit, students continue to build on the concept of area as they represent and solve problems involving the multiplication of two fractions. Students also use their understanding of the relationship between multiplication and division to divide a whole number by a unit fraction and unit fraction by a whole number. Because the prerequisite knowledge for this unit is developed in the previous unit, we do not recommend additional lessons.
Add-in Lessons	• none
5.3 Lessons to Combine or Skip	• none
Prior-grade Practice and Fluency	• none

Extension and Exploration	• Center: Rolling for Fractions Stages 4-8
	• Center: Compare Expressions Stage 6
	• IM Tasks:
	    ○ New Park
	    ○ Salad Dressing
	    ○ Banana Pudding
	    ○ How Many Marbles?

Assessment	• none

Table of Contents

Extension and Exploration Resources

Center: Rolling for Fractions Stage 4

Narrative
In this center, students apply their understanding of multiplication or division to work with fractions.

Stage Number 4: Multiply a whole number by a fraction

Addressing CCSS:
- 5.NF.B.4

Learning Goals
- Use area understanding to represent the multiplication of a whole number by a fraction (5.2.C)

Required Material
- number cubes
- pencils

Blackline Master
- Rolling for Fractions: Stage 4 Recording Sheet

Stage Narrative

In this stage, students relate the multiplication of a whole number by a fraction to area. They draw an area diagram to represent multiplication expressions, and find the area.

Play Rolling for Fractions with 2 players

1. Each player rolls 1 number cube. The player with the greatest number goes first.
2. Player 1: Roll 3 number cubes. Use the numbers to complete the expression and find the product.
3. Player 2: Roll 3 number cubes. Use the numbers to complete the expression and find the product.
4. Each player draws an area diagram and labels the area and side lengths.
5. Check one another's diagram and values for accuracy. Determine the amount of points each player receives:
   a. 2 points for generating the smallest area
   b. 5 points for generating the biggest area
   c. 10 points for generating an area equivalent to a whole number
6. Play 5 rounds. The player with the most points wins.

Center: Rolling for Fractions Stage 5
Narrative In this center, students apply their understanding of multiplication or division to work with fractions.
Stage Number 5: Multiply unit fractions
Addressing CCSS: ● 5.NF.B.4.A
Learning Goals ● Recognize that $\frac{a}{b}$ x $\frac{c}{d}$ = $\frac{a \times c}{b\,x\,d}$ and use this generalization to multiply fractions numerically (5.3.A)
Required Material ● number cubes ● pencils
Blackline Master ● Rolling for Fractions: Stage 5 Recording Sheet
Stage Narrative In this stage, students multiply a unit fraction by a unit fraction. Play Rolling for Fractions with 2 players 1. Each player rolls 1 number cube. The player with the greatest number goes first. 2. Player 1: Roll 2 number cubes. Use the numbers to complete the expression and find the product. 3. Player 2: Roll 2 number cubes. Use the numbers to complete the expression and find the product. 4. Check one another’s equation for accuracy. Determine the amount of points each player receives: a. 5 points to the player that rolls the largest product b. 3 points to the player that rolled the smallest product 5. Play 5 rounds. The player with the most points wins.

Center: Rolling for Fractions Stage 6
Narrative In this center, students apply their understanding of multiplication or division to work with fractions.
Stage Number 6: Multiply a unit fraction by a fraction
Addressing CCSS: ● 5.NF.B.4.A
Learning Goals ● Recognize that $\frac{a}{b}$ x $\frac{c}{d}$ = $\frac{a \times c}{b\,x\,d}$ and use this generalization to multiply fractions numerically (5.3.A)
Required Material ● number cubes ● pencils
Blackline Master ● Rolling for Fractions: Stage 6 Recording Sheet

Stage Narrative
In this stage, students multiply a unit fraction by a fraction.
Play Rolling for Fractions with 2 players
1. Each player rolls 1 number cube. The player with the greatest number goes first.
2. Player 1: Roll 3 number cubes. Use the numbers to complete the expression and find the product.
3. Player 2: Roll 3 number cubes. Use the numbers to complete the expression and find the product.
4. Check one another’s equation for accuracy. Determine the amount of points each player receives:
    a. 5 points to the player that rolls the largest product
    b. 3 points to the player that rolled the smallest product
5. Play 5 rounds. The player with the most points wins.

Center: Rolling for Fractions Stage 7

Narrative In this center, students apply their understanding of multiplication or division to work with fractions.
Stage Number 7: Multiply two fractions
Addressing CCSS: ● 5.NF.B.4.A
Learning Goals ● Recognize that $\frac{a}{b}$ x $\frac{c}{d}$ = $\frac{a \times c}{b\,x\,d}$ and use this generalization to multiply fractions numerically (5.3.A)
Required Material ● number cubes ● pencils
Blackline Master ● Rolling for Fractions: Stage 7 Recording Sheet
Stage Narrative In this stage, students multiply a fraction by a fraction. Play Rolling for Fractions with 2 players 1. Each player rolls 1 number cube. The player with the greatest number goes first. 2. Player 1: Roll 4 number cubes. Use the numbers to complete the expression and find the product. 3. Player 2: Roll 4 number cubes . Use the numbers to complete the expression and find the product. 4. Check one another's equation for accuracy. Determine the amount of points each player receives: a. 5 points to the player that rolls the largest product b. 3 points to the player that rolled the smallest product 5. Play 5 rounds. The player with the most points wins.

Center: Rolling for Fractions Stage 8
Narrative

In this center, students apply their understanding of multiplication or division to work with fractions.
Stage Number 8: Division involving a whole number and unit fraction
Addressing CCSS: ● 5.NF.B.7
Learning Goals ● Divide a whole number by a unit fraction using whole-number division concepts (5.3.B) ● Divide a unit fraction by a whole number using whole-number division concepts (5.3.B)
Required Material ● number cubes ● pencils
Blackline Master ● Rolling for Fractions: Stage 8 Recording Sheet

Stage Narrative
In this stage, students divide unit fractions by whole numbers and whole numbers by unit fractions.
Play Rolling for Fractions with 2 players
1. Each player rolls 1 number cube. The player with the greatest number goes first.
2. Player 1: Roll 2 number cubes. Use the numbers to complete the expression and find the quotient.
3. Player 2: Roll 2 number cubes. Use the numbers to complete the expression and find the quotient.
4. Check one another’s equation for accuracy. Determine the amount of points each player receives:
    a. 5 points to the player that rolls the largest quotient
    b. 3 points to the player that rolled the smallest quotient
5. Play 6 rounds. The player with the most points wins.

