Illustrative Mathematics®

Illustrative Mathematics®

GRADE 5

Unit

2

Teacher Adaptation Pack

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K5_Beta

IM K–5 MATH™ BETA

Grade 5 Unit 2 Teacher Adaptation Pack

Directions for Use

1. Read the current grade level unit standards and prior-grade connections.
2. Ask prior grade level teachers if students were taught the topics last year or show students a problem on the prior-grade level topic and anonymously ask students if they know how to solve the problem.
   a. If yes, start the current grade level section without the add-in lessons.
   b. If not, teach the prior grade level add-in lessons.
3. In the last add-in lesson, give the mini-assessment as the cool-down.
   a. If students get the questions correct, start the current grade level section.
   b. If students get some things correct and some not, still start the current grade level section, and use the ongoing practice materials to support students.

Recommended Implementation

Add-In *Grade 4 Unit 5*	Mini Assess	Section A *Grade 5 Unit 2*	Add-In *Grade 4 Unit 3*	Mini Assess	Section B *Grade 5 Unit 2*	Section C *Grade 5 Unit 2*
Ongoing Practice and Fluency Support						

Alternate Implementation

Add-In *Grade 4 Unit 5*	Add-In *Grade 4 Unit 3*	Mini Assess	Section A *Grade 5 Unit 2*	Section B *Grade 5 Unit 2*	Section C *Grade 5 Unit 2*
Ongoing Practice and Fluency Support					

Grade 5 Unit 2: Fractions as Quotients and Fraction Multiplication		
	Section A (fractions as division)	**Sections B and C (fraction multiplication and area)**
Standards	• 5.NF.B.3	• 5.NF.B.4.A, 5.NF.B.4.B, 5.NF.B.6, 5.OA.A
Prior-Grade Connections	• 3.OA.A.2, 3.NF.A.1, 3.NF.A.2, 4.OA.A.3	• 4.NF.B.4
Rationale	In 5.2 Section A, students extend their understanding of whole-number division to include quotients that are fractions or mixed numbers. This work relies on students' prior knowledge of whole-number division that results in whole-number quotients and remainders. Students should also be familiar with the relationship between multiplication and division, and have a conceptual understanding of fractions as numbers. Since much of this prior knowledge is developed in grade 3, the prior grade work in this resource will include whole-number division with remainders, and the meaning of the numerator and denominator in fractions. In Sections B and C, students multiply a whole number by a fraction and find the area of rectangles with one whole number and one fractional or mixed-number side-length. Success in these sections requires an understanding of the multiplication of a fraction by a whole number, developed in grade 4.	
Add-in Lessons	Before Section A: • 4.5 Lesson 20 • 4.2 Lessons 1–2	Before Section B: • 4.3 Lessons 1–5
5.2 Lessons to Combine or Skip	None	
Prior-grade Practice and Fluency	• Grade 4 whole-number division Number Talks • Grade 4 fraction multiplication Number Talks • Grade 4 fraction multiplication True or False • Center: Race to Zero (Stages 1–4)	

<table>
<tr><td>Extension and Exploration</td><td colspan="2">Center: Rolling for Fractions (Stages 1–2)
IM Task: Sugar in Six Cans of Soda
4.8 Lessons 3, 5, and 12</td></tr>
<tr><td>Assessment</td><td>Mini-Assessment 1

If students need ongoing practice:
Rolling for Fractions Center: Stage 3
5.2 Practice problems</td><td>Mini-Assessment 2

If students need ongoing practice:
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Fraction Minute Center: Stage 4</td></tr>
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4.5 Lesson 20: Finding All Kinds of Quotients	
Standards Alignment Addressing CCSS**:** 4.NBT.B.6, 4.OA.A.3	
Teacher-facing Learning Goals ● Represent and solve word problems that involve division of numbers up to four digits by single-digit divisors. ● Interpret the result and remainder of division in context.	
Lesson Purpose The purpose of this lesson is for students to represent and solve contextual problems that involve dividing a whole number of up to four digits by a single-digit divisor, resulting in a number with or without a remainder. Students also interpret the result and remainder in context.	
Access for Students with Disabilities Activity 1: Action and Expression	**Access for English Learners** Activity 2: MLR8 Discussion Supports
Materials to Gather ● tools for creating a visual display: 1 per student, or 1 for every 2 students	**Materials to Copy** ● none
Cool-down: Miscounting?	
Student-facing Task Statement Mai is reciting multiples of 6. The last number she calls out is 194. Clare says, "I think you may have made a mistake." Do you agree with Clare? Explain or show your reasoning.	
Student Responses Yes, I agree with Clare. Sample reasoning: ● 194 is not a multiple of 6. I know that 6 x 30 = 180, and 194 is 14 away from 180. Because 14 is not a multiple of 6, then 194 is also not a multiple of 6. ● Six is not a factor of 194. I divided 194 by 6 and got 32 with a remainder of 2. If Mai counted correctly, she would have called out 192 and then 198.	
Teacher Reflection Question What productive and unproductive beliefs did students show when they were solving problems today? How might you amplify the productive beliefs and address the unproductive ones?	

Warm-up: True or False: Multiples

Time: 10 minutes

Standards Alignment
Building on CCSS: 4.OA.B.4
Addressing CCSS: 4.NBT.B.6

Warm-up Narrative
The purpose of this warm-up is to elicit strategies and understandings students have for identifying multiples of small numbers, primarily by looking for and making use of structure (MP7). These understandings will be helpful later when students solve division problems that involve distinguishing quotients with and without a remainder.

Task Statement

Is each statement true or false? Be prepared to explain your reasoning.

1. 105 is a multiple of 2.
2. 105 is a multiple of 3.
3. 105 is a multiple of 5.
4. 105 is a multiple of 15.

Teacher Directions

Launch/Activity
- Display one statement.
- “Give me a signal when you know whether the equation is true and can explain how you know.”
- 1 minute: quiet think time
- Share and record answers and strategy.
- Repeat with each statement.

Synthesis
- How can you use the second and third statements to help you reason about the fourth statement? (Because 105 is a multiple of 3 and 5, it is also a multiple of 15.)
- How do you know if a number is a multiple of a different number? (When you divide a multiple by that number, it will result in a whole-number quotient without a remainder.)

Student Responses
1. False, because all multiples of 2 have an even number in the ones place.
2. True, because 105 ÷ 5 = 21 with no remainder.

3. True, because all multiples of 5 have 5 or 0 in the ones place.
4. True, because 105 = 150 – 45 and both 150 and 45 are multiples of 15.

Activity 1: Muffins and Seats

Time: 15 minutes

Standards Alignment
Addressing CCSS: 4.NBT.B.6, 4.OA.A.3

Activity Narrative
This activity encourages students to interpret the quantities in situations, represent them mathematically, use their models to find solutions, and then interpret their solutions in context. The dividends here are limited to three-digit numbers.

The work engages students in quantitative and abstract reasoning (MP2).

SwD Support Tags

- Action and Expression

SwD Support Text

Action and Expression: Develop Expression and Communication. Synthesis: Identify connections between strategies that result in the same outcomes but use differing approaches.
Supports accessibility for: Conceptual Processing.

Task Statement

1. Two bakers at a bakery made 378 muffins. The muffins are put in boxes of 4.

   - The first baker says they will need 94 boxes for all the muffins.
   - The second baker says 95 boxes are needed.

   Who do you agree with? Explain or show your reasoning.

2. An auditorium seats 258 people. The seats are arranged in rows of 9, but there is one short row with fewer than 9 seats.

   How many regular rows are there? How many seats are in the shorter row?

Teacher Directions

Launch/Activity

- Groups of 2
- 5–6 minutes: quiet work time
- 2–3 minutes: partner discussion
- Monitor for the strategies students use to divide the numbers and the different ways they interpret the remainder in each question.

Synthesis

- Invite students to share their responses and reasoning.
- For the first question, highlight that both 94 and 95 boxes are plausible if they could be defended and the assumptions are made clear. (For example, Students might say that the bakers could have two leftover muffins rather than trying to sell them in boxes, so 94 boxes are enough.)
- For the second question, ask: "What equation could we write to describe the relationship between the number of rows and the number of seats?" (One possible equation: (68 x 9) + 6 = 258.)

Student Responses

1. Sample responses:

- I agree with the first baker. 378 ÷ 4 is 94 with a remainder of 2. The bakers can't just put the 2 extra muffins in a box and sell half a box, so they only need 94 boxes.
- I agree with the second baker. If all the muffins are to be put in a box, there will be 94 boxes of 4 and 1 box with only 2 muffins. This means they will need 95 boxes.

2. There are 68 rows of 9 and 1 row of 6. Sample reasoning: 258 divided by 9 is 68 with a remainder of 6.

Activity 2: Save for a Garden

Time: 20 minutes

Standards Alignment
Addressing CCSS: 4.NBT.B.6, 4.OA.A.3

Activity Narrative
In this activity, students continue to solve contextual problems that involve division. Here, the dividends extend to four-digit numbers and the problems demand a greater lift, prompting students to persevere in making sense of them (MP1).

In the second half of the activity, students are asked to reason in the opposite direction: given a division expression, they are to invent a situation that it can represent and interpret the value of the expression in context.

MLR Tags
- MLR8

EL Support Text
MLR8 Discussion Supports. Synthesis: Display sentence frames to support partner discussion: "I agree because . . .", and "I disagree because . . . ." when explaining why the quotient is correct or incorrect.
Advances: Speaking, Conversing

Task Statement	Teacher Directions
1. A school needs $1,270 to build a garden. After saving the same amount each month for 8 months, the school is still short by $6. How much did they save each month? Explain or show your reasoning. 2. Choose one of the following division expressions.	**Launch/Activity** • 3–4 minutes: quiet work time for the first question • Invite students to share responses and reasoning. • "What equation(s) can we write to represent the relationship between the monthly

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711 ÷ 3   3,128 ÷ 8
a. Write a situation to represent the expression.
b. Find the quotient. Show your reasoning.
c. Explain what the value means in the situation you invented.
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saving, the number of months of saving, and the amount needed for the garden?” (One possible equation: (8 x 158) + 6 = 1,270)
● 5–7 minutes: quiet work time for the second question
● Give access to tools for creating a visual display for the second question.

Synthesis
● Students find a partner who chose a different expression and take turns presenting their display. The person listening should consider whether the response makes sense and check if the quotient is correct.
● If time permits, find another partner to present their display.
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Student Responses

1. $158 a month. Sample reasoning:
    a. 1,270 ÷ 8 = 158 with a remainder of 6. This means the family has saved up $1,264.
    b. 1,270 – 6 = 1,264 and 1,264 ÷ 8 = 158

2. Sample response for 711 ÷ 3:
    a. Three siblings collected 711 pennies and are dividing them equally. The quotient is the number of pennies each sibling gets.
    b. 711 ÷ 3 = 237
    c. Each sibling gets 237 pennies.

    Sample response for 3,128 ÷ 8:

    a. A family is making a road trip that is 3,128 miles long over 8 days. The quotient is the distance traveled each day if they travel the same distance per day.
    b. 3,128 ÷ 8 = 391
    c. The family travels 391 miles a day.

Lesson Synthesis

“Today we solved problems that involved division. What strategies did you find yourself using to divide numbers? Did you:

- Use partial products?
- Use partial quotients?
- Draw diagrams?
- Divide by place value (thousands, hundreds, tens, and ones)?
- Write a series of equations?
- Estimate first?
- Use other ways?"

4.2 Lesson 1: Representations of Fractions (Part 1)

Standards Alignment

Building On CCSS:
- 3.NF.A.1

Building Toward CCSS:
- 4.NF.A.1

Teacher-facing Learning Goals
- Make sense of the numerator and denominator of unit fractions that have denominators 2, 3, 4, 5, 6, 8, 10, and 12.
- Use physical and visual representations to reason about fractions.

Student-facing Learning Goals
Let's name some fractions and represent them visually.

Lesson Purpose
The purpose of this lesson is for students to make sense of unit fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12, using physical and visual representations.

Lesson Narrative
In grade 3, students were introduced to fractions. They learned to name and represent fractions, to recognize equivalent fractions, and to compare fractions with like and unlike denominators (limited to 2, 3, 4, 6, and 8). They used area diagrams, tape diagrams, and number lines to support their reasoning with fractions.

In this unit, students build on the work they began in grade 3. They continue to represent fractions, reason about equivalence, and compare fractions—using fraction strips, tape diagrams, and number lines to support their thinking. Students also move toward generalizing the processes for determining equivalence and for making comparisons.

This first lesson activates students' prior knowledge of unit fractions but includes fractions with denominators 5, 6, 10, and 12. Students revisit the meaning of numerator and denominator, name unit fractions, create representations for them, and recall some strategies and tools for reasoning about fractions.

The idea of equivalence may naturally come up (and will help to prepare students for upcoming work), but it is not the focus of this lesson.

Access for Students with Disabilities
Activity 1: Engagement

Instructional Routines

- What Do You Know About _____?, MLR1: Stronger and Clearer Each Time

Materials to Gather	**Materials to Copy**
• 4 strips of equal-size paper per group of 2 students (cut lengthwise from letter-size or larger paper), or cut from 4.2.A1 Blackline Master • 1 straightedge per student	• 4.2.A1.Blackline Master, if needed to create fraction strips
Lesson Timeline **Warm-up** 10 minutes **Activity 1** 20 minutes **Activity 2** 20 minutes **Lesson Synthesis** 5 minutes **Cool-down** 5 minutes	**Teacher Reflection Question** What did you learn about each student and their foundational understanding of fractions based on their work today?

Cool-down: What Do the Diagrams Show?

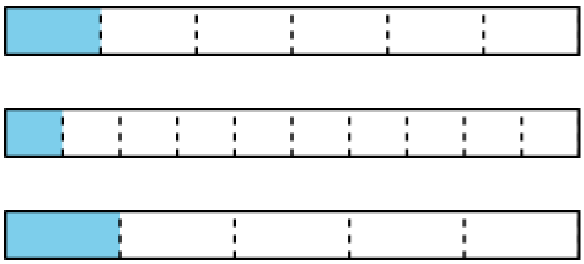
Standards Alignment:

- 4.NF.A.1

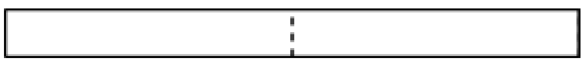
Student-facing Task Statement

Each tape diagram represents 1.

1. What fraction does each shaded part represent?



2. Explain or show how you could use this tape diagram to represent sixths.



Student Responses

1. $\frac{1}{6}$, $\frac{1}{10}$, and $\frac{1}{5}$.
2. Sample response: Split each half into 3 equal parts so there will be a total of 6 parts. Each part is a sixth.

Warm-up: What Do You Know About $\frac{1}{2}$?

Time: 10 minutes

Standards Alignment

Building On CCSS:

- 3.NF.A.1

Building Toward CCSS:

- 4.NF.A.1

Materials to Gather

- none

Materials to Copy

- none

Instructional Routines

- What Do You Know About _____?

Warm-up Narrative
The purpose of this warm-up is to invite students to share what they know about the number $\frac{1}{2}$ and elicit ways in which it can be represented. It gives the teacher the opportunity to hear students' understandings about and experiences with fractions, $\frac{1}{2}$ in particular. The fraction $\frac{1}{2}$ is familiar to students and will be central in the first activity.

Student-facing Task Statement	Teacher Directions
What do you know about $\frac{1}{2}$?	**Launch** • Groups of 2 • Display the number $\frac{1}{2}$. • "What do you know about this number?" • 1 minute: quiet think time **Activity** • "Discuss your thinking with your partner." • 2 minutes: partner discussion • Share and record responses. **Synthesis** • "What different ways can we represent $\frac{1}{2}$?" (Cut an object, a rectangle, a shape into two equal pieces, mark the middle point between 0 and 1 on a number line.)

Student Responses
Sample responses:
- It is a fraction.
- I shared half of my sandwich with my friend.
- Split something into two pieces.
- We can "halve" something.
- Dividing by 2.
- It is halfway between 0 and 1 on a number line.
- It is less than 1.
- It is a number.

Activity 1: Fraction Strips
Time: 20 minutes
Standards Alignment
Building On CCSS: ● 3.NF.A.1
Building Toward CCSS: ● 4.NF.A.1
Materials to Gather ● 4 strips of equal-size paper per group of 2 students (cut lengthwise from letter-size or larger paper), or cut from 4.2.A1.Blackline Master ● 1 straightedge per student
Materials to Copy ● 4.2.A1.Blackline Master, if needed to create fraction strips
Activity Narrative The purpose of this activity is for students to use fraction strips to represent halves, fourths, and eighths. The denominators in this activity are familiar from grade 3. The goal is to remind students of the relationships between fractional parts in which one denominator is a multiple of another. Students should notice that each time the unit fractions on a strip are folded in half, there are twice as many equal-size parts on the strip and that each part is half as large. In the discussion, use the phrases “number of parts” and “size of the parts” to reinforce the meaning of a fraction.
SwD Support Tags ● Engagement
SwD Support Text *Engagement: Provide Access by Recruiting Interest.* Launch: Provide choice and autonomy. Provide access to different colored strips of paper students can use to differentiate each fraction. *Supports accessibility for: Organization; Visual-Spatial Processing.*

Student-facing Task Statement

Your teacher will give you strips of paper. Each strip represents 1.

1. Use the strips to represent halves, fourths, and eighths.

   Use one strip for each fraction and label the fractional parts.

2. What do you notice about the number of pieces or the size of the pieces? Make at least two observations.

Teacher Directions

Launch

- Groups of 2
- 4 paper strips and a straightedge for each group
- Hold up one strip for all to see.
- “For this activity, each strip represents 1.”
- Label that strip with the number 1 and tell students to do the same on one of their strips.
- “Take a new strip. How would you fold it to show halves?”
- 30 seconds: partner think time
- Share responses.
- “Think about how you would show fourths on the third strip and eighths on the last strip.”

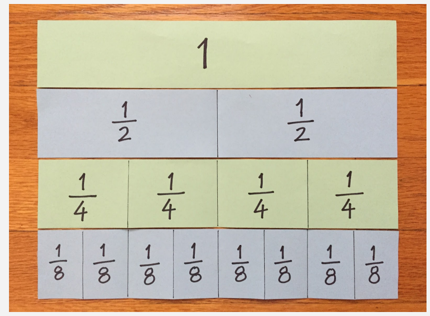
Activity

- “Work with your partner to complete the task.”
- 10 minutes: partner work time
- Monitor for students who notice that each denominator is twice of the next smaller denominator.

Synthesis

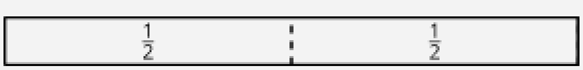
- Select a group to share their paper strips and how they identified those fractional parts. Ask if others in the class also found them the same way.

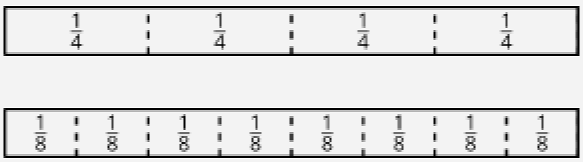
- Display one set of completed strips, as shown.



- Invite students to share what they noticed about the number and size of the parts on the strips. Highlight the ideas noted in Student Responses.
- If no students mentioned the relationships between the fractions on different strips, encourage them to work with a partner to look for some.
- If the terms “numerator” and “denominator” did not arise during discussion, ask students about them.
- Remind students that the **denominator**, the number at the bottom of a fraction, tells us the number of equal-size parts in 1 whole, and the **numerator**, the number at the top of a fraction, refers to how many of those parts are being described. Consider displaying these terms and their meanings for students to reference.
- Ask students to save the fraction strips for a future lesson.

Student Responses

1. 



2. Sample responses:
    - Each time we folded, there were more pieces.
    - Each time we folded, the pieces got smaller.
    - The $\frac{1}{8}$ pieces are each half the size of the $\frac{1}{4}$ pieces.
    - The $\frac{1}{4}$ pieces are each half of the $\frac{1}{2}$ pieces.

Supporting Student Thinking

Some students may have trouble identifying the relationships between fractions (or may arrive at wrong conclusions) if they did not fold the paper strips with enough precision and ended up looking at unequal parts. Prompt them to fold as precisely as possible and to create crisp creases with each fold. Consider preparing extra strips, in case needed.

Activity 2: Fractions, Represented

Time: 20 minutes

Standards Alignment

Building On CCSS:
- 3.NF.A.1

Building Toward CCSS:
- 4.NF.A.1

Materials to Gather
- 1 straightedge per student

Materials to Copy
- none

Instructional Routines
- MLR1: Stronger and Clearer Each Time

Activity Narrative

Previously, students revisited some benchmark fractions (halves, fourths, and eighths) and recalled the meaning of numerators and denominators. The purpose of this activity is for students to revisit the meaning of unit fractions with some other denominators (3, 5, 6, 10, and 12) and be reminded of how to name and represent them.

