Student: _____ Class: _____ Date: _____

Analyzing Numerical Data: Indices Using Weighted Sums and Averages

I.C Student Activity Sheet 7: Slugging Averages

One example of a *weighted average* in sports is a batter's *slugging average* (or *percentage*) in baseball. The slugging average (***SLG***) is calculated using the following equation:

$$SLG = \frac{(1 \cdot S) + (2 \cdot D) + (3 \cdot T) + (4 \cdot HR)}{AB}$$

where S = singles, D = doubles, T = triples, HR = home runs, and AB = total at-bats.

Each single has a weight of 1, each double a weight of 2, each triple a weight of 3, and each home run a weight of 4. An at-bat without a hit has a weight of 0.

In his first season with the New York Yankees, Babe Ruth set a record for slugging average that stood for more than 80 years. In 1920, Ruth pounded 172 hits in 458 at-bats. His hits consisted of 73 singles, 36 doubles, 9 triples, and 54 home runs, resulting in a total base count of (73 • 1) + (36 • 2) + (9 • 3) + (54 • 4) = 388. When his total number of bases (388) is divided by his total at-bats (458), the result is .847, his slugging percentage for the season. This record was broken in 2001 by Barry Bonds, who had 411 total bases in 476 at-bats for a slugging average of .863. (Statistics from www.baseball-almanac.com)

1. Find the slugging average for a player with the following statistics:

$S = 68$
$D = 40$
$T = 4$
$HR = 16$
$AB = 320$

2. **REFLECTION:** Is it possible to have a slugging average of more than 1?

   Theoretically, what is the highest possible value for the slugging average? Could a player ever achieve this value during a baseball season? Give an example or explain why none exist.

3. A slugging average of .500 or higher is considered a sign of an excellent player. Suppose a player had 4 triples and a batting average of .300 in 400 at-bats (batting average = hits/at-bats). Determine a combination of singles, doubles, and home runs that gives this player a slugging average higher than .500.

   Still using 400 at-bats, what is the maximum number of singles this player could have with a slugging average between .500 and .700? Justify your answer.

Slugging Percentage

In baseball there are many common statistics used batting average and slugging percentage. Batting average is just simply the percentage of the number of "at bats" for which the player gets a hit. If a player has 20 hits in 50 at bats then their batting average would be, $\frac{20}{50} = 0.400$. Their slugging percentage however, weights the type of hit where doubles count double, triples count triple, and homeruns count quadruple and would be given by the formula:

$$Slugging\ Percentage = \frac{(singles) + 2 \cdot (doubles) + 3 \cdot (triples) + 4 \cdot (home\ runs)}{(at\ bats)}$$

8. Calculate Freddie's Slugging Percentage if he has 42 singles, 12 doubles, 4 triples, and 14 home runs in 230 at bats.

9. Can you use the above information to determine Freddie's current batting average?

10. If Andrelton has a slugging percentage of 0.450 and has 12 doubles, 3 triples, and 8 homeruns in 200 at bats then how many singles must he have?

11. Can you determine Andrelton's current batting average?

4. **EXTENSION:** Research the slugging averages of some current-day professional baseball players. Then research the slugging averages of some college players. Based on your findings, which college players should professional scouts be watching? Support your answer with statistics.

Student: _____ Class: _____ Date: _____

Analyzing Numerical Data: Indices Using Weighted Sums and Averages

I.C Student Activity Sheet 8: Quarterback Ratings

The National Football League (NFL) rates quarterbacks for statistical purposes against a fixed performance standard based on the statistical achievements of all qualified pro passers since 1960. This system allows passing performances to be compared from one season to the next.

The following categories are used to compute the quarterback rating:

- percent of completions per attempt (***%COMP***)
- percent of touchdown passes per attempt (***%TD***)
- percent of interceptions per attempt (***%INT***)
- average yards gained per attempt (***YD***)

(From www.nfl.com/help/quarterbackratingformula)

The following is the formula for compiling the quarterback rating (***QR***):

$$QR = \frac{25 + 10(\%COMP) + 40(\%TD) - 50(\%INT) + 50(YD)}{12}$$

(***Note***: This formula is subject to a few conditions discussed after Question 2.)

