Illustrative Mathematics®

Illustrative Mathematics®

GRADE 5

Unit

1

Teacher Adaptation Pack

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K5_Beta

Grade 5 Unit 1 Teacher Adaptation Pack

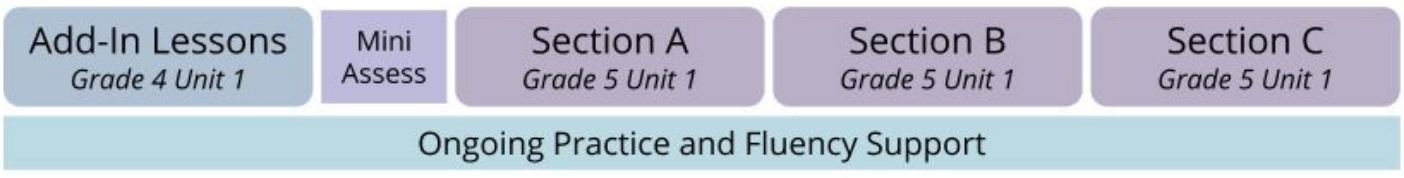
Guiding Questions

- How do we invite students back into our class in a way that is welcoming, supportive, and centered around community?
- How do we formatively assess student thinking and make decisions in ways that don't perpetuate the problems often caused by grouping students?
- How do we create a coherent learning experience for students so each day's learning feels connected despite the missed months of school?

Directions for Use

1. Read the current grade level unit standards and prior-grade connections.
2. Ask prior grade level teachers if the unit topic was taught last year or show the students a problem on the unit topic and anonymously ask them if they know how to solve it.
    a. If yes, start the current grade level section without the add-in lessons, however if there is any doubt, we recommend teaching the add-in lessons.
    b. If not, teach the prior grade level add-in lessons.
3. After the add-in lessons, give the mini-assessment.
    a. If students got the questions correct, start the current grade level section.
    b. If students got some things correct, start the current grade level section and use the ongoing practice materials to support students.

Recommended Implementation



Grade 5 Unit 1: Finding Volume	
Standards	• 5.MD.C.3, 5.MD.C.4, 5.MD.C.5
Prior-Grade Connections	• 4.MD.A.3
Rationale	Unit 5.1 introduces students to the concept of volume by building on previous understandings of area and multiplication. Since the prior understandings were introduced in grade 3 and applied in grade 4, there are only 2 add-in lessons. These add-in lessons are not required to start the unit on volume, but offer an opportunity to formatively assess students' understanding of area and multiplication fluency.
Add-in Lessons	• 4.1 Lesson 1 • 4.1 Lesson 2
5.1 Lessons to Combine or Skip	• Skip Lesson 12
Prior-grade Practice and Fluency	• Center: Rectangle Rumble • Center: Multiplication Card Sort • Center: Product Game

Extension and Exploration	• Grade 4, Unit 1
	  ○ Lesson 7
	  ○ Lesson 8

Mini-assessment	• Grade 4 Unit 1, End-of-Unit Assessment Problems

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4.1 Lesson 1: Build Rectangles

Standards Alignment
Building On: 3.OA.A.1, 3.MD.C.7.A
Addressing: 4.OA.B.4

Teacher-facing Learning Goals

- Find areas of different rectangles with a given a side length.
- Understand that the area of a rectangle is a multiple of each of its side lengths.

Student-facing Learning Goals

- Let's build some rectangles.

Lesson Purpose
The purpose of this lesson is for students to apply their understanding of area to explore multiples.

Lesson Narrative
In previous grades, students learned how to find the area of a rectangle by tiling to show that this is the same area found by multiplying the side lengths.

The purpose of this lesson is for students to apply their understanding of area and multiplication to build rectangles and find their area. As students consider the areas of rectangles with a given side length, they explore **multiples**. Students learn that a **multiple** is the result of multiplying any whole number by another.

While students are introduced to the term multiple in this lesson, they will work more with them in upcoming lessons. They do not need to have a formal understanding of the term in this lesson. In upcoming lessons, students also explore and learn the terms factor and factor pair. In this lesson they refer to them as side lengths within the context of area.

Math Community:
Prepare a space, such as a piece of poster paper, titled "Mathematical Community" and a T-chart with the headers "Doing Math" and "Norms." Partition each of the columns into two sections: students and teacher. The two sections encourage the students and teacher to be mindful that both respective parties are responsible for the way math is done in the classroom.

Mathematical Community	
Doing Math	*Norms*
Students	Students
Teacher	Teacher

Access for Students with Disabilities (SwD) Activity 2: Representation	**Access for English Learners (EL)** Activity 2: MLR7 Connect and Compare
Instructional Routines How Many Do You See?, MLR8	
Materials to Gather • 36 inch tiles for each group of 2 students • grid paper	**Materials to Copy** • 4.1.A.1 Blackline Master
Lesson Timeline **Warm-up** 10 minutes **Activity 1** 20 minutes **Activity 2** 15 minutes **Lesson Synthesis** 10 minutes **Cool-down** 5 minutes	**Teacher Reflection Question** In grade 3, students fluently multiplied and divided within 100 and related area to multiplication and addition. How is that prior understanding supporting students in understanding multiples in this lesson?
Cool-down: Area and Multiples **Standards Alignment:** 4.OA.B.4	
Student-facing Task Statement If a rectangle is 6 tiles wide, what are 3 possible areas of that rectangle? Explain or show your reasoning.	

Student Responses

1. 12, 18, and 24, because 6 x 2 = 12, 6 x 3 = 18, 6 x 4 = 24

Warm-up: Which One Doesn't Belong: All Kinds of Area

Time: 10 minutes

Standards Alignment
Building On: 3.OA.A.1
Addressing: 4.OA.B.4

Warm-up Narrative
This warm-up prompts students to carefully analyze and compare the area of different figures. In making comparisons, students have a reason to use language precisely (MP6) as they describe the area of different figures. The activity also enables the teacher to hear the terminologies students know and how they talk about characteristics of shapes that support strategies for determining different areas.

Math Community:

- After the warm-up, ask students to reflect on both individual and group actions while considering the questions: "What does it look and sound like to do math together as a mathematical community? What am I doing? What are you doing?"
- Record and display their responses under the "Doing Math" header. Students might mention things such as: we talked to each other and to the teacher, we had quiet time to think, we shared our ideas, we thought about the math ideas and words we knew, you were writing down our answers, you were waiting until we gave the answers.

Student-facing Task Statement
Which one doesn't belong?

A	B

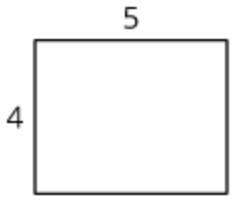
Launch

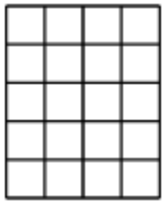
- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.
- 30 seconds: quiet think time

Activity

- Display the image.
- 1 minute: partner discussion
- Record responses.
- Repeat for each image.