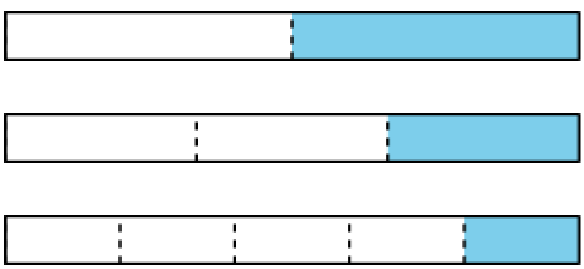
As they draw diagrams to represent these fractions, students have opportunities to look for structure and make use of the relationships between the denominators of the fractions (MP7).

To support students in drawing straight lines on the tape diagrams, provide access to a straightedge or ruler. Students should not, however, use rulers to measure the location of a fraction on any diagram.

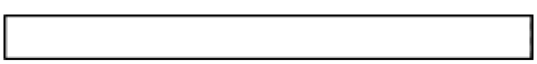
This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: reading, writing

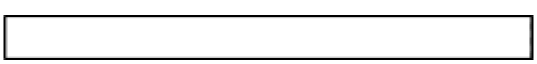
Student-facing Task Statement

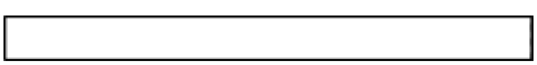
1. If the entire diagram represents 1, what fraction does each shaded part represent?

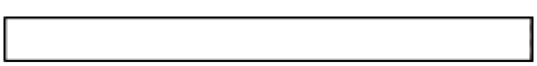


2. Here are four other blank diagrams. Each diagram represents 1. Partition each diagram and shade one part so that the shaded part represents the given fraction.

a. $\frac{1}{6}$ 

b. $\frac{1}{8}$ 

c. $\frac{1}{10}$ 

d. $\frac{1}{12}$ 

Teacher Directions

Launch

- Groups of 2
- Straightedge for each student
- "Let's look at some other fractions and draw diagrams to represent them. Consider using a straightedge when you draw."

Activity

- 7–8 minutes: independent work time
- "Discuss your responses with your partner. Be sure to talk about how you created diagrams for $\frac{1}{6}$, $\frac{1}{10}$, and $\frac{1}{12}$."
- 2–3 minutes: partner discussion
- Monitor for students who:
    - notice the relationship of thirds, sixths, and twelfths, and of fifths and tenths
    - use the given tape diagrams to help partition the other diagrams

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3. Suppose you are creating a representation of $\frac{1}{20}$ using the same tape diagram.

   Would the shaded part be larger or smaller than the shaded part in the diagram of $\frac{1}{10}$? Explain how you know.

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Synthesis

- "How did you know how to partition the diagrams in the second question?"
- Select students who made use of the relationships between denominators or the given tape diagrams to display their diagrams and share their reasoning or drawing process.
- "What relationships do you see between the fractions in this activity?" Sample responses:
  - As the denominator gets larger, each fractional part gets smaller.
  - Halves, fourths, and eighths are related in that twice 2 is 4 and twice 4 is 8.
  - Thirds, sixths, and twelfths are related in that a third is 2 sixths and a sixth is 2 twelfths. Fifths and tenths are related in the same way.

MLR1 Stronger and Clearer Each Time

- "Share your response to the last question with your partner. Take turns being the speaker and the listener. If you are the speaker, share your response. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.

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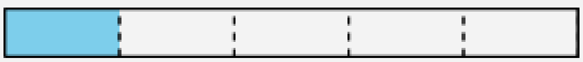
- "Revise your initial response based on the feedback from your partners."
- 2–3 minutes: independent work time

Student Responses

1. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$
2. Sample response:

a.



b.

c.

d.

3. It will be smaller, because the tape will be partitioned into 20 equal parts and each part will be smaller (half the size) of the parts representing tenths.

Lesson Synthesis

"Today we refreshed our memory about fractions like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and so on. We used fraction strips and diagrams to represent these fractions."

Based on students' work during the lesson, choose the questions that need more discussion:

- "In general, what does the denominator in a fraction represent?" (The number of equal parts in 1 whole.)
- "What does the fraction $\frac{1}{5}$ tell us?" (The size of one part if 1 whole is split into 5 equal parts.)
- "What did you notice about the size of a fraction as the denominator gets larger?" (The size of the fraction gets smaller.) "Why might that be?" (There are more equal parts in 1 whole, so each part gets smaller.)

• “What relationships did we see between the fractions that we studied today?” (The denominators of some fractions are multiples of other fractions. A representation of one fraction can be split into two or three to represent another fraction.)

Response to Student Thinking Next Day Students do not use precise language to explain and describe relationships of fractional parts in the fraction strips.	**Next Day Support** • During the synthesis of the warm-up, highlight the terms “parts,” “partition,” “numerator,” and “denominator” to support vocabulary from grade 3.
Response to Student Thinking Students do not understand the meaning of numerator and denominator.	**Prior-unit Support** • Grade 3, Unit 5, Section A

4.2 Lesson 2: Representations of Fractions (Part 2)
Standards Alignment
Building On CCSS: • 3.NF.A.1
Building Toward CCSS: • 4.NF.A.1
Teacher-facing Learning Goals • Make sense of the numerator and denominator of non-unit fractions (including those greater than 1) that have denominators 2, 3, 4, 5, 6, 8, 10, and 12. • Use tape diagrams to represent fractions.
Student-facing Learning Goals Let’s name some other fractions and represent them with tape diagrams.
Lesson Purpose The purpose of this lesson is for students to make sense of non-unit fractions (including those greater than 1) that have denominators 2, 3, 4, 5, 6, 8, 10, and 12, using tape diagrams to do so.
Lesson Narrative In the previous lesson, students made sense of the meaning of numerator and denominator in unit fractions. They identified fractions represented by tape diagrams, and created diagrams to represent

given fractions. In this lesson, they reason in similar ways—using numerical and visual representations—about non-unit fractions and fractions that are greater than 1.

Students are reminded of what they learned in grade 3: that a non-unit fraction $\frac{a}{b}$ can be understood as *a* parts of a unit fraction $\frac{1}{b}$, and that fractions with different numerators and denominators can have the same representation. Unlike in grade 3, the denominators they see here now include 5, 10, and 12.

Access for Students with Disabilities
Activity 1: Representation

Access for English Learners
Activity 2: MLR2 Collect and Display

Instructional Routines
- Which One Doesn't Belong

Materials to Gather
- 1 straightedge per student

Materials to Copy
- none

Lesson Timeline

Warm-up 10 minutes
Activity 1 20 minutes
Activity 2 15 minutes
Lesson Synthesis 10 minutes
Cool-down 5 minutes

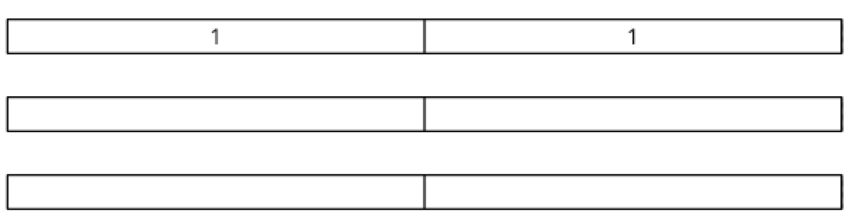
Teacher Reflection Question
Who participated in math class today? What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class?

Cool-down: What Do the Diagrams Show?

Standards Alignment:
- 4.NF.A.1

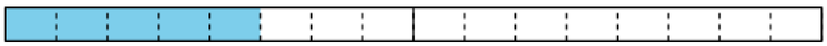
Student-facing Task Statement

Use the blank diagrams to create representations for $\frac{5}{8}$ and $\frac{9}{8}$.



Student Responses

$\frac{5}{8}$



$\frac{9}{8}$

Warm-up: Which One Doesn't Belong: All Cut Up

Time: 10 minutes

Standards Alignment

Building On CCSS:

- 3.NF.A.1

Building Toward CCSS:

- 4.NF.A.1

Materials to Gather

- none

Materials to Copy

- none

Instructional Routines

- Which One Doesn't Belong

Warm-up Narrative

This warm-up prompts students to carefully analyze and compare the features of four shapes that are each partitioned. It enables the teacher to hear the terminologies students know and how they talk about fractions and fractional parts. In making comparisons, students have a reason to use language precisely (MP6).

Student-facing Task Statement

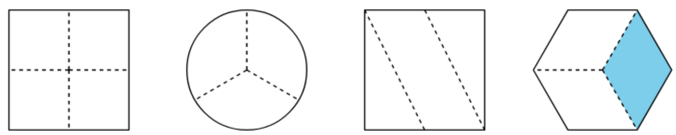
Which one doesn't belong?

A B C D

Teacher Directions

Launch

- Groups of 2
- Display the image.



- “Pick one that doesn’t belong. Be ready to share why it doesn’t belong.”
- 1 minute: quiet think time

Activity
- “Discuss your thinking with your partner.”
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis
- “What does the shaded part in D represent?” ($\frac{1}{3}$ or one-third of the shape)
- Shade one part of B and C.
- “Is each shaded part one-third of the shape as well?” (Yes for B, no for C.)
- “Why is the shaded part not one-third of the square in C?” (The parts aren’t equal in size.)
- Shade one part of A. “Is it a third of the square?” (No, it is $\frac{1}{4}$ or one-fourth.)

Student Responses
Sample response:
- A is the only one that is not partitioned into 3 parts.
- B is the only one that does not have straight edges.
- C is the only one that is not partitioned into equal parts.
- D is the only one whose parts are not all clear or unshaded.

Activity 1: Matching: A Diagram for Each Fraction

Time: 20 minutes

Standards Alignment

Building On CCSS:
- 3.NF.A.1

Building Toward CCSS:
- 4.NF.A.1

Materials to Gather
- none

Materials to Copy
- none

Activity Narrative
The purpose of this activity is to activate what students know about the meaning and size of non-unit fractions. Students match a set of fractions with tape diagrams that represent them. In doing so, they recall that a non-unit fraction $\frac{a}{b}$ can be understood as *a* parts of a unit fraction $\frac{1}{b}$. They also recall that fractions with different numerators and denominators can have the same representation. The reasoning they do and the representations they use here echo their work on equivalence from grade 3.

SwD Support Tags
- Representation

SwD Support Text
Representation: Internalize Comprehension. Use visual details such as color or arrows to illustrate connections between representations. For example, use the same color for the numerator, and the shaded portion of the corresponding diagram.
Supports accessibility for: Visual-spatial Processing

Student-facing Task Statement

Match each fraction with a diagram that represents it.

Two of the fractions are not represented. Create a representation for each of them.

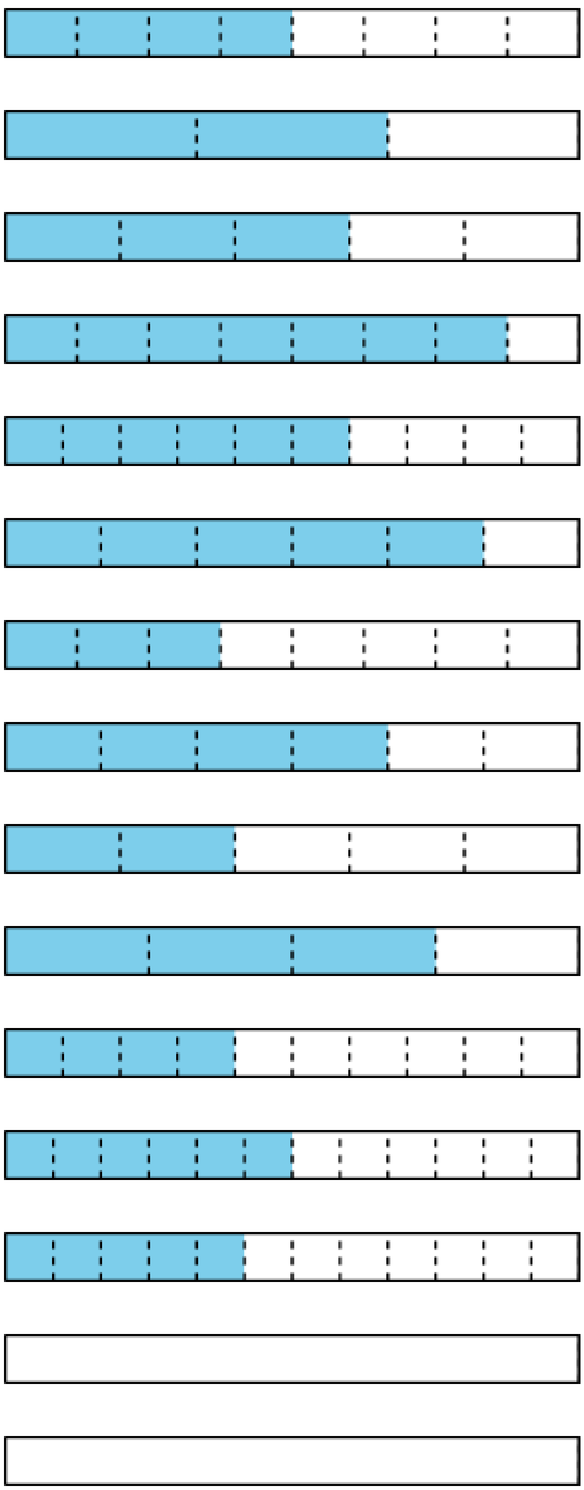
$\frac{2}{3}$ $\frac{3}{8}$ $\frac{4}{6}$ $\frac{4}{10}$ $\frac{6}{6}$

$\frac{3}{5}$ $\frac{4}{8}$ $\frac{6}{12}$ $\frac{6}{10}$ $\frac{3}{4}$

$\frac{5}{12}$ $\frac{7}{10}$ $\frac{7}{8}$ $\frac{5}{6}$ $\frac{2}{5}$

Teacher Directions

Launch
- Groups of 2
- Straightedge for each student
- Record and display the fraction $\frac{1}{4}$ for all to see.
- “Describe to your partner what the fraction strip would look like for this fraction.”
- 30 seconds: partner discussion



A.
B.
C.
D.
E.
F.
G.
H.
I.
J.
K.
L.
M.
N.
O.

- Record and display the fraction $\frac{2}{4}$ for all to see.
- "Describe what the fraction strip would look like for this fraction."
- 30 seconds: partner discussion
- Share responses.
- "In an earlier lesson, we looked at fractions with 1 for the numerator. Now let's look at fractions with other numbers for the numerator."
- Draw students' attention to the list of fractions in the task. As a class, read aloud the word name of each fraction.

Activity

- "Take a minute to think quietly about how you might match each fraction with a diagram that represents it."
- 1 minute: quiet think time
- "Work with a partner to match each fraction with a diagram. Two of the fractions have no matching diagrams. Use the blank tapes to create representations for them."
- 10 minutes: group work time

Synthesis

- To add movement to the activity, students can check their matches with other groups in the room before the synthesis.
- Invite students to share how they went

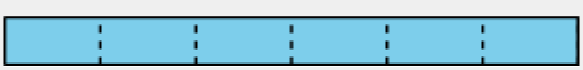
about making the matches.

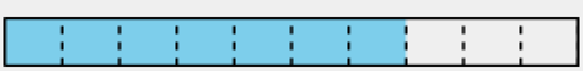
- Highlight explanations that emphasize the meaning of numerator and denominator in a fraction.
- Ask students if they noticed that some diagrams have the same amount shaded but the fractions they represent have different numbers. "Which diagrams show this?" (A and L, B and H, C and E, I and J)
- "What does it mean that the diagrams representing those fractions are the same?" (The fractions are the same size. The term "equivalent" may or may not come up at this point.)

Student Responses

$\frac{2}{3}$: B	$\frac{3}{8}$: G	$\frac{4}{6}$: H	$\frac{4}{10}$: I	$\frac{6}{6}$: N
$\frac{3}{5}$: C	$\frac{4}{8}$: A	$\frac{6}{12}$: L	$\frac{6}{10}$: E	$\frac{3}{4}$: K
$\frac{5}{12}$: M	$\frac{7}{10}$: O	$\frac{7}{8}$: D	$\frac{5}{6}$: F	$\frac{2}{5}$: J

The missing diagrams for $\frac{6}{6}$ and $\frac{7}{10}$:

N. 

O. 

Activity 2: Diagrams for Some Other Fractions

Time: 15 minutes

Standards Alignment

Building On CCSS:
- 3.NF.A.1

Building Toward CCSS:
- 4.NF.A.1

Materials to Gather
- 1 straightedge per student
- fraction strips student created in the previous lesson

Materials to Copy
- none

Activity Narrative
The purpose of this activity is to extend students' reasoning about the meaning of numerator and denominator and the size of non-unit fractions to include fractions greater than 1. Students are prompted to reason both ways: to match fractions to tape diagrams that represent them, and to create tape diagrams that represent fractions.

Some students may benefit from having physical manipulatives to help them conceptualize fractions that are greater than 1. Consider using fraction strips to support them—for instance, by asking them to fold as many strips as needed to represent, say, 4 halves or 5 fourths.

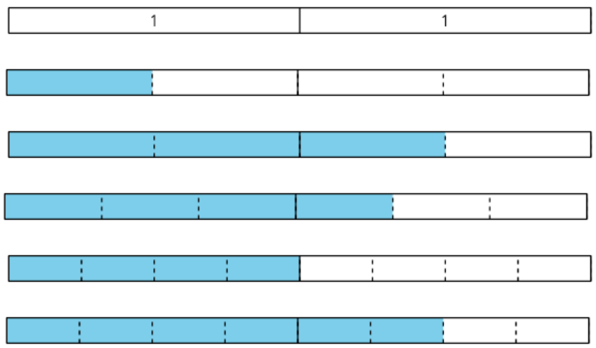
As in the previous lesson, rulers can be provided to help students draw, extend, or align partition lines, but should not be used to measure the location of a fraction on any diagram.

MLR Tags
- MLR2

EL Support Text
MLR2 Collect and Display. Collect the language students use to reason about fractions greater than one. Display words and phrases such as: fraction, numerator, denominator, 1 whole, greater than, equal-size parts, and so on. During the activity, invite students to suggest ways to update the display: "What are some other words or phrases we should include?", and so on. Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Student-facing Task Statement	Teacher Directions
1. What fraction does each shaded part represent?	**Launch**

a.
b.
c.
d.
e.



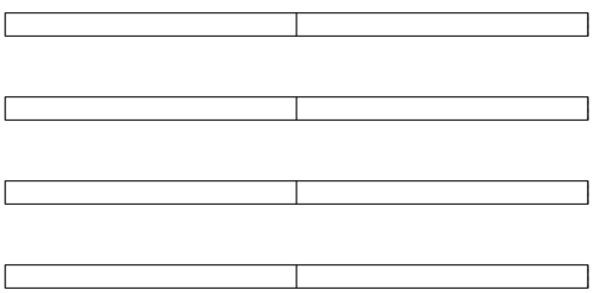
2. Here are four fractions and four blank diagrams. Partition each diagram and shade one part so that the shaded part represents the fraction.

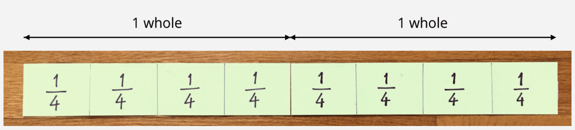
a. $\frac{2}{2}$

b. $\frac{4}{2}$

c. $\frac{5}{4}$

d. $\frac{4}{8}$



- Groups of 2
- Ask students to retrieve the fraction strips they created in a previous lesson.
- “How can you show $\frac{3}{4}$ with a fraction strip?” (Find the strip showing fourths, highlight 3 parts of fourths.)
- “How can you show $\frac{8}{4}$?”
- 30 seconds: partner discussion
- If students say that they don’t have enough strips that show fourths, ask them to work with another group and combine their strips.
- Ask groups to share how they used strips to represent $\frac{8}{4}$. Students may think of different ways (including using fourths and halves, or fourths and eighths).
- Highlight one thing all representations share: to show $\frac{8}{4}$, two strips are used.
- Display two strips that show fourths side by side, as shown.
- 

- Count the fourths: “$\frac{1}{4}$, $\frac{2}{4}$, . . . , $\frac{8}{4}$.”
- “How many fourths did we count?” (8) “How many wholes was that?” (2)
- Straightedge for each student

	Activity ● "Take a couple of quiet minutes to work on the first question. Afterward, discuss your responses with your partner." ● "Both you and your partner should be prepared to explain how you know what fractions the diagrams represent." ● 2–3 minutes: independent work time on the first question ● 2 minutes: partner discussion ● Monitor for students who reference the length of 1 whole on the tape diagram (rather than the entire length, which is 2) to determine the size of the fractional parts. ● Pause for a discussion. Invite students to share their responses and reasoning for the first question. ● "How did you determine the fractions the diagrams represent?" (Count the number of parts in 1, and then count the number of shaded pieces.) ● Display students' work, or display and annotate the tape diagrams as they explain. ● Consider labeling each fractional part with the corresponding unit fraction and counting each shaded part aloud ("1 half, 2 halves, 3 halves," "1 third, 2 thirds, 3 thirds, 4 thirds," and so on) before writing the value of the shaded parts as $\frac{3}{2}$ or $\frac{4}{3}$.