1. For the first two games of the 2008 season, Dallas Cowboys quarterback Tony Romo completed 45 passes in 62 attempts for a total of 632 yards, with 4 touchdowns and 2 interceptions. Verify that Romo's quarterback rating for those games is approximately 113. Round each value to the nearest tenth.

   Percent of completions per attempt (***%COMP***) =

   Percent of touchdown passes per attempt (***%TD***) =

   Percent of interceptions per attempt (***%INT***) =

   Average yards gained per attempt (***YD***) =

2. As of 2009, Steve Young has the highest career quarterback rating in NFL history. He completed 2,059 passes in 3,192 attempts for a total of 25,479 yards, with 174 touchdowns and 85 interceptions. Find Young's career quarterback rating.

Analyzing Numerical Data: Indices Using Weighted Sums and Averages

I.C Student Activity Sheet 8: Quarterback Ratings

The actual formula for the quarterback rating is subject to some conditions, represented below.

$$QR = \left[\frac{5(\%COMP - 30)}{6}\right] + \left[\frac{10(\%TD)}{3}\right] + \left[\frac{25(19 - 2(\%INT))}{12}\right] + \left[\frac{25(YD - 3)}{6}\right]$$

where it is understood that each item in brackets [ ] is truncated to be no smaller than 0 and no larger than 475/12 (giving a maximum rating of 475/3 or 158.3). This rating is rounded to the nearest tenth.

These truncated values mean there is a minimum and maximum value for each component of the rating.

For example, if $\frac{10(\%TD)}{3} \leq \frac{475}{12}$, then $\%TD \leq \frac{475 \cdot 3}{12 \cdot 10} = 11.875\%$.

3. Show that $30\% \leq \%COMP \leq 77.5\%$.

4. If $c \leq YD \leq d$, find the values of c and d.

5. If $0 \leq \%INT \leq e$, find the value of e.

6. Explain why the actual formula and the simplified formula are the same when each component is between its minimum and maximum possible value.

$$\left[\frac{5(\%COMP - 30)}{6}\right] + \left[\frac{10(\%TD)}{3}\right] + \left[\frac{25(19 - 2(\%INT))}{12}\right] + \left[\frac{25(YD - 3)}{6}\right] =$$

$$\frac{10(\%COMP) - 300 + 40(\%TD) + 25 \cdot 19 - 50(\%INT) + 50(YD) - 150}{12}$$

7. **REFLECTION:** An index is a numerical scale. Characteristics of an index can be used for the following:

   - to compare variables with one another or a reference number,
   - to give information about general trends, and
   - to help make comparisons and judgments.

   It is often calculated as a weighted sum of various factors resulting in a single summary number. How can the quarterback rating system be thought of as an index?

Student: _____ Class: _____ Date: _____

Analyzing Numerical Data: Indices Using Weighted Sums and Averages

I.C Student Activity Sheet 9: Fan Cost Index

An *index* is a numerical scale. Characteristics of an index can be used for the following:

- to compare variables with one another or a reference number,
- to give information about general trends, and
- to help make comparisons and judgments.

It is often calculated as a weighted sum of various factors resulting in a single summary number.

The Fan Cost Index (FCI), compiled by Team Marketing Report, tracks the cost for a family of four to attend a professional sporting event. The FCI includes the prices of 2 average-price adult tickets, 2 average-price child tickets, 4 small soft drinks, 2 small other drinks, 4 regular-size hot dogs, parking for 1 car, 2 game programs, and 2 least expensive, adult-size adjustable caps. The Average Ticket Price in the following tables represents the average cost of a ticket for each member of the family.

The FCI for each Texas team in professional baseball, basketball, and football for 2006 and 2007 is shown in the following two tables. The Soft Drink column includes the price of 1 drink and its size in ounces.