Synthesis





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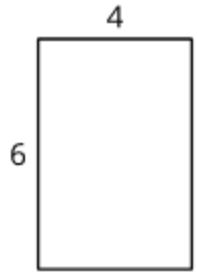

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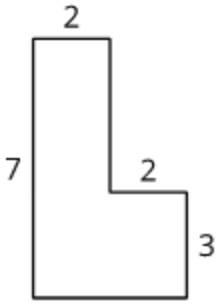


- "How might we determine the area of each figure?" (We can use multiplication for most of them or count the units in one of them.)
- Consider saying: "Let's find at least one reason why each one doesn't belong."

Student Responses

- A is the only figure that doesn't rest on a length of 4.
- B is the only one that doesn't have side lengths labeled.
- C is the only one that doesn't have an area of 20.
- D is the only figure that doesn't have all of the corners sticking out.

Activity 1: Build and Find Area

Time: 20 minutes

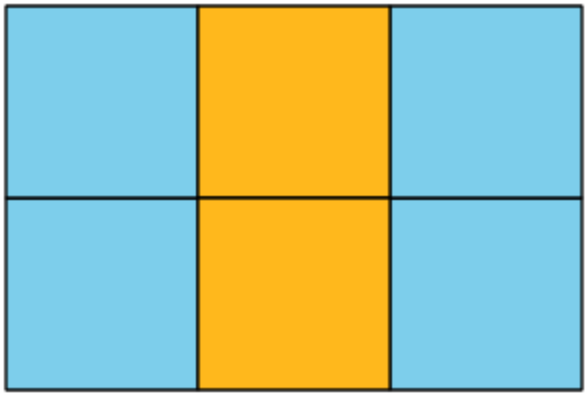
Standards Alignment
Addressing: 3.MD.C.7.A

Activity Narrative
The purpose of this activity is for students to find the area of a rectangle by tiling and recall that this is the same area found by multiplying the side lengths. Students use inch tiles to build rectangles with given dimensions and find the area of those rectangles. During the activity, students work together to

compare and explain the strategies used to determine the area of rectangles and make connections between strategies.

Student-facing Task Statement

Launch



1. Build 5 different rectangles with the given width. Record the area of each rectangle in the table below.

	Area of Rectangle				
2 tiles wide					
3 tiles wide					
4 tiles wide					

2. Discuss with a partner what you notice about areas in each row of the table.
3. Predict the area of another rectangle for each width and explain your reasoning.

Launch

- Display the launch image.
- "Look at the rectangle on your page and describe it to a partner."
- Give each group 10 tiles.
- "Build all the rectangles you can using all 10 tiles. Describe them to a partner."
- 2 minutes: partner discussion
- "Who has a rectangle that is 2 tiles wide, 5 tiles wide, 10 tiles wide?"
- Draw each rectangle on the board after a volunteer shares a response.

Activity

- Give students more inch tiles.
- "Take a minute to think about how you would build rectangles with these areas."
- 1 minute: quiet think time
- "Talk with your partner about your strategies and then work through the problems together."
- 5–7 minutes: partner work time
- Monitor for students who build one row or column and repeat the same number of tiles over again to build the area, those who skip count or multiply to determine the area of each rectangle, and those who combine skip counting with another counting strategy.

Synthesis

- Collect predictions for areas of rectangles with a width of 2. (18, 14, 20, 30)
- "How might we know each prediction was a possible rectangle without building?" (each number is even and a number we

say when counting by twos)
- "Is this also true for a width of 3? Are all the prediction numbers we say when multiplying with 3 or counting by 3? What about a width of 4?"
- 2 minutes: partner discussion
- Share and record responses. (All the predictions for rectangles with a width of 3 and 4 were numbers you get when you multiply with 3 or 4.)

Student Responses

1.

	Area of Rectangle				
2 tiles wide	12	6	8	4	10
3 tiles wide	6	18	12	15	9
4 tiles wide	8	12	20	24	16

2. Each row in the column has numbers that are multiples of the width of each rectangle.
3. Answers will vary. Sample responses:

   2 tiles wide: 12

   3 tiles wide: 18

   4 tiles wide: 24

Advancing Student Thinking

Students may count tiles using a variety of strategies to determine the area of the rectangles they build. Consider asking, "Is there a different, maybe faster way to determine the area than counting tiles?"

Activity 2: What Areas Can You Build?

Time: 15 minutes

Standards Alignment
Addressing: 4.OA.B.4

Activity Narrative
The purpose of this activity is for students to explore multiples through an area context to learn that a multiple is the result of multiplying any whole number by another whole number. As students build rectangles given 1 dimension and find the area, they see that every area is a multiple of each of the side lengths of a rectangle.

SwD Support Tags
- Representation

MLR Tags
- MLR7

EL Support Text
MLR7 Compare and Connect. Synthesis: Invite students to prepare a visual display that shows the strategy they used to build rectangles with a width of 3 units and an area of 30 square units or less. Encourage students to include details that will help others interpret their thinking. For example, drawings and labels. Give students time to investigate each other's work. During the whole-class discussion, ask students, "What did the different representations have in common?" and "How were they different?"
Advances: Representing, Conversing

SwD Support Text

Representation: Develop Language and Symbols. Synthesis: Maintain a visible display to record new vocabulary. Invite students to suggest details (words, pictures, or equations) that will help them remember the meaning of the terms. In this lesson, include the terms multiple, even, and odd. Throughout the unit, add the terms factor/factor pair, composite, and prime.
Supports accessibility for: Language, Memory.

Student-facing Task Statement

1. Elena is building rectangles with a width of 3 units and an area of 30 square units or less.
    a. Build the rectangles Elena could make and draw the rectangles on the grid paper. Label the area and the side lengths of each rectangle.
    b. What is the area of each rectangle you built?