Center: Compare Expressions Stage 6
Narrative

In this center, students evaluate and compare numerical expressions. This center focuses on the procedural skills needed to solve single- and multi-step word problems.

Play Compare Expressions with 2 players.

1. Shuffle the cards and place them face down.
2. Player 1 draws two cards and determines which of the two has the greatest value.
3. Player 2 gives feedback. If correct, player 1 keeps the cards otherwise the player must return the cards.
4. Take turns and continue steps 2-3 until no cards are left.
5. The player with the most cards wins.

Stage Number 6: Multiply and Divide with Fractions

Addressing CCSS:
- 5.NF.B.6, 5.OA.A.1

Learning Goals
- Solve problems involving fraction multiplication and division (5.3.C)

Required Material
- scratch paper

Blackline Master
- Compare Expressions: Stage 6 Cards

Stage Narrative

In this stage, students multiply and divide to evaluate single- and multi-step numerical expressions involving fractions. Some expressions include parentheses.

IM Task: New Park

Task
Part 1:
There are two design proposals for a new rectangular park in town.

- In design one, $\frac{3}{4}$ of the area of the park is going to be a rectangular grass area and $\frac{1}{2}$ of the grass area will be a rectangular soccer field.
- In design two, only $\frac{1}{2}$ of the park is going to be a rectangular grass area and $\frac{3}{4}$ of the grass area will be a rectangular soccer field.

Which design (one or two) will have a bigger soccer field? Explain your answer. Draw a diagram that can be used to compare the size of the soccer field in the two designs. Label the values $\frac{1}{2}$ and $\frac{3}{4}$ on the diagram.

Part 2:
Presley and Julia are cutting 1 ft. square poster board to make a sign for the new park. Presley cut her poster so that the length of the top and bottom are $\frac{1}{2}$ ft and the length of the sides are $\frac{3}{4}$ ft. Julia cut her poster so that the lengths of the top and bottom are $\frac{3}{4}$ ft and the length of the sides are $\frac{1}{2}$ ft.

Draw a diagram of each poster board. Label the values on the diagram.
How are their poster boards similar and different? Justify your reasoning.

IM Commentary
Part 1 of this task is designed to elicit student thinking about multiplication of fractions and the commutative property. An entry-level task into the concept, students can solve the problem without recognizing that they can multiply the fractions to find the fraction of the park that is the soccer field. However, they should have seen prior to this task that $\frac{3}{8}$ can be interpreted as one-half of three-fourths of a whole, so that they can make the connection between the two designs and the expressions $\frac{3}{4} \times \frac{1}{2}$ and $\frac{1}{2} \times \frac{3}{4}$. Through discussion of the task and student solutions, students can extend and apply their previous understandings of multiplication of whole numbers and their growing understanding of the meaning of fraction multiplication to see that the commutative property still holds when multiplying fractions.

After students have solved Part 1, the class could have a discussion of the solution methods. Once both designs have been analyzed, the teacher could ask what the students notice about the solutions (they are the same). Either the students or the teacher should represent this discovery symbolically:

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{4} \times \frac{1}{2}$

The fractions $\frac{3}{4}$ and $\frac{1}{2}$ were selected so that students should have an understanding of both values, providing students with multiple entry points and opportunities to reason and make sense of the problem. Considering how the fractions might be partitioned and represented was also taken into account. Substituting other fractions within the problem may elicit different opportunities for learning.

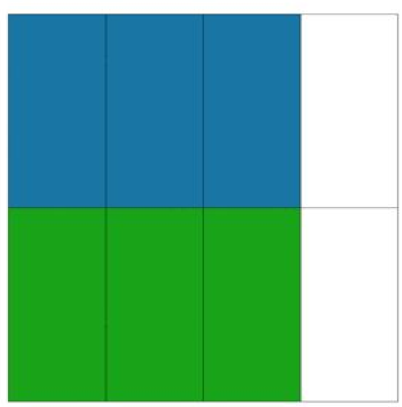
Part 2 of the task uses the area of a rectangle to help students understand why the commutative property always holds. In part 1, students see that $\frac{1}{2}$ of $\frac{3}{4}$ gives the same results as $\frac{3}{4}$ of $\frac{1}{2}$, and Part 2 asks students to focus on a rectangle with side-lengths $\frac{1}{2}$ and $\frac{3}{4}$. A teacher led-discussion that helps students see the relationship between the diagrams from Part 1 and Part 2, in particular, that you can see the answer in part 1 corresponds to a rectangle like those drawn in part 2. Note that the difference in the contexts and the diagrams for parts 1 and 2 are subtly different, but seeing this difference is very important for students' future mathematical lives. If we analyze the two parts using units, we might write $\frac{1}{2}$ x $\frac{3}{4}$ square units = $\frac{3}{8}$ square units for the first one and $\frac{1}{2}$ ft. x $\frac{3}{4}$ ft = $\frac{3}{8}$ square ft for part two.

Solutions

Part 1:

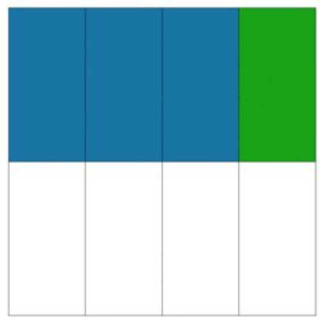
The two designs have the same size area. Students may use a variety of representations to model the soccer fields, one of which might include a diagram like the one below.

Design 1:



Green represents the grass and blue represents the soccer field. $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$.

