

MATH 1

UNIT 3

Geometry

Mathematics I – Unit 3: Geometry

INTRODUCTION:

The study of geometry requires an understanding of the way we think. Inductive and deductive reasoning has been introduced in unit one. In this unit students will explore, understand, and use the formal language of reasoning and justification. Students will be presented with the opportunity to use logical reasoning and proofs. Students will prove conjectures through multiple forms of justification. Students will explore angles, triangle inequalities, congruencies, and points of concurrency; and apply properties to determine special quadrilaterals.

ENDURING UNDERSTANDINGS:

- Properties of angles, triangles, quadrilaterals, and polygons are connected.
- Geometric ideas are significant in all areas of mathematics such as: algebra, trigonometry, and analysis.
- Geometric ideas are appropriate for describing many aspects of our world.

KEY STANDARDS ADDRESSED:

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

- a. Determine the sum of interior and exterior angles in a polygon.
- b. Understand and use the triangle inequality, the side-angle inequality, and the exterior-angle inequality.
- c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).
- d. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.
- e. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

RELATED STANDARDS ADDRESSED:

MM1G2. Students will understand and use the language of mathematical argument and justification.

- a. Use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof as appropriate
- b. Understand and use the relationships among a statement and its converse, inverse, and contrapositive.

MM1P1. Students will solve problems (using appropriate technology).

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

MM1P2. Students will reason and evaluate mathematical arguments.

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

MM1P3. Students will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

MM1P4. Students will make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- c. Recognize and apply mathematics in contexts outside of mathematics.

MM1P5. Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

UNIT OVERVIEW:

A great deal of geometry is part of the Mathematics 6 – 8 Georgia Performance Standards. The middle school curriculum has fostered the development of geometric thinking and investigated measurement geometry. In Mathematics I, students deepen and expand these geometric ideas. In order to achieve this goal, the tasks in this unit begin with the concrete and progress to the abstract. Students learn with a greater depth of understanding and retain more of the concepts when they are actively engaged in the learning process. The tasks in this unit are designed to motivate the students and to give them the opportunity to be actively involved in the construction of their own mathematics.

The concepts in this unit should build on the rich background students already have. In seventh grade, students thoroughly investigated basic geometric constructions including angle bisectors, perpendicular bisectors, and parallel and perpendicular lines. This unit will build upon that knowledge by using these constructions as they apply to triangles and polygons. Students also spent time in middle school investigating what it means to be congruent. In this unit, the focus is on the minimum information necessary to conclude that triangles are congruent. Building on prior knowledge of quadrilaterals, students conjecture and prove or disprove properties that allow classification of quadrilaterals. Students are expected to make and appropriately justify their conclusions. That justification may be through paragraph proofs, flow proofs, two-column proofs, and other forms of communicating mathematical ideas. The emphasis of this unit is the mathematics and the communication of it, not on the rote production of two-column proofs.

The tasks in this unit lend themselves to extensive use of manipulatives. All students may not have had access to the same manipulatives or technology during middle school so students may need an opportunity to investigate the use of other tools. Graphing calculators and dynamic geometry software are valuable tools and should be available to all students. In addition to technological tools, MIRAs, patty paper, a compass, and a straight edge can be powerful tools for investigating concepts in geometry. Students should not be "re-taught" those basic constructions from the seventh grade curriculum but rather be expected to explore relationships using them.

FORMULAS AND DEFINITIONS:

- Centroid: The point of concurrency of the medians of a triangle.
- Circumcenter: The point of concurrency of the perpendicular bisectors of the sides of a triangle.
- Incenter: The point of concurrency of the bisectors of the angles of a triangle.
- Orthocenter: The point of concurrency of the altitudes of a triangle.
- Sum of the measures of the interior angles of a convex polygon: $180^\circ(n - 2)$.
- Measure of each interior angle of a regular n-gon: $\frac{n-2}{n} \cdot 180^\circ$
- Exterior angle of a polygon: an angle that forms a linear pair with one of the angles of the polygon.
- Measure of the exterior angle of a polygon: equals the sum of the measures of the two remote interior angles.
- Remote interior angles: the two angles that are non-adjacent to the exterior angle.
- Exterior Angle Inequality: an exterior angle is greater than either of the remote interior angles.

THEOREMS:

- Exterior Angle Sum Theorem: If a polygon is convex, then the sum of the measure of the exterior angles, one at each vertex, is 360° . The Corollary that follows states that the measure of each exterior angle of a regular n-gon is $360^\circ/n$.
- If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.
- If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- If two sides of one triangle are equal to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle is larger than the included angle of the second.
- If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent. (SSS)

- If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. (SAS)
- If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent. (ASA)
- ASA leads to the AAS corollary. If two angles on a triangle are known, then the third is also known.
- If two triangles are congruent, then the corresponding parts of the two congruent triangles are congruent.
- Converse of the Pythagorean Theorem: If c is the measure of the longest side of a triangle, a and b are the lengths of the other two sides, and $c^2 = a^2 + b^2$, then the triangle is a right triangle.
- If two legs of one right triangle are congruent to the corresponding legs of another right triangle, the triangles are congruent. (LL)
- If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and corresponding angle of another right triangle, then the triangles are congruent. (HA)
- If one leg and an acute angle of a right triangle are congruent to the corresponding leg and angle of another right triangle, then the triangles are congruent. (LA)
- If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent. (HL)
- If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.
- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
- If each diagonal bisects a pair of opposite angles, then the parallelogram is a rhombus.
- If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
- If three parallel lines cut off equal segments on one transversal, then they cut off equal segments on every transversal.

Student Learning Map for Math 1 Unit 3

Topic: Geometry

Unit Enduring Understandings:

1. Properties of angles, triangles, quadrilaterals, and polygons are connected.
2. Geometric ideas are significant in all areas of mathematics such as: algebra, trigonometry, and analysis.
3. Geometric ideas are appropriate for describing many aspects of our world.