Launch
- Groups of 2
- Give each group inch tiles and the Blackline Master.
- "I am thinking of a rectangle that is 2 tiles wide. What is the area of my rectangle?"
- 1 minute: partner discussion
- Share and record responses.
- "How do we know all of these are possible areas?" (We can multiply by

c. What do you notice about the areas? 2. Why is 28 square units not a possible area for a rectangle with a width of 3 units? 3. If the area was larger than 30 square units, find 2 other numbers that could be the area of the rectangle in square units. Explain your reasoning. 4. What is another area that is not possible for a rectangle with a width of 3 units? Explain your reasoning.	another number by 2 to get those numbers.) **Activity** ● "In this activity we are going to think about areas we could build if we only knew 1 side length of a rectangle. Work with your partner to answer these questions." ● 5–7 minutes: partner work time ● Monitor for students who notice that areas you can build are a result of multiplying 3 by another possible side length. **Synthesis** ● Display a set of student-generated areas for question 1. ● "What did you notice about the areas you built in question 1?" ● Ask students to share their responses to question 2. ● "What other numbers did you find for questions 3 and 4? How did you know they could or could not be an area for this rectangle with width 3 units?" ● "We can have an area of 12 square units when the width of our rectangle is 3 units. That is because 12 is a **multiple** of 3. A multiple is the result of multiplying a number by a whole number." ● "Look back at your work. Which numbers are multiples of 3? Which numbers are not multiples of 3?" (3, 6, 9, 12, 15, 18, 21, 24, 27, and 30 are all multiples of 3, and 29, 28, 26, 25, 23, 22, 20, 19, 17, 16, 14, 13, 11, 10, 8, 7, 5, 4, 2, 1 are not multiples of 3) ● 2 minutes: partner discussion

	• Share responses.

Student Responses

1. a. Students may build a 3 by 4, 3 by 5, and 3 by 6 rectangle and record the areas of 12, 15, and 18.

   3 x 1, 3 x 2, 3 x 3, 3 x 4, 3 x 5, 3 x 6, 3 x 7, 3 x 8, 3 x 9, 3 x 10

   b. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 square units

   c. Answers vary. Sample responses:

   - We noticed that the areas went up by 3 for each rectangle when we added a number to the length.
   - We noticed that the areas were alternating odd, even, odd, even.

2. 28 square units is not possible because there is nothing we can multiply 3 by to get 28.
3. Answers vary. Sample responses:

   33 square units since 33 ÷ 3 = 11

   45 square units since 15 x 3 = 45

   75 square units since 3 x 25 = 75

4. Answers vary. Sample responses:

- 16 square units since we cannot multiply any number by 3 to get 16.
- 35 square units since we cannot multiply any number by 3 to get 35.

Lesson Synthesis

"Today we built rectangles and learned about multiples. How can we decide if a number is a multiple of another number?"

Share and record responses.

"How would you decide whether 28 is a multiple of 4?" (I think about whether there is a number I can multiply 4 by to get 28.)

"What is a number that would not be a multiple of 4? How do you know?" (Responses vary. Twenty-five is not a multiple of 4 because I cannot divide 25 evenly by 4.)

Math Community:

After the cool-down, revisit the "Doing Math" list of actions. Ask students to discuss with a partner where they saw evidence of the actions during the rest of the day's lesson. As a whole group, add any missing actions and revise earlier ideas.

Response to Student Thinking • Student counts to determine the area of each rectangle.	**Next Day Support** • During the synthesis of the next lesson, connect strategies showing a progression from counting to multiplication.
Response to Student Thinking • Students do not connect multiplication with area and given the side lengths of a rectangle, use addition or counting as a primary strategy.	**Prior-unit Support** • Grade 3, Unit 2, Section A

4.1 Lesson 2: Factor Pairs

Standards Alignment
Building On: 3.MD.C.7.B
Addressing: 4.OA.B.4

Teacher-facing Learning Goals
- Find side lengths of different rectangles with a given area.
- Understand that the side length of a rectangle is a factor of its area.

Student-facing Learning Goals
- Let's learn about factor pairs.

Lesson Purpose
The purpose of this lesson is for students to learn what factor pairs are by building rectangles with a specified area.

Lesson Narrative
In the previous lesson, students learned that a multiple is the result of multiplying a number by a whole number.

The purpose of this lesson is for students to understand that a factor pair is two numbers that multiply to result in another number. Students continue to work within the context of area as they create posters of all rectangles with a given area. During the gallery walk, students understand and recognize factor pairs.

Math Community:
Tell students they will have an opportunity to revise their "Mathematical Community" ideas at the end of this lesson, so as they work today, they should think about actions that may be missing from the current list.

Access for Students with Disabilities (SwD) Activity 1: Action and Expression	
Instructional Routines • Number Talk • MLR7: Compare and Connect	
Materials to Gather • grid paper, 2 sheets for 8 groups • 16 sheets of poster paper, 2 for each group • scissors • glue or tape • inch tiles, as needed	**Materials to Copy** • 4.1.A.1 Blackline Master
Lesson Timeline **Warm-up** 10 minutes **Activity 1** 30 minutes **Activity 2** 15 minutes **Lesson Synthesis** 10 minutes **Cool-down** 5 minutes	**Teacher Reflection Question** What did you say, do, or ask during the lesson synthesis that helped students be clear on the learning of the day?
Cool-down: The Dimensions of Rectangles **Standards Alignment:** 4.OA.B.4	
Student-facing Task Statement 1. What are all the possible dimensions of a rectangle with an area of 21 square units? 2. What are all the possible dimensions of a rectangle with an area of 50 square units?	
Student Responses 1. 1 x 21, 3 x 7 2. 1 x 50, 2 x 25, 5 x 10	

Warm-Up
Time: 10 minutes
Standards Alignment

Addressing: 3.OA.C.7
Building Toward: 4.OA.B.4

Warm-up Narrative
The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying within 100. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to find factor pairs of numbers.

As students use earlier problems to find the new products, they look for and make use of structure (MP7) and use repeated reasoning (MP8).

Student-facing Task Statement
Find the value of each product mentally.

2 x 7
4 x 7
3 x 7
7 x 7

Launch
- Display one problem.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity
- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis
- "How did the first 3 problems help you find 7 x 7?" (The 7 breaks apart into 3 and 4 so I could multiply in parts and add them together.)
- Consider asking:
  - "Who can restate _____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"
  - "Does anyone want to add on to _____'s strategy?"

Student Responses
- 14: I just know it. It's 2 groups of 7.
- 28: 4 is double 2, so it's double 14 or 28.

- 21: It's one less group of 7 than 4 x 7.
- 49: 7 is 4 + 3, so I could add the answers to the previous two problems 28 + 21 = 49.

Activity 1: How Many Rectangles?

Time: 20 minutes

Standards Alignment
Addressing: 4.OA.B.4

Activity Narrative
The purpose of this activity is for students to find all the possible pairs of whole-number side lengths given the area of a rectangle. Each group is assigned 2 areas in which to draw all the possible rectangles. They draw and cut out the possible rectangles with that area to display for the class.

Areas to assign (in square units):

Group A: 11, 27	Group B: 25, 5	Group C: 16, 8	Group D: 9, 18
Group E: 24, 12	Group F: 14, 28	Group G: 15, 30	Group H: 19, 20

This activity uses *MLR7 Compare and Connect*. Advances: representing, conversing

SwD Support Tags
- Action and Expression

SwD Support Text

Action and Expression: Internalize Executive Functions. Synthesis: Invite students to plan a strategy, including the tools they will use, for listing the factor pairs for their assigned areas. If time allows, invite students to share their plan with their partner or another group before they begin.