Design 2:

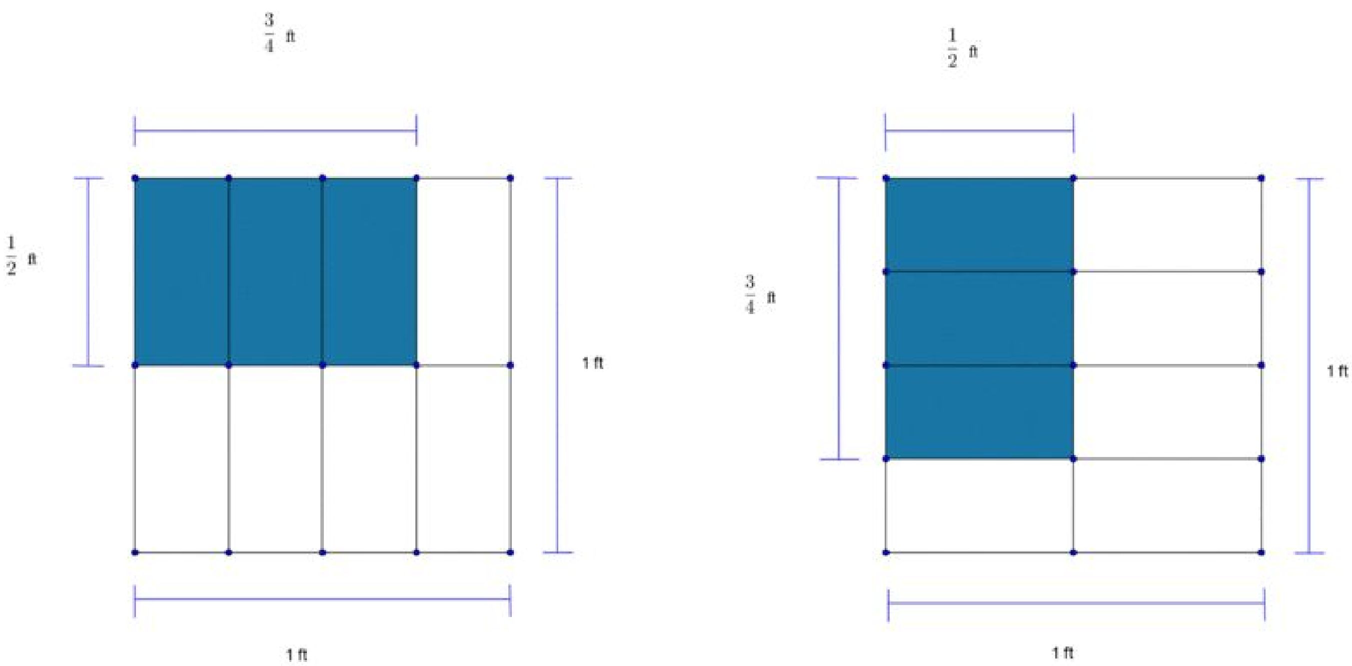


Green represents the grass and blue represents the soccer field. $\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$.

When discussing this part of the task, these diagrams should be the ones that are the focus of a whole-class discussion, as they tie into the diagrams for Part 2 in fundamental ways.

Students may use equations to represent and solve the problem. Such equations would include $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ or $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$. They may identify the commutative property as a way to justify the soccer fields being the same size.

Part 2:



Note that the only difference between the two rectangles is that one is a rotation of the other. In particular, they have the exact same area. Note that the area also corresponds to the same areas that we saw in Part 1 by finding $\frac{1}{2}$ of $\frac{3}{4}$ and $\frac{3}{4}$ of $\frac{1}{2}$.

IM Task: Salad Dressing

Task

Aunt Barb's Salad Dressing Recipe

- $\frac{1}{3}$ cup olive oil
- $\frac{1}{6}$ cup balsamic vinegar
- a pinch of herbs
- a pinch of salt

Makes 6 servings

a. How many cups of salad dressing will this recipe make? Write an equation to represent your thinking. Assume that the herbs and salt do not change the amount of dressing.
b. If this recipe makes 6 servings, how much dressing would there be in one serving? Write a number sentence to represent your thinking.

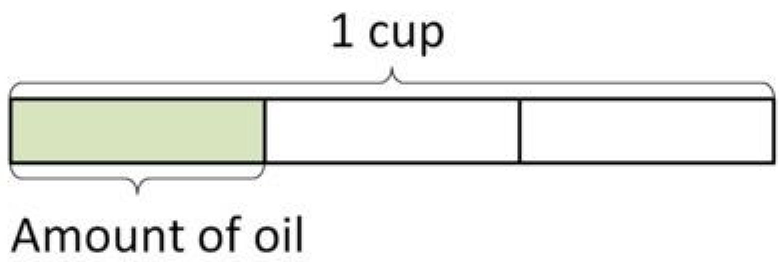
IM Commentary

The purpose of this task is to have students add fractions with unlike denominators and divide a unit fraction by a whole number. This accessible real-life context provides students with an opportunity to apply their understanding of addition as joining two separate quantities. Additionally, the context presents a "how many groups" division problem where a unit fraction should be divided into 6 equal groups.

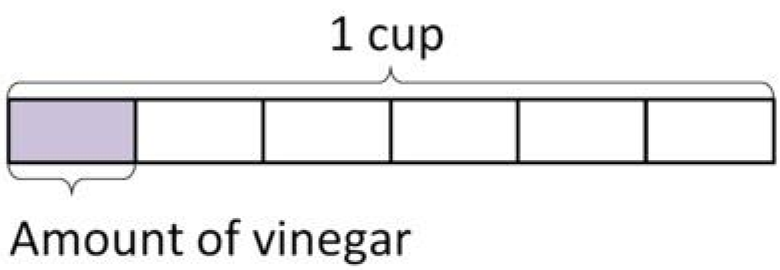
This particular task helps illustrate Mathematical Practice Standard 4, Model with mathematics. Students apply the mathematics they know to solve problems arising in everyday life. For this problem, fifth graders apply their understanding of addition as joining two separate quantities along with their understanding of division of a unit fraction by a whole number. The addition of fractions with unlike denominators can be shown symbolically or by creating a bar diagram depicting $\frac{1}{3}$ cup of olive oil and $\frac{1}{6}$ cup of balsamic vinegar. In order to make sense of these quantities, students will subdivide the $\frac{1}{3}$ cup of olive oil into sixths. Then they can visually see that the two ingredients total $\frac{3}{6}$ of a cup which is $\frac{1}{2}$ of one whole cup. This same method may be used to divide the $\frac{1}{2}$ cup of salad dressing into 6 equal portions. In each instance, students use the mathematics they know to find a solution pathway and then capture their thinking in an equation. By using the mathematical model of an equation to represent the real life situation described in the task students exercise their ability to model with the mathematics that they know.

