Unit 6 TASK 2: Walk Like a Mathematician Learning Task:

Matrices allow us to perform many useful mathematical tasks which ordinarily require large number of computations. Some types of problems which can be done efficiently with matrices include solving systems of equations, finding the area of triangles given the coordinates of the vertices, finding equations for graphs given sets of ordered pairs, and determining information contained in vertex edge graphs. In order to address these types of problems, it is necessary to understand more about matrix operations and properties; and, to use technology to perform some of the computations.

Matrix operations have many of the same properties as real numbers. There are more restrictions on matrices than on real numbers, however, because of the rules governing matrix addition, subtraction, and multiplication. Some of the real number properties which are more useful when considering matrix properties are listed below.

Let a, b, and c be real numbers	ADDITION PROPERTIES	MULTIPLICATION PROPERTIES
COMMUTATIVE	a + b = b + a	ab = ba
ASSOCIATIVE	(a + b) + c = a + (b + c)	(ab)c = a(bc)
IDENTITY	There exists a unique real number zero, 0, such that a + 0 = 0 + a = a	There exists a unique real number one, 1, such that a * 1 = 1 * a = a
INVERSE	For each real number a, there is a unique real number - a such that a + (-a) = (-a) + a = 0	For each nonzero real number a, there is a unique real number $\frac{1}{a}$ such that $a(\frac{1}{a}) = (\frac{1}{a})a = 1$

The following is a set of matrices without row and column labels. Use these matrices to complete the problems.

$$D = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 2 & 4 & 1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix} \quad F = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \quad G = \begin{bmatrix} 0 & -1 \\ 1/3 & 2/3 \end{bmatrix} \quad H = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad J = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} \quad K = \begin{bmatrix} 3 & 4 & -1 \\ 0 & -2 & 3 \\ 5 & 1 & 2 \end{bmatrix} \quad L = \begin{bmatrix} 5 & -2 \\ -1 & 0 \end{bmatrix}$$

1. In this problem, you will determine whether matrix addition and matrix multiplication are commutative by performing operations and comparing the results.

   a. Does $[D]+[J]=[J]+[D]$?

   b. Does $[E]+[K]=[K]+[E]$?

   c. Is matrix addition commutative? Why or why not?

   d. Does $[D]*[J]=[J]*[D]$?

   e. Does $[E]*[H]=[H]*[E]$?

   f. Is matrix multiplication commutative? Why or why not?

2. Are matrix addition and matrix multiplication associative?

a. Does $([D]+[J])+[L]=[D]+([J]+[L])$?

b. Is matrix addition associative? Why or why not?

c. Does $([D]*[J])*[L]=[D]*([J]*[L])$?

d. Is matrix multiplication associative? Why or why not?

3. Is there a **zero** or **identity, 0,** for addition in matrices? If so, what does a **zero matrix** look like? Provide an example illustrating the additive identity property of matrices.

4. Do matrices have a **one** or an **identity, I,** for multiplication? If so what does an **identity matrix** look like; is it unique; and, does it satisfy the property $a * 1 = 1 * a = a$?

a. Multiply $[E]*[I]$ and $[I]*[E]$. Describe what you see.

5. Find [D]*[G] and [G]*[D]. Describe what you see.

D and G are called **inverse matrices**.

In order for a matrix to have an inverse, it must satisfy two conditions.
1. The matrix must be a square matrix.
2. No row of the matrix can be a multiple of any other row.
Both D and G are 2x2 matrices; and, the rows in D are not multiples of each other. The same is true of G.
The notation normally used for a matrix and its inverse is D and D^{-1} or G and G^{-1}.

6. Multiply G by 3 and look at the result. Can you see any relationship between D and the result?

The following formula can be used to find the inverse of a 2x2 matrix. Given matrix A where the rows of A are not multiples of each other:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{then} \quad A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

a. Find the inverse of matrix J from the matrices listed above. Verify that J and J^{-1} are inverses.

b. Now your teacher will show you how to use technology to find the inverse of this matrix.

7. A unique number associated with every square matrix is called the determinant. Only square matrices have determinants.

A. To find determinants of 2x2 matrices by hand use the following procedure.

$$\text{determinant}(A) = \det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

a. Find $|D|$

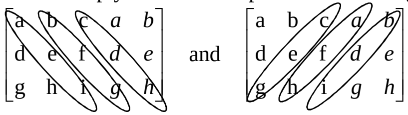
b. Find $|J|$

c. Can you find $|F|$? Why or why not?

B. One way to find determinants of 3x3 matrices use the following procedure: given matrix

$$B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \text{ rewrite the matrix and repeat columns 1 and 2 to get } \begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix}.$$

Now multiply and combine products according to the following patterns.



= det(B) = aei + bfg + cdh – ceg – afh – bdi.

a. Find $|E|$

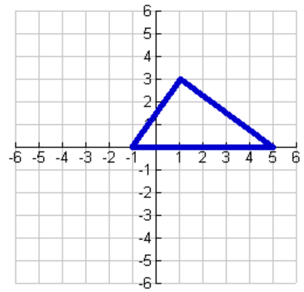
b. Find $|K|$

c. Now your teacher will show you how to use technology to find the determinants of these matrices.

8. The determinant of a matrix can be used to find the area of a triangle. If (x_1, y_1), (x_2, y_2), and (x_3, y_3) are vertices of a triangle, the area of the triangle is

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|.$$

a. Given a triangle with vertices (-1, 0), (1, 3), and (5, 0), find the area using the determinant formula. Verify that area you found is correct using geometric formulas.



b. Suppose you are finding the area of a triangle with vertices (-1, -1), (4, 7), and (9, -6). You find half the determinant to be -52.5 and your partner works the same problem and gets +52.5. After checking both solutions, you each have done your work correctly. How can you explain this discrepancy?

c. Suppose another triangle with vertices (1, 1), (4, 2), and (7, 3) gives an area of 0. What do you know about the triangle and the points?

d. A gardener is trying to find a triangular area behind his house that encloses 1750 square feet. He has placed the first two fence posts at (0, 50) and at (40, 0). The final fence post is on the property line at y = 100. Find the point where the gardener can place the final fence post.