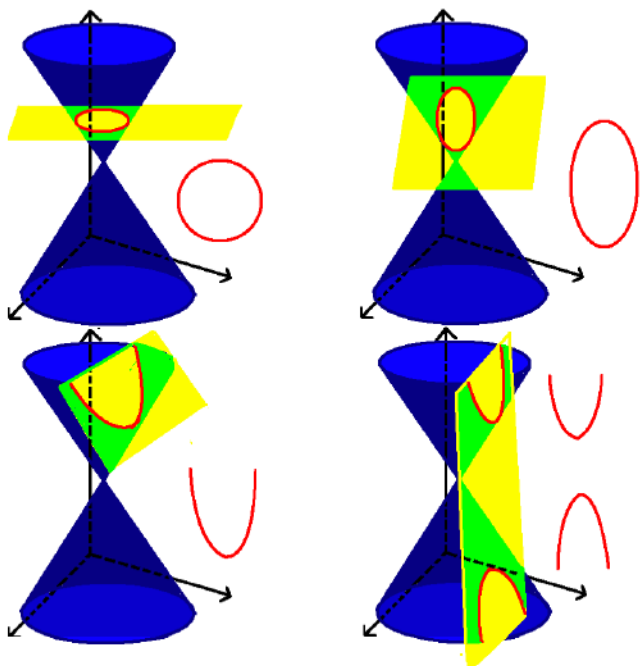
Pre-Calculus Unit 6 Task 1: Our Only Focus: Circles & Parabolas Review

Name: ____ **Date:** _______________ **Period:** ____

For most students, you last learned about conic sections in Analytic Geometry, which was a while ago. Before we begin looking back over the first two types of conic sections that you have already discovered, let's take a look at the geometric meaning of a conic section. First, why "conic"? Conic sections can be defined several ways, and what we'll focus on in this unit is deriving the formulas for the last two types of conic sections from special points called foci. But for the purpose of (re)introduction, the geometric meaning of "conic" comes into focus.



The reason we call these graphs conic sections is that they represent different slices of a double-napped cone. The diagram above shows how the different graphs can be "sliced off" of the figure. You should notice right away that the vast majority of these graphs are relations, not functions – in fact, only one case of parabolas (those featured in your past learning about quadratic functions) are actually functions. That doesn't limit the usefulness of these special planar graphs, however.

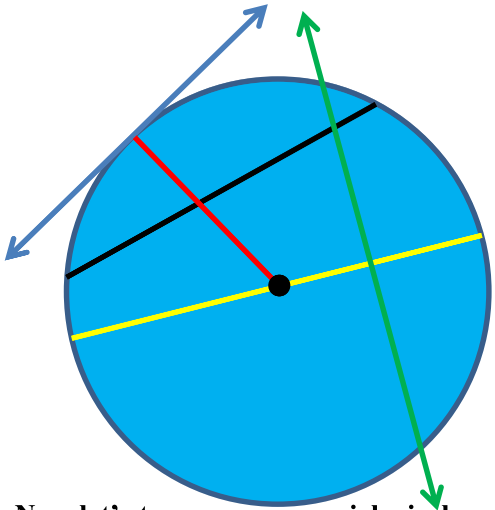
The General Form of a Quadratic Relation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \text{ where } A, B, C \text{ cannot all be zero}$$

The graphs of all conic sections follow this same equation. It should be noted that for our more basic purposes, the B coefficient will always be zero.

Consider…the Circle

Circles should be old news to you, but just as a quick review let's see if you can remember their important parts.



For the circle to the left, label the following features on the diagram:

Center
Diameter
Radius
Chord
Secant Line
Tangent Line
Point of Tangency

Now let's try some very quick circle review using a few of these terms.

(a) What relationship do the points making up the graph of a circle have to the center?

(b) What relationship do the radius and the diameter of a circle have?

(c) What is a tangent line and what relationship does it have to the radius that it meets at the point of tangency?

(d) If a circle has a diameter with endpoints $(-2, -5)$ and $(3, 4)$, what is...

(i) the length of the diameter of the circle?

(ii) the center of the circle?

(iii) the length of the radius of the circle?

(iv) the slope of the radius from the center to $(3, 4)$?

(v) the equation of the tangent line that intersects the circle at the point $(-2, -5)$?