Solutions

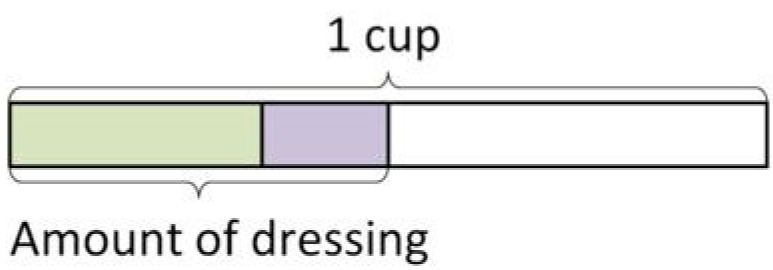
a. The total amount of dressing can be found by adding the oil and vinegar (since the other two ingredients won't change the amount). Here is a picture that represents the amount of oil:



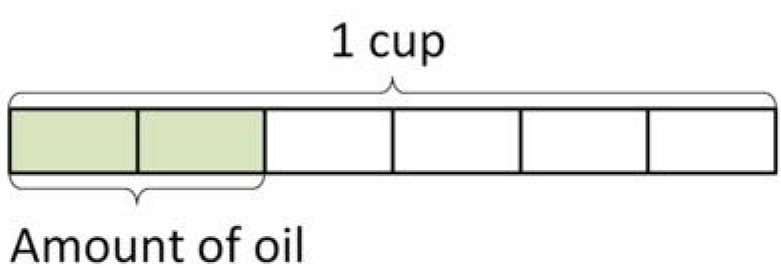
and here is a picture that represents the amount of vinegar:



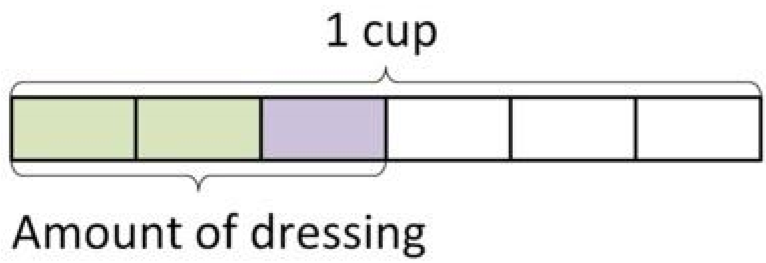
We can show the sum like this:



It is hard to tell by looking at it what fraction of the whole cup this is, but if we divide each of the thirds into two pieces, we can see how much a third is in terms of sixths:



Now we can show the sum in a way that makes it much easier to see what fraction of the whole cup it is:

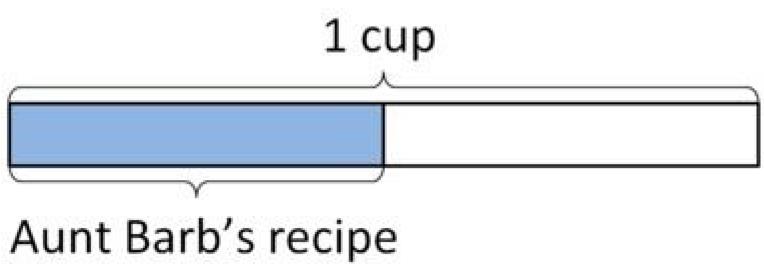


If we look carefully at the picture, we can see that $\frac{3}{6} = \frac{1}{2}$.

We can write this whole process up symbolically like this:

$$\begin{aligned}\tfrac{1}{3} + \tfrac{1}{6} &= \tfrac{2}{6} + \tfrac{1}{6} \\ &= \tfrac{3}{6} \\ &= \tfrac{1}{2}\end{aligned}$$

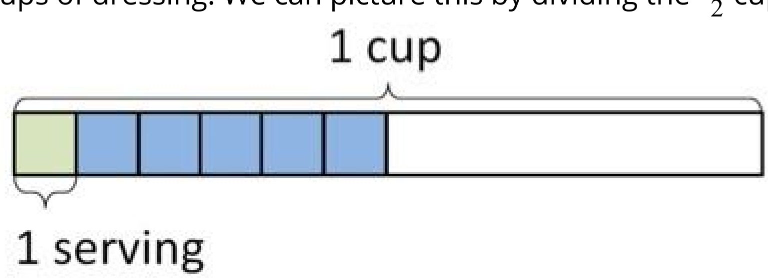
So the total amount of dressing will be $\frac{1}{2}$ cup. There is $\frac{1}{2}$ cup of dressing.



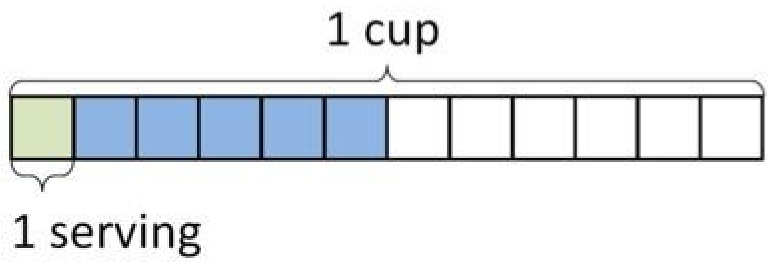
and 6 servings, so each serving is:

$$\tfrac{1}{2} \div 6$$

cups of dressing. We can picture this by dividing the $\frac{1}{2}$ cup of dressing into 6 equal parts:



To see what fraction of a cup this is, we should divide the other $\frac{1}{2}$ cup into 6 equal parts:



so the whole cup is divided into 12 equal parts. From the picture, we can see:

$$\frac{1}{2} \div 6 = \frac{1}{12}$$

So each serving will be $\frac{1}{12}$ cup of salad dressing.

Attached Resources

Cooking Time – Lesson 4

Objective:
1. Students will use equivalent fractions as a strategy to add and subtract fractions.
2. Students will divide unit fractions by non-zero whole numbers.

Overview: In this lesson, students will use equivalent fractions to add and subtract fractions, and will divide unit fractions by non-zero whole numbers, in the real-world context of food and cooking. The teacher will demonstrate, either through the use of models or the use of actual food, how different measurements (i.e. different fractions) of liquid can be combined, and how a fractional whole can be divided into an equal number of parts.