Supports accessibility for: Conceptual Processing, Organization.

Student-facing Task Statement

Your teacher will assign your group 2 numbers. Each number represents the area of a rectangle.

1. On grid paper, draw all the possible rectangles that have the given area and side

Launch
- 8 groups
- Give students Blackline Master.
- "You are going to be given 2 numbers. Each number represents the area of a rectangle. With your group, draw all the possible rectangles with that area and

lengths. Label the area and the side lengths.

Use each side length only once. (For example, if you draw a rectangle with 4 units across by 6 units down, you don't need to also draw a rectangle with 6 units across and 4 units down because they have the same pair of side lengths.)

2. When you think you've drawn all the possible rectangles, cut out your rectangles and put them on a poster for each area you were assigned.

3. Display your poster in the room for all to see.

create a poster for each area. Inch tiles are available if you find them helpful."

Activity

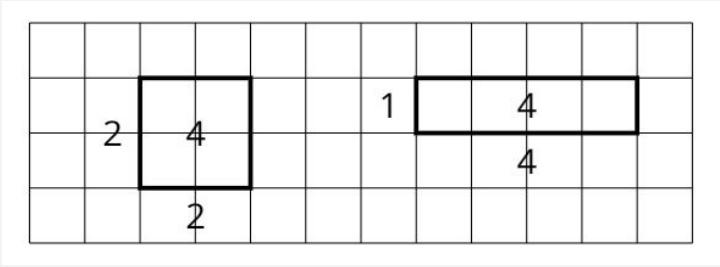
MLR7 Compare and Connect

- "Create a visual display that shows the rectangles with your assigned area. Include details such as area and side lengths to help others understand your thinking."
- Assign each group 2 area values and provide grid paper, poster paper, scissors, and glue.
- 15 minutes: small-group work time

Synthesis

- "We are going to take a gallery walk to observe the posters."
- "Before we walk around and look at all the posters, take a minute to reflect on the numbers you worked with. What did you notice and wonder as you worked on this activity?"
- 1–2 minutes: quiet think time

Student Responses



Student Misconceptions

Students may omit some rectangles because they did not recall all the factor pairs for a given multiple. Prompt students by asking, "Are there any other rectangles you can draw?" or "How can you be sure that you have drawn all the possible rectangles?"

Activity 2: How Many Rectangles Gallery Walk

Time: 15 minutes

Standards Alignment
Addressing: 4.OA.B.4

Activity Narrative
The purpose of this activity is for students to examine the rectangles drawn by their classmates during the previous activity as motivation for introducing factor pairs. Students recognize the side lengths of each rectangle as a factor pair of its area.

This activity uses *MLR7 Compare and Connect*. Advances: representing, conversing

Student-facing Task Statement

Gallery Walk: As you visit each poster, discuss with your partner:

1. Describe what you notice during the gallery walk. Use the following sentence frames when you share:
   a. I notice some of the posters _____ .
   b. I notice the posters for numbers _____ and _____ are similar because....
2. How do you know that all possible rectangles have been found for the given area?

Launch

- Groups of 2
- 5–7 minutes: gallery walk

Activity

- Monitor for students who discuss that you know you've found all the possible rectangles because those are the only numbers that multiply together to give that area.

Synthesis
MLR7 Compare and Connect

- "What is the same and what is different between the rectangles on the posters?"
- 30 seconds: quiet think time
- 1 minute: partner discussion
- "How do you know that all possible rectangles have been found for the given area?" (We could not make any more rectangles and we do not know any other numbers that multiply together to make the area.)
- Share and record responses.
- Display the rectangles for 21: 1 by 21 and 3 by 7.
- "Are there any more rectangles we can

	draw? Why or why not?" (No, because there are no more whole-number factors of 21. Or, no, because to get 21 we can multiply 1 x 21, 3 x 7, 7 x 3, and 21 x 1.) • "We call 1 and 21 and 3 and 7 **factor pairs** of 21 because when multiplied, the product will be 21." • "Work with your partner to write down the factor pairs for the areas you were assigned." • 2 minutes: partner work time

Student Responses

1. Sample responses:
   a. I notice some posters only have 2 rectangles.
   b. I notice posters for even numbers are similar because they each have 2 as a side length.
2. We know that there are no more rectangles for a given area because there are no more numbers that multiply together to give that area as their product.

Lesson Synthesis

"Today we learned about factor pairs of whole numbers. What are the factors pairs of 24?" (1 and 24, 2 and 12, 3 and 8, and 4 and 6)

"How do we know if we have found all the factors of 24?" (We went in order. When we reached 4 and 6, there are no more pairs between 4 and 6, so we can stop there. Or, we used multiplication to see how many facts we could pair to make 24. Or, we used division, and these were all the numbers that we could divide equally.)

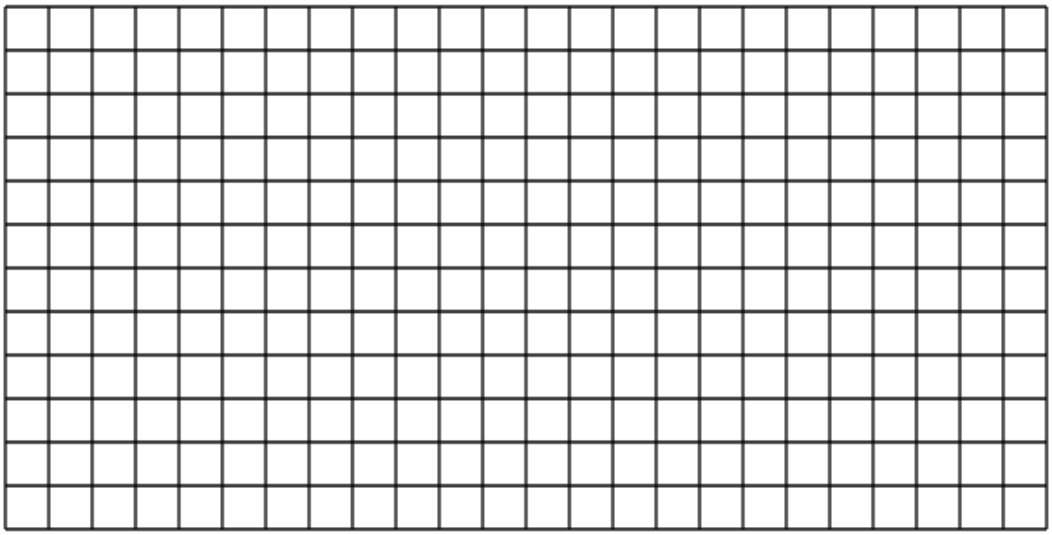
"Does this reasoning always work? Will it work for 45?" (Yes, 1 and 45, 3 and 15, 5 and 9. There are no more factors between 5 and 9, so I have found all the factor pairs.)