Unit Essential Questions:

1. How can you determine relationships between angles and sides of triangles?
2. How can you find and use points of concurrency?
3. How can you prove two triangles are congruent?
4. What are the properties of the special quadrilaterals?

Instructional Tools Needed for Unit 3:

1. Elmo or Overhead Projector
2. Graphing Calculators
3. Geometer's Sketchpad
4. MIRA
5. Compass
6. Straightedge
7. Protractor
8. Patty Paper

Concept 1: Measures of interior and exterior angles of polygons Lesson Essential Question 1. How do you find the sum of the measures of all the interior or exterior angles of a polygon? 2. How do you find the measure of one interior or of one exterior angle of a regular polygon? 3. What is the relationship between an exterior angle of a triangle and the remote interior angles? Vocabulary 1. Convex Polygon 2. Concave Polygon 3. Remote Interior Angles Notes: Vocabulary to Maintain: Interior and Exterior Angles Diagonal of a Polygon Regular Polygon	Concept 2: Triangle Inequalities Lesson Essential Questions 1. What are the relationships between the sides and angles of a triangle? 2. How do you use the Pythagorean Theorem to determine the type of triangle you are given? Vocabulary 1. Triangle Inequality 2. Side-angle Inequality 3. Side-side Inequality 4. Exterior Angle Inequality Notes:	Concept 3: Triangle Points of Concurrency Lesson Essential Questions 1. How do you find the points of concurrency in a triangle? 2. How can you solve problems using points of concurrency? Vocabulary 1. Incenter 2. Orthocenter 3. Circumcenter 4. Centroid Notes:	Concept 4: Congruency Theorems for Triangles Lesson Essential Question How do you determine and justify the congruence of two triangles? Vocabulary 1. SSS 2. SAS 3. ASA 4. AAS 5. LL 6. HA 7. LA 8. HL Notes: Corresponding parts of congruent triangles are congruent (CPCTC)	Concept 5: Relationships Between Special Quadrilaterals Lesson Essential Questions 1. What characteristics differentiate the following quadrilaterals: parallelogram, rectangle, square, rhombus, trapezoid, and kite? 2. If three parallel lines cut off equal segments of one transversal, how would the lengths of the segments of any transversal be related? Vocabulary: 1. Kite Notes: Vocabulary to maintain: 1. Quadrilateral 2. Parallelogram 3. Rectangle 4. Square 5. Rhombus 6. Trapezoid 7. Diagonal
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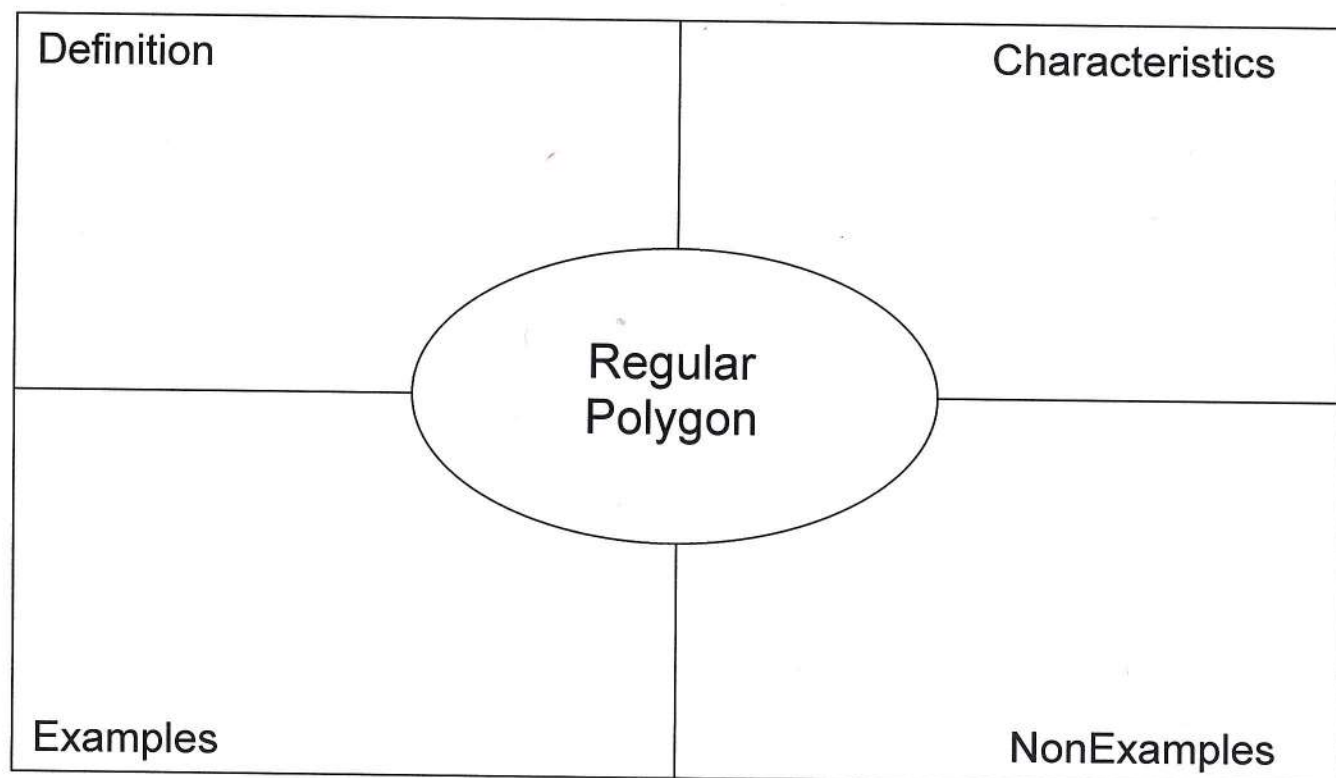
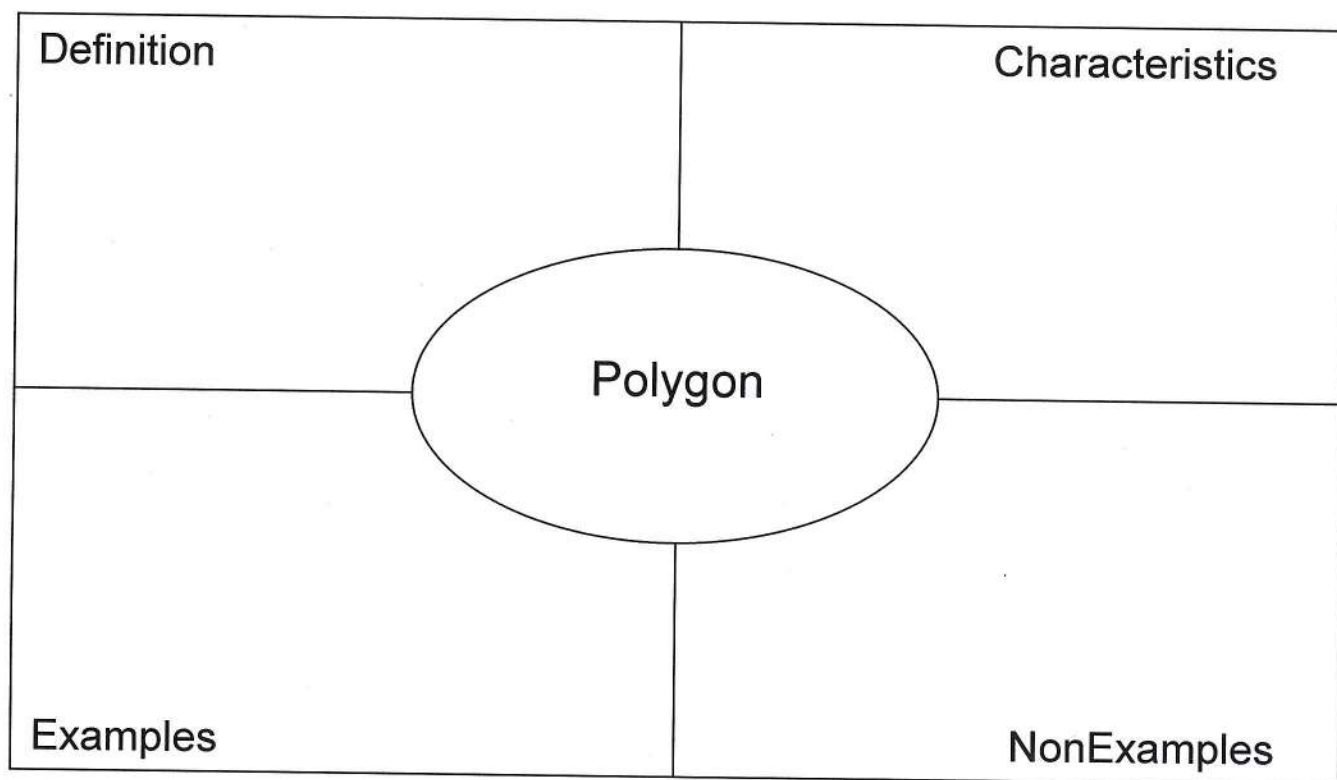
VOCABULARY