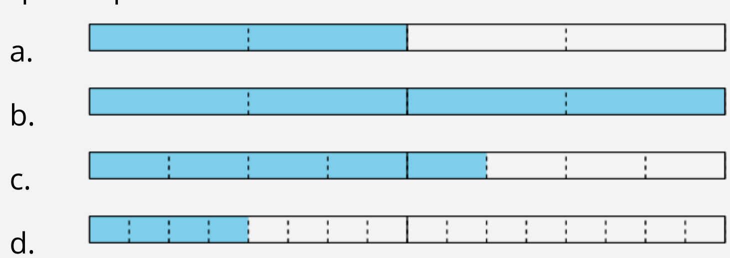
- Ask students to work with their partner to create the diagrams in the second question.
- "You may use a straightedge to help you draw your diagrams."
- 5–7 minutes: partner work time

Synthesis

- Select students to share their completed diagrams.
- "How did you know how many pieces to partition each diagram and how many parts to shade?" (Cut each 1 whole portion into as many equal parts as the number in the denominator. Shade as many parts as the number in the numerator.)
- "How did you partition a diagram into 4 equal parts?" (Split each 1 whole into 2 parts, and then split each half into 2 parts again.)
- "How did you partition a diagram into 8 equal parts?" (Split each fourth into 2 parts.)

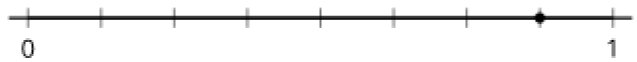
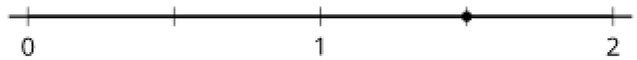
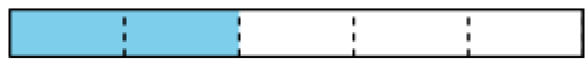
Student Responses

1. a. $\frac{1}{2}$ b. $\frac{3}{2}$ c. $\frac{4}{3}$ d. $\frac{4}{4}$ e. $\frac{6}{4}$
2. Sample response:



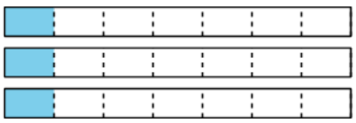
Supporting Student Thinking	
Some students may identify fractional parts correctly but may not be sure how to name or write fractions greater than 1. Encourage them to count aloud each fractional part on a diagram and keep counting even when the parts extend beyond 1 (for example, "3 fourths, 4 fourths, 5 fourths," and so on).	
Lesson Synthesis "Did you notice anything interesting about the last two diagrams you created and the fractions they represent?" (Students may or may not refer to equivalency. Sample responses: ● The shaded parts are the same size. They have the same amount of shading. ● The numerator and denominator in one fraction are twice the numerator and denominator in the other. ● The fractions are equivalent.)	
Response to Student Thinking Next Day Students do not accurately read and represent fractions greater than 1.	**Next Day Support** ● Add this cool-down to Activity 1 to review how to represent fractions greater than 1.
Response to Student Thinking Students claim that fractions are always less than 1.	**Prior-unit Support** ● Grade 3, Unit 5, Section B

Mini-Assessment 1	
Item Narrative	Students perform division to solve a problem with a context of equal sharing. They may write equations or draw a tape diagram or use partial products. The answer has a remainder and students will need to interpret that in terms of the context, that is, as the toothpicks that remain after dividing the rest evenly into 6 equal groups.
Item Statement	There are 1,420 toothpicks in a bag. Six students share the toothpicks equally for an art project. 1. How many toothpicks does each student get? Explain your reasoning. 2. How many toothpicks are left over?
Item Solution	1. Sample response: 236. First, each student gets 200 toothpicks and that leaves 220 toothpicks. Then each student gets 30 more and that leaves 40 toothpicks. Each student gets 6 and there are 4 toothpicks left.

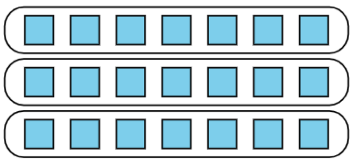
	2. 4
Item Statement	Name the fraction represented by the point on each number line and the shaded portion of the tape diagram. Explain your reasoning. 1.  2.  3.  The entire tape is one whole.
Item Solution	1. $\frac{7}{8}$, because there are 8 equal parts in 1 and the point is on the seventh tick. 2. $\frac{3}{2}$, because there are 2 tick marks in each whole and the point is on the third tick. 3. $\frac{2}{5}$, because there are 5 equal parts in 1 whole and 2 parts are shaded.
Item Learning Goals	Identify fractions on the number line and in tape diagrams.

4.3 Lesson 1: Representing Equal Groups of Unit Fractions
Standards Alignment
Addressing CCSS: ● 4.NF.B.4.A
Teacher-facing Learning Goals ● Interpret the two factors of a multiplication expression as the number of groups and the number of objects in each group in a way that makes sense to them. ● Match diagrams and situations that represent a fraction multiplied by a whole number to expressions.
Student-facing Learning Goals ● Let's identify equal groups of fractions.
Lesson Purpose The mathematical purpose of this lesson is to represent situations involving equal groups of unit

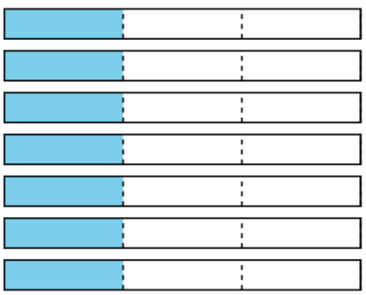
fractions and understand the product is a multiple of that unit fraction.	
Lesson Narrative In grade 3, students represented multiplication with expressions, area diagrams, and equal groups pictures. In unit 2, students used tape diagrams to represent and compare fractions. The purpose of this lesson is for students to extend their understanding of multiplication as equal groups of whole number objects to situations that involve equal groups of unit fractions.	
Access for Students with Disabilities Activity 1: Representation	**Access for English Learners** Activity 2: MLR8 Discussion Supports
Instructional Routines Notice and Wonder, Card Sort	
Materials to Gather Activity 1: none Activity 2: none	**Materials to Copy** Activity 1: none Activity 2: 4.3.A.1 Expressions and Diagrams Cards
Lesson Timeline **Warm-up** 10 minutes **Activity 1** 20 minutes **Activity 2** 15 minutes **Lesson Synthesis** 10 minutes **Cool-down** 5 minutes	**Teacher Reflection Question** Which question did you ask today that best supported students' understanding of multiplication of a fraction by a whole number? What did students say or do that showed the question was effective?
Cool-down: More Snacks	
Standards Alignment: ● 4.NF.B.4.A	
Student-facing Task Statement There were 7 bags left over after snack time. Each bag had $\frac{1}{3}$ of an orange in it. Select the diagram that best matches the amount of leftover orange in all the bags. A.	



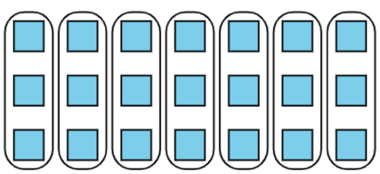
B.



C.



D.



Student Responses

- C, because it shows 7 groups with $\frac{1}{3}$ in each.

Warm-up: How Many Do You See: Oranges

Time: 10 minutes

Standards Alignment

Addressing CCSS:

- 4.NF.B.4.b
- 4.NF.B.4.c

Instructional Routines

- How Many Do You See?

Warm-up Narrative

The purpose of this How Many Do You See is for students to use grouping strategies to describe the images they see. The image is designed to elicit ideas about equal groups of fractions of objects. This will be useful when students work with equal groups of fractions in this lesson. While students may notice and wonder many things about these situations, ideas around describing the groups as equal and having fractional parts are the important discussion points. Students may notice the halves of oranges and describe them with whole numbers. If this happens, consider asking: “How might we describe the amount of fruit using fractions?”

Students explain their reasoning as they describe the way they see the image. They also make sense of the approaches of others and identify connections between different approaches (MP1).

Student-facing Task Statement

How many do you see? How do you see them?

Teacher Directions

Launch

- Groups of 2
- “How many do you see? How do you see them?”
- Display the image.
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.
- Repeat for each image.

Synthesis

- “How might you describe this image to a friend?” (There are three plates with $\frac{1}{2}$ an orange on each plate.)
- “How many groups do you see?” (I see 3 plates or 3 groups.)
- Describe any groups you see and how you see them. (3 groups of orange slices, 3 groups of $\frac{1}{2}$)

Student Responses

- Three plates with $\frac{1}{2}$ orange on each
- One whole orange and $\frac{1}{2}$ of another orange
- Three $\frac{1}{2}$ pieces of oranges

Activity 1: Equal Groups of Fractions

Time: 20 minutes

Standards Alignment

Addressing CCSS:

- 4.NF.B.4.A

Activity Narrative

The purpose of this activity is for students to interpret and represent situations involving equal groups of a fractional amount, and to connect such situations to multiplication of a whole number by a fraction. Students create diagrams and expressions to describe how they show "groups of" situations and then compare them. Encourage students to use any representation that makes sense to them. In grade 3, students used equal-groups diagrams when they learned multiplication. This lesson provides an opportunity for the teacher to see with which representations students are already familiar.

Focus the lesson synthesis on connecting equal-groups situations with fractions and those with whole numbers. Students notice that both situations involve equal groups and describe how they know the groups are equal and also use expressions to represent equal-groups situations.

SwD Support Tags

- Representation

SwD Support Text

Representation: Access for Perception. Launch: Use pictures (or actual oranges, if possible) to represent the situation. Ask students to identify correspondences between this concrete representation and the diagrams they create or see.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing.

Student-facing Task Statement	Teacher Directions
1. Lin and Andre brought fruit to share with their class. Draw a diagram to represent each situation.	**Launch** • Groups of 2 • 1 minute: share responses • "If you could bring fruit to share with the

Lin brought 6 bags with 4 oranges in each.	Andre brought 6 bags with $\frac{1}{4}$ orange in each.

2. Describe the groups you see and how you see them.
3. Which parts of each diagram are the same?
4. Which parts of the diagram changed and how did you change them?
5. What expressions could you write for each situation?
6. Draw a diagram for each situation.
    a. Clare brought 3 bags with 6 strawberries in each.
    b. Diego put $\frac{1}{2}$ apple into each of 8 bags.
    c. Noah had 7 bags with $\frac{1}{3}$ banana in each.
    d. Priya packed $\frac{1}{8}$ of a watermelon into 5 different bags.

class, what fruit would you bring?"

- Collect student responses. Allow students to describe fruits that may be unfamiliar to the class.
- "Let's look at what Lin and Andre brought to class."

Activity

- Students complete problems 1–5.
- 5 minutes: independent work time
- 2 minutes: partner discussion
- Monitor for students who use the same type of representation for equal groups of whole numbers and fractions.
- Select 1–2 students to share responses to problems 3 and 4.
- 2 minutes: group discussion
- Discuss problem 5. "What expression would we use to represent Lin's oranges? Why?" (6 x 4, because there are 6 groups of 4 oranges.)
- "What expressions would we use to represent Andre's oranges? Why?" (6 x $\frac{1}{4}$, because there are 6 groups of $\frac{1}{4}$ of an orange. We can use the same thinking as we used for whole groups.)
- Students complete problem 6.
- Monitor for students who have equal-groups drawings with expressions to match.

Synthesis

- Select previously identified students to describe the equal groups in the diagrams they created.
- "What expression could we use to represent the fruit Noah brought to class? Why?" (7 x $\frac{1}{3}$, because seven

	groups of $\frac{1}{3}$ would be written as 7 x $\frac{1}{3}$). ● "Let's continue to think about these diagrams as we work on the next activity."

Student Responses

1. Image 1 shows 6 groups of 4. Image 2 shows 6 groups of $\frac{1}{4}$ in each.
2. I see 6 groups of 4 in Lin's situation. I see 6 groups of $\frac{1}{4}$ in Andre's situation.
3. The number of groups stays the same. There are 6 groups in each diagram.
4. The number in each group changes from 4 to $\frac{1}{4}$.
5. 6 x 4, 6 x $\frac{1}{4}$
6. Diagrams vary. Sample responses:
    a. Image shows 3 groups of 6.
    b. Image shows 8 groups of $\frac{1}{2}$.
    c. Image shows 7 groups of $\frac{1}{3}$.
    d. Image shows 5 groups of $\frac{1}{8}$.

Advancing Student Thinking

If a student doesn't draw equal groups of fractions, consider asking, "How could you represent one of the fractions?" Then follow up by asking, "How can we use this reasoning to help us represent _____ groups of that fraction?" Students may also need to be asked these questions with a whole number first and then with fractions.

Activity 2: Expressions and Diagrams

Time: 15 minutes

Standards Alignment

Addressing CCSS: 4.NF.B.4.A

Materials to Copy

- 4.3.A.1 Expressions and Diagrams Cards

Instructional Routines

- Card Sort K5

Activity Narrative

The purpose of this activity is for students to interpret multiplication expressions and diagrams as the number of groups and amount in each group. After students have matched the expressions and diagrams in problem 1, pause for a whole-class discussion for students to explain how they decided on the matches before they move to problems 2 and 3.

MLR Tags

- MLR8

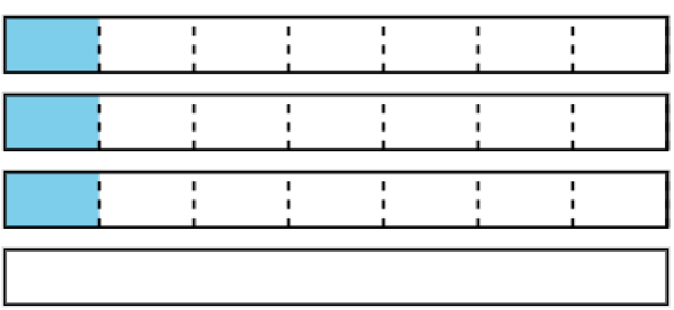
EL Support Text

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed _____ , so I matched . . .” Encourage students to challenge each other when they disagree.
Advances: Speaking, Conversing

Student-facing Task Statement

Your teacher will give you a set of cards with expressions and diagrams.

1. Match each expression with a diagram that represents the same quantity.
2. Record each expression without a match. Draw a matching diagram.
3. Han started drawing a diagram to represent $7 \times \frac{1}{8}$ and did not finish. Complete his diagram.



Teacher Directions

Launch

- Groups of 2
- Review directions to the task.

Activity

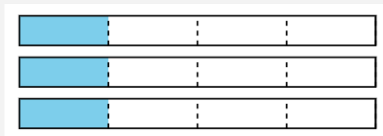
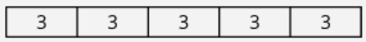
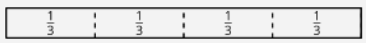
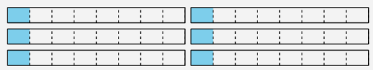
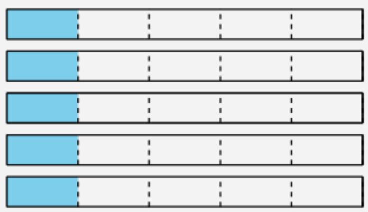
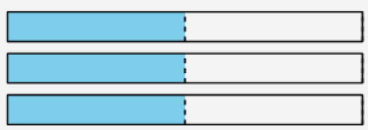
- 5 minutes: partner work time
- Monitor for students who reason about the number of groups and amount in each group as they match or draw.

Synthesis

- Select 2–3 students to describe the groups they see and how they see them.
- Consider asking different students to echo reasoning by asking:
    - “Who can say that differently?”
    - “Who can restate what _____ just said?”

Student Responses

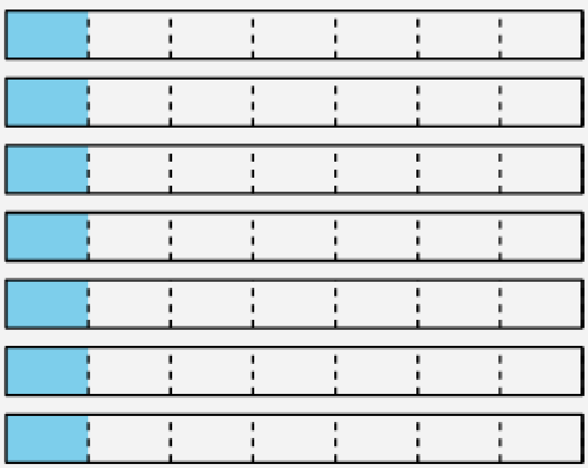
1.

$3 \times \frac{1}{4}$	
5×3	
$4 \times \frac{1}{3}$	
$6 \times \frac{1}{8}$	
$5 \times \frac{1}{5}$	
$3 \times \frac{1}{2}$	
$6 \times \frac{1}{6}$	no match
$8 \times \frac{1}{2}$	no match
3×4	no match
$2 \times \frac{1}{12}$	no match

2.

$6 \times \frac{1}{6}$	Image of 6 groups of $\frac{1}{6}$
$8 \times \frac{1}{2}$	Image of 8 groups of $\frac{1}{2}$
3 x 4	Image of 3 groups of 4
$2 \times \frac{1}{12}$	Image of 2 groups of $\frac{1}{12}$

3.



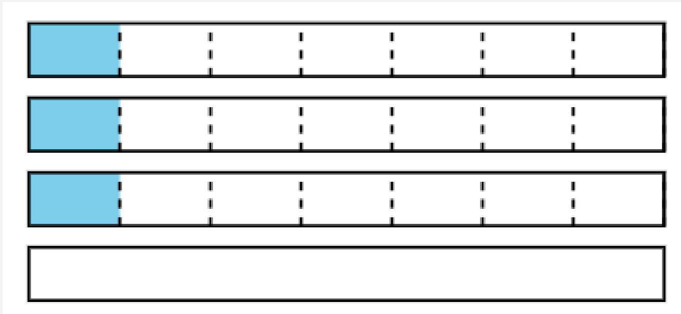
Advancing Student Thinking

If students are not yet matching expressions to appropriate diagrams, consider asking students to compare the diagrams for 5 x 3 and $5 \times \frac{1}{3}$ and reason about the number of groups and the size of each group. Consider asking, "What is the same and what is different?"

Lesson Synthesis

"Today we matched diagrams for multiplication expressions and drew diagrams of equal-group situations."

- "What was missing from Han's diagram? How do you know?" (4 more groups of $\frac{1}{8}$ were missing, because $7 \times \frac{1}{8}$ means 7 groups of $\frac{1}{8}$ and there are only 3 in Han's diagram.)



- If the expression was for 7 groups of $\frac{1}{3}$ instead of $\frac{1}{8}$, how would Han's diagram change? (Each tape would have 3 equal parts with 1 shaded for 7 groups of $\frac{1}{3}$.)

Response to Student Thinking Next Day Students select an incorrect diagram.	**Next Day Support** • Before the warm-up, have students work in partners to discuss a correct response to this cool-down.
Response to Student Thinking Students do not yet interpret linear diagrams as equal group representations.	**Prior-unit Support** • Grade 4, Unit 2, Section A

4.3 Lesson 2: Name That Expression

Standards Alignment

Addressing CCSS:
- 4.NF.B.4.A

Teacher-facing Learning Goals
- Write expressions for diagrams and situations that represent a unit fraction multiplied by a whole number.

Student-facing Learning Goals
- Let's write multiplication expressions from diagrams and situations.

Lesson Purpose
The purpose of this lesson is for students to represent the multiplication of a unit fraction by a whole number with expressions, given diagrams and situations.

Lesson Narrative

In previous lessons, students matched diagrams to multiplication expressions and drew their own representations for multiplication expressions with unit fractions.

The purpose of this lesson is for students to write expressions for situations and diagrams that represent multiplication of unit fractions and whole numbers. While students may begin to evaluate expressions as they discuss situations, this lesson focuses on interpreting diagrams and situations as equal groups of unit fractions to write multiplication expressions.

Access for Students with Disabilities	**Access for English Learners**
Activity 1: Representation	Activity 2: MLR8 Discussion Supports

Instructional Routines
Choral Count

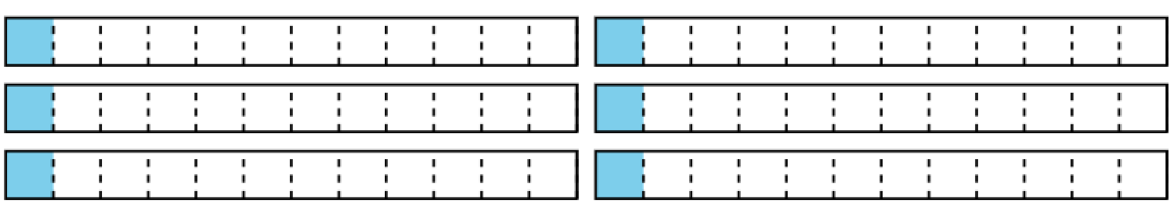
Materials to Gather	**Materials to Copy**
Activity 1: none Activity 2: none	Activity 1: none Activity 2: none

Lesson Timeline	**Teacher Reflection Question**
Warm-up 10 minutes **Activity 1** 20 minutes **Activity 2** 15 minutes **Lesson Synthesis** 10 minutes **Cool-down** 5 minutes	Revisit class norms and routines. Are all students contributing to the conversation? Do some students' ideas seem to hold more value in the dynamics of the group? Are there any adjustments you might make so that all students do math tomorrow?