If you answered (a) correctly, you know that the locus of points making up a circle are equidistant from the circle's center. This leads to an important idea about the center – it serves as the focus of the circle. The points making up the circle are all entirely dependent upon the location of that important focal point.

Now let's review the standard form of the equation describing a circle.

Standard Form of a Circle:

$$(x-h)^2+(y-k)^2=r^2 \ \ with\ center\ at\ (h,k)\ and\ radius\ r$$

This is the most useful form of a circle in terms of recognizing important pieces and for graphing and was the emphasis of your previous work with circles.

Let's try writing a few equations in standard form.

1. Write the equation for the circle with a diameter containing the endpoints $(-3,0)$ and $(3,0)$.

2. Write the equation for the entire set of points that are 4 units away from $(1,-5)$.

3. Write the equation of the circle with a radius from the center at $(2,7)$ to an endpoint at $(6,5)$.

And now let's review how to take a circle in a different form and change it to the more useable standard form. For example, let's look at the following:

$$x^2+y^2+6x-2y+1=0$$

Notice that this circle is presented in the **general form** $Ax^2+Cy^2+Dx+Ey+F=0$ where $A=1, C=1, D=6, E=-2,$ and $F=1$. As you work through the next set of problems, see if you recognize any patterns in the coefficients for general form, and then see if you can find other patterns using the general form equations for other conic sections. In any case, this general form is not useful in terms of graphing, or picking out the radius, diameter, or center. So we need to put the equation into standard form. To do this by completing the square, first group like variables together and move the constant to the other side of the equation.

$$x^2+y^2+6x-2y+1=0$$
$$x^2+6x+y^2-2y=-1$$

Once we've gotten like variables together and sent the constant to the other side, we have to complete the square by taking the coefficient of the linear term for both variables, dividing it by 2, and squaring the quotients. Add both of these squares to both sides of your equation.

$$\left(x^2+6x+\left(\frac{6}{2}\right)^2\right)+\left(y^2-2y+\left(\frac{-2}{2}\right)^2\right)=-1+\left(\frac{6}{2}\right)^2+\left(\frac{-2}{2}\right)^2$$

$$(x^2 + 6x + 9) + (y^2 - 2y + 1) = 9$$

Now, all we must do is factor our two perfect square trinomials and we'll have standard form.

$$(x + 3)^2 + (y - 1)^2 = 9$$

Now we know that the circle has a center of $(-3, 1)$ and a radius of 3, facts not obvious from the original general form. **Put the following equations into standard form**.

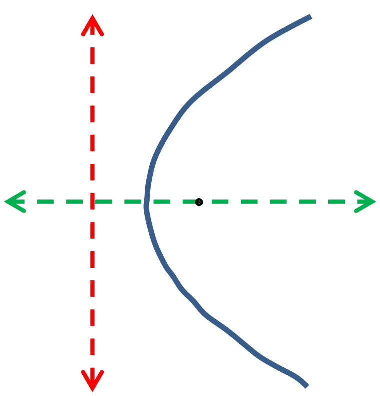
1. $x^2 + y^2 - 4x + 12y - 6 = 0$

2. $x^2 - 6x = y - y^2 + 7$

(Re)Presenting the…Parabola

The conic parabolas you learned about in Analytic Geometry were either functions (opened either up or down) or relations (opened either left or right). Let's do a little parabola review and see what you remember about these.

The conic parabolas you learned about in Analytic Geometry were either functions (opened either up or down) or relations (opened either left or right). Let's do a little parabola review and see what you remember about these.



Label the following features on the sketch to the left.

Vertex
Focus
Axis of Symmetry
Directrix
The directed distance p (label 2 different places)

Now let's see if you can answer some basic questions.

(a) What relationship does the locus of points forming a parabola have with the focus and directrix?

(b) What relationship does the vertex have with the focus and the directrix?

(c) What relationship does the directed distance p have with the focus and the directrix?

Just as with circles, the most useable form for parabolas is standard form. Therefore, we need to know the following:

Standard Form of a Parabola and Related Information
With vertex (h, k) and directed distance from the vertex to the focus p:

$$Vertical\ Axis\ of\ Symmetry: (x-h)^2 = 4p(y-k)$$

If p is positive, the parabola opens up; if p is negative, the parabola opens down.

$$Horizontal\ Axis\ of\ Symmetry: (y-k)^2 = 4p(x-h)$$

If p is positive, the parabola opens to the right; if p is negative, the parabola opens to the left.