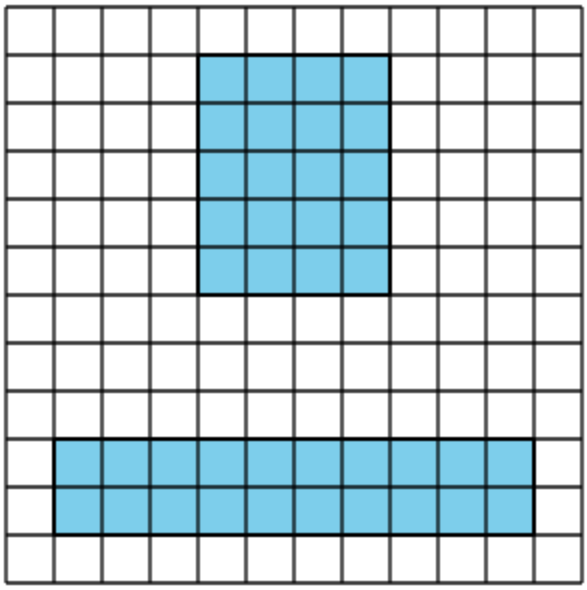
Math Community

- After the cool-down, give students 2–3 minutes to discuss any revisions to the "Doing Math" actions in small groups.
- Share ideas as a whole group and record any revisions.

Response to Student Thinking	**Next Day Support**

Students may not list all the possible dimensions for the given areas. It may be helpful for students to use tiles to build the rectangles in order to find all the possible dimensions.	During the launch of the first activity in the next lesson, remind students about available math tools for the lesson.
Response to Student Thinking Students do not conceptualize that the area is the number of unit squares within a figure.	**Prior-unit Support** Grade 3, Unit 2, Section A

Mini-Assessment	
Item 1 Narrative	Students find different rectangles with a given area on a grid. In the lessons, students were not expected to list the same dimensions twice. For example, if a student writes 4 units by 5 units, they need not also write 5 units by 4 units, as they represent the same rectangle with a different orientation.
Item Statement	1. What are the possible dimensions of a rectangle with area 20 square units? 2. Sketch two different examples. 
Item Solution	1. Sample responses: 4 units by 5 units, 5 units by 4 units, 2 units by 10 units, 1 unit by 20 units

	2. Sample response: 

Item 2 Narrative	Students find factors of numbers within 100. This item mainly assesses student understanding of the word “factor” and the calculations should be relatively quick.
Item Statement	List all of the factors of the following numbers. a. 25 b. 18
Item Solution	a. 1, 5, 25 b. 1, 2, 3, 6, 9, 18

Prior-grade Practice and Fluency Resources

Rectangle Rumble

Narrative
In this center, students understand concepts of area and relate area to multiplication and to addition. They use this understanding to represent areas on grid paper.

Play Rectangle Rumble with 2 players:

1. Choose a color for your rectangles that is different from your partner's.
2. Spin the spinner twice.
3. Shade in a rectangular area to represent the product of the two numbers.
4. Keep taking turns until the grid can't fit any more rectangles.
5. Add up your total area.
6. The player with the most total square units wins!

Stage Number 3: Generate products using the numbers 1, 2, 3, 4, 5, 6, 8, and 10

Addressing CCSS:
- 3.MD.C.7

Building Toward CCSS: (only for K)

Learning Goals (Section goal)
- Solve problems involving the area of rectangles (3.2.B).

Required Material
- 2 different color crayons
- paper clips

Blackline Master
- Rectangle Rumble: Stage 3 Spinner
- Rectangle Rumble: Stage 3 Grid

Stage Narrative

In this stage, students spin with the numbers 1, 2, 3, 4, 5, 6, 8, and 10 to generate products and fill a 20 x 20 grid

Multiplication Card Sort
Narrative In this center, students reflect on and develop their fluency with multiplication. Options for play: ● Independent: Students independently develop multiplication strategies. ● Partners: Students share their multiplication strategies with each other.
Stage Number 4: Factors of 1–12
Addressing CCSS: ● 4.OA.B.4
Learning Goals ● Apply multiplication fluency within 100 to find rectangles with given side lengths or a given area (4.1.A).
Required Material ● pencils
Blackline Master ● Multiplication Card Sort: Stage 1–4 Sort Table ● Multiplication Card Sort: Stage 4 Factor Cards
Stage Narrative Students sort products with factors of 1–12 into 3 categories: know it right away, can find it quickly, and don't know it yet. Then, students review the strategies they have for multiplication and practice the products they categorized as "don't know it yet."

Product Game
Narrative In this center, students strategically choose factors to get 4 products in a row. 1. Partner 1 chooses two factors to multiply and places a counter on the product. 2. Partner 2 moves one of the paper clips to a different factor, multiplies the factors, and places a counter on the product. 3. Continue to take turns moving one paper clip and placing a chip on the product. (It is possible to put both paper clips on the same number.)

4. The winner is the first one to get four chips in a row. Rows can be horizontal, vertical, or diagonal.

Stage Number 3: The Product Game; Two-digit Factors

Addressing CCSS:
- 4.NBT.B.5

Learning Goals
- Multiply a whole number of up to four digits by a one-digit whole number, and two two-digit numbers using strategies based on place value and the properties of operations.

Required Material
- 2 paper clips per group
- counters

Blackline Master
- Product Game: Stage 3 Game Board

Stage Narrative

In this stage, students choose from the factors 6, 7, 8, 9, 10, 12, and 15 to get four products in a row.

Extension and Exploration Resources

4.1 Lesson 7: Find Factors and Multiples	
Standards Alignment: Addressing: 4.OA.B.4	
Teacher-facing Learning Goals ● Find all factor pairs of a given whole number from 1–100. ● Determine whether a number from 1–100 is a multiple of another number.	
Student-facing Learning Goal ● Let's find factors and multiples of whole numbers from 1–100.	
Lesson Purpose The purpose of this lesson is for students to find factors and multiples of a given whole number from 1–100.	
Lesson Narrative In previous lessons, students learned about factor pairs, multiples, and prime and composite numbers. The purpose of this lesson is to systematically use the language of factors and multiples to describe numbers within 100. Students look for all factors of numbers and decide whether a given number is prime or composite. Students are encouraged to find patterns in composite numbers which help to identify a factor. For example, if the last digit of a number is 0 then 2, 5, and 10 are all factors of that number.	
Access for Students with Disabilities (SwD) Activity 2: Engagement	Access for English Learners (EL) Activity 1: MLR8 Discussion Supports
Instructional Routines Number Talk	
Materials to Gather ●	Materials to Copy ● Blackline Master Factor or Multiple Recording Sheet

Lesson Timeline	Teacher Reflection Question
Warm-up 10 minutes Activity 1 15 minutes Activity 2 15 minutes Lesson Synthesis 10 minutes Cool-down 5 minutes	As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. How have you seen each student grow as a young mathematician throughout this work? How have you seen yourself grow as a teacher? What will you continue to do and what will you improve on in the next unit?