Cool-down: Equal Groups of Fractions

Standards Alignment:
- 4.NF.B.4

Student-facing Task Statement
1. Write a multiplication expression to represent the shaded parts of the diagram. Show or explain your reasoning.



2. One batch of Priya's slime recipe calls for $\frac{1}{8}$ cup of glitter. She is making 4 batches of slime. Write an expression to represent the amount of glitter that Priya needs.

Student Responses

1. 6 x $\frac{1}{12}$, because there are 6 equal groups of $\frac{1}{12}$.
2. 4 x $\frac{1}{8}$, because there is $\frac{1}{8}$ cup of glitter in one batch, and there are 4 groups of $\frac{1}{8}$ cup of glitter in 4 batches.

Warm-up: Choral Count

Time: 10 minutes

Standards Alignment

Addressing CCSS: 4.NF.B.4.A

Instructional Routines

- Choral Count K5

Warm-up Narrative

The purpose of this Choral Count is to invite students to practice counting by a unit fraction and notice patterns in the count. These understandings help students develop fluency and will be helpful later in this lesson when students are to recognize that every fraction can be written as the product of a whole number and unit fraction.

When students describe the number of groups of $\frac{1}{4}$ and $\frac{2}{4}$, they are noticing repeated structure and making use of this structure to determine the next number in the counting sequence (MP7).

Student-facing Task Statement	Teacher Directions
Count by $\frac{1}{4}$ starting at 0. Count by $\frac{2}{4}$ starting at 0.	**Launch** • "Count by $\frac{1}{4}$ starting at 0." **Activity** • Record as students count. • Stop counting and recording at $\frac{11}{4}$.

- Repeat with $\frac{2}{4}$.

Synthesis

- "What patterns do you notice?" (The numerators go up by 1 and 2, and denominators stay the same.)
- "How many groups of $\frac{1}{4}$ do we have?" (11)
- "Where do you see them?" (Each count represents a new group of $\frac{1}{4}$.)
- "How might we represent $\frac{9}{4}$ with an expression?" (9 groups of $\frac{1}{4}$ is 9 x $\frac{1}{4}$.)

Consider asking:

- "Who can restate the pattern in different words?"
- "Does anyone want to add an observation as to why that pattern is happening here?"
- "Do you agree or disagree? Why?"

Student Responses

- Start recording $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$ . . . and stop at $\frac{11}{4}$.
- Start recording $\frac{2}{4}$, $\frac{4}{4}$, $\frac{6}{4}$, $\frac{8}{4}$ . . . , start a new column after $\frac{10}{4}$, and continue recording until you reach $\frac{22}{4}$.

Activity 1: Writing Expressions

Time: 20 minutes

Standards Alignment

Addressing CCSS:

- 4.NF.B.4.A

Activity Narrative

In this activity, students use expressions to represent "groups of" unit fractions. Allow students to write

<table>
<tr><td colspan="2">both $\frac{1}{b}$ x n ($\frac{1}{b}$ of n) and n x $\frac{1}{b}$ (n groups of $\frac{1}{b}$). For upcoming work with fractions in this grade and grade 5, it will be helpful for them to interpret an expression n x $\frac{1}{b}$ (b not equal to zero) as n groups with $\frac{1}{b}$ objects in each group. This can be addressed in the synthesis if needed.

The synthesis asks students to describe equal groups of unit fractions in diagrams and use expressions to describe these equal groups in terms of the problem situations.</td></tr>
<tr><td colspan="2">SwD Support Tags
Representation</td></tr>
<tr><td colspan="2">SwD Support Text
Representation: Access for Perception. Launch: Show students measuring cups and spoons. Then invite students who are assigned Problem Set A (consider assigning strategically) to briefly act out each scenario. Provide a visual display that reminds all students to ask themselves as they work: How many groups are there? What is the size of each group?
Supports accessibility for: Conceptual Processing, Memory, Attention.</td></tr>
<tr><td>Student-facing Task Statement

Five students made slime using this recipe:

3-Ingredient Slime


2 bottles of white liquid glue
1 tablespoon baking soda
2–3 tablespoons of saline solution (contact lens solution)</td><td>Teacher Directions

Launch
Groups of 2
“Think of something your family likes to make for breakfast, dinner, or a snack.”
“Share the recipe or describe to a partner the ingredients in the dish.”
1 minute: partner discussion
“Sometimes we make recipes for things we don’t eat.”
“The students in Mai’s class are working together to make slime.”
“Take a look at this recipe for slime.”
“Review each problem or diagram in the set you have been assigned and write an expression for each one.”
Activity
2 minutes: quiet think time
3–5 minutes: partner work time
Monitor for students who describe the equal groups in situations, diagrams, and expressions.</td></tr>
</table>

Optional Ingredients:

- 2 tablespoons of glitter
- 1–3 drops of food coloring

Write an expression to answer each question and identify a matching diagram.

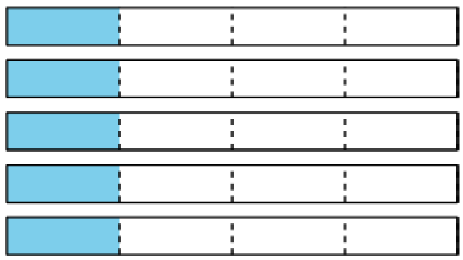
1. Each of the 5 students got $\frac{1}{4}$ pound of slime. How much slime did the students make?
2. Each person added $\frac{1}{3}$ teaspoon of food coloring to their slime. How much food coloring did the students use?
3. Next, each person added $\frac{1}{2}$ tablespoon of glitter to their slime. How much glitter did the students use?
4. The students didn't think there was enough slime, so they made more. This time, each person got $\frac{1}{8}$ pound of the newly made slime. How much slime did they make the second time?

Diagrams A–D

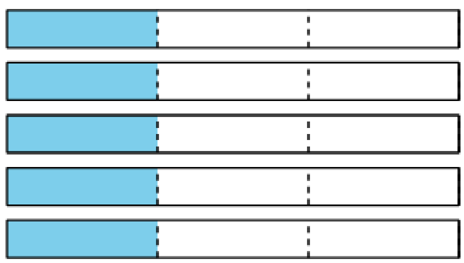
A.

Synthesis

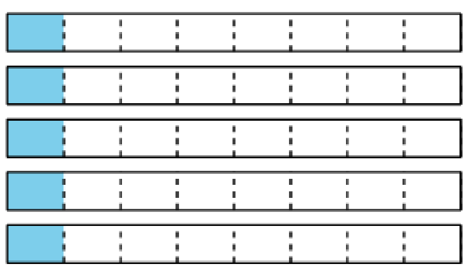
- Select 1–2 students to describe the equal groups in their expressions, situations, and diagrams.
- "Let's look at problem 3. Which diagram matches this problem?" (The one with 5 groups of $\frac{1}{2}$.)
- "What part of the diagram represents the students in the group?" (There are 5 rows that are the same. Each row represents a student because there are 5 students.)
- "What part of the diagram represents the tablespoons of glitter?" (The shaded portion of each row is $\frac{1}{2}$ of the row. Each shaded region represents $\frac{1}{2}$ tablespoon of glitter.)
- "What expression should we write to represent 5 groups of $\frac{1}{2}$? Why?" 5 x $\frac{1}{2}$ because there are 5 groups of $\frac{1}{2}$.)
- If some students write expressions with the factors switched, take time to explain that while the product is the same, we will be reading the expressions as "_____ groups of _____" in this activity, and the fraction is the second factor.



B.



C.



D.

Student Responses

1. 5 x $\frac{1}{4}$, diagram showing 5 groups of $\frac{1}{4}$
2. 5 x $\frac{1}{3}$, diagram showing 5 groups of $\frac{1}{3}$
3. 5 x $\frac{1}{2}$, diagram showing 5 groups of $\frac{1}{2}$
4. 5 x $\frac{1}{8}$, diagram showing 5 groups of $\frac{1}{8}$

Advancing Student Thinking

If students write expressions that don’t yet represent the situation, consider asking a related question but replace the unit fraction with a whole number. For example, to support a student with problem 1, ask, “What expression could you write if Kiran packed 8 scoops of slime into each of 5 containers?” Then follow up by asking, “How might we use this reasoning to help us think about $\frac{1}{8}$ cup of slime in each container?”

Activity 2: Complete the Story

Time: 15 minutes

Standards Alignment

Addressing CCSS:

- 4.NF.B.4.A

Activity Narrative
The purpose of this activity is for students to notice and describe the number of groups and the size of each group in situations involving the multiplication of a fraction by a whole number. Students generate situations, diagrams, and write expressions.

MLR Tags

- MLR8

EL Support Text
MLR8 Discussion Supports. Synthesis. Display sentence frames to agree or disagree. "I agree because . . .” and "I disagree because . . . ”
Advances: Speaking, Conversing

Student-facing Task Statement

1. Use numbers to complete each problem so it matches the diagram. Write an expression to represent the quantity in each problem.

    a. Jada made 7 cups of red slime. She added _____ tablespoon of blue food coloring to each cup to make purple slime. How much food coloring did she use?

Teacher Directions

Launch

- Groups of 2
- 5 minutes: independent work time

Activity

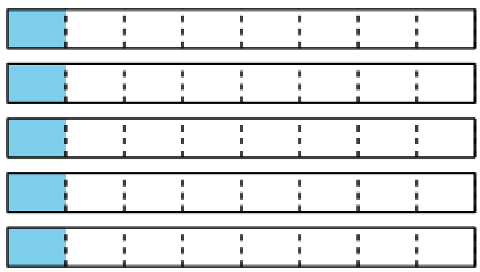
- “Compare the expressions, diagrams, and situations with a partner.”
- 5 minutes: partner discussion
- “Discuss any expressions that are not the same.”

Synthesis

- Select 2–3 students to share their responses for problem 1.

- “How did you decide which numbers to place in the blank for each situation?” (In

b. _____ students each had _____ cup of slime. They combined their slime to make a large slime ball. How many cups of slime is that ball?



2. A batch of slime takes $\frac{1}{4}$ hour to make. How many hours would it take to make 8 batches of slime?

   Draw a diagram and write an expression to match the problem.

3. Clare gave _____ friends a slime ball. Each slime ball weighed _____ pounds. How much slime did Clare give away?

   Use a whole number and a unit fraction to complete the problem. Draw a diagram and write an expression to answer the question.

part a, the diagram shows groups of $\frac{1}{2}$, and in part b, the diagram shows 5 groups of $\frac{1}{8}$.)

- Discuss problem 3 during the lesson synthesis.

Student Responses

1. a. $\frac{1}{2}$, 7 x $\frac{1}{2}$

   b. 5, $\frac{1}{8}$, 5 x $\frac{1}{8}$
2. 8 x $\frac{1}{5}$ and an image of 8 groups of $\frac{1}{5}$
3. Answers vary. Sample response: 4 x $\frac{1}{6}$ and an image of 4 groups of $\frac{1}{6}$.

Lesson Synthesis

“Today we wrote expressions to represent diagrams and problems.”

MLR7 Compare and Connect • "Place the diagram for problem 3 on your desk for others to see." • "Review the diagrams on 2–3 desks and pay attention to: ○ what is the same and what is different as you visit different desks" • 1 minute: silent review discussion • 1 minute: partner share • "Describe how you see the groups represented in each student's diagram." (The whole number represents the groups and the fraction represents the number in each group in each diagram.)	
Response to Student Thinking Next Day Students may use fractions to represent the number of groups when writing expressions or describing diagrams or situations.	**Next Day Support** • To avoid confusing situations involving multiplication of a whole number by a fraction, Launch warm-up or Activity 1 by highlighting important notation from previous lessons.
Response to Student Thinking Students do not yet represent or interpret equal groups situations.	**Prior-unit Support** • Grade 3, Unit 2, Section 1

4.3 Lesson 3: Multiplication Patterns
Standards Alignment
Addressing CCSS: • 4.NF.B.4.A, 4.NF.B.4.C
Teacher-facing Learning Goals • Evaluate multiplication expressions and recognize that $n \times \frac{1}{b} = \frac{n}{b}$.
Student-facing Learning Goals • Let's look at patterns in multiplication of a fraction by a whole number.
Lesson Purpose The purpose of this lesson is to evaluate multiplication expressions of a unit fraction by a whole number and for students to understand that every fraction can be written as the product of a whole number and unit fraction.
Lesson Narrative In this lesson students notice how a product changes when either the number of groups or amount in

each group is kept constant. The patterns that emerge in the series of problems offer students the opportunity to see any fraction as a multiple of a whole number and unit fraction.

Access for Students with Disabilities
Activity 1: Representation

Materials to Gather	**Materials to Copy**
Activity 1: none Activity 2: none	Activity 1: none Activity 2: none

Lesson Timeline	**Teacher Reflection Question**
Warm-up 10 minutes **Activity 1** 20 minutes **Activity 2** 15 minutes **Lesson Synthesis** 10 minutes **Cool-down** 5 minutes	In tomorrow's lesson, students multiply a non-unit fraction by a whole number, such as 5 x $\frac{2}{3}$. How can students apply their understanding from today to reason about these problems tomorrow?

Cool-down: Fraction Multiplication

Standards Alignment:
- 4.NF.B.4

Student-facing Task Statement

1. Complete the equation to make it true. Show or explain your reasoning using words or diagrams.

   5 x $\frac{1}{8}$ = _____

2. Write $\frac{8}{9}$ as the product of a whole number and unit fraction.

   $\frac{8}{9}$ = _____ x _____

Student Responses
1. $5 \times \frac{1}{8} = \frac{5}{8}$ 2. $8 \times \frac{1}{9}$

Warm-up: Choral Count	
Time: 10 minutes	
Standards Alignment	
Addressing CCSS: 4.NF.B.4	
Instructional Routines ● Choral Count K5	
Warm-up Narrative The purpose of this Choral Count is to invite students to practice counting by a unit fraction and notice patterns in the count. These understandings help students develop fluency and will be helpful later in this lesson when students are to recognize that every fraction can be written as the product of a whole number and unit fraction. When students describe the number of groups of $\frac{1}{8}$, they notice repeated structure and use this structure to determine the next number in the counting sequence (MP7).	
Student-facing Task Statement Count by $\frac{1}{8}$ starting at 0.	**Teacher Directions** **Launch** ● "Count by _____ , starting at _____." **Activity** ● Record as students count. ● Stop counting and recording at _____. **Synthesis** ● "How would our count change if we counted by $\frac{3}{8}$?" (Each numerator would

	be a multiple of 3.) Consider asking: ● "Who can restate the pattern in different words?" ● "Does anyone want to add an observation as to why that pattern is happening here?" ● "Do you agree or disagree? Why?"

Student Responses

- Start recording $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, . . . and stop at $\frac{18}{8}$.

Activity 1: Describe the Pattern

Time: 10 minutes

Standards Alignment

Addressing CCSS:

- 4.NF.B.4, 4.NF.B.4.A

Activity Narrative

The purpose of this task is for students to notice that the product of a whole number and unit fraction has a numerator that is the same as the whole-number factor. Students may use diagrams to support their reasoning.

SwD Support Tags

- Representation

SwD Support Text

Representation: Develop Language and Symbols. Launch: Provide students with access to a chart that shows definitions and examples of the terms that will help them articulate the patterns they see, including whole number, fraction, numerator, denominator, unit fraction, and product.
Supports accessibility for: Language, Memory.

Student-facing Task Statement	**Teacher Directions**
1. Find the value of each expression. Use a diagram if it is helpful. Set A ● $1 \times \frac{1}{5}$	**Launch** ● Groups of 2 ● "You and your partner will each solve a different set of problems and then look

- 2 x $\frac{2}{5}$
- 3 x $\frac{3}{5}$
- 4 x $\frac{4}{5}$
- 5 x $\frac{5}{5}$
- 6 x $\frac{6}{5}$

Set B

- 2 x $\frac{1}{4}$
- 2 x $\frac{1}{5}$
- 2 x $\frac{1}{6}$
- 2 x $\frac{1}{7}$
- 2 x $\frac{1}{8}$
- 2 x $\frac{1}{9}$

2. Describe patterns in the products of Sets A and B.
3. Complete each equation to make it true.
   a. 4 x _____ = $\frac{4}{5}$
   b. 6 x _____ = $\frac{6}{10}$
   c. _____ x $\frac{1}{12}$ = $\frac{7}{12}$
   d. _____ x $\frac{1}{4}$ = $\frac{3}{4}$

for patterns when you compare your work."

Activity

- 5 minutes: partner work time
- Monitor for the language students use to explain patterns:
  - The whole number in each expression is only being multiplied by the numerator of each fraction.
  - Language describing patterns in the denominator of the product (The denominator in the product is the same as the unit fraction each time.)
  - "Groups of" language to justify or explain patterns (The number of groups of each unit fraction is going up each time because it is one more group.)

Synthesis

- Select 1–2 students to share patterns.
- "How did you use what you noticed in Sets A and B to reason about the expression in problem 2?" (I knew that each blank was a whole number and a fraction and the whole number should be the same as the numerator of the product.)
- Write $\frac{3}{4}$ on the board.
- "Can you write any fraction as a multiplication expression using its unit fraction?" (Yes, because the numerator is the number of groups and the denominator represents the size of each

	unit.) ● "What would it look like to write $\frac{3}{4}$ as a multiplication expression using a whole number and a unit fraction?" ($\frac{3}{4}$ = 3 x $\frac{1}{4}$)

Student Responses

1. Set A:
   - $\frac{1}{5}$
   - $\frac{4}{5}$
   - $\frac{9}{5}$
   - $\frac{16}{5}$
   - $\frac{25}{5}$
   - $\frac{36}{5}$

   Set B:
   - $\frac{2}{4}$
   - $\frac{2}{5}$
   - $\frac{2}{6}$
   - $\frac{2}{7}$
   - $\frac{2}{8}$
   - $\frac{2}{9}$

2. The numerator is going up by one every time because it is 1 more group. The size of the piece is staying the same throughout each set. Set A has a product that is more than 1 and Set B does not.
3.
   a. 4 x $\frac{1}{5}$ = $\frac{4}{5}$
   b. 6 x $\frac{1}{10}$ = $\frac{6}{10}$
   c. 7 x $\frac{1}{12}$ = $\frac{7}{12}$
   d. 3 x $\frac{1}{4}$ = $\frac{3}{4}$

Advancing Student Thinking

Students only notice that the factors are increasing in Set A or the factor of 2 for Set B. If so, consider asking, “How are the numerators changing in each expression?”, “Why do you think this is happening?”, and “What do you think would be the next expression in this series and what would the answer be for this one?”

Activity 2: Write, Solve, and Draw

Time: 25 minutes

Standards Alignment

Addressing CCSS:
- 4.NF.B.4.A, 4.NF.B.4.C

Instructional Routines
- *MLR7 Compare and Connect*

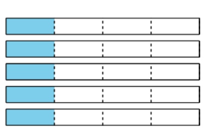
Activity Narrative

The purpose of this activity is to use the pattern noticed in the previous activity to solve problems. In the previous task, students noticed regularity between the numerator and the whole number being multiplied in each expression. This activity uses a “carousel” structure in which students complete a rotation of tasks. In this task, they use this generalized understanding to evaluate expressions without using patterns and use this to describe in general terms what happens when a fraction number is multiplied by a whole number.

This activity uses *MLR7 Compare and Connect*. Advances: representing, conversing

Student-facing Task Statement

1. Complete the table.

diagram	expression	product
		
	$6 \times \frac{1}{3}$	
		$\frac{7}{2}$

Teacher Directions

Launch
- Groups of 2
- “Turn to a partner and explain what you are supposed to do to complete problem 1.”
- Ask students to complete problem 1 independently.
- 5 minutes: independent work time

Activity

2. In your group of 4, complete the following steps. After each step, pass your paper to your right. ● Step 1: Choose a fraction. Write it down. ● Step 2: Write the fraction you received as a multiplication expression using a whole number and a unit fraction. ● Step 3: Find the value of the expression you received. ● Step 4: Draw a diagram to represent the expression. ● Step 5: Discuss what's on each paper and make revisions if needed.	● 1–2 minutes: whole-class discussion ● "How did you complete the last row of the table?" (The whole number was the same as the number in the numerator and the fraction has the same denominator as the product.) ● "Let's explore the pattern we are noticing and try to explain what is happening." ● Combine two groups of 2 to form groups of 4. ● Demonstrate the 5 steps of the carousel using $\frac{7}{2}$ from problem 1 for the first step. ● Read each step aloud and complete a practice round as a class. "What questions do you have about the task before you begin?" ● 5–7 minutes: group work time **Synthesis** ● Use lesson synthesis to summarize the ideas from this activity.

Student Responses

1.
   a. $5 \times \frac{1}{4}$, $\frac{5}{4}$
   b. A diagram showing 6 groups of $\frac{1}{3}$, $\frac{6}{3}$
   c. A diagram showing 7 groups of $\frac{1}{2}$, $7 \times \frac{1}{2}$
2. Answers vary.

Advancing Student Thinking

Students may be unsure about how to begin writing expressions for fractions. Remind students that the fraction will be written as a whole number times a unit fraction. Consider asking, "How might this help to write the expression?"