VOCABULARY

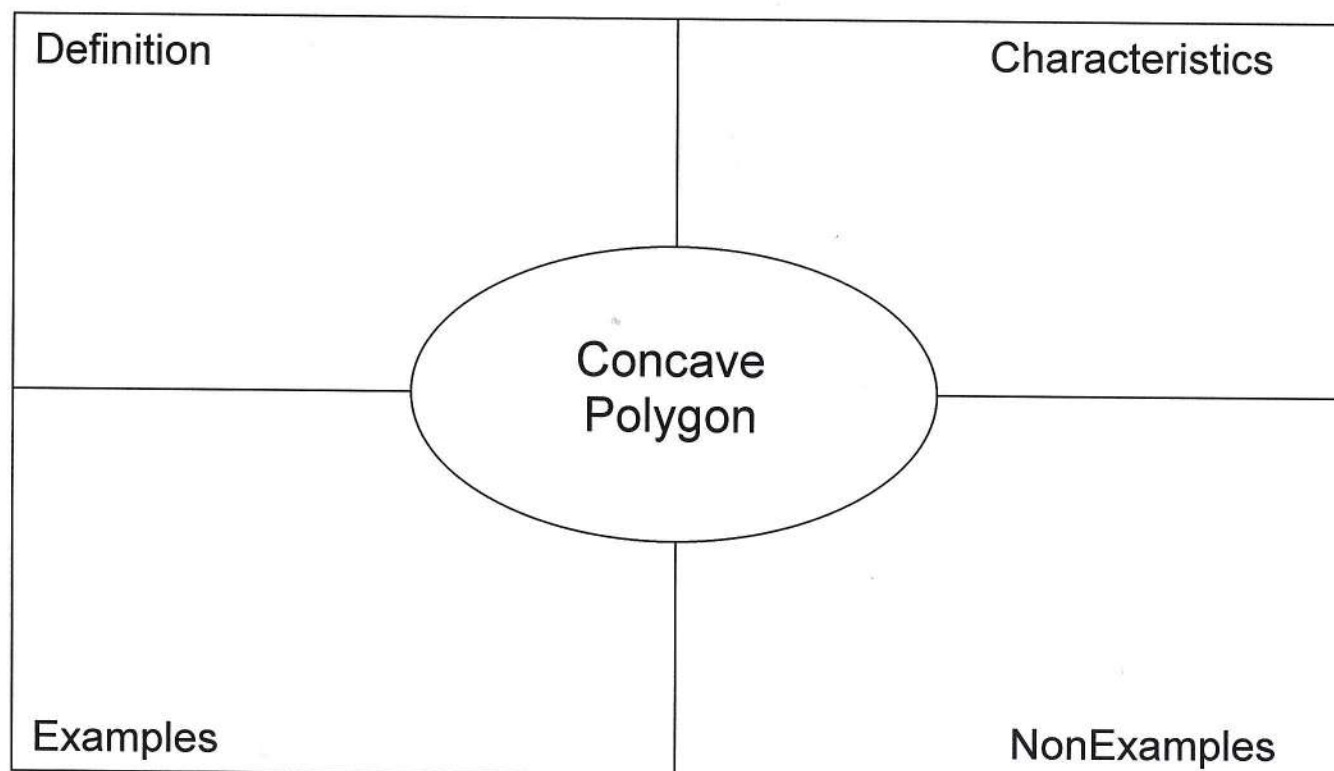
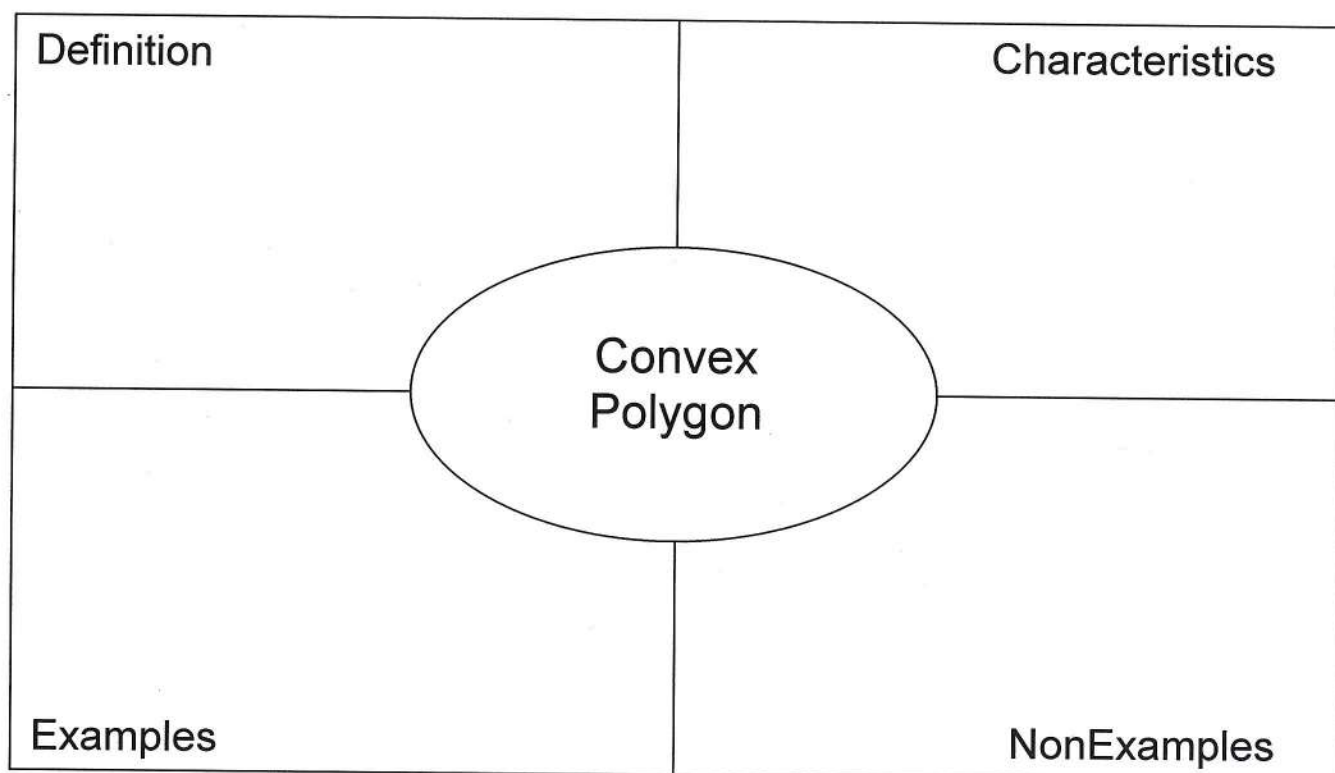
Polygons Graphic Organizer/Summarizer

What is a polygon?	
Examples	Non-examples
What is a regular polygon?	
What is a concave polygon?	What is a convex polygon?
What is the sum of the interior angles of a polygon? Interior Angle Sum Theorem	
What is the measure of an interior angle of a regular polygon?	
What is the sum of the exterior angles of a convex polygon? Exterior Angle Sum Theorem	
What is the measure of an exterior angle of a regular polygon?	

Frayer Models



Frayer Models

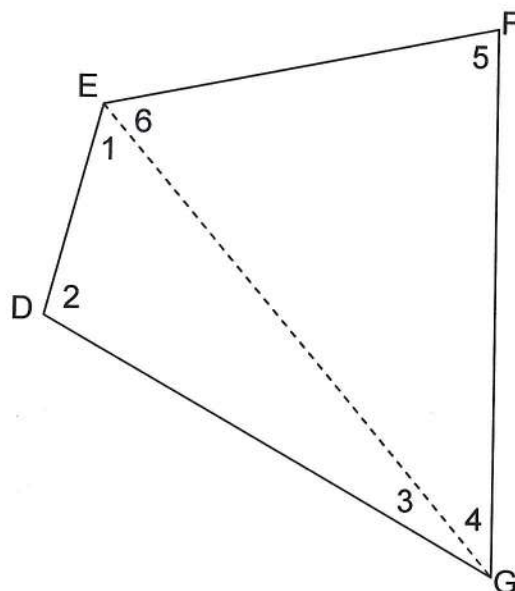


Interior Angle Investigation

Consider the quadrilateral to the right.
Draw diagonal EG (the dotted line).
A diagonal is a segment connecting a vertex with a nonadjacent vertex.

The quadrilateral is now divided into two triangles.
What are they?