Cool-down: Complete the Statements

Standards Alignment:
- 4.OA.B.4

Student-facing Task Statement
Complete the statements for each number.

number	factor	multiple
11	_____ is a factor of _____ because...	_____ is a multiple of _____ because...
24	_____ is a factor of _____ because...	_____ is a multiple of _____ because...

Student Responses
Answers vary. Possible responses:

number	factor	multiple
11	11 is a factor of 55 because...11 x 5 = 55.	11 is a multiple of 1 because... 11 x 1 = 11.
24	8 is a factor of 24 because... 8 x 3 = 24.	24 is a multiple of 8 because... 8 x 3 = 24.

Warm-up

Time: 10 minutes

Standards Alignment:
Addressing: 3.OA.B.5, 3.OA.C.7

Instructional Routines: Number Talk

Warm-up Narrative
The purpose of this Number Talk is to elicit strategies and understandings students have for dividing within 100. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to find factor pairs of numbers.

Student-facing Task Statement
Find the value of each quotient mentally.

12 ÷ 3
30 ÷ 3
60 ÷ 3
72 ÷ 3

Launch
- Display one problem.
- "Give me a signal when you have an answer and can explain how you got it."

Activity
- 1 minute: quiet think time
- Record answers and strategy.
- Keep problems and work displayed.
- Repeat for each problem.

Synthesis
- "How does knowing the first and third quotients help you find the last quotient?" (Since 12 + 60 is 72, we can add the answers to those quotients to get the answer to the last problem.)
- Consider asking:
  - "Who can restate _____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"
  - "Does anyone want to add on to _____'s strategy?"

Student Responses
- 4. I know 4 groups of 3 is 12.
- 10. I know that 10 groups of 3 is 30.
- 20. That's double 30 and double 10 is 20.
- 24. 72 is 60 and 12 so that's 20 and 4 groups or 24 groups of 3.

Activity 1: Factor and Multiple Statements

Time: 15 minutes

Standards Alignment

Addressing: 4.OA.B.4

Activity Narrative
The purpose of this activity is for students to find factors and multiples of a given number and make statements that use the language of factors and multiples. Students can generate many different statements for each number and use the given number in either blank in the sentence stem. Students should be encouraged to justify their choices as they share statements with their partner.

MLR Tags
- MLR8

EL Support Text
MLR8 Discussion Supports. Launch: Use multi-modal examples to show the meaning of factor and multiple. Invite students to use verbal descriptions along with gestures, drawings, or concrete objects to show factors of 10 and multiples of 10.
Advances: Listening, Representing

SwD Support Text

Student-facing Task Statement
1. Complete a statement using the word factor and a statement using the word multiple for each number.

number	factor	multiple
10	___ is a factor of ___ because...	___ is a multiple of ___ because...
7	___ is a factor of ___ because...	___ is a multiple of ___ because...
50	___ is a factor of ___ because...	___ is a multiple of ___ because...
16	___ is a factor of ___ because...	___ is a multiple of ___ because...
35	___ is a factor of ___ because...	___ is a multiple of ___ because...
20	___ is a factor of ___ because...	___ is a multiple of ___ because...

Launch
- Groups of 2
- "We are going to practice using the words factor and multiple in preparation for a game we are going to play. Take some time to complete the statements on your own."
- 3–5 minutes: independent work time

Activity
- "Now share your statements with your partner. Be sure to ask any questions you have as you justify each of your statements together."
- 3–5 minutes: partner discussion

Synthesis
- "What was your favorite statement you came up with? Why was it your favorite?"
- "What did you and your partner notice and wonder about this activity?"

19	___ is a factor of ___ because...	___ is a multiple of ___ because...
6	___ is a factor of ___ because...	___ is a multiple of ___ because...

2. As you compare statements with your partner, discuss one thing you notice and one thing you wonder.

Student Responses

1. Answers may vary.
   - 10 is a factor of 20 because 10 x 2 = 20. 10 is a multiple of 5 because you can count by 5s and say 10.
   - 5 is a factor of 10 because 5 x 2 =10. 30 is a multiple of 10 because 10 x 3 = 30.
2. Sample responses:
   - Students may notice:
     - If a number was larger, we could think about what we could divide it by to find factors of the number.
     - If the number was smaller, we multiplied it by another number to find a multiple.
     - Prime numbers like 7 and 19 only have 2 factors.
     - 10 is a factor of a number and a number is a multiple of 10 when the last digit of the number is 0.
     - 5 is a factor of a number and a number is a multiple of 5 when the last digit of the number is 5 or 0.
     - Even numbers are multiples of 2.
   - Students may wonder:
     - Is every number a factor or multiple of another number?
     - Is 1 a factor of every number?

Activity 2: Factor and Multiple Game

Time: 20 minutes

Standards Alignment
Addressing: 4.OA.B.4

Materials to Copy

- Blackline Master Factor or Multiple Recording Sheet

Activity Narrative

The purpose of this activity is for students to practice finding factors and multiples of numbers. Students can play multiple rounds of the game as time allows and should be encouraged to use the ideas surfaced from the previous activity if needed. The game also provides another opportunity for students to use factor and multiple vocabulary. If necessary, play a round against the class to demonstrate the game.

SwD Support Tags
- Engagement

SwD Support Text
Engagement: Develop Effort and Persistence. Launch: Invite students to generate a list of shared expectations and possible language to use during group work, especially when playing a game that has a winner. Encourage students to discuss how they might support their partner's learning or collaborate to find solutions, even though they are on opposing teams. Record responses on a display and keep it visible during the activity.
Supports accessibility for: Language, Social-Emotional Functioning

Student-facing Task Statement
Directions:
1. Partner 1: Choose an even number that is less than 50, and cover it with a counter and record the number on the recording sheet.
2. Partner 2: Choose a number to cover. The number must be a factor or multiple of the first number.
3. Take turns covering and recording. On each turn, choose a number that is a factor or multiple of the number just covered by your partner.
4. If there are no factors or multiples left to cover on your turn, your partner wins the round.
5. If a partner forgets to name a factor or multiple and you say it, you earn a bonus win.
6. Keep track of who wins each round on your recording sheet and repeat as time allows.

Launch
- Groups of 2
- "Now you're going to use your knowledge of factors and multiples to play a game against your partner."