2006 Fan Cost Index

Team	Avg. Ticket Price	Soft Drink (size in oz)	Other Drink	Hot Dog	Parking	Program	Cap	FCI
Rangers	15.81	2.75 (16)	6.00	2.50	8.00	5.00	10.00	134.24
Astros	26.66	4.00 (21)	7.00	4.00	10.00	4.00	11.00	192.64
Spurs	45.88	3.75 (24)	6.25	4.00	10.00	5.00	18.00	283.02
Mavericks	54.24	2.75 (12)	4.00	4.00	20.00	6.00	15.00	313.96
Rockets	38.64	4.00 (22)	5.75	4.25	15.00	–	20.00	254.06
Cowboys	66.12	3.50 (32)	5.00	3.50	12.00	5.00	10.00	344.48
Texans	56.97	3.25 (21)	6.00	5.00	15.00	5.00	20.00	337.88

(Compiled from www.teammarketing.com)

Analyzing Numerical Data: Indices Using Weighted Sums and Averages

I.C Student Activity Sheet 9: Fan Cost Index

2007 Fan Cost Index

Team	Avg. Ticket Price	Soft Drink (size in oz)	Other Drink	Hot Dog	Parking	Program	Cap	FCI
Rangers	16.47	3.00 (20)	6.50	2.75	8.00	5.00	11.00	141.88
Astros	26.90	4.00 (21)	7.00	4.25	10.00	4.00	12.00	196.60
Spurs	51.24	2.50 (24)	6.00	4.00	10.00	6.00	21.00	306.96
Mavericks	60.58	2.75 (15)	4.00	4.00	20.00	6.00	15.00	339.32
Rockets	41.98	4.00 (22)	7.00	4.50	15.00	–	20.00	270.92
Cowboys	84.12	3.50 (32)	5.00	3.50	12.00	5.00	10.00	416.48
Texans	62.41	3.25 (21)	6.00	5.00	15.00	5.00	20.00	359.64

(Compiled from www.teammarketing.com)

1. Complete the 2006 table by calculating the missing FCI values.

2. There are seven components in the FCI. Find the largest percent increase in a single component from 2006 to 2007 by giving the team, component, and percent increase.

3. Which of the Dallas-area teams (Rangers, Mavericks, Cowboys) had the following during 2006 or 2007:

   a. Most expensive soft drink in terms of price per ounce?

   b. Least expensive soft drink in terms of price per ounce?

4. **EXTENSION:** Create a personal cost index (PCI) in which you select only the items and the number of those items that you and three friends would buy (for example, 4 tickets, 8 hot dogs, no soft drinks, no caps). In 2007, which team is the best buy according to your PCI? Why?

Analyzing Numerical Data: Indices Using Weighted Sums and Averages

I.C Student Activity Sheet 9: Fan Cost Index

The following table contains the average FCI for all teams in Major League Baseball (MLB), the National Basketball Association (NBA), and the National Football League (NFL) for 2002 through 2007.

FCI Average

	2002	2003	2004	2005	2006	2007
MLB	145.26	148.66	155.52	164.43	171.19	176.55
NBA	254.86	261.26	263.44	267.37	274.67	281.90
NFL	290.41	301.75	321.62	329.91	346.16	367.31

This table contains the rates of inflation from 2003 through 2007.

Year	2003	2004	2005	2006	2007
Inflation Rate (%)	2.27	2.68	3.39	3.24	2.85

Use this information and additional calculations to answer the following questions.

5. Did the rate of increase of each sport's average FCI exceed the rate of inflation for all these years? Find the sport, year, and rate of increase for all cases in which the average FCI rate increase did not exceed the inflation rate.

6. Find the sport and year in which the average rate of increase exceeded the inflation rate by the greatest amount.

7. **REFLECTION:** Why do you think the FCI is highest for the NFL and lowest for MLB?

8. **EXTENSION:** There are several indices related to consumer spending. Two of them are the Consumer Price Index (CPI) and the Consumer Confidence Index (CCI). Find out how they are calculated and how they are used.