Angles 1, 2, and 3 represent the interior angles of triangle DEG and angles 4, 5, and 6 represent the interior angles of triangle FEG.



$$m\angle 1 + m\angle 2 + m\angle 3 = \underline{\hspace{2cm}}.$$

$$m\angle 4 + m\angle 5 + m\angle 6 = \underline{\hspace{2cm}}.$$

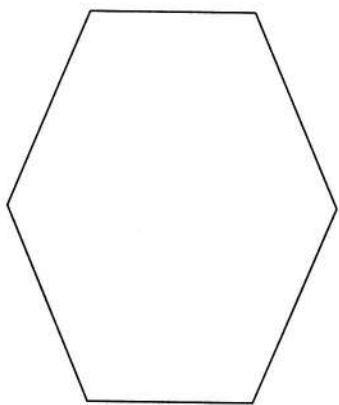
$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = \underline{\hspace{2cm}}.$$

What is the relationship between the sum of the angles in the quadrilateral and the sum of the angles in the two triangles?

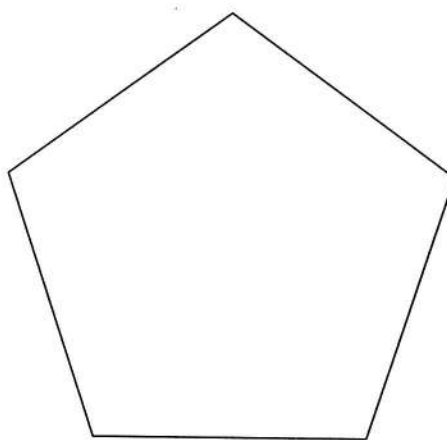
This procedure can be used to determine the sum of the interior angles in any polygon.

1. Draw the polygon.
2. Select one vertex.
3. Draw all possible diagonals from that vertex.
4. Determine the number of triangles formed.
5. Multiply the number of triangles by the sum of the interior angles of any triangle.

Try it with the polygons below.



Sum of the interior angles of a
hexagon is $\underline{\hspace{2cm}}$.



Sum of the interior angles of a
pentagon is $\underline{\hspace{2cm}}$.

Graphic Organizer #1 Interior Angle Sum

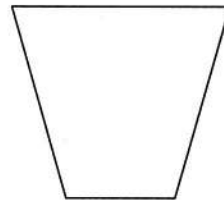
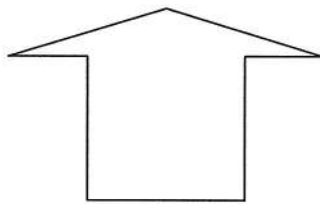
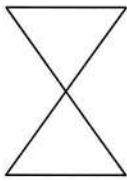
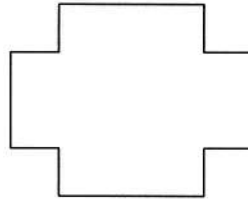
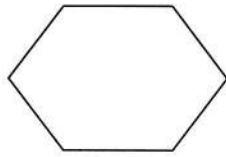
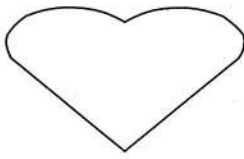
Polygon	Sketch	Number of Sides of Polygon	Number of Triangles Formed	Number of Degrees in a Triangle	Interior Angle Sum
Triangle					
Quadrilateral					
Pentagon					
Hexagon					
Heptagon					
Octagon					
Decagon					
Dodecagon					
n-gon					

Consider the table on the previous page and answer the following questions:

1. What patterns do you notice in the table? When you know the number of triangles formed, how do you find the sum of the interior angles of the triangle?
2. In the last row of the table you should have developed a formula for finding the sum of the interior angles of a polygon. Use this formula to find the sum of the interior angles of a 20-gon.
3. Write a sentence explaining how to find the sum of the interior angles of a polygon.
4. The measures of the angles in a convex quadrilateral are $2x$, $2(2+1)$, $x-5$, and $3(x-2)$.
 - a. Sketch and label the figure.
 - b. What is the sum of the interior angles of a convex quadrilateral?
 - c. Find x
 - d. Find each angle measure.
5. The measures of the angles in a convex pentagon are 120° , $2y-5$, $3y-25$, $2y$, and $2y$.
 - a. Sketch and label the figure.
 - b. What is the sum of the interior angles of a convex quadrilateral?
 - c. Find y
 - d. Find each angle measure.

Problems for Practice:

1. Classify the following as either a polygon or not a polygon. If it is a polygon, further classify it as convex or concave.



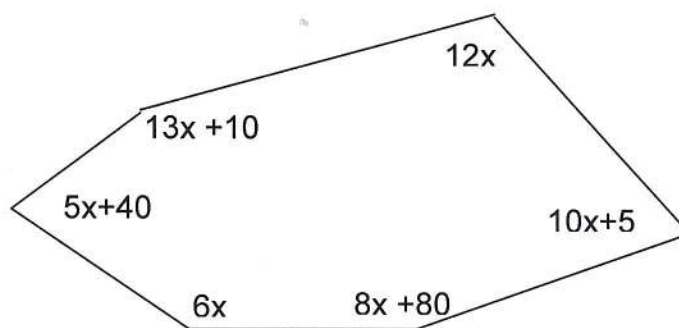
2. Find the sum of the interior angles of the following convex polygons:

24-gon:

13-gon:

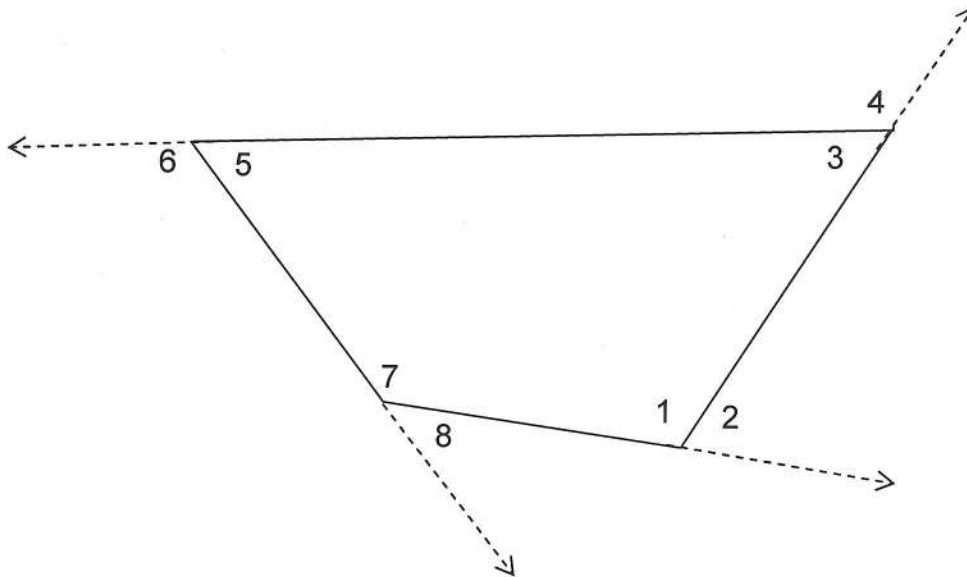
3. The four interior angles of a quadrilateral measure $x-5$, $3(x+8)$, $3x+6$, and $5x-1$. Find the measures of the four angles.

4. Find each angle measure in the figure below.



Exterior Angle Exploration

An exterior angle of a polygon is formed by extending a side of the polygon outside the figure.



1. How many exterior angles does the quadrilateral have? Name them.
2. Using your protractor, find the measure of $\angle 1$.
3. What kind of angles are $\angle 1$ and $\angle 2$? What is their sum?
4. Find the measure of $\angle 2$.
5. Find the measure of each of the remaining exterior angles in the same way.
6. What is the sum of the exterior angles?

Angles of Regular Polygons

Regular Polygon	Interior Angle Sum	Measure of Each Interior Angle	Exterior Angle Sum	Measure of Each Exterior Angle
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
Heptagon				
Octagon				
Decagon				
Dodecagon				
n-gon				

Find the measure of one interior angle of a regular 18-gon.

Find the measure of one exterior angle of a regular 14-gon.

Guided Practice Problems

1. An exterior angle of a regular polygon measures 72° . What is the measure of its corresponding interior angle?
2. What is the measure of one interior angle of a regular 40-gon?
3. What is the measure of one exterior angle of a regular 40-gon?
4. If the exterior angles of a quadrilateral measure 93° , 78° , 104° , and x° , find the value of x .
5. If an exterior angle of a regular polygon measures 45° , how many sides does the polygon have?
6. If an interior angle of a regular polygon measures 160° , how many sides does the polygon have?

Summarizer

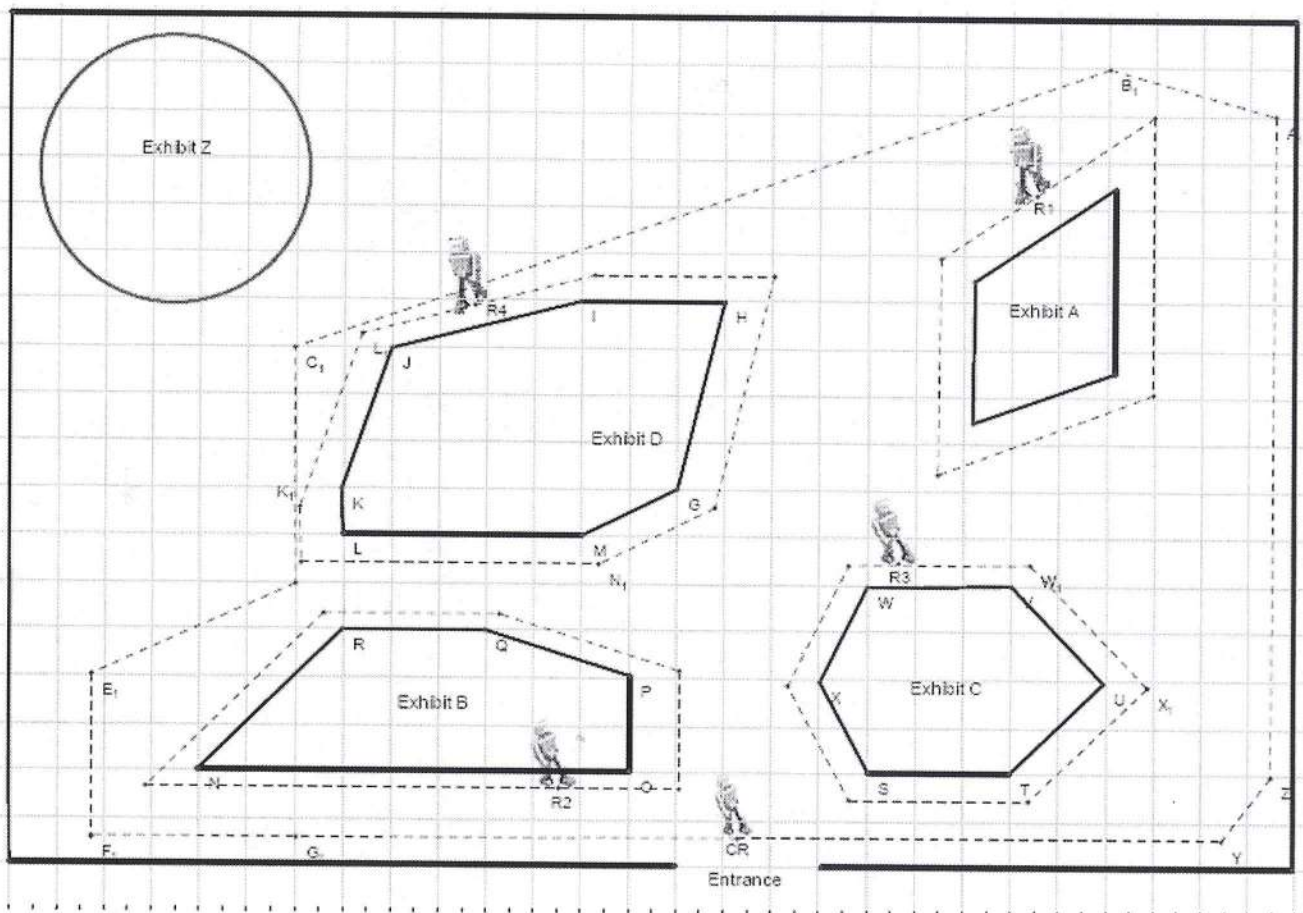
Sum of the Interior Angles of <u>any</u> polygon:	Measure of Each Interior Angle of any <u>regular</u> polygon:
Sum of the Exterior Angles of <u>any</u> polygon:	Measure of Each Exterior Angle of any <u>regular</u> polygon

Robotic Gallery Guards Learning Task

The Asimov Museum has contracted with a company that provides Robotic Security Squads to guard the exhibits during the hours the museum is closed. The robots are designed to patrol the hallways around the exhibits. The robots are equipped with cameras and sensors that detect motion.

Each robot is assigned to patrol the area around a specific exhibit. They are designed to maintain a consistent distance from the wall of the exhibits. Since the shape of the exhibits change over time, the museum staff must program the robots to turn the corners of the exhibit.

Below, you will find a map of the museum's current exhibits. One robot is assigned to patrol each exhibit. There is one robot, Captain Robot, CR, who will patrol the entire area.



1. What angles will R1 need to turn? What is the total of these turns?
2. What angles will R2 need to turn? What is the total of these turns?

3. What angles will R3 need to turn? What is the total of these turns?
4. What angles will R4 need to turn? What is the total of these turns?
5. What angles will CR need to turn? What is the total of these turns?
6. What do you notice about the sum of the angles? Do you think this will always be true?
7. Determine the measure of the interior angles of the polygons formed by Exhibits A – D route.
8. Look at the sum of the angles in the polygons. Do you notice a pattern?
9. How could you determine the sum of the angles in the exhibits without using a protractor?
10. Can you generalize your results from #9 so that you could find the sum of the interior angles of a decagon? Dodecagon? An n -gon?
11. Choose one of the exhibits from the map and look at the interior and exterior angle found at a vertex. What do you notice about the sum of these two angles? Find this sum at each vertex. What do you notice?

12. Looking at your results from #10 and #11, can you find a way to prove your conjecture about the sum of the exterior angles from #6?
13. The museum intends to create regular polygons for its next exhibition, how can the directions for the robots be determined for a regular pentagon? Hexagon? Nonagon? N-gon?
14. A sixth exhibit was added to the museum. The robot patrolling this exhibit only makes 15° turns. What shape is the exhibit? What makes it possible for the robot to make the same turn each time?
15. Robot 7 makes a total of 360° during his circuit. What type of polygon does this exhibit create?
16. The sum of the interior angles of Robot 7's exhibit is $3,420^\circ$. What type of polygon does the exhibit create?
17. All of the current exhibits and the robots are convex polygons. Would your generalizations hold for a concave polygon?
18. The museum is now using Exhibit Z. Complete a set of instructions for RZ that will allow the robot to make the best circuit of this circular exhibit. Defend why you think your instructions are the best.

VOCABULARY

VOCABULARY

VOCABULARY

Triangular Inequality

Anticipation Guide

Name: _____ Date: _____ Period: _____

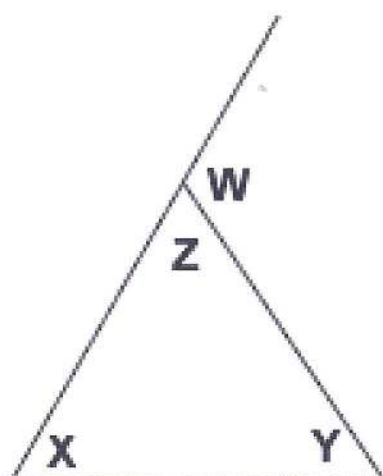
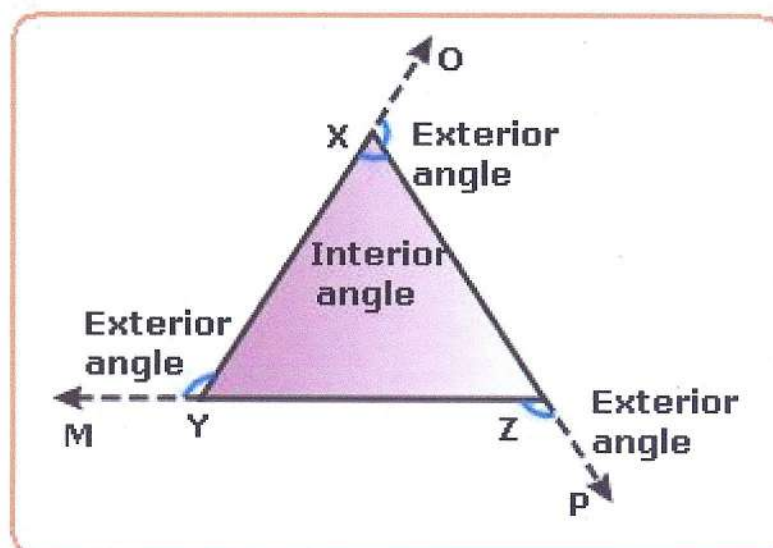
TRUE or FALSE

- _____ 1. The lengths of the sides of a triangle affects the size of the angles of the triangle.
- _____ 2. Any 3 segments can form a triangle.
- _____ 3. A triangle can have 2 right angles.
- _____ 4. In a right triangle, the hypotenuse is always the side with the greatest measure.
- _____ 5. An exterior angle of a triangle is always greater than either of its remote interior angles.
- _____ 6. The sum of 2 sides of a triangle will be less than the length of the third side.
- _____ 7. The Pythagorean Theorem can show if a triangle is acute.
- _____ 8. The smallest angle in a triangle is opposite the longest side of the triangle.
- _____ 9. Obtuse triangles have more degrees in the triangle than acute triangles.
- _____ 10. The largest angle in a triangle is opposite the longest side of the triangle.

UNIT 3 LESSON 2

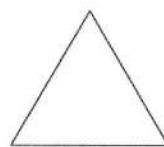
STANDARD: UNDERSTAND THE USE OF TRIANGLE INEQUALITY, THE SIDE-ANGLE INEQUALITY, AND THE EXTERIOR ANGLE INEQUALITY

E.Q.: WHAT ARE THE RELATIONSHIPS BETWEEN THE LENGTHS OF THE SIDES AND THE MEASURES OF THE ANGLES OF A TRIANGLE?



MATCH THE TRIANGLES

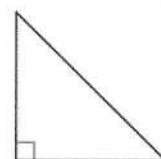
ACUTE TRIANGLE



SCALENE TRIANGLE



RIGHT TRIANGLE



EQUILATERAL TRIANGLE



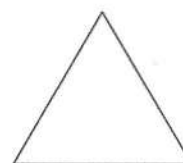
ISOSCELES TRIANGLE

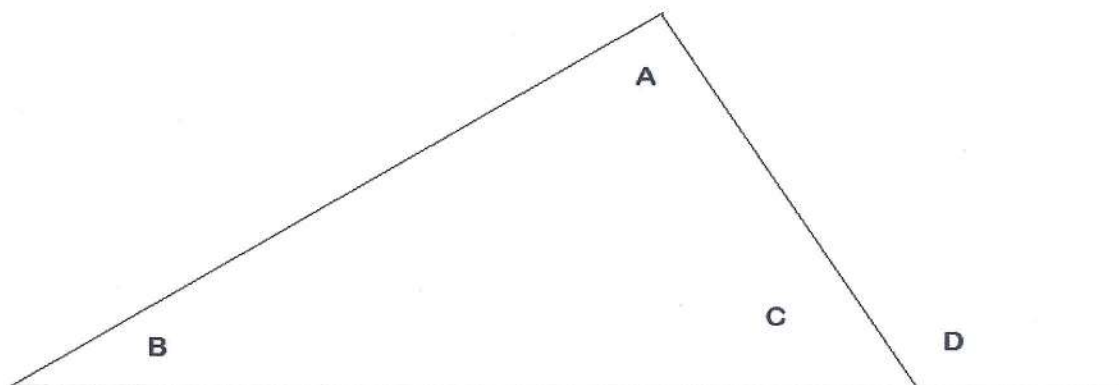


EQUIANGULAR TRIANGLE



OBTUSE TRIANGLE





MEASURE INTERIOR ANGLES A, B & C.

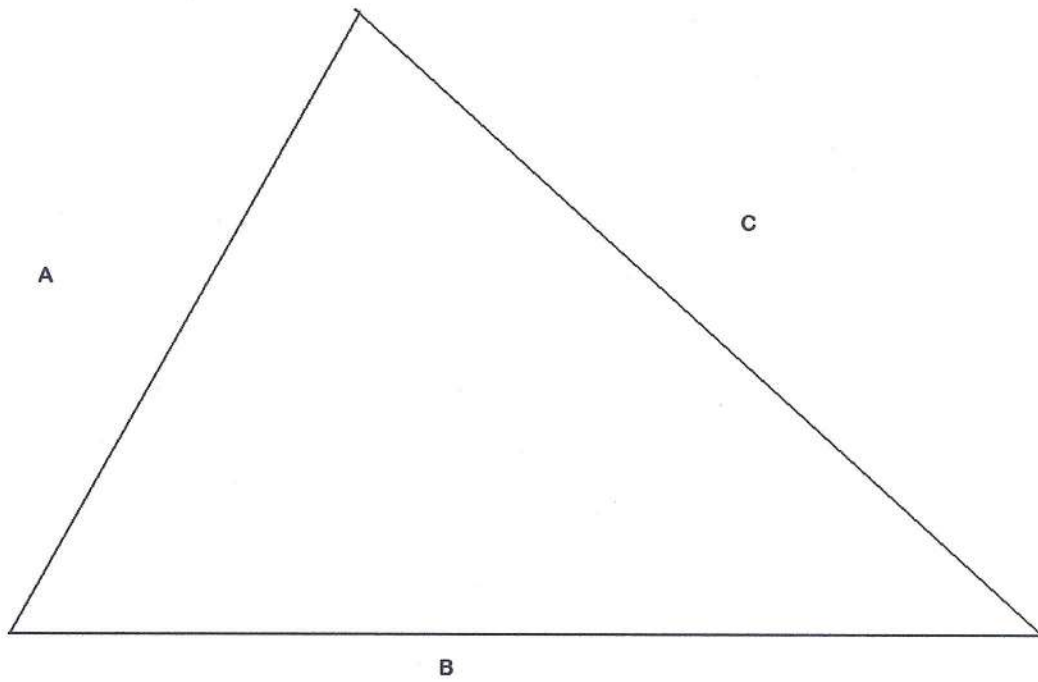
MEASURE EXTERIOR ANGLE D

FILL IN THE FOLLOWING:

$$A + B + \underline{\hspace{1cm}} = 180$$

$$A + \underline{\hspace{1cm}} = C$$

$$180 - C = \underline{\hspace{1cm}}$$



MEASURE THE SIDES OF THE TRIANGLE TO THE NEAREST MILLIMETER.

TRUE OR FALSE:

____ $A + B = C$

____ $A + C = B$

____ $A + C > B$

____ $B - A = C$

____ $A + C < B$

Activator #1: Soda Straw Experiment

These combinations of straws will form a triangle.

Length of Shortest Straw (cm)	Length of Middle Straw (cm)	Length of Longest Straw (cm)