Activity
- 15 minutes: small-group work time
- Monitor for students who strategically choose numbers to win the round.

Synthesis
- "What was your strategy for choosing numbers as you played the game?" (I wanted to keep the round going as long as possible, so I liked choosing numbers that I knew had a lot of factors. I tried to find a number that was prime that I could use so that my partner would have a harder time choosing a number.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student Responses

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student A selects 24
Student C selects 12

Lesson Synthesis
“In today’s lesson we used the terms factors and multiples to describe numbers within 100. We learned that:

- 1 is a factor of every number and every number is a multiple of 1, and 2 is a factor of each even number.
- Each even number is a multiple of 2.

- A number is a multiple of 5 when the last digit of the number is 5 or 0, 10 is a factor of a number and a number is a multiple of 10 when the last digit of the number is 0.

"Take turns using each number to complete each statement with a partner: 1, 2, 5, 10."
How do you know if _____ is a factor of a number?
How do you know if a number is a multiple of _____?

"For example, how do you know if 2 is a factor of a number? How do you know if a number is a multiple of 2?"

Share and record responses.

Student Lesson Summary
In this section we identified factors and multiples of numbers between 1–100. We learned that numbers can share factors and multiples. For example, 6 and 8 share a factor of 2 and are both multiples of 24. We also used what we learned about prime numbers to predict numbers that have very few or only 2 factors during the locker problem. We explained the difference between factors and multiples by completing factor and multiple statements about the same number.

Response to Student Thinking	Next Day Support
• Students confuse the terms factor and multiple.	• Launch warm-up or Activity 1 by highlighting key vocabulary from previous lessons.
Response to Student Thinking	Prior-unit Support
• The student is not using multiplication to determine unknown factors for given multiples and may be struggling to see the relationship between multiplication and division.	• Grade 3, Unit 4, Section A

4.1 Lesson 8: Mondrian Art

Standards Alignment:
Building On: 3.MD.C.7.b
Addressing: 4.OA.B.4

Teacher-facing Learning Goals
- The mathematical purpose of this lesson is for students to apply their understanding of the area of rectangles and factor pairs to analyze student generated Mondrian inspired art.

Student-facing Learning Goals
- Let's make Mondrian art.

Lesson Purpose	
Lesson Narrative In previous lessons, students used the concept of area as a context for understanding factors, multiples, prime, and composite numbers. In this lesson, students make a unique piece of artwork using the area of rectangles and the multiplication facts within 100. At the beginning of the lesson, students learn about Piet Mondrian and see a number of his abstract paintings. Students are encouraged to notice that his compositions are filled with rectangles: some with the same and some with different areas. When students describe the art in the context of mathematics, they model with mathematics (MP4). In the first activity, they outline their own composition by dividing a 20 by 20 grid into rectangular spaces with certain requirements. When students adhere to these constraints, they model with mathematics (MP4). In the second activity, students observe another student's artwork and identify rectangles with areas that are the same, prime, or composite. Activity 3 is optional. If it is omitted, consider giving students time during activity 1 or 2 to color their art pieces.	
Access for Students with Disabilities Activity 1: Action and Expression	Access for English Learners Activity 2: MLR8 Discussion Supports
Materials to Gather Warm-up: Additional Mondrian artwork (optional) Visit MoMA's site for a virtual installation of Mondrian's work or the Tate Gallery site and search for Piet Mondrian. Activity 1: straight edge, black marker or crayon for each student Activity 3: tools for coloring, adhesive, sticky note	Materials to Copy Warm-up: Virtual Mondrian artwork by visiting MoMA's website for Piet Modrian's installation (optional) Activity 1: 4.1.ModelLesson Blackline Master Modrian Image
Lesson Timeline Warm-up 10 minutes Activity 1 20 minutes Activity 2 15 minutes Activity 3 20 minutes (optional) Lesson Synthesis 5 minutes	Teacher Reflection Question What part of the lesson went really well today in terms of students' learning? What did you do that made that part go well?

Warm-up: Notice and Wonder: Piet Mondrian's Art
Time: 10 minutes

Building Toward CCSS: 4.OA.B.4

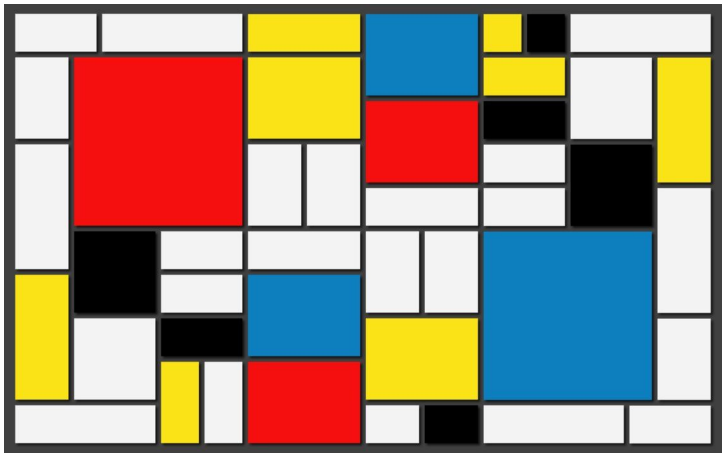
Materials to Gather
- virtual Mondrian artwork (optional)

Instructional Routines
- Notice and Wonder

Warm-up Narrative
The purpose of this task is to introduce students to the artwork of Piet Mondrian. Students may notice that the compositions of his most famous painting style are comprised wholly of various sized rectangles. Students will create their own versions of Mondrian art in the first activity.

To show students additional art pieces by Mondrian, visit MoMA's site to see the Piet Mondrian installation virtually or visit the Tate Gallery site.

Student-facing Task Statement
What do you notice? What do you wonder?

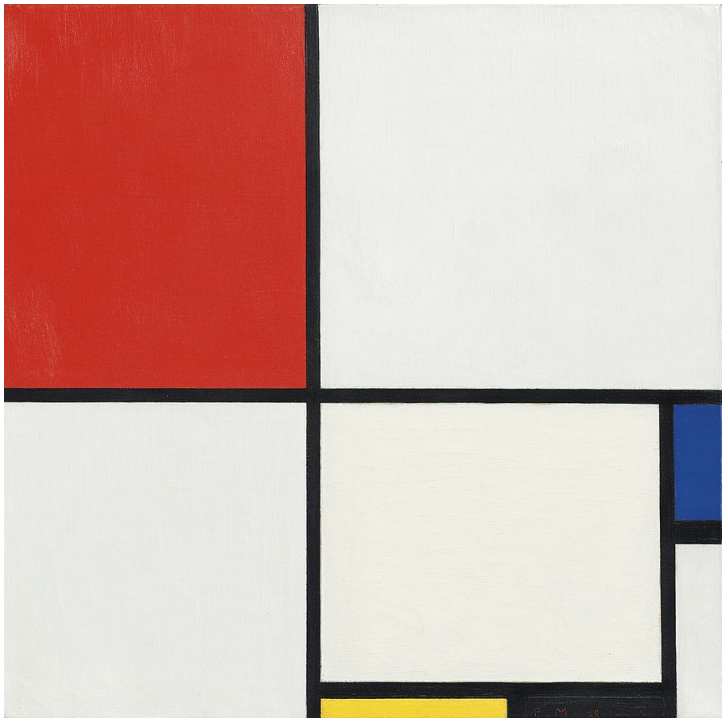


Teacher Directions

Launch
- Groups of 2
- Display the images.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis
- "These are images of famous paintings by a Dutch artist named Piet Mondrian. He lived from 1872–1944. A little more than 100 years ago, he became known for painting in a style that connects to the math we have been studying. Many of his paintings hang in museums all around the world."
- "How do you think his art connects with what we've been studying? Why are we looking at it during math class?" (He uses