Lesson Synthesis

- "Today we looked at multiplication expressions in two different ways. In the first activity, the number of groups changed while the unit fraction stayed the same like this:
  - 1 x $\frac{1}{5}$
  - 2 x $\frac{2}{5}$
  - 3 x $\frac{3}{5}$
  - 4 x $\frac{4}{5}$
  - 5 x $\frac{5}{5}$
  - 6 x $\frac{6}{5}$ "
- "Then we looked at a situation in which the unit fraction changed and the number of groups stayed the same, like this:
  - 2 x $\frac{1}{5}$
  - 2 x $\frac{1}{3}$
  - 2 x $\frac{1}{4}$
  - 2 x $\frac{1}{2}$ "
- "In both cases, we discussed how we could write any fraction as a product of a whole number and unit fraction. Tell a partner about how we could write $\frac{7}{3}$ as a product of a whole number and a fraction."
- Solicit students' responses and as students share, write: $\frac{7}{3}$ = 7 x $\frac{1}{3}$.

Suggested Centers

- Rolling for Fractions, Stage 1

Response to Student Thinking Next Day

Students may write incorrect expressions that include familiar factors—for example $\frac{8}{9}$ = 2 x 4.

Next Day Support

- Before the warm-up, pass back the cool-down and work in small groups to make corrections, being sure to use diagrams to support reasoning.

Response to Student Thinking Students do not understand factors and multiples.	**Prior-unit Support** • Grade 4, Unit 1, Sections A–B

4.3 Lesson 4: Groups with More Things

Standards Alignment

Addressing CCSS:

- 4.NF.B.4.A
- 4.NF.B.4.C

Teacher-facing Learning Goals

- Use diagrams to represent multiplication expressions of a non-unit fraction by a whole number.
- Recognize that $n \times (\frac{a}{b}) = \frac{n \times a}{b}$.

Student-facing Learning Goals

- Let's multiply any fraction by a whole number.

Lesson Purpose

The purpose of this lesson is to apply understandings from previous lessons to multiply a non-unit fraction by a whole number. Students generalize a method for multiplying any fraction by a whole number and use what they have learned about multiplication situations, representations, and equations to explain why their method works.

Lesson Narrative

In previous lessons, students learned that any unit fraction multiplied by a whole number results in multiples of that unit fraction, and that any fraction can be written as a multiplication expression of a unit fraction by a whole number.

In this lesson, students generalize a method of multiplying any fraction by a whole number by reasoning about the number of groups and amount in each group. Students notice and begin to articulate that they can multiply the numerator by the whole number to find the number of parts and the denominator remains the same because the size of each part is the same, or $n \times (\frac{a}{b}) = \frac{n \times a}{b}$.

Access for Students with Disabilities

Activity 1: Engagement

Instructional Routines

Notice and Wonder

Materials to Gather Activity 1: none Activity 2: none	**Materials to Copy** Activity 1: none Activity 2: none
Lesson Timeline **Warm-up** 10 minutes **Activity 1** 15 minutes **Activity 2** 20 minutes **Lesson Synthesis** 10 minutes **Cool-down** 5 minutes	**Teacher Reflection Question** Who got to do math today in class? How do you know? What norms or routines allowed those students to engage in mathematics? How can you adjust these norms and routines so all students do math tomorrow?

Cool-down: What's the Value?

Standards Alignment:

- 4.NF.B.4.B, 4.NF.B.4.C

Student-facing Task Statement

Find the value of each expression. Explain your reasoning. Use a diagram if it is helpful.

1. $6 \times \frac{2}{4}$
2. $8 \times \frac{3}{10}$

Student Responses

1. $\frac{12}{4}$, because 6 groups of $\frac{2}{4}$ is $\frac{12}{4}$.
2. $\frac{24}{10}$, because 8 groups of $\frac{3}{10}$ is $\frac{24}{10}$.

Warm-up: Notice and Wonder: Thirds

Time: 10 minutes

Standards Alignment

Addressing CCSS: 4.NF.B.4

Instructional Routines

- Notice and Wonder K5

Warm-up Narrative

The purpose of this warm-up is for students to compare multiplication situations that involve a unit and non-unit fraction. The understandings elicited in this Notice and Wonder allow students to discuss the relationship between the product of a whole number and a unit fraction and the product of a whole number and a non-unit fraction with the same denominator.

Student-facing Task Statement

What do you notice? What do you wonder?

Teacher Directions

Launch

- Groups of 2
- Display the image.

Activity

- “What do you notice? What do you wonder?”
- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- If no students notice or wonder about equal groups, ask, “What groups do you see and how do you see them?” (4 groups with tapes in each, each tape has $\frac{2}{3}$ shaded)
- “How many thirds do you see?” (8 thirds)
- “We have been using diagrams to show groups of unit fractions. Today, we will think about how diagrams can show groups of fractions that are not unit fractions.”

Student Responses

Students may notice:

- 4 groups of tapes, or 4 rectangles, each partitioned into 3 equal parts
- Each tape has 2 parts shaded.
- There are 4 groups of $\frac{2}{3}$ in each rectangle.

Students may wonder:

- Are we going to work with fractions?
- Will we work with fractions with more than 1 in the numerator?

Activity 1: Diagrams for Groups of Many Fractions

Time: 15 minutes

Standards Alignment

Addressing CCSS:
- 4.NF.B.4.C

Activity Narrative

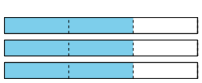
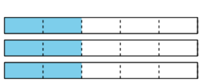
The purpose of this activity is for students to generate and evaluate expressions to match diagrams that represent a fraction multiplied by a whole number. Students also begin to describe the regularity in the expressions and products when the number of groups changes as the size of the group remains the same. They use the regularity they notice to evaluate expressions that follow this same pattern (MP8).

SwD Support Tags
- Engagement

SwD Support Text
Engagement: Provide Access by Recruiting Interest. Launch: Optimize meaning and value. Invite students to share real-life examples of multiplying a fraction by a whole number.
Supports accessibility for: Conceptual Processing, Attention, Social-Emotional Functioning.

Student-facing Task Statement

1. Write an expression to match each diagram. Evaluate each expression.

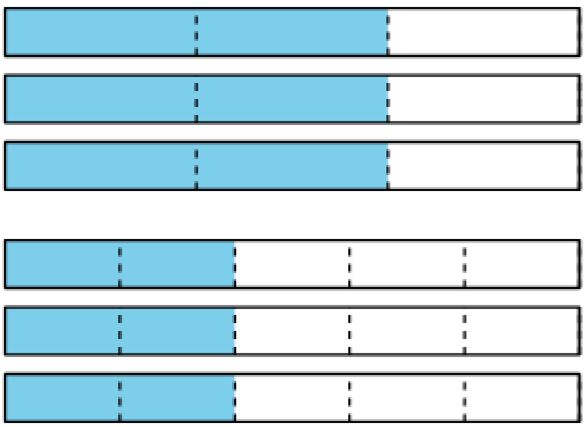
diagram	expression	product
		
		

Teacher Directions

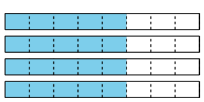
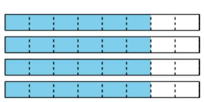
Launch
- Groups of 2
- "Read the directions for your task and explain to a neighbor what the task is asking you to do."

Activity
- 5 minutes: independent work time
- "Describe to a partner the pattern you see in the first set of expressions." (The numerators are both 6 because there are

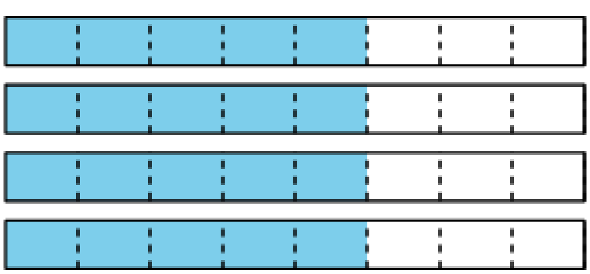
2. Explain the patterns you notice.



3. Write an expression to match each diagram. Find the value of each expression.

diagram	expression	product
		
		

4. Explain how the diagrams in problem 1 are different from the diagrams in problem 2.

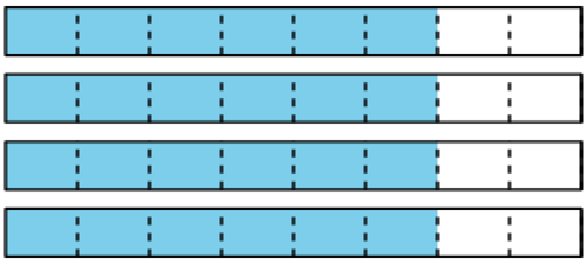


3 groups with 2 in each. In one, they are groups of two thirds and in the other, they are two fifths.)

- 2 minutes: partner discussion
- "Describe to a partner how the two sets of diagrams are different." (One pair shows 3 groups of 2 different fractions resulting in the same numerator and different denominators. The next pair of diagrams shows 4 and 5 groups of the same fraction, resulting in the same denominator and different numerators.)
- 2 minutes: partner discussion
- "Let's see if we can use the patterns we noticed to complete the rest of the task."
- 5–7 minutes: partner work

Synthesis

- Ask students to share the pattern they used to evaluate the second set of expressions.
- "Why does it make sense to multiply the numerator by the whole number to get the product?" (because that's the size of each group)
- "Why does the denominator stay the same?" (because the size of the unit piece or the unit fraction does not change)

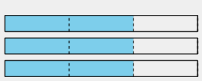
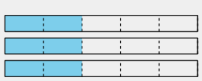


5. Use the pattern to evaluate each expression. Explain your reasoning. Draw a diagram if it is helpful.

- $2 \times \frac{1}{8}$
- $2 \times \frac{3}{8}$
- $2 \times \frac{5}{8}$
- $5 \times \frac{3}{12}$
- $6 \times \frac{3}{12}$
- $7 \times \frac{3}{12}$

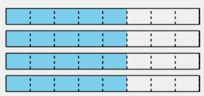
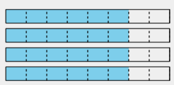
Student Responses

1.

diagram	expression	product
	$3 \times \frac{2}{3}$	$\frac{6}{3}$
	$3 \times \frac{2}{5}$	$\frac{6}{5}$

2. Sample response: I notice that the whole number and numerator is the same in the expression, and they both have 6 as a numerator in the product (3 groups of 2 parts shaded). I also notice that in each case the denominators remain the same. They do not change with the multiplication. More groups does not change the partitions. In one, there are 3 groups of two-thirds, and in the other, there are 3 groups of two-fifths.

3.

diagram	expression	product
	$4 \times \frac{5}{8}$	$\frac{20}{8}$
	$4 \times \frac{6}{8}$	$\frac{24}{8}$

4. Each diagram shows 4 groups, but the size of each group is different. Both are partitioned into eighths, but one has 5 eighths and the other has 6 eighths shaded. The number of groups does not change the size of the parts, so the denominator in the product is the same as the expression. The numerators change with the number of groups because there are more parts shaded.

5. $2 \times \frac{1}{8} = \frac{2}{8}$, because 2 groups of $\frac{1}{8}$ is $\frac{2}{8}$.
   $2 \times \frac{3}{8} = \frac{6}{8}$, because 2 groups of $\frac{3}{8}$ is $\frac{6}{8}$.
   $2 \times \frac{5}{8} = \frac{10}{8}$, because 2 groups of $\frac{5}{8}$ is $\frac{10}{8}$.
   $\frac{15}{12}$, because 5 x 3 = 15 and the units are twelfths.
   $\frac{18}{12}$, because 6 x 3 = 18 and the units are twelfths.
   $\frac{21}{12}$, because 7 x 3 = 21 and the units are twelfths.

Advancing Student Thinking

To support precision of language when describing patterns in expression and products, consider asking students to use the diagrams to describe how the numerators or number of groups are changing. Circle back and ask them to explain their reasoning after revising.

Activity 2: Mai's Big Discovery

Time: 20 minutes

Standards Alignment

Addressing CCSS:
- 4.NF.B.4.A
- 4.NF.B.4.C

Instructional Routines
- 5 Practices K5

Activity Narrative
This is a 5 Practices activity. The purpose of this activity is for students to evaluate multiplication expressions and begin to articulate the general idea behind multiplying any fraction by a whole number. As students work, monitor and select students who:

- attempt to draw tape diagrams for all problems. These students may or may not recognize that there could be a more efficient approach.
- draw tape diagrams for only some of the expressions, and for others use a numerical approach (multiply the numerator by the whole number, and keep the denominator the same) to get the product
- use a numerical approach for each expression

Sequence student explanations in the order provided in the synthesis. The synthesis offers questions that will be used to connect the ideas of each student as they share. Before each student begins, consider clearly stating the idea and part of the student work to be shared with the class.

Student-facing Task Statement	Teacher Directions
1. Evaluate each expression. Use a diagram if it is helpful. a. $3 \times \frac{3}{4}$ b. $8 \times \frac{3}{5}$ c. $5 \times \frac{6}{8}$ d. $10 \times \frac{2}{3}$ e. $9 \times \frac{12}{5}$ 2. Mai said she can multiply any fraction by a whole number by multiplying the whole number by the numerator and keeping the denominator. Do you agree with Mai? Why or why not?	**Launch** • Groups of 2 • 5–7 minutes: independent work **Activity** **Five Practices:** • As students work, consider asking questions that assess how they are making sense of the problems and invite them to think more deeply about their process: ○ For students using diagrams as the only strategy for solving: “How might you solve these without a diagram?”

	○ For students using diagrams for some expressions and multiplication to solve others: “How do you choose which expressions to use multiplication for?” ○ For students using multiplication to solve: “How can you be sure that your answer is correct?” **Synthesis** ● Select a student who chose to draw diagrams for each expression. ● “Where is the answer in your diagram? How do you know?” (There are 3 groups of $\frac{3}{4}$, so there are 9 fourths shaded, and 3 x $\frac{3}{4}$ = $\frac{9}{4}$.) ● Select a student who drew a diagram for some expressions, and used a numerical approach for others. ● “Why did you choose to draw a diagram for some expressions and to do something else for others?” (Sample response: After drawing some diagrams, I realized that I would have to draw a lot of groups or partitions for some of the expressions. I multiplied the numbers in each expression together when the groups got too big.) ● Select the student who chose to use a numerical approach to evaluate each expression. ● “I noticed that you did not use diagrams at all. What strategy did you use to evaluate the expressions?” (I noticed that when we multiply fractions by whole numbers, we can multiply the whole number and the numerator of the

	fraction and keep the denominator the same to get the product.) ● If no students articulate the idea that when multiplying a whole number and a fraction, we multiply the whole number and the fraction and the denominator stays the same, use Mai's thinking in problem two to investigate the idea as a whole class.

Student Responses

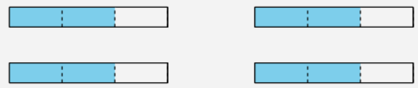
1. Responses may include diagrams.
   a. $\frac{9}{4}$
   b. $\frac{24}{5}$
   c. $\frac{30}{8}$
   d. $\frac{20}{3}$
   e. $\frac{108}{5}$
2. I agree because the total number of pieces is found by multiplying the numerator by the whole number and the size of the piece stays the same.

Lesson Synthesis

"Today we multiplied fractions by whole numbers. Mai said she can multiply any fraction by a whole number by multiplying the whole number by the numerator and keeping the denominator."

"Let's discuss Mai's reasoning using the expression $4 \times \frac{2}{3}$ and the diagram from today's warm-up."

$$4 \times \frac{2}{3} = \frac{8}{3}$$



- “Why can we multiply 4 x 2 to get the numerator of the product?” (The numerator in the product is 8. The diagram shows 4 groups of 2 thirds, or 8 thirds total.)
- “Why is the denominator of the product the same as the fraction in the expression?” (The denominator represents the size of the equal pieces in each group. The size of the group does not change when you increase the number of groups.)

Share and record responses.

Suggested Centers

- Rolling for Fractions, Stage 2

Response to Student Thinking Next Day	**Next Day Support**
Students get an incorrect product as a result of multiplying the whole number by the denominator, or treat the numerator as a whole number and record a whole number product.	• Launch warm-up or Activity 1 by highlighting important notation and vocabulary from previous lessons.
Response to Student Thinking	**Prior-unit Support**
Students make errors in multiplication within 100.	• Grade 3, Unit 2, Section C

4.3 Lesson 5: Relate Multiplication Expressions

Standards Alignment

Addressing CCSS:

- 4.NF.B.4.B, 4.NF.B.4.C

Teacher-facing Learning Goals

- Write equivalent expressions for the multiplication of a fraction by a whole number.

Student-facing Learning Goals

- Let's write multiplication expressions in different ways.

Lesson Purpose The purpose of this lesson is for students to extend patterns for multiplying a unit fraction by a whole number to multiply any fraction by a whole number.	
Lesson Narrative In previous lessons, students have multiplied unit and non-unit fractions by a whole number and represented their reasoning with diagrams and expressions. In this lesson, students apply these understandings to explain how two multiplication expressions are equivalent. Students use what they know about multiple groups of unit fractions to explain how two different expressions result in the same product.	
Access for Students with Disabilities Activity 1: Action and Expression	**Access for English Learners** Activity 2: MLR8 Discussion Supports
Materials to Gather Activity 1: none Activity 2: none	**Materials to Copy** Activity 1: none Activity 2: 4.3.A.5 Equivalent Expressions Cards
Lesson Timeline **Warm-up** 10 minutes **Activity 1** 15 minutes **Activity 2** 20 minutes **Lesson Synthesis** 10 minutes **Cool-down** 5 minutes	**Teacher Reflection Question** What did you say, do, or ask during the lesson synthesis that helped students be clear on the learning of the day? How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?
Cool-down: Explain Why It is True	
Standards Alignment: • 4.NF.B.4.A, 4.NF.B.4.B, 4.NF.B.4.C	
Student-facing Task Statement Explain or show why each equation is true. You can use a diagram if it is helpful. 1. $6 \times \frac{1}{9} = 3 \times \frac{2}{9}$ 2. $4 \times \frac{1}{5} = 2 \times 2 \times \frac{1}{5}$	

Student Responses

1. For each side of the equation, multiplying the numerator and the whole number gives 6, and the denominators are both 9. So each side is equal because they are both $\frac{6}{9}$.
2. 4 = 2 x 2, so the two expressions are equal because each one shows 4 multiplied by $\frac{1}{5}$.

Warm-up: How Many Do You See?

Time: 10 minutes

Standards Alignment

Addressing CCSS:

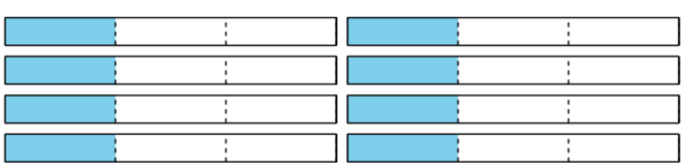
- 4.NF.B.4.b
- 4.NF.B.4.c

Instructional Routines

- How Many Do You See? K5

Warm-up Narrative

The purpose of this How Many Do You See is for students to use grouping strategies to describe the images they see. Students' descriptions are recorded using equations and expressions to support the goal of creating equivalent expressions. The synthesis encourages students to think about why two expressions can represent the same amount.

Student-facing Task Statement	Teacher Directions
How many thirds do you see? How do you see them? 	**Launch** ● Groups of 2 ● "How many thirds do you see? How do you see them?" **Activity** ● Display the image. ● 1 minute: quiet think time **Synthesis** ● For each way that students see thirds, ask: "What expression should we use to

	represent the groups of thirds that _____ saw?” • If students do not suggest it, ask, “How might someone see 4 groups of $\frac{2}{3}$ in the diagram?” (By combining 2 of the thirds from each strip, we can make 4 groups of $\frac{2}{3}$.) • Write 8 x $\frac{1}{3}$ = 4 x $\frac{2}{3}$, and ask students if they agree or disagree with the statement. • “We are going to revisit this question at the end of our lesson today. Let’s see if our thinking changes in any way.” Consider asking: • “Who can restate the way _____ saw the image in different words?” • “Did anyone see the dots the same way but would explain it differently?” • “Does anyone want to add an observation to the way _____ saw the image?”

Student Responses

- 8 groups of $\frac{1}{3}$, 8 x $\frac{1}{3}$
- 4 groups of $\frac{2}{3}$, 4 x $\frac{2}{3}$
- 2 groups of $\frac{4}{3}$, 2 x $\frac{4}{3}$

Activity 1: Complete the Equation

Time: 15 minutes

Standards Alignment

Addressing CCSS:

- 4.NF.B.4.B

- 4.NF.B.4.C

Activity Narrative
The purpose of this activity is for students to think of different ways of using multiplication expressions to represent a product. Students informally use the associative property as they work toward generalizing that $n \times \frac{a}{b} = \frac{n \times a}{b} = n \times a \frac{1}{b}$.

SwD Support Tags
- Action and Expression

SwD Support Text
Action and Expression: Develop Expression and Communication. Activity: Provide access to fraction strips or pre-formatted tape diagrams, including sevenths, fifths, and tenths.
Supports accessibility for: Visual-Spatial Processing, Fine Motor Skills.

Student-facing Task Statement

1. Here are two sets of numbers:
   - Set A: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
   - Set B: $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}$

   a. Choose a number from Set A and a number from Set B to complete this equation and make it true:

   $\frac{6}{7}$ = _____ x _____

   b. Choose a different number from Set A and a number from Set B to complete the equation to make it true.