Let's try writing a few equations in standard form.

1. Write the equation of the parabola with a vertex at the origin and a focus at $(5, 0)$.

2. Write the equation of the parabola with focus at $(-3, 3)$ and directrix at $y = 9$.

3. Write the equation of the parabola that opens to the left, contains a distance of 5 between the focus and the directrix, and contains a vertex at $(9, 6)$.

Just as with circles, often you will be given either an equation for a parabola that is not in standard form and you'll need to convert the equation to standard form. Consider the following equation of a parabola:

$$5y^2 - 6x + 10y - 7 = 0$$

This parabola has been written in general form. Using what we know about the coefficients from general form, we have $C = 5, D = -6, E = 10,$ and $F = -7$. It's easy to see that the y term is squared, so either the parabola will open left or right, but beyond this, it's difficult to tell anything else about the relation. Therefore, once again, we will have to convert to standard form by manipulating terms and completing the square:

$$5y^2 - 6x + 10y - 7 = 0$$

$$5y^2 + 10y = 6x + 7$$

Completing the Square!!

$$5\left(y^2 + 2y + \left(\frac{2}{2}\right)^2\right) = 6x + 7 + 5\left(\frac{2}{2}\right)^2$$

$$5(y^2 + 2y + 1) = 6x + 12$$

$$5(y+1)^2 = 6(x+2)$$

$$(y+1)^2 = \frac{6}{5}(x+2)$$

So what do we now know? Well, we know the vertex of the parabola is at. $(-2,-1)$. We know the parabola opens to the right because p is positive. How do we know it's positive? Let's see…

Standard Form: $(y-k)^2 = 4p(x-h)$

so $$4p = \frac{6}{5} \; so \; p = \frac{6}{20} = \frac{3}{10}$$

Therefore the focus is at $\left(-2+\frac{3}{10},-1\right) = \left(\frac{-17}{10},-1\right)$ and the directrix would be at $x = -2 - \frac{3}{10}$ which simplifies to. $x = -\frac{23}{10}$

Convert the following equations of parabolas into standard form.

1. $x^2 + x - y = 5$

2. $2y^2 + 16y = -x - 27$

3. $x = -y^2 + 6y - 5$

Two more things before we go…

Circles and parabolas are from the past – they're not our focus now. But the next two conic sections are built upon your knowledge of these simplest of conics. Therefore, think about (and answer!) these two questions.

1. We know that the general form of a quadratic relation is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

What relationship do the coefficients A and C have for a circle? For a parabola?

2. Why was this activity named "our only focus"?

Conics Homework Set 1

Find the center and radius of the circle with the given equation.

1. $3x^2+3y^2-12x-24y+12=0$

2. $(x+3)^2+(y-4)^2=25$

3. $x^2+y^2+6x-2y=6$

Find the standard form of the equation of the circle with the given radius and center.

4. Center (10,-5) and radius = 10

5. Center (0,0) and radius = 2

6. Center (-8,-5) and radius = $2\sqrt{5}$

Graph the circle with the equation given.

7. $2x^2+2y^2=32$

8. $(x+1)^2+(y-1)^2=9$

9. $x^2+y^2+4x+6y+9=0$

10. $3x^2+3y^2-12x-24y+12=0$

Conics Homework Set 2

Write the standard equation of the parabola with the given characteristics.

1. Focus (-2,1) and directrix x = 3

2. Focus (-2,3) and directrix y = -3

3. Vertex (3, -5) and directrix x= 2

4. Focus (-2,0) and vertex at the origin

Write the standard equation of the ellipse with the given characteristics.

5. Vertices (-8,1), (0,1), (-4,7), (-4,5), and center (-4,1)

6. Center at the origin , foci (±2,0), and vertices (±5,0)

7. Vertices (±8,0), foci(±5,0)

8. Vertices (0,±7), foci(0,±2)

Graph the conic with the following equation.

9. $4x^2+5y^2=20$

10. $\frac{(x+3)^2}{25}+\frac{(y+2)^2}{36}=1$

11. $13x^2-8y-9=0$

12. $(y-2)^2=-3(x+4)$

Change from general to standard form.

13. $x^2+9y^2-6x+90y=-225$

14. $4x^2+y^2-8x-2y+1=0$

15. $x^2+6x+4y+5=0$

16. $y^2+12x+2y=-25$