Lesson Plan:

1. Prior to lesson, gather materials:
    a. Olive oil (at least $\frac{1}{3}$ cup)
    b. Vinegar (at least $\frac{1}{6}$ cup)
    c. Assorted herbs (e.g. oregano, thyme; at least a “pinch”)
    d. Salt (at least a “pinch”)
    e. Measuring tools:
        i. A $\frac{1}{3}$ cup is a must; two such cups would be even better.
        ii. At least one large, clear measuring cup (at least 1 cup)
        iii. Teaspoon
        iv. If possible, six small condiment/”prep” cups

2. Tell students that you have a wonderful recipe for salad dressing, and share it with them. Ask if students can tell us what operation we would use to calculate the total amount of dressing this recipe will make (answer: addition). Then have students model this as an equation (answer: $\frac{1}{3} + \frac{1}{6}$).

3. Ask students to demonstrate possible solutions. Solutions may include converting $\frac{1}{3}$ to $\frac{2}{6}$, so that both fractions have like denominators, or using fraction strips or another model to demonstrate that $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$.

4. Ask the students if the dash of herbs and salt will change the total volume appreciably (answer: no).

5. Add the $\frac{1}{3}$ cup of olive oil to the clear measuring cup. Tell students that the recipe calls for $\frac{1}{6}$ cup of vinegar, but you do not have a $\frac{1}{6}$ measuring cup. Ask students to brainstorm ways that we might be able to measure it out. Ideas could include looking for other equivalent measurements, and approximating half of the $\frac{1}{3}$ cup. Teacher's Note: 8 teaspoons = $\frac{1}{6}$ cup.

6. Once an acceptable solution for measuring $\frac{1}{6}$ cup has been agreed upon, add the vinegar to the measuring cup. Confirm that the total is approximately $\frac{1}{2}$ cup. If it is slightly more or less, discuss with students why that might be (answer: human error). Add the herbs and salt.

7. Now tell students that you want to know how much of the dressing each person will get. Remind them that there are 6 servings per batch. Have students write a number sentence demonstrating this: ($\frac{1}{2} \div 6$). For students having difficulty, ask them first what operation they will need to use when putting things into equal groups (answer: division), and ask them what the total amount that is being divided (answer: $\frac{1}{2}$ cup).

8. Ask students to predict what an answer to $\frac{1}{2} \div 6$ might be. Have students explain their predictions (note: there may be many incorrect predictions, and/or no correct predictions).

9. Ask students how we could solve $\frac{1}{2} \div 6$. To demonstrate how to solve this problem by modeling, have students graphically represent dividing $\frac{1}{2}$ cup into six equal parts. Then, have students show that 1 cup divided into parts of the same size will have 12 equal parts. Thus, one serving size is $\frac{1}{12}$ cup, and $\frac{1}{2} \div 6 = \frac{1}{12}$.

10. Do this step if the six small condiment bowls are available. Demonstrate that $\frac{1}{2} \div 6 = \frac{1}{12}$ by dividing the dressing into six equal groups. Ask students how much should go into each bowl, if we are putting one serving in each (answer: $\frac{1}{12}$ cup). Tell students that $\frac{1}{12}$ cup = 4 teaspoons, and divvy up the dressing into 6 equal groups.

11. Consider having students practice dividing other fractions by whole numbers, with or without physical models.

Assessment:
Tell students that in your family, you really love salad dressing, and so this recipe really only is enough for 4 people. Ask students to write an equation for how much dressing each person gets (Answer: $\frac{1}{2} \div 4 = \frac{1}{8}$ cup).

Allow students to practice dividing several unit fractions by whole numbers. Have them solve pictorially and use “invert and multiply”. For example:

$\frac{1}{4} \div 3$ $\frac{1}{6} \div 2$ $\frac{1}{3} \div 5$

Differentiation:
Students could be challenged to calculate the number of teaspoons in $\frac{1}{6}$ cup and in $\frac{1}{12}$ cup, given that 48 tsps = 1 cup (answer: 8 and 4 teaspoons, respectively). This form of proportional reasoning is a feature of the middle school standards, but might be appropriate for some fifth grade students.

Some students may need terms explained to them, and students may also need it explained that olive oil and vinegar are major components of many types of salad dressings. English language learners or students with limited vocabularies may benefit from a word web of common types of herbs (e.g. oregano, thyme, rosemary, basil) or salad dressing types.

Students may be taught the “invert and multiply” algorithm for dividing with fractions. That is, they could be taught that an efficient way for dividing $\frac{1}{2} \div 6$ is to “invert and multiply” the 6. Thus, $\frac{1}{6} \div 2 = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$. Encourage students to focus on reasoning through the division problem, rather than relying on the rote memorization of the algorithm. The algorithm will be more fully addressed in sixth grade mathematics.

Commentary:

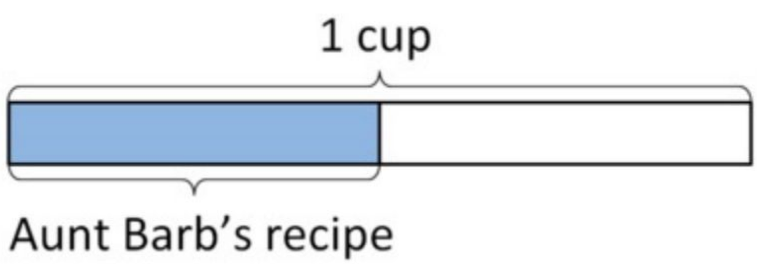
This lesson could be extended by having the students make enough dressing for the entire class, and having the teacher bring in salad for them to eat. An additional question would be, “If this recipe makes 6 servings, how many batches do we need to have enough?” Students would also calculate how the ingredients would change (i.e. a class of 24 would increase the oil and vinegar quantities by a factor of 4, and make a judgment call on the amount of herbs and spices). Students would then carefully measure out the single serving they calculated as part of the main lesson. A classroom discussion would ensue: is $\frac{1}{12}$ c of dressing enough? Too much? How many teaspoons is that? Where do “they” come up with serving size? If we were to make the recipe again, how would we change it? This extension has the non-mathematical advantage of modeling healthy eating.