   $\frac{6}{7}$ = _____ x _____

2. Find the missing number to make each equation true. Draw a diagram if it is helpful.

   a. $\frac{12}{5}$ = 12 x _____

Teacher Directions

Launch

- Groups of 2
- 3 minutes: independent work time on the first set of problems

Activity
- 4 minutes: partner discussion
- "Did you choose the same numbers as your partner? If not, are both equations correct?"

- 7 minutes: independent work time
- Monitor for students who use the factors of 12 to complete the equations in the second problem, and factors of 8 in the last problem.

Synthesis

- Select 2–3 partners to share the following equations from the first problem:

b. $\frac{12}{5}$ = 6 x _____ c. $\frac{12}{5}$ = 4 x _____ d. $\frac{12}{5}$ = 3 x _____ e. $\frac{12}{5}$ = 2 x _____ f. $\frac{12}{5}$ = 1 x _____ 3. Write 3 different expressions that make each equation true: ○ $\frac{8}{10}$ = _____ x _____ ○ $\frac{8}{10}$ = _____ x _____ ○ $\frac{8}{10}$ = _____ x _____	○ $\frac{6}{7}$ = 6 x $\frac{1}{7}$ ○ $\frac{6}{7}$ = 3 x $\frac{2}{7}$ ○ $\frac{6}{7}$ = 2 x $\frac{3}{7}$ ○ $\frac{6}{7}$ = 1 x $\frac{6}{7}$ ● "Why are these all true equations?" (To get to $\frac{6}{7}$, we can multiply the whole number by the numerator and then keep the denominator the same for each equation. When we multiply the whole number and the numerator, it equals 6 each time. There are different ways to multiply to get 6 as a numerator: 6 x 1, 3 x 2, 2 x 3, and 1 x 6.) ● "How did you know what fractions to use to complete each equation in the second problem?" (We looked for the numbers to multiply to get $\frac{12}{5}$ or $\frac{8}{9}$. The denominators stay the same, so we find out what is going to be the whole number and numerator by knowing the factor pairs of 12 or 8.)

Student Responses

1. Sample responses for parts a and b:
   - ○ $\frac{6}{7}$ = 6 x $\frac{1}{7}$
   - ○ $\frac{6}{7}$ = 3 x $\frac{2}{7}$
   - ○ $\frac{6}{7}$ = 2 x $\frac{3}{7}$
   - ○ $\frac{6}{7}$ = 1 x $\frac{6}{7}$

2.
   a. $\frac{12}{5}$ = 12 x $\frac{1}{5}$
   b. $\frac{12}{5}$ = 6 x $\frac{2}{5}$

c. $\frac{12}{5} = 4 \times \frac{3}{5}$
d. $\frac{12}{5} = 3 \times \frac{4}{5}$
e. $\frac{12}{5} = 2 \times \frac{6}{5}$
f. $\frac{12}{5} = 1 \times \frac{12}{5}$

3. Sample responses:
    - $\frac{8}{9} = 8 \times \frac{1}{9}$
    - $\frac{8}{9} = 4 \times \frac{2}{9}$
    - $\frac{8}{9} = 2 \times \frac{4}{9}$

Advancing Student Thinking

It may not occur to students to identify factor pairs of the numerator of each fraction and use them to complete the equations. Ask students to draw a diagram for 12 groups of $\frac{1}{5}$, then circle different equal-size groups that they see.

Activity 2: Equivalent Expressions

Time: 20 minutes

Standards Alignment

Addressing CCSS:
- 4.NF.B.4.B
- 4.NF.B.4.C

Materials to Copy
- 4.3.A.5 Equivalent Expressions Cards

Instructional Routines
- Card Sort K5

Activity Narrative
The purpose of this activity is for students to write equivalent expressions that match different representations of the same amount. Students will use the results of a card sort to reason about equivalent expressions. Each fraction may not have the same number of matching cards.

MLR Tags

- MLR8

EL Support Text
MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: "I noticed ____ , so I matched . . . " Encourage students to challenge each other when they disagree.
Advances: Speaking, Conversing

Student-facing Task Statement

Your teacher will give you and your partner a set of cards that have expressions.

Match each card to one of the following fractions.

$\frac{4}{5}$	$\frac{10}{12}$	$\frac{6}{10}$	$\frac{8}{9}$

Complete each equation to make it true. Use the diagrams to support your reasoning.

1. $\frac{4}{5}$ = _____ x _____ = _____ x _____
2. $\frac{10}{12}$ = _____ x _____ = _____ x _____
3. $\frac{6}{10}$ = _____ x _____ = _____ x _____
4. $\frac{8}{9}$ = _____ x _____ = _____ x _____

Teacher Directions

Launch
- Groups of 2
- Fraction Cards

Activity
- "Match cards that represent the same fraction. Each fraction may not have the same number of matching cards."
- "When you are done matching, write equivalent expressions for each fraction."
- As students work, monitor for students who describe the non-unit fractions as multiples of the unit fraction in their reasoning for problems 1–4.

Synthesis
- Ask selected students to share different expressions for problem 2.
- "How can we explain why these expressions are equivalent?"
- Display the solution to the final problem: $\frac{8}{9} = 8 \times \frac{1}{9} = 2 \times 4 \times \frac{1}{9} = 4 \times 2 \times \frac{1}{9} = 2 \times \frac{4}{9}$

Student Responses
1.

$\frac{4}{5}$	$\frac{10}{12}$	$\frac{6}{10}$	$\frac{8}{9}$
$2 \times 2 \times \frac{1}{5}$ $4 \times \frac{1}{5}$	$5 \times 2 \times \frac{1}{12}$	$6 \times \frac{1}{10}$ $3 \times 2 \times \frac{1}{10}$	$2 \times \frac{4}{9}$

2. $\frac{4}{5} = 4 \times \frac{1}{5} = 2 \times 2 \times \frac{1}{5} = 2 \times \frac{2}{5}$
3. $\frac{10}{12} = 10 \times \frac{1}{12} = 5 \times 2 \times \frac{1}{12} = 5 \times \frac{2}{12}$
4. $\frac{6}{10} = 6 \times \frac{1}{10} = 3 \times 2 \times \frac{1}{10} = 3 \times \frac{2}{10} = 2 \times \frac{3}{10}$
5. $\frac{8}{9} = 8 \times \frac{1}{9} = 2 \times 4 \times \frac{1}{9} = 4 \times \frac{2}{9} = 4 \times 2 \times \frac{1}{9} = 2 \times \frac{4}{9}$

Advancing Student Thinking

Students do not recognize expressions as equivalent. Use diagrams from card sort to illustrate various groupings within the same diagram, by circling and labeling groups while asking, “Where do we see _____ (whole number) groups of _____ (fraction)?”

Lesson Synthesis

“Today we looked at different expressions to represent the same fraction.”

Display the following equations:

$\frac{6}{8} = 6 \times \frac{1}{8}$

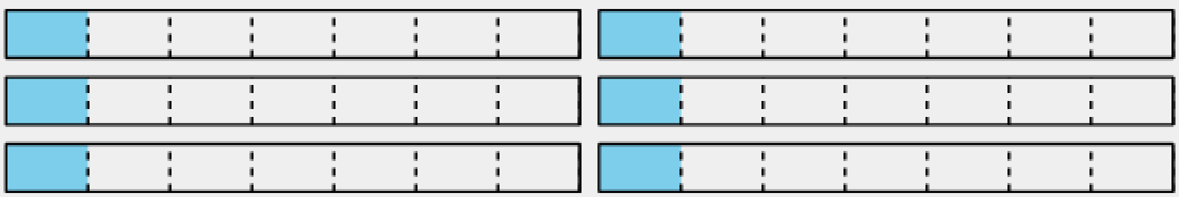
$\frac{6}{8} = 3 \times \frac{2}{8}$

$\frac{6}{8} = 2 \times \frac{3}{8}$

$\frac{6}{8} = 3 \times 2 \times \frac{1}{8}$

“Talk with your neighbor to explain how the first equation is related to the second equation, and how that one is related to the third equation.” (I know they are equivalent because they both equal $\frac{6}{8}$, or I know they are equivalent because there are 3 groups but 2 eighths in each group, so there are still $\frac{6}{8}$.)

Share and record responses. Encourage students to use a diagram to support their explanation. If needed, display the following diagram for students to use in their explanation.

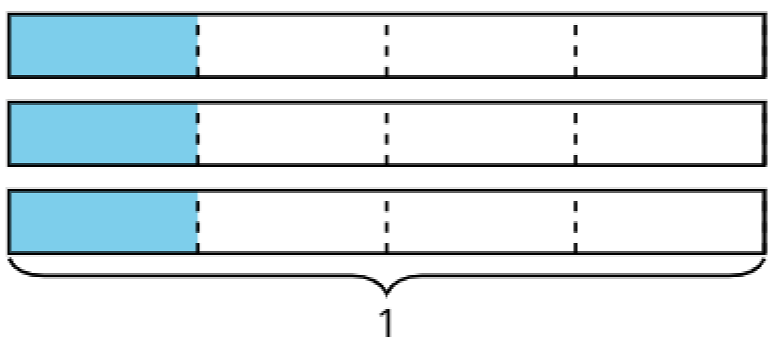


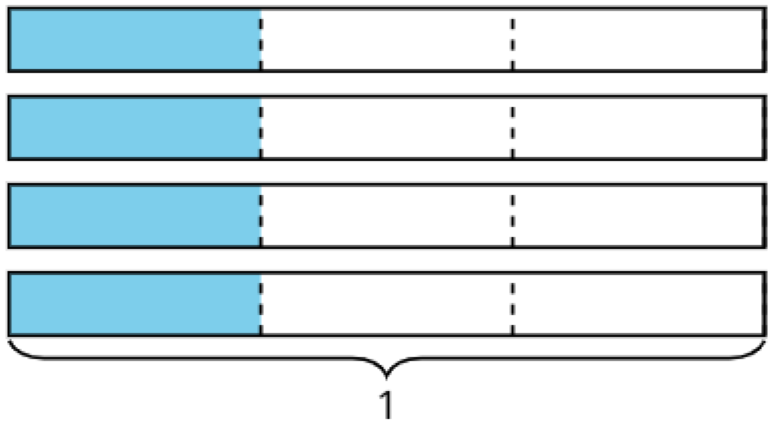
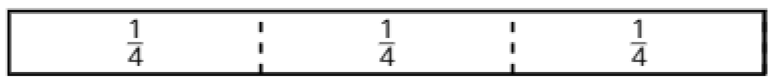
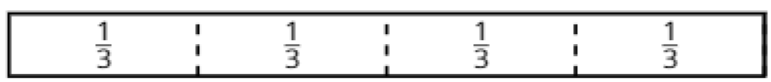
Suggested Centers

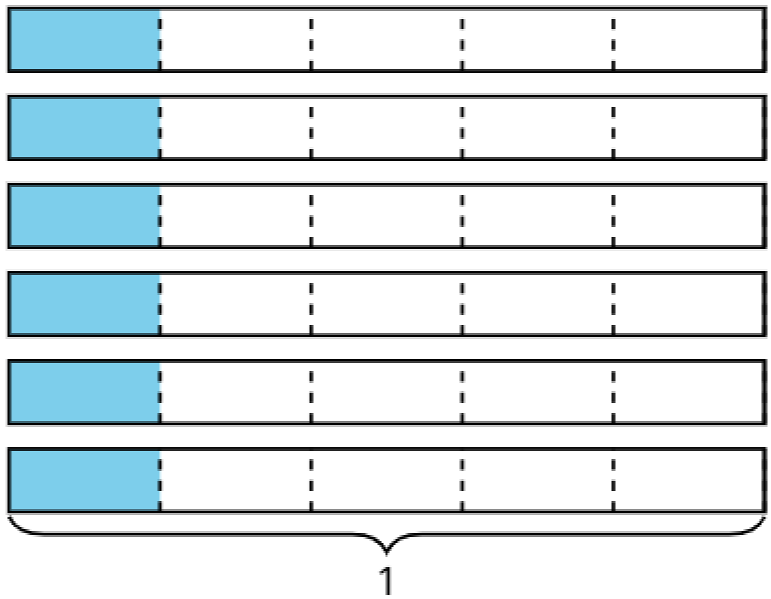
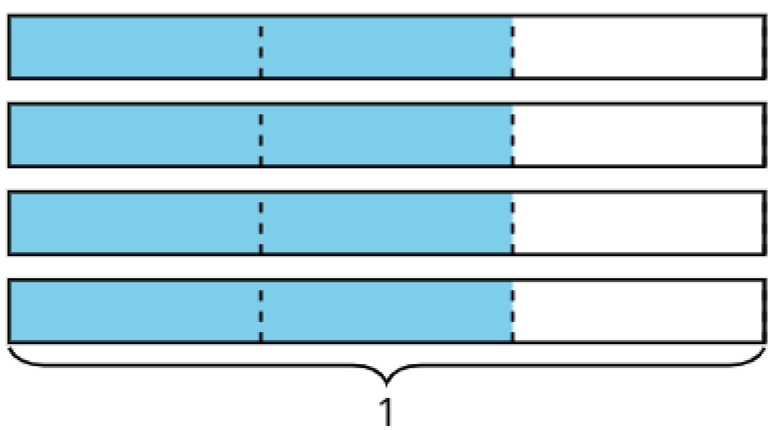
- Rolling for Fractions, Stage 2

Response to Student Thinking Next Day	**Next Day Support**
Students generate only one equivalent expression.	• During the Activity Syntheses, connect diagrams to expressions or equations.
Response to Student Thinking Students do not identify and use factor pairs of the numerators of the products.	**Prior-unit Support** • Grade 4, Unit 1, Sections A–B

Mini-Assessment 2	
Item Statement	Select all the expressions that are equivalent to $\frac{8}{5}$. A. 5 x $\frac{1}{8}$ B. 8 x $\frac{1}{5}$ C. 2 x $\frac{4}{5}$ D. 4 x $\frac{2}{5}$ E. 2 x $\frac{6}{5}$
Item Solution	B, C, D
Item Learning Goals	Write equivalent expressions for the multiplication of a fraction by a whole number.

Item Statement	Select all diagrams that show 4 x $\frac{1}{3}$.  A.

	B.  C. Number line with 0 and 1 marked, all fifths with tick marks, and $\frac{4}{5}$ marked with a dot.  D.  E. 
Item Solution	B, E
Item Learning Goals	Match diagrams and situations that represent a fraction multiplied by a whole number to expressions.
Item Statement	Each cup holds $\frac{1}{5}$ liter of water. There are 6 cups. 1. Draw a diagram representing the situation. 2. How many liters of water do the 6 cups hold?

Item Solution	 1. 2. $\frac{6}{5}$ liters
Item Learning Goals	Write and evaluate expressions from diagrams and situations that represent a fraction multiplied by a whole number.
Item Statement	1. Draw a diagram showing 4 x $\frac{2}{3}$. 2. Use the diagram to calculate 4 x $\frac{2}{3}$.
Item Solution	 1. 2. $\frac{8}{3}$, as there are 4 x 2 or 8 small shaded rectangles and each one is $\frac{1}{3}$ of a rectangle.

Item Learning Goals	Use diagrams to represent multiplication expressions of a fraction by a whole number.

Prior-grade Practice and Fluency Resources

Grade 4 Number Talk: Dividing by 3

Time: 10 minutes

Addressing CCSS: 4.NBT.B.6

Warm-up Narrative

This Number Talk encourages students to look for and make use of the structure of numbers in base-ten to mentally solve division problems (MP7). The reasoning elicited here will be helpful later in the lesson when students divide large numbers using increasingly more abstract strategies.

In explaining their reasoning, students practice being precise in their word choice and use of language (MP6).

Task Statement	Launch/Activity
Find each quotient mentally. 90 ÷ 3 96 ÷ 3 960 ÷ 3 954 ÷ 3	● Display one expression. ● "Give me a signal when you have an answer and can explain how you got it." ● 1 minute: quiet think time ● Record answers and strategy. ● Keep expressions and work displayed. ● Repeat with each expression. **Synthesis** ● "How did each expression help you find the next one?" Consider asking: ● "Who can restate _____'s reasoning in a different way?" ● "Did anyone have the same strategy but would explain it differently?" ● "Did anyone approach the problem in a different way?" ● "Does anyone want to add on to _____'s strategy?"

Student Responses

- 30: I know that 90 is 10 x 9 and 9 ÷ 3 is 3, so 90 ÷ 3 is 10 x 9 ÷ 3, which is 30.
- 32: Ninety is 30 x 3 and 6 is 2 x 3, so 96 is (30 x 3) + (2 x 3), which is (30 + 2) x 3 or 32 x 3.
- 320: I know that 96 ÷ 3 is 32, so 960 ÷ 3 is 10 x 96 ÷ 3 or 10 x 32, which is 320.
- 318. I know that 954 is 960 – 6, so it is (320 x 3) – (2 x 3), which is (320 – 2) x 3 or 318 x 3.

Grade 4 Number Talk: Dividing by 4

Time: 10 minutes

Addressing CCSS: 3.OA.C.7

Warm-up Narrative

This Number Talk encourages students to use multiples of 4, to compose or decompose them, and to rely on properties of operations to mentally solve problems. The ability to compose and decompose numbers will be helpful when students divide multi-digit numbers. It also promotes the reasoning that is useful when finding multiples of a number, or when deciding if a number is a multiple of another number.

To find quotients of larger numbers, students need to look for and make use of structure (MP7). In explaining their reasoning, students practice being precise in their word choice and use of language (MP6).

Task Statement

Find the value of each quotient mentally.

40 ÷ 4
80 ÷ 4
16 ÷ 4
96 ÷ 4

Launch/Activity

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did the first three expressions help us find the value of the last expression?"

Consider asking:

- "Who can restate _____'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"

	• "Did anyone approach the problem in a different way?" • "Does anyone want to add on to _____'s strategy?"

Student Responses

- 10: There are 10 groups of 4 in 40 (or 10 x 4 = 40).
- 20: Eighty is 2 times 40, so there are 20 groups of 4 in 80 (or 20 x 4 = 80).
- 4: There are 4 groups of 4 in 16 (or 4 x 4 = 16).
- 24: Ninety-six is 80 plus 16, so there are 20 plus 4 groups of 4 in 96 (or 96 = (20 x 4) + (4 x 4)).

Grade 4 Number Talk: Dividing by 7

Time: 10 minutes

Addressing CCSS: 4.OA.B.6

Warm-up Narrative

This Number Talk encourages students to use multiples of 7, to compose or decompose them, and to rely on properties of operations to mentally solve problems. The ability to compose and decompose numbers will be helpful when students divide multi-digit numbers. It also promotes the reasoning that is useful when finding multiples of a number, or when deciding if a number is a multiple of another number.

To find quotients of larger numbers, students need to look for and make use of structure (MP7). In explaining their reasoning, students practice being precise in their word choice and use of language (MP6).

Task Statement	Launch/Activity
Find each quotient mentally. 21 ÷ 7 35 ÷ 7 140 ÷ 7 196 ÷ 7	• Display one expression. • "Give me a signal when you have an answer and can explain how you got it." • 1 minute: quiet think time • Record answers and strategy. • Keep expressions and work displayed. • Repeat with each expression. **Synthesis** • "What do the expressions have in common?" (They all involve division by 7. The dividends are all multiples of 7.

<table>
<tr><td></td><td>
The results have no remainders.)
• “How did the first three expressions help us find the value of the last expression?”
Consider asking:
• “Who can restate _____’s reasoning in a different way?”
• “Did anyone have the same strategy but would explain it differently?”
• “Did anyone approach the problem in a different way?”
• “Does anyone want to add on to _____’s strategy?”
</td></tr>
<tr><td colspan="2">
Student Responses
• 3: Three times 7 is 21.
• 5: Five times 7 is 21.
• 20: Two times 7 is 14, so 20 times 7 is 140.
• 28: Sample reasoning:
  ○ 196 = 140 + 56, and 56 = 21 + 35. This means 196 = 140 + 21 + 35, so 196 = (20 x 7) + (3 x 7) + (5 x 7).
  ○ Three times 7 is 21, so 30 times 7 is 210. 196 is 14 less than 210, so it is 2 x 7 less than 30 x 7.
</td></tr>
</table>

<table>
<tr><td colspan="2">Grade 4 Number Talk: Multiplying by Thirds</td></tr>
<tr><td colspan="2">Time: 10 minutes</td></tr>
<tr><td colspan="2">Addressing CCSS: 4.NF.B.4.B</td></tr>
<tr><td colspan="2">Warm-up Narrative
This Number Talk encourages students to use multiplicative reasoning and to rely on properties of operation to mentally find products of a whole number and a fraction. The reasoning elicited here will be helpful later in the lesson when students find the perimeter of a figure with fractional side lengths.

Students practice looking for and making use of structure (MP7) as they decompose factors to facilitate multiplication. In explaining their strategies, students need to be precise in their word choice and use of language (MP6).</td></tr>
<tr><td>Task Statement

Find the value of each product mentally.</td><td>Launch/Activity
• Display one expression.
• “Give me a signal when you have an answer and can explain how you got it.”</td></tr>
</table>

$6 \times \frac{1}{3}$ $30 \times \frac{1}{3}$ $60 \times \frac{2}{3}$ $90 \times \frac{2}{3}$	• 1 minute: quiet think time • Record answers and strategy. • Keep expressions and work displayed. • Repeat with each expression. **Synthesis** • “What do these expressions have in common?” (The first number in each sequence is a multiple of 3 and a multiple of 6. The second number is a fraction with 3 in the denominator.) • “How did these observations about the numbers help you find each product?” Consider asking: • “Who can restate _____’s reasoning in a different way?” • “Did anyone have the same strategy but would explain it differently?” • “Did anyone approach the problem in a different way?” • “Does anyone want to add on to _____’s strategy?” •

Student Responses

- 2, because 3 groups of $\frac{1}{3}$ is 1, so 6 groups of $\frac{1}{3}$ is 2.
- 10, because:
  - $3 \times \frac{1}{3}$ is 1, so $30 \times \frac{1}{3}$ is 10 x 1.
  - $30 \times \frac{1}{3}$ is $5 \times (6 \times \frac{1}{3})$, so it is 5 x 2.
- 40, because:
  - $6 \times \frac{2}{3}$ is 4, so $60 \times \frac{2}{3}$ is 10 x 4.
  - 60 is 2 x 30, and $\frac{2}{3}$ is $2 \times \frac{1}{3}$, so $60 \times \frac{2}{3}$ is $2 \times 2 \times (30 \times \frac{1}{3})$ or 4 x 10.
- 60, because:
  - $9 \times \frac{2}{3}$ is 6, so $90 \times \frac{2}{3}$ is 10 x 6.
  - 90 is 1.5 times 60, so $90 \times \frac{2}{3}$ is 1.5 times $60 \times \frac{2}{3}$ or 1.5 x 40.

Grade 4 Number Talk: Multiply a Fraction by a Whole Number

Time: 10 minutes	
Addressing CCSS: 4.NF.4	
Warm-up Narrative This Number Talk encourages students to think about equivalent fractions as they review strategies for multiplying unit fractions and non-unit fractions by a whole number. The patterns elicited here will be helpful in the activities that follow when students think about equivalence when creating line plots using halves, fourths, and eighths.	
Task Statement Find the value of each product mentally. $5 \times \frac{1}{4}$ $5 \times \frac{2}{8}$ $5 \times 3 \times \frac{1}{4}$ $3 \times 5 \times \frac{2}{8}$	**Launch/Activity** ● Display one problem at a time. ● Give students quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. ● Keep all problems displayed throughout the talk. ● Follow with a whole-class discussion. **Synthesis** ● "How do these problems help you think about equivalent fractions?"
Student Responses ● $\frac{4}{4}$ ● $\frac{10}{8}$ ● $\frac{15}{4}$ ● $\frac{30}{8}$	

Grade 4 True or False: Multiplication Expressions
Time: 10 minutes
Addressing CCSS: 4.NF.B.4
Warm-up Narrative The purpose of this True or False is to elicit strategies and understandings students have for comparing groups of fractions without calculating products by reasoning about the number of groups or amount

in each group. These understandings will be helpful as students evaluate multiplication expressions in this lesson.

Task Statement	**Launch/Activity**
Is each statement true or false? Be prepared to explain your reasoning. $4 \times \frac{1}{2} = 5 \times \frac{1}{2}$ $6 \times \frac{1}{4} = 3 \times 2 \times \frac{1}{4}$ $3 \times \frac{1}{5} = 3 \times \frac{1}{8}$	• Display one problem. • "Give me a signal when you know whether the equation is true and can explain how you know." • 1 minute: quiet think time • Record answers and strategy. • Repeat with each problem. **Synthesis** Focus Question: • "How can you justify your answer without solving each expression?"

Student Responses

- False. Sample response: Both have groups of $\frac{1}{2}$, but one has more groups.
- True. Sample response: 3 x 2 is the same as 6 and each has a group of $\frac{1}{4}$.
- False. Sample response: Both have 3 groups, but the group size is different, so they can't be the same.

Center: Race to Zero
Teacher-facing Learning Goals • Find whole-number quotients and remainders with dividends of up to four digits and one-digit divisors.
Addressing CCSS: 4.NBT.B.6

Stage Descriptions
Stage Descriptions • **Stage 1:** Students divide within 100. (after Lesson 13) • **Stage 2:** Students divide beyond 100. (after Lesson 14) • **Stage 3:** Students divide within 100 and write a multiplication equation. (after Lesson 19) • **Stage 4:** Students divide beyond 100 and write a multiplication equation. (after Lesson 19) **Look Fors** • Students can divide using strategies that relate multiplication to division. • Students can divide using strategies based on properties of operations.
Required Materials • number cubes • stage-specific recording sheet for each student

Student-facing Directions	**Teacher Interactions**
Stages 1–2: Race to Zero	Groups of 2
Task Statement 1. Roll the number cube. 2. Choose a number to divide by the number you rolled. 3. Write the quotient in the correct column. 4. Write a multiplication equation. It may or may not include a remainder. 5. Take turns rolling the number cube and dividing. 6. Person closest to 0 wins.	Questions to ask during the center activity: • Why did you choose that number to divide? • How did you figure out this quotient? • Do you know a multiplication fact that could help you find the quotient? • Could you decompose the dividend to make some quotients that are easier to find?
Stages 3–4: Race to Zero	Groups of 2

Task Statement	Questions to ask during the center activity:
1. Roll the number cube. 2. Choose a number to divide by the number you rolled. 3. Write a matching division expression in the correct column. 4. Write a multiplication equation. It may or may not include a remainder. 5. Take turns rolling the number cube and dividing. 6. Person closest to 0 wins.	• How did you figure out this quotient? • Will there be a remainder? How do you know? • How does the multiplication equation help you understand the remainder?

Extension and Exploration Resources

Center: Rolling for Fractions 1
Narrative In this center, students apply their understanding of multiplication or division to work with fractions.
Stage 1: Compare Fraction Multiplication Expressions to 1
Addressing CCSS: • 4.NF.B.4
Building Toward CCSS: (only for K)
Learning Goals (Section Goal) • Represent and explain that a fraction $\frac{a}{b}$ is a multiple of $\frac{1}{b}$, namely a x $\frac{1}{b}$. (4.3.A) • Recognize that n x $\frac{a}{b}$ = $\frac{(n \times a)}{b}$. (4.3.A)
Required Materials • number cubes • pencils
Blackline Master

• Rolling for Fractions: Stage 1 Recording Sheet
Stage Narrative In this stage, students multiply a whole number by a fraction and compare the value of the expression to 1. Play Rolling for Fractions with 2 players 1. Each player rolls 1 number cube. The player with the greatest number goes first. 2. Player 1: Roll 2 number cubes. Use the numbers to complete the expression and find the product. 3. Player 2: Roll 2 number cubes. Use the numbers to complete the expression and find the product. 4. Check one another's equation for accuracy. Determine the amount of points each player receives: a. 2 points for creating an expression less than 1 b. 5 points for creating an expression greater than 1 c. 10 points for creating an expression that is equal to 1 5. Play 5 rounds. The player with the most points wins.

Center: Rolling for Fractions 2
Narrative In this center, students apply their understanding of multiplication or division to work with fractions.
Stage 2: Compare Fraction Multiplication Expressions to 0 or 10
Addressing CCSS: • 4.NF.B.4
Building Toward CCSS: (only for K)
Learning Goals (Section Goal) • Represent and explain that a fraction $\frac{a}{b}$ is a multiple of $\frac{1}{b}$, namely a x $\frac{1}{b}$. (4.3.A) • Recognize that n x $\frac{a}{b}$ = $\frac{(n \times a)}{b}$. (4.3.A)
Required Materials • number cubes • pencils

Blackline Master
- Rolling for Fractions: Stage 2 Recording Sheet

Stage Narrative

In this stage, students multiply a whole number by a fraction and determine whether the value is closer to 0 or 10.

Play Rolling for Fractions with 2 players

1. Each player rolls 1 number cube. The player with the greatest number goes first.
2. Player 1 chooses whether the target is to get closer to 0 or closer to 10.
3. Player 1: Roll 3 number cubes. Use the numbers to complete the expression and find the product.
4. Player 2: Roll 3 number cubes. Use the numbers to complete the expression and find the product.
5. Check one another's equation for accuracy. Determine the amount of points each player receives:
   a. 2 points for creating an expression that is closer to the target
   b. 1 point each if players are the same distance from the target
6. Player 2 chooses a new target in the next round.
7. Play 5 rounds. The player with the most points wins.

IM Task: Sugar in Six Cans of Soda

Task

For a certain brand of orange soda, each can contains $\frac{4}{15}$ cup of sugar.

a. How many cups of sugar are there in six cans of this orange soda?
b. Draw a picture representing the answer to (a).

IM Commentary

This task provides a familiar context allowing students to visualize multiplication of a fraction by a whole number.

Notice in the picture for part (b) that the second cup of sugar has been cut into fifteen equal pieces even though only nine of these are filled up by the sugar from the soda. It is important to show this division of the cup into fifteen equal pieces, even though not all of them are filled with sugar, because this tells us that the equal amounts of sugar represented in the picture are each fifteenths of a cup.

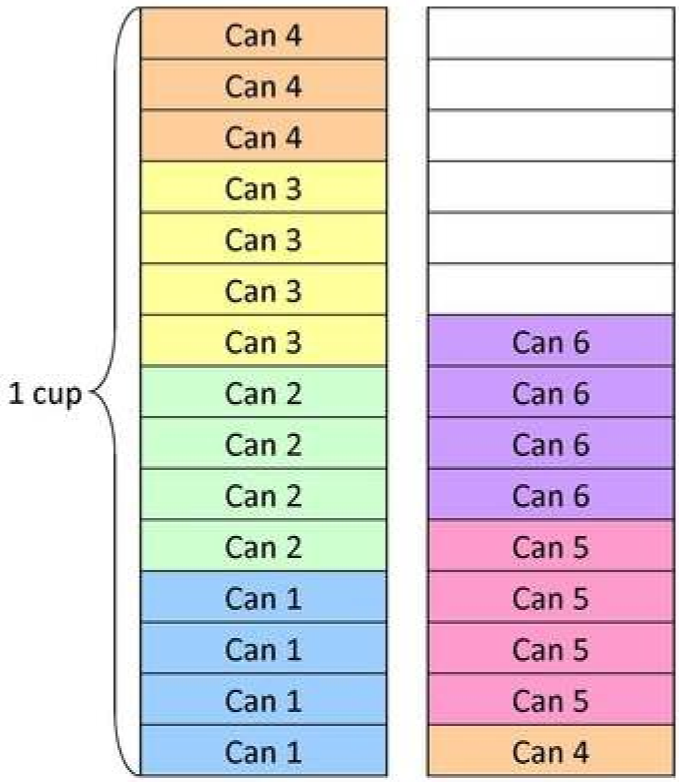
This particular task helps illustrate Mathematical Practice Standard 4, Model with Mathematics. Students apply the mathematics they know to solve problems arising in everyday life. For this problem, fourth-graders might apply their understanding of multiplication as repeated addition and multiply 4/15 cups of sugar 6 times, draw a picture depicting the sugar in 6 cans of soda, or create a bar diagram with each of the six sections representing 4/15 of sugar. As students think about the situation in real life, they have to select the mathematical thinking that will help them solve the problem.

Solutions

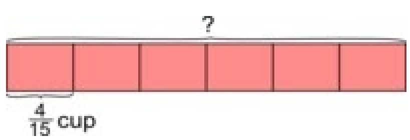
Solution: 1

a. There are six cans of soda and each one contains 4/15 cup of sugar. In the six cans of soda, there are 6 groups of $\frac{4}{15}$ cup of sugar. To find out how much sugar there is in the six cans, this is a multiplication problem: 6 groups of $\frac{4}{15}$ cups of sugar is 6 x $\frac{4}{15}$ cups of sugar. To find the solution, 6 x $\frac{4}{15}$ = 6 x $\frac{4}{15}$ = $\frac{24}{15}$ = 1 $\frac{9}{15}$. So there are $\frac{24}{15}$ = 1 $\frac{9}{15}$ cups of sugar in the six cans of orange soda. To see the reasoning behind the first equation in the sequence above, 6 groups of $\frac{4}{15}$ cups is the same as adding $\frac{4}{15}$ to itself six times: so the denominator of the result will be 15 (all quantities being added are fifteenths of a cup) while the numerator is a sum of six fours or 6 x 4.

b. Below is a picture showing the sugar in the 6 cans of soda. There are two cups, each of which has been divided into 15 equal pieces. Each can of soda contains $\frac{4}{15}$ cup of sugar: this is why each can is listed four times, once for each $\frac{1}{15}$ of a cup of sugar contained in that can. We see from the picture that the six cans of soda contain one full cup of sugar plus part of another. Counting the parts in the second cup, we see that $\frac{9}{15}$ of the second cup has been filled with the sugar from the soda. So, in total, the six cans of orange soda contain 1 $\frac{9}{15}$ or $\frac{24}{15}$ cups of sugar.



Solution: Using a bar diagram



1 can: $\frac{4}{15}$ cup

6 cans: 6 x $\frac{4}{15}$ = 6 x $\frac{4}{15}$ = $\frac{24}{15}$ = 1 $\frac{9}{15}$ = 1 $\frac{3}{5}$ cups

There would be 1 $\frac{3}{5}$ cups of sugar in 6 cans of soda.

4.8 Lesson 3: Operating with Fractions

Standards Alignment

Addressing CCSS: 4.NF.B.3, 4.NF.B.4

Teacher-facing Learning Goals

- Solve problems involving addition and subtraction of fractions.
- Solve problems involving multiplication of a fraction by a whole number.

Lesson Purpose

The purpose of this lesson is for students to represent and solve problems involving fraction operations. Students also reason about equivalence to compare fractions to whole numbers.

Materials to Gather	Materials to Copy
• none	• none

Cool-down: Compare to 2

Student-facing Task Statement

$\frac{15}{10}$	$\frac{13}{100}$
$\frac{53}{100}$	$\frac{9}{10}$

1. Select two fractions that have a sum greater than 2.
2. Use all four fractions to write an expression that has a value greater than 1 but less than 2.

Student Responses

Sample response:

1. $\frac{15}{10} + \frac{53}{100} = \frac{203}{100}$
2. $\frac{15}{10} + \frac{53}{100} - \frac{9}{10} + \frac{13}{100}$ or $\frac{15}{10} + \frac{9}{10} - \frac{53}{100} - \frac{13}{100}$

Teacher Reflection Question

What evidence from today's lesson indicates students are thinking flexibly as they add, subtract, and multiply fractions?

<table>
<tr><td colspan="2">Warm-up: Number Talk: Fluency Talk</td></tr>
<tr><td colspan="2">Time: 10 minutes</td></tr>
<tr><td colspan="2">Addressing CCSS: 4.NBT.4</td></tr>
<tr><td colspan="2">Warm-up Narrative
This Number Talk encourages students to think flexibly about numbers to multiply. The understandings elicited here will be helpful throughout this unit as students build toward fluency with multiplying whole numbers and fractions.

To flexibly adjust numbers, students need to look for and make use of structure (MP7). In describing strategies, students need to be precise in their word choice and use of language (MP6).</td></tr>
<tr><td>Task Statement

Find the value of each difference mentally.

● 5 x $\frac{10}{5}$
● 8 x $\frac{11}{4}$
● 9 x $\frac{6}{3}$
● 6 x $\frac{12}{10}$</td><td>Teacher Directions
Launch/Activity
● Display one expression.
● “Give me a signal when you have an answer and can explain how you got it.”
● 1 minute: quiet think time
● Record answers and strategy.
● Keep expressions and work displayed.
● Repeat with each expression.

Synthesis
● “How is the last expression different from the others?” (It is the only one that did not have a whole-number product.)

● “How does this change the pattern of products we are getting?” (When the whole number is a multiple of the denominator, the product is also a whole number. Otherwise, the product is a mixed number.)

Consider asking:

● “Who can restate _____’s reasoning in a different way?”
● “Did anyone have the same strategy but would explain it differently?”
● “Did anyone approach the problem in a</td></tr>
</table>

different way?"
- "Does anyone want to add on to _____'s strategy?"

Student Responses
- $\frac{50}{5}$ or 10
- $\frac{88}{4}$ or 22
- $\frac{54}{3}$ or 18
- $\frac{72}{10}$ or $7\frac{2}{10}$

Activity 1: Let's Make Head Wraps

Time: 15 minutes

Addressing CCSS: 4.NF.B.4

Activity Narrative
The purpose of this activity is to multiply whole numbers and fractions to solve problems. Students use reasoning about denominators to compare fractions.

Task Statement	Teacher Directions
Jada and Lin are making head wraps out of African wax print fabric. Jada stitches together 5 pieces of fabric that each have a length of $\frac{2}{6}$ yard. 1. Write an equation to show the total length of fabric Jada used. Show or explain your reasoning. Lin stitches together 3 pieces of fabric that are each $\frac{2}{3}$ yard long. 2. Write an equation to show the total length of fabric Lin used. 3. Who used more fabric? Show or explain	**Launch/Activity** • Display:  • "What do you notice? What do you wonder?" • "In many African cultures, women wrap their heads with colorful fabric when they dress for the day." • "We will be thinking about this in today's tasks."

<table>
<tr><td>your reasoning.</td><td>

- 5 minutes: independent work time
- 5 minutes: partner work
- Monitor for tape diagrams and multiplication equations that represent each situation.

Synthesis

- Select 2–3 students to share their equations and reasoning.

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Student Response

- $5 \times \frac{2}{6} = \frac{10}{6}$,

$\frac{2}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{2}{6}$

- $3 \times \frac{2}{3} = \frac{6}{3}$,

$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

- Jada used more fabric because her head wrap is more than 2 yards long and Lin’s is exactly 2 yards.

</td></tr>
<tr><td colspan="2">Activity 2: Make 2 Yards of Fabric</td></tr>
<tr><td colspan="2">Time: 10 minutes</td></tr>
<tr><td colspan="2">Addressing CCSS: 4.NF.B.4</td></tr>
<tr><td colspan="2">

Activity Narrative
The purpose of this activity is to practice adding and subtracting fractions. Students reason about different combinations of fractions to make 2.

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Task Statement
Jada and Priya’s moms taught the class how to combine and use fabric pieces for head wraps. The lengths of each piece of fabric are listed below. What are 4 different combinations of fabric that could be used to make 2 yards? Write an equation for each combination. Each piece

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Teacher Directions

Launch/Activity

- 5 minutes: independent work
- Monitor for students using multiplication to combine groups of units.
- 5 minutes: partner share

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</table>

may only be used one time.

$\frac{2}{6}$ yard	$\frac{2}{6}$ yard	$\frac{2}{6}$ yard	$1\frac{2}{5}$ yards	$\frac{11}{10}$ yard
$\frac{9}{10}$ yard	$\frac{2}{6}$ yard	$\frac{6}{12}$ yard	$\frac{3}{6}$ yard	$\frac{2}{6}$ yard
$\frac{2}{6}$ yard	$\frac{12}{12}$ yard	$\frac{2}{6}$ yard	$\frac{3}{5}$ yard	$\frac{2}{6}$ yard

- Monitor for:
  - combinations that include addition and multiplication equations
  - students who reason about fractions equivalent to 2

Synthesis:

- Select 2–3 students to share their equations and their reasoning.
- "How did you know when your fraction was equivalent to 2?" (Answers vary. Sample response: When the numerator is twice as big as the denominator, the fraction is equivalent to 2.)
- "Why can we use multiplication to represent the combination $\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}$?" (There are 6 groups of $\frac{2}{6}$ or 6 x $\frac{2}{6}$. Both expressions are equal to $\frac{12}{6}$, or 2.)

Student Response

Answers vary. Sample responses:

- $\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6} = 2$ or $6 \times \frac{2}{6}$
- $1\frac{2}{5}+\frac{3}{5} = 2$
- $\frac{6}{12}+\frac{3}{6}+\frac{12}{12} = 2$
- $\frac{11}{10}+\frac{9}{10} = 2$

Activity 3: Fraction Puzzles

Time: 10 minutes

Addressing CCSS: 4.NF.B.4

Activity Narrative

The purpose of this activity is to practice adding and subtracting fractions. Students reason about different combinations of fractions, including fractions greater than 1, and the relationship between addition and subtraction.

Task Statement	**Teacher Directions**
Solve each puzzle. | $\frac{15}{12}$ | $\frac{7}{12}$ | | $\frac{21}{12}$ | $\frac{18}{12}$ | 1. What is the sum of all the fractions in the puzzle? 2. Select two fractions from the puzzle with a difference that is less than $\frac{1}{3}$. Show or explain your reasoning. 3. Shade the row or column of the puzzle that has a sum greater than 3. Show or explain your reasoning. Use the numbers in each puzzle below to make 1. • Use addition or subtraction or both to solve each puzzle. • Each number may only be used one time. • All numbers must be used. • Compare solutions with a partner and try to find a different way to solve. 4. | $\frac{5}{12}$ | $\frac{8}{12}$ | 5. | $\frac{15}{10}$ | $\frac{13}{100}$ |	**Launch/Activity** • 5 minutes: independent work • 5 minutes: partner work • Monitor for students who: ◦ use benchmarks such as $\frac{1}{2}$ and whole numbers to reason about comparisons **Synthesis** Display: | $\frac{15}{10}$ | $\frac{13}{100}$ | | $\frac{53}{100}$ | $\frac{9}{10}$ | • "How can we use our understanding of equivalence to help us make a total of 1 using these fractions?" ($\frac{15}{10} = \frac{150}{100}$ which is greater than 1, so we will need to use subtraction. $\frac{9}{10} = \frac{90}{100}$ which is close to 1, so we may need to subtract twice to get to 1.)

$\frac{3}{12}$	$\frac{2}{12}$		$\frac{53}{100}$	$\frac{9}{10}$	

Student Response

1. $\frac{61}{12}$
2. $\frac{18}{12}$ - $\frac{15}{12}$ = $\frac{3}{12}$, and $\frac{3}{12}$ is equivalent to $\frac{1}{4}$ which is less than $\frac{1}{3}$
3. $\frac{21}{12}$ + $\frac{18}{12}$ = $\frac{39}{12}$, 3 = $\frac{36}{12}$ so $\frac{39}{12}$ is greater than 3
4. $\frac{5}{12} - \frac{3}{12} + \frac{2}{12} + \frac{8}{12}$ = 1
5. $\frac{15}{10} - \frac{9}{10} + \frac{53}{100} - \frac{13}{100}$ = 1

Lesson Synthesis

"Today we added, subtracted, and multiplied fractions to solve problems."

"Why is it important to understand fraction equivalence while operating with fractions?" (Sometimes we will need to compare products, sums, and differences to whole numbers. If we understand when a fraction is equivalent to a whole number, we can determine which ones are greater or less than that number. We can also use benchmarks such as $\frac{1}{2}$ and $\frac{1}{3}$ to help us reason about our responses.)

4.8 Lesson 5: Multiplying Fractions and Whole Numbers Part 1

Standards Alignment

Addressing CCSS: 4.OA.A.2, 4.NF.B.4

Teacher-facing Learning Goals

- Solve problems involving multiplicative comparison.
- Solve problems involving the multiplication of a fraction by a whole number.

Lesson Purpose

The purpose of this lesson is to solve multiplicative comparison word problems involving both fractions and whole numbers.

Materials to Gather	**Materials to Copy**
• game boards • number cubes (3 per group of 2)	• none

Cool-down: Spiced Ginger Ale
Student-facing Task Statement Jada is using the following recipe to make spiced ginger ale. Cinnamon: $\frac{3}{8}$ oz Ginger: $3\frac{1}{4}$ oz Lemon juice: $6\frac{1}{2}$ oz Sugar: $\frac{3}{6}$ cup Seltzer water: 7 oz If Jada is serving 4 times as many people as the recipe serves, how much of each ingredient should she purchase?
Student Responses Cinnamon: $4 \times \frac{3}{8} = \frac{12}{8}$ or $1\frac{4}{8}$ or $1\frac{1}{2}$ oz Ginger: $4 \times 3\frac{1}{4}$, $4 \times 3 = 12$, $4 \times \frac{1}{4} = 1$ and $12 + 1 = 13$ oz Lemon juice: $4 \times 6\frac{1}{2} = 26$ oz Seltzer water: $4 \times 7 = 28$ oz
Teacher Reflection Question Which representations were used the most in today's lesson? How were students able to describe how representations helped solve problems?

Warm-up: Number Talk
Time: 10 minutes
Addressing CCSS: 4.NBT.4
Warm-up Narrative

This Number Talk encourages students to think about less familiar combinations that make 10 and100, and to rely on the structure of making tens and multiples of ten or familiar combinations to mentally solve problems. The understandings elicited here will be helpful throughout this unit as students add and subtract whole numbers fluently and add and subtract fractions with the same denominator.

To identify parts that make 10 and 100, students need to look for and make use of structure (MP7). In describing strategies, students need to be precise in their word choice and use of language (MP6).

Task Statement

799 + 698
798 + 699 + 100
798 + 689 + 110
798 + 697 + 100

Teacher Directions

Launch/Activity

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- How did you use the expression before to help solve each new expression?

Consider asking:

- "Who can restate _____'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone approach the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"

Student Responses

- 800 + 700 = 1,500, 1,500 - 3 = 1,497
- 1,497 + 100 = 1,597
- 700 + 600 + 100 = 1,400, 98 + 99 = 197, 1,400 + 197 = 1,597

- Two less than the second expression, so 1,597 - 2 = 1,595.

Activity 1: Native American Fried Bread

Time: 20 minutes

Addressing CCSS: 4.OA.A.2, 4.NF.B.4

Activity Narrative
The purpose of this lesson is to multiply fractions and whole numbers to solve multiplicative comparison problems. At this point in the year, most students will not use tape diagrams to represent problems. The synthesis reviews “times and much” language and solving problems when only part of the information is made available.

The purpose of this lesson is to add, subtract, and multiply fractions to solve problems. Students have an opportunity to create questions as well. They reason about familiar structures in story problems with whole numbers and how fractions fit in these structures.

Task Statement	Teacher Directions
Part 1: Clare's uncle is making Native American fried bread for two classes. The recipe he has will only serve 4 students. Native American fried bread recipe: 1 cup flour $1\frac{1}{2}$ teaspoons of baking powder $\frac{1}{4}$ teaspoon salt $\frac{1}{2}$ cup milk 2 cups vegetable oil 1. How many times as much of each ingredient will Clare's uncle need? 2. If Clare's uncle has $8\frac{2}{3}$ cups of flour, does he have enough flour? If not, how much more flour is needed? 3. Adjust each ingredient to make sure the recipe will serve exactly 48 students.	**Launch/Activity** • Groups of 2 • 5–7 minutes: independent work time • 5 minutes: partner discussion **Synthesis** • “Has anyone ever tried fried bread?” (Answers vary.) • “How does Clare's uncle need to adjust the ingredients in the recipe to make bread for 2 classes?” (Multiply each ingredient by 12.)

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4. Compare strategies with a partner.

Part 2:
Andre wants to create a family bread recipe. Use the following guidelines to write a recipe for Andre:

Between $\frac{1}{6}$ and 1 cup of flour: _____
Between 0 and $\frac{3}{8}$ cup of milk: _____
More than $1\frac{1}{2}$ teaspoon of baking powder: _____
$\frac{3}{4}$ cup of sugar
Between $1\frac{1}{4}$ and $3\frac{1}{2}$ sticks of butter: _____
3 tablespoons of coconut oil

1. After baking the bread, Andre had $2\frac{1}{4}$ cups of sugar left over. How much sugar did he start with?

2. Write a question about milk that would require addition or subtraction to solve.

3. Trade papers with a different group and work together to solve the problems they created.
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Student Response
Part 1
1. Flour: 12 x 1 cup
Baking powder: 12 x 1 $\frac{1}{2}$ teaspoon
Salt: 12 x $\frac{1}{4}$ teaspoon
Milk: 12 x $\frac{1}{4}$ salt
Oil: 12 x 2 cups
2. $12 - 8\frac{2}{3}$
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Part 2 1. Sample response: Flour: $\frac{3}{6}$ cup Milk: $\frac{3}{8}$ cup Baking powder: 2 teaspoons Butter: 2 sticks 2. Andre used 4 times as much flour to make chocolate chip cookies. How much flour did Andre use? 3. $2\frac{1}{4} - \frac{2}{4} = 1\frac{3}{4}$ 4. Andre has 4 cups of milk and the recipe requires $\frac{3}{8}$. How much milk will he have left after making the recipe?
Activity 2: Rolling for Fractions
Time: 15 minutes
Addressing CCSS: 4.NF.B.4, 4.NF.A.2
Activity Narrative The purpose of this activity is to evaluate multiplication expressions where fractions are multiplied by whole numbers, accurately reason about the size of fractions relative to benchmarks, and reason strategically about how the placement of a digit will affect the value of the expression.

Task Statement	**Teacher Directions**
Use the directions to play Rolling for Fractions with a partner. Who's Closer to 0 or 10? 1. Each player rolls 1 number cube. The player with the highest number goes first. 2. Player 1 chooses whether the target is to get closer to 0 or closer to 10. 3. Player 1: Roll 3 number cubes. Use the numbers to complete the expression and find the product.	**Launch/Activity** • Groups of 2 • Roll the number cube. • Decide if the target number is closer to 0 or 10. • Display: $\square \times \frac{\square}{\square}$ • Roll 3 number cubes. Use the numbers to complete the expression and find the product.

4. Player 2: Roll 3 number cubes. Use the numbers to complete the expression and find the product.
5. Check one another's equation for accuracy. Determine the amount of points each player receives:
    a. 2 points for creating an expression that is closer to the target.
    b. 1 point each if players are the same distance from the target.
6. Rotate Player 1 and Player 2. Player 1 may choose a new target in the next round.
7. Play 5 rounds. The player with the most points wins.

Round	Equation	Points
1	$\square \times \frac{\square}{\square} =$ ____	
2	$\square \times \frac{\square}{\square} =$ ____	
3	$\square \times \frac{\square}{\square} =$ ____	
4	$\square \times \frac{\square}{\square} =$ ____	
5	$\square \times \frac{\square}{\square} =$ ____	
Total		

- "Will we be able to use this roll to make 1? How do you know?"
- 1 minute: quiet think time
- Ask students to share.
- Player 2 repeat.
- "How are you deciding where to place each fraction?"
- Calculate the number of points each player receives:
    - 2 points for creating an expression that is closer to the target
    - 1 point each if players are the same distance from the target
- "The player with the most points after 2 rounds wins."
- Give students center materials.
- "Play Rolling for Fractions with your partner."
- 10–12 minutes: partner work time

Synthesis

- "What strategies were helpful as you played Rolling for Fractions?" (Using smaller digits for the denominator to be closer to 10.)

Student Responses

Answers vary.
Lesson Synthesis "Today we solved recipe problems and wrote problems of our own. Let's look at the problem _____ created." ● Discuss the problem and how it represents either subtraction or addition.

4.8 Lesson 12: What's the Quotient?	
Standards Alignment	
Addressing CCSS: 4.NBT.B.6	
Teacher-facing Learning Goals ● Practice using the partial quotients method to divide multi-digit numbers by a single-digit divisor. ● Analyze strategies for finding quotients and identify ways to improve efficiency in dividing.	
Student-facing Learning Goal: Let's find some quotients of multi-digit numbers.	
Lesson Purpose The purpose of this lesson is to reinforce students' understanding of the partial quotients method and build their fluency in using it to divide multi-digit numbers by a single-digit divisor. Students also consider different strategies for dividing and their merits.	
Lesson Narrative In an earlier unit, students learned to use the partial quotients algorithm to divide whole numbers up to four digits by single-digit divisors. This lesson deepens their understanding of the partial quotients method and allows them to continue building their fluency in finding quotients. Students also analyze different ways to divide whole numbers and consider how to improve their efficiency.	
Materials to Gather ● grid paper, to support students in aligning the digits when using partial quotients to divide multi-digit numbers	**Materials to Copy** ● none
Cool-down: Divide Like a Pro	

Student-facing Task Statement

1. Here are two different ways to start finding 8,435 ÷ 7. Choose one way and complete the calculation.

A.

```
        5
7 ) 8,435
  –    35
    8,400
```

B.

```
    1,000
7 ) 8,435
  – 7,000
    1,435
```

2. Find the value of 1,038 ÷ 6. Try to use as few steps as possible.

Student Responses

1. Sample responses:

```
    [1,205]
        200
      1,000
          5
7 ) 8,435
  –    35
    8,400
  – 7,000
    1,400
  – 1,400
        0
```

```
    [1,205]
          5
        100
        100
      1,000
7 ) 8,435
  – 7,000
    1,435
  –   700
      735
  –   700
       35
  –    35
        0
```

2. Sample response:

```
    [173]
        3
       70
      100
6 ) 1,038
  –   600
      438
  –   420
       18
  –    18
        0
```

Teacher Reflection Question

Which questions did you ask today that were effective in prompting students to think strategically or structurally? Which ones might have pushed them toward a particular method or process?

Warm-up: Number Talk: Divide by 3 and by 6

Time: 10 minutes

Addressing CCSS: 4.NBT.B.6

Warm-up Narrative

This Number Talk encourages students to think about relying on the structure of numbers in base ten (MP7) and properties of operations to mentally solve division problems. The reasoning elicited here will be helpful later in the lesson when students find quotients of multi-digit numbers.

In explaining their reasoning strategies, students practice being precise in their word choice and use of language (MP6).

Task Statement

Find the value of each quotient mentally.

48 ÷ 3
480 ÷ 3
528 ÷ 3
5,280 ÷ 6

Teacher Directions

Launch/Activity

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How is each expression related to the one before it?"

Consider asking:

- "Who can restate ____'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone approach the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"

Student Responses

Sample reasoning:

- 16, because 48 is 30 + 18. Dividing 30 by 3 gives 10, and dividing 18 by 3 gives 6, and 10 + 6 = 16.
- 160, because 480 is 10 times 48, so the result of 480 ÷ 3 is 10 times the result of 48 ÷ 3.
- 176, because 528 is 480 + 48, so 528 ÷ 3 is (480 ÷ 3) + (48 ÷ 3) or 160 + 16.
- 1,760, because 5,280 is 10 times 528, so the result of 5,280 ÷ 3 is 10 times the result of 528 ÷ 3.

Activity 1: Unfinished Divisions

Time: 15 minutes

Addressing CCSS: 4.NBT.B.6

Activity Narrative
Previously, students saw that there are many ways to find products of multi-digit numbers. In this activity, students are reminded that there are many ways to use partial quotients to divide numbers and see that some strategies are more practical or efficient than others.

Task Statement

Here are four calculations to find the value of 7,465 ÷ 5, but each one is unfinished.

A

```
         200
          80
          13
   5 ) 7,465
     –    65
       7,400
     –   400
       7,000
     – 1,000
```

B

```
         400
       1,000
   5 ) 7,465
     – 5,000
       2,465
     – 2,000
```

C

```
    5 ÷ 5 =     1
   60 ÷ 5 =    12
5,000 ÷ 5 = 1,000
```

D

7,465 is a little less than 7,500.

7,500 ÷ 5 = 1,500
35 ÷ 5 = 7

Teacher Directions

Launch/Activity
- Groups of 2–4
- "Choose at least two calculations to finish. Make sure each calculation is completed by someone in your group."
- 3–4 minutes: quiet work time
- 2 minutes: group discussion

Synthesis
- Compare and contrast the four calculations. "How are the four strategies alike? How are they different?" (The first three are similar in that they involve partial quotients. The last one involves estimation.)
- "Which strategy or strategies do you find easy to follow? Hard to follow?"
- "Which strategy seems the most efficient? The least efficient?" (Sample

Complete at least two of the unfinished calculations. Be prepared to explain how you know what to do to complete the work.	response: Calculation B could be shortened by using larger multiples of 3. The last seems most efficient.)

Student Response

A

1,493
200
1,000
200
80
13
5) 7,465
− 65
7,400
− 400
7,000
− 1,000
6,000
− 5,000
1,000
− 1,000
0

B

1,493
13
80
400
1,000
5) 7,465
− 5,000
2,465
− 2,000
465
− 400
65
− 65
0

C

5 ÷ 5 = 1
60 ÷ 5 = 12
5,000 ÷ 5 = 1,000
2,000 ÷ 5 = 400
400 ÷ 5 = 80

1,000 + 400 + 80 + 12 + 1 = 1,493

D

7,465 is a little less than 7,500.

7,500 ÷ 5 = 1,500
35 ÷ 5 = 7
1,500 – 7 = 1,493

Activity 2: Where Do We Begin?

Time: 20 minutes

Addressing CCSS: 4.NBT.B.6

Activity Narrative

This activity serves two goals. First, it prompts students to consider whether the order in which parts of the dividend are divided makes a difference in the process or in the result. Second, it deepens students' understanding of the structure of the partial quotients algorithm.

Students first explain why different initial steps could be equally productive for starting a division process. Next, they analyze and complete some partial quotients calculations with missing numbers. (The missing numbers could be partial quotients, parts of the dividend being removed, or results of subtraction.) To find the missing numbers, students need to recognize and make use of the structure of the algorithm (MP7). Lastly, students use the algorithm to find a quotient, being mindful of their starting move and of the efficiency of their process.

Task Statement

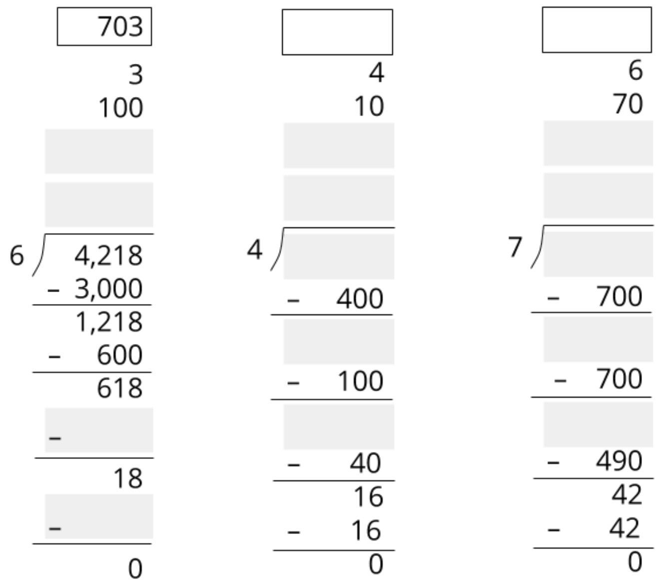
1. Jada and Noah are finding 3,681 ÷ 9. Jada says to start by dividing 81 by 9. Noah says start by dividing 3,600 by 9.
    a. Explain why each suggestion is helpful for finding the quotient.
    b. Find the quotient. Show your reasoning.
2. Can you find the missing numbers such that each calculation shows a correct division calculation?

    a. b. c.

Teacher Directions

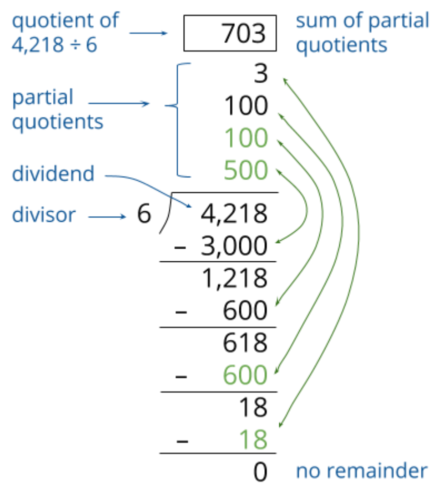
Launch/Activity

- Groups of 2
- 6–8 minutes: independent work time on the first two sets of questions
- 2–3 minutes: partner discussion
- Monitor for students who:
    - can clearly explain why Jada and Noah's initial steps are both effective
    - recognize the structure of the partial quotients algorithm and can articulate how it helps to find the missing numbers
- Pause for a discussion before the last question. Select students to share responses and reasoning.
- When discussing the second set of questions, ask: "How did you determine what the missing numbers are?" Display the incomplete calculations to facilitate students' explanations.



3. Consider the expression 5,016 ÷ 8.
    a. What would you do to start finding the quotient?
    b. Show how you'd find the quotient with as few steps as possible.

- Consider annotating the calculations to clarify the structure (for instance, by drawing arrows between partial quotients and the corresponding parts of the dividend being subtracted, labeling the parts, and so on).



- 3–4 minutes: independent work time on the last question
- Monitor for the different first steps taken to divide 5,016 by 8.

Synthesis

- See lesson synthesis.

Student Response

1. Sample response:
    a. Jada's suggestion is helpful because 81 is a familiar multiple of 9 (9 x 9 = 81) and removing it from 3,681 leaves 3,600, which is also a familiar multiple of 9.

       Noah's suggestion means removing the largest partial quotient first, which is helpful because then only 81 is left to divide, and it is a familiar multiple of 9.
    b. 409. Sample reasoning: 3,600 ÷ 9 = 400 and 81 ÷ 9 = 9, and 400 + 9 = 409.
2.

a.

```
      703
        3
      100
      100
      500
6 ) 4,218
  – 3,000
    1,218
  –   600
      618
  –   600
       18
  –    18
        0
```

b.

```
      276
        6
       70
      100
      100
7 ) 1,932
  –   700
    1,232
  –   700
      532
  –   490
       42
  –    42
        0
```

c.

```
    139
      4
     10
     25
    100
4 ) 556
  – 400
    156
  – 100
     56
  –  40
     16
  –  16
      0
```

3. Sample response:
    a. Divide 16 by 8 to get a partial quotient of 2.
    b.

```
      627
       25
      600
        2
8 ) 5,016
  –    16
    5,000
  – 4,800
      200
    – 200
        0
```

Lesson Synthesis

“Today we studied different ways to divide multi-digit numbers and single-digit divisors.“

Select students who took different initial steps to find 5,016 ÷ 8 to share their calculations and reasoning. Discuss:

- “Why did you decide to start with that number?”
- “How did you determine the next chunk to divide and remove?”
- “Can you think of a way to find the quotient with fewer steps?”