a lot of rectangles; his art looks mathematically inspired.)
- If it isn't mentioned by students, highlight that some of the lines go from edge to edge of the painting, while others are shorter, and that some rectangles have the same area.
- Considering showing students additional pieces by Mondrian.

Student Responses
Students may notice:
- There are lots of rectangles.
- The rectangles are all different sizes.
- The paintings weren't of anything.
- There are only 3 or 4 colors.
- There are black lines around all the rectangles.

Student may wonder:
- Are any of the rectangles the same size?
- What is the area of the white rectangles?
- Which colors fill more space on the paintings?

Activity 1: My Mondrian Outline

Time: 20 minutes

Building Toward CCSS: 4.OA.B.4

Materials to Gather
- black marker or crayon, straight edge, coloring tools

Materials to Copy
- 4.1.ModelLesson Blackline Master Modrian Image

Activity Narrative
The purpose of this activity is for students to create an outline for their artwork. In this activity, students draw lines on graph paper, marking out rectangular areas that will be the basis for their Mondrian-inspired artwork.

SwD Support Tags
- Action and Expression

Student-facing Task Statement
Create an outline for art in the Mondrian style.
Your artwork should:
- be divided into at least 12 rectangles
- include two different rectangles that have the same area
- include at least one rectangle whose area is a prime number

Try at least one of these challenges. Make a design for which:
- all but two of the rectangles have a prime number area
- no rectangles "line up" with each other (no two rectangles share a side)

Teacher Directions

Launch
- "We are going to create our own art pieces that are inspired by Mondrian's work."
- Distribute 20 by 20 grid, straight edge, and black marker or crayon.

Activity
- "Use your straight edge and pencil to divide your grid up, filling it with at least 12 rectangles. Include at least one of the challenges. Once you are happy with your design, trace it with the black marker or crayon."
- 15 minutes: independent work time
- Monitor for students who attempt or accomplish one or more of the challenges.

Synthesis
- "Turn and talk: What is similar or different about your outlines?" (They are similar because they're only rectangles, but the way they are arranged and their sizes are different.)
- "How can you determine whether any of the rectangles have the same area?" (See if the sides are factor pairs of the same number. Use the side lengths and multiply them.)

	• "Did you accomplish one or both of the challenges? What was your strategy?" (Answers vary.)
Student Responses Students have created artwork that meets all required constraints.	
Activity 2: Analyze the Rectangles	
Time: 15 minutes	
Addressing CCSS: • 4.OA.B.4	
Activity Narrative The purpose of this activity is for students to use their understaning of factor pairs, prime, and composite numbers to analyze their peers' artwork.	
MLR Tags • MLR8	
EL Support Text MLR8 Discussion Supports. Synthesis: Display the following sentence frames to support whole-class discussion: "To find rectangles with the same area I looked for . . .", "To find rectangles with an area that is prime, I looked for . . .", and "To find rectangles with an area that is composite, I looked for . . ." Advances: Speaking, Representing	
Student-facing Task Statement Switch with your partner. Using your partner's artwork, determine: 1. the rectangles that have the same area 2. the rectangles with an area that is prime 3. the rectangles with an area that is composite 4. which challenge they completed Explain or show your reasoning.	Teacher Directions Launch • Groups of 2 • "Switch artwork with your partner." Activity • 5–7 minutes: independent work time • 2–3 minutes: partner check-in • Monitor for students who consider factor pairs in their reasoning. Synthesis • Invite previously selected students to share how they found rectangles with the

	same area. (The sides are factor pairs of the same number.) ● "Is it possible to make all the rectangles have an area that is prime?" (No, because we had to make two that have the same area, so those two would have to be composite.) ● "What strategies did you use to find which challenge was completed?" (I looked for rectangles that have areas that are prime numbers. I could see none of the rectangles lined up.)
Student Responses 1. Students identify rectangles that have the same area. They have the same area or they are factor pairs of the same number. 2. Students identify the rectangles with an area that is prime. They have a side of 1 unit. 3. Students identify the rectangles with an area that is composite. The area can be found using more than one set of factor pairs. 4. Students identify which challenge they completed. If all but 2 rectangles had a side length or if none of the rectangles lined up, then the challenge was completed.	
Activity 3 Title: Gallery Walk	
Time: 20 minutes	
Addressing CCSS: ● 4.OA.B.4	
Building Toward CCSS:	
Materials to Gather ● color tools, tape or adhesive, sticky note	
Student-facing Task Statement Color your artwork with 3–4 colors. Gallery Walk Reflection Question: What question would you ask the artist about how they chose to create their design?	Teacher Directions Launch ● Distribute sticky notes and adhesive. ● "After coloring your artwork, you'll hang it on the wall like in a gallery." ● Display reflection question for gallery walk.

	• Consider asking students for examples if needed. (Why did you choose these colors? Why did you arrange the rectangles like this?)
	
	Activity
	• 10 minutes: independent work time
	• Give students time to hang their artwork.
	• "Pick one art piece, then write a question for the artist on the sticky note about their artwork. Stick it on the side of their work."
	• Monitor for questions related to:
	  ○ artist intent
	  ○ mathematics
	
	Synthesis
	• Invite previously selected students to have an opportunity to answer questions.

Student Responses
Sample response:
- Why did you choose these colors?
- Why did you arrange the rectangles like this?
- If you had to redo this, what would you do differently?

Lesson Synthesis
"Today you had a chance to create artwork and display it like in an art gallery."

"What was the most challenging part about creating the artwork?" (I was limited to only using rectangles. I had to make sure two rectangles had the same area.)
"What connections do you see between the mathematics and art we experienced today?" (The art we made today uses rectangles and we can use factor pairs to figure out when they have the same area or not. Even though we had the same requirements, our art came out differently, which shows that our work is unique.)