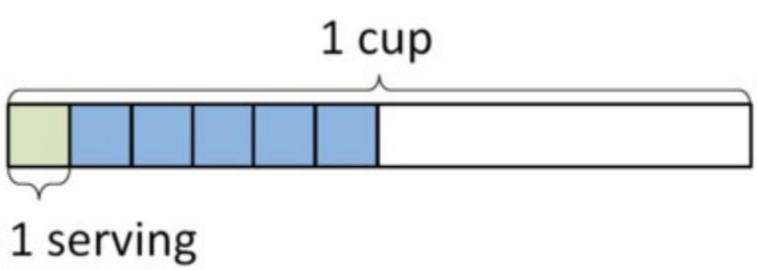
The images below may be helpful for the teacher to visualize step 8, and may also be reproduced for student use. If applicable, include worksheets, diagrams, etc. at end.

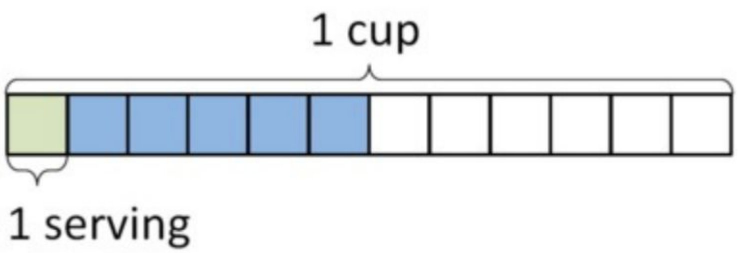
Aunt Barb’s Salad Dressing Recipe

- $\frac{1}{3}$ cup olive oil
- $\frac{1}{6}$ cup balsamic vinegar
- a pinch of herbs
- a pinch of salt

Makes 6 servings







IM Task: Banana Pudding
Task

Carolina's Banana Pudding Recipe

- 2 cups sour cream
- 5 cups whipped cream
- 3 cups vanilla pudding mix
- 4 cups milk
- 8 bananas

Carolina is making her special banana pudding recipe. She is looking for her cup measure, but can only find her quarter cup measure.

a. How many quarter cups does she need for the sour cream? Draw a picture to illustrate your solution, and write an equation that represents the situation.
b. How many quarter cups does she need for the milk? Draw a picture to illustrate your solution, and write an equation that represents the situation.
c. Carolina does not remember in what order she added the ingredients but the last ingredient added required 12 quarter cups. What was the last ingredient Carolina added to the pudding? Draw a picture to illustrate your solution, and write an equation that represents the situation.

IM Commentary

The purpose of this task is to provide students with a concrete situation they can model by dividing a whole number by a unit fraction. For students who are just beginning to think about the meaning of division by a unit fraction (or students who have never cooked), the teacher can bring in a $\frac{1}{4}$ cup measuring cup so that students can act it out. If students can reason through parts (a) and (b) successfully, they will be well-situated to think about part (c) which could yield different solution methods.

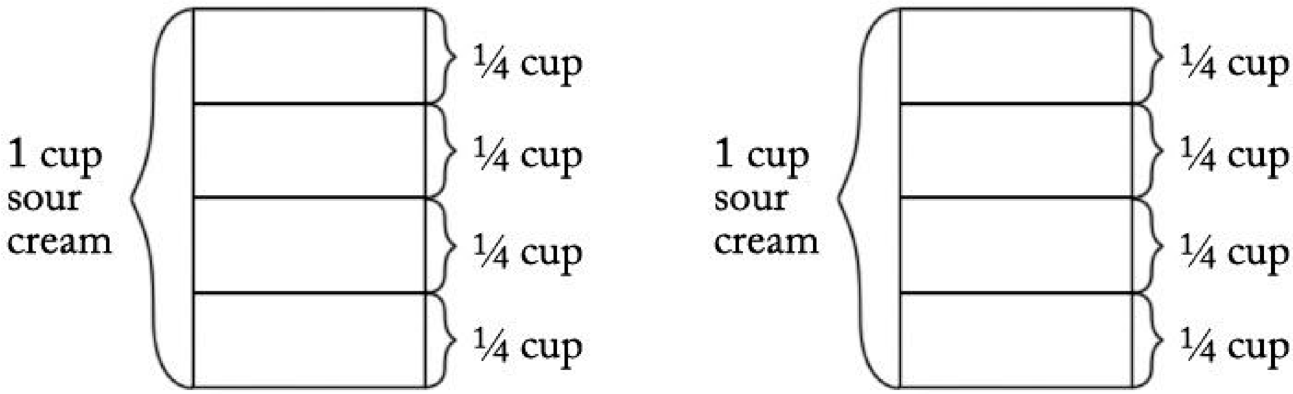
Students may need a great deal of practice seeing the connection between the visual representation and the more abstract equations. This task provides an excellent opportunity for teachers to emphasize the relationship between multiplication and division, and if students do not automatically see the connection, the teacher should draw their attention to it.

The approaches described in the solution can be used to make the connection between dividing by a whole number and multiplying by its reciprocal. If students are just beginning to understand the meaning of dividing by a unit fraction, such a conversation should be postponed. However, if they feel comfortable solving these kinds of problems with pictures, this task provides a perfect opportunity to see that dividing a whole number by a unit fraction is the same as multiplying by the reciprocal of the unit fraction. This, in turn, prepares them for the more general result that dividing by any fraction is the same as multiplying by its reciprocal, which students will see in 6th grade.

Solutions

a.

Carolina needs 8 quarter cups of sour cream because there are 4 quarter cups in 1 cup, and so it takes 2 x 4 = 8 quarter cups to make 2 cups:



This is a "how many groups?" division problem because it asks "How many quarter cups are in 2 cups?" There are two correct equations:

$$2 \div \frac{1}{4} = ?$$

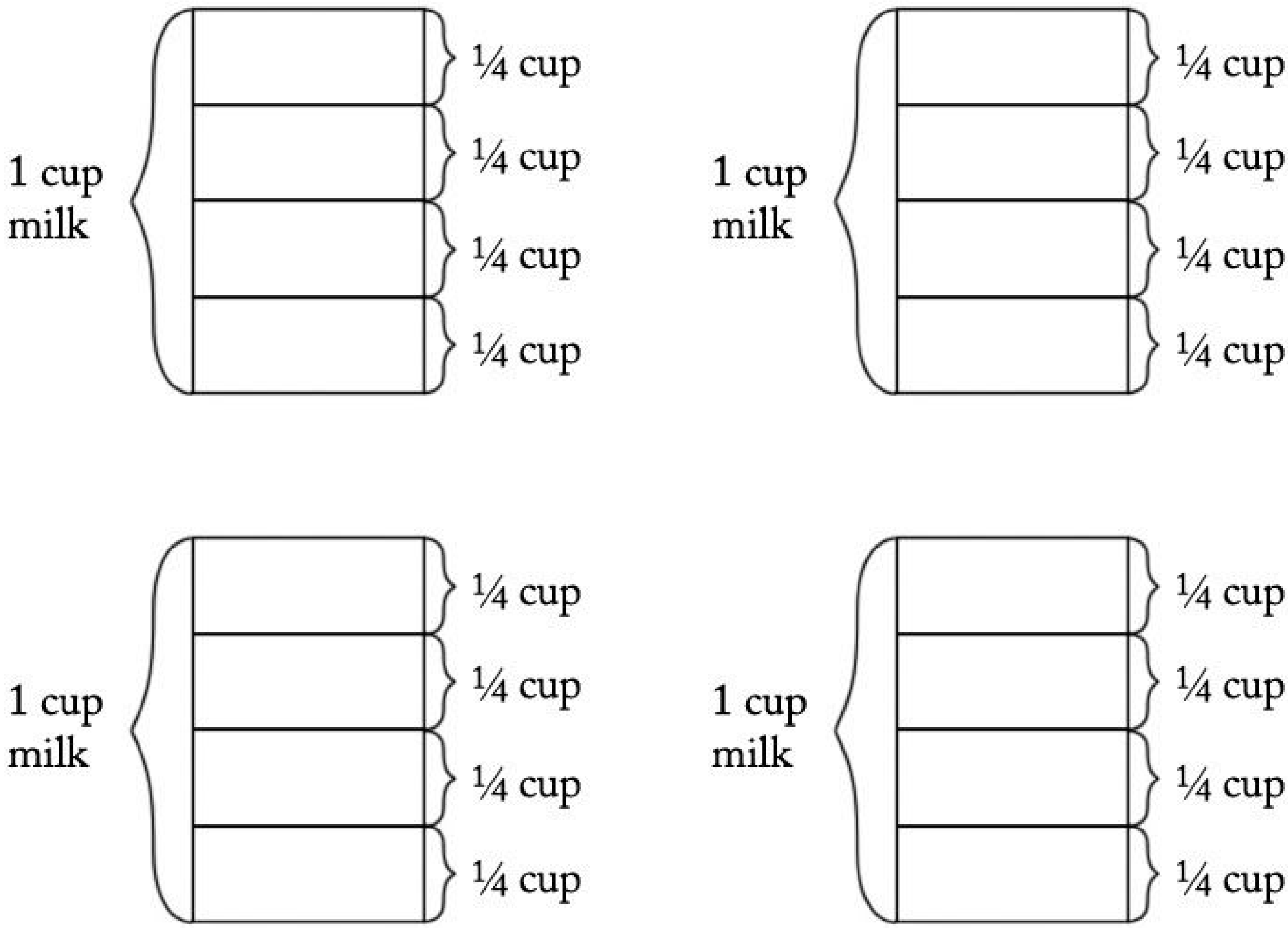
or equivalently:

$$? \times \frac{1}{4} = 2$$

We can verify that 8 is the correct solution by noting that $8 \times \frac{1}{4} = \frac{8}{4} = 2$ (4.NF.4c).

b.

Carolina needs 16 quarter cups of milk because there are 4 quarter cups in 1 cup, and so 4 × 4 quarter cups in 4 cups, as we see in this picture:



Again, this is a "how many groups?" division problem because it asks "How many quarter cups are in 4 cups?" There are two correct equations:

$$4 \div \frac{1}{4} = ?$$

or equivalently,

$$? \times \frac{1}{4} = 4$$

We can verify that 16 is the correct solution by noting that $16 \times \frac{1}{4} = \frac{16}{4} = 4$.

c.

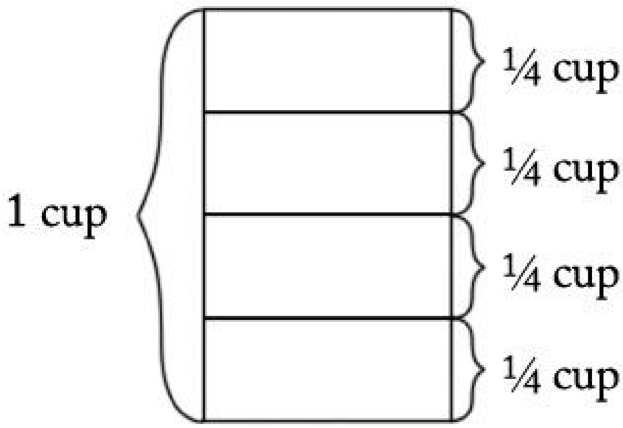
We can think of this problem several ways. First, we can ask, "How many cups did I start with if I ended up with 12 quarter cups?" We could write an equation for this:

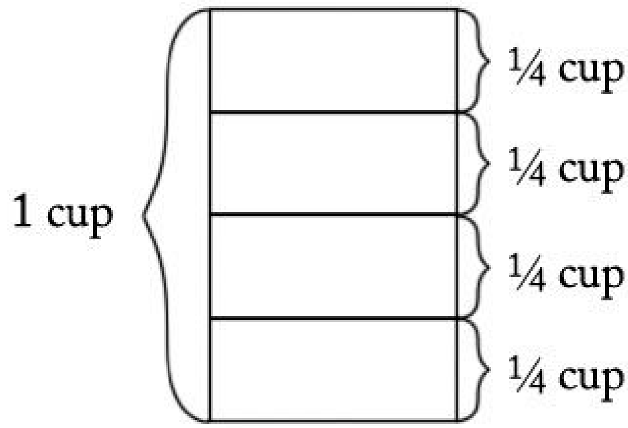
$$? \div \frac{1}{4} = 12$$

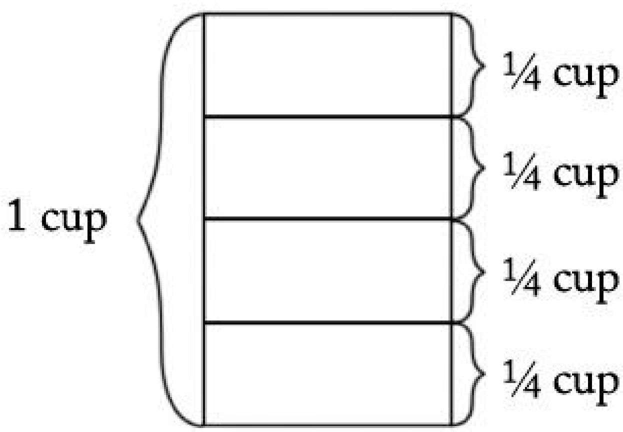
We could also say, "I have 12 quarter cups and I want to know how many cups this is, so I can multiply 12 and $\frac{1}{4}$ to find the number of cups I started with." This can be represented by the following equation:

$$12 \times \frac{1}{4} = ?$$

Notice that these two equations are equivalent and can both be interpreted in terms of the following picture:





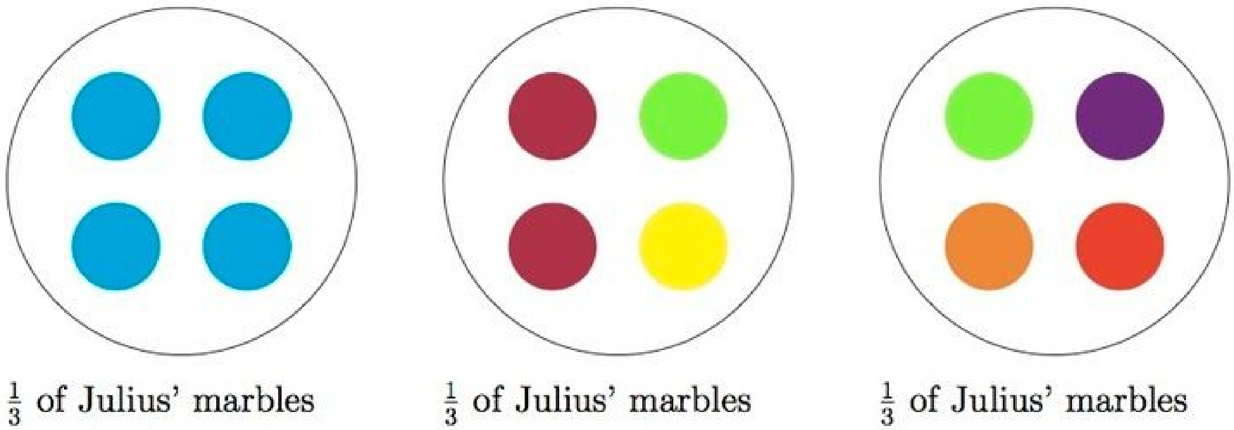


Here is yet another approach: If we note that there are 4 quarter cups in 1 cup, we can also think of this as a "how many groups?" division problem. We know that there are a total of 12 quarter cups and that there are 4 quarter cups in a cup (a group), and we want to know how many cups (or how many groups) this makes. Here the equation is:

$$12 \div 4 = ?$$

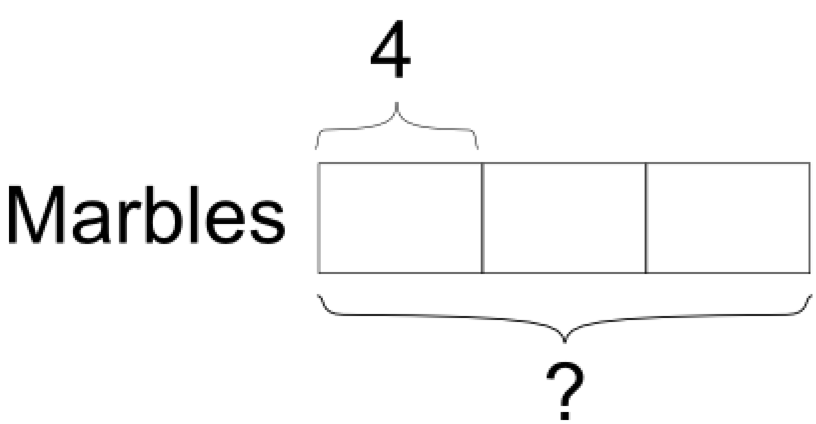
Since $12 \div 4 = 3$, she must have started with the vanilla pudding mix, as that is the only ingredient that requires 3 cups.

IM Task: How Many Marbles?
Task Julius has 4 blue marbles. If one third of Julius' marbles are blue, how many marbles does Julius have? Draw a diagram and explain.
IM Commentary This task is intended to complement "5.NF How many servings of oatmeal?" and "7.RP Molly's run.'' All three tasks address the division problem $4 \div \frac{1}{3}$ but from different points of view. "How many servings of oatmeal'' presents a how many groups version of $4 \div \frac{1}{3}$ while "Molly's run" approaches this division problem from the point of view of rates. This task provides a how many in each group version of $4 \div \frac{1}{3}$. To recall in this setting the difference between a how many groups version and a how many in each group version of the equation $4 \div \frac{1}{3} = 12$ notice that there are two different multiplication equations which can give rise to this division equation: $12 \times \frac{1}{3} = 4$ $\frac{1}{3} \times 12 = 4.$ In the first of these equations 12 groups of $\frac{1}{3}$ make 4 and so 12 is the answer to how many thirds there are in 4. In the second equation $\frac{1}{3}$ a group of 12 make 4 and so 12 is the answer to how much is in a group if one third of that group is 4.
Solutions Solution: 1 If one third of Julius' marbles are blue, this means that $\frac{2}{3}$ of his marbles are colors other than blue. This means that Julius has two times as many marbles of colors other than blue than he has blue marbles. Since Julius has 4 blue marbles, this means that he has 8 marbles of other colors and so he has 12 marbles total. Below is a picture representing Julius' marbles:



Since $\frac{1}{3}$ of Julius' marbles are blue, we can also record this as a missing factor problem: $\frac{1}{3}$ × ____ =4. This missing factor problem is equivalent to the division problem: 4 ÷ $\frac{1}{3}$ = ____. In either case, the answer is 12 marbles, as we saw in the picture.

Solution 2: Using a tape diagram



The number of blue marbles is represented by 1 unit. We can find the total using the missing factor sentence:

4 = $\frac{1}{3}$ × ____.

This missing factor sentence is equivalent to the division problem:

4 ÷ $\frac{1}{3}$ = ____.

As the diagram shows, this is equivalent to solving:

$4 \div \frac{1}{3} = 4 \times 3 = 12$.

Julius has 12 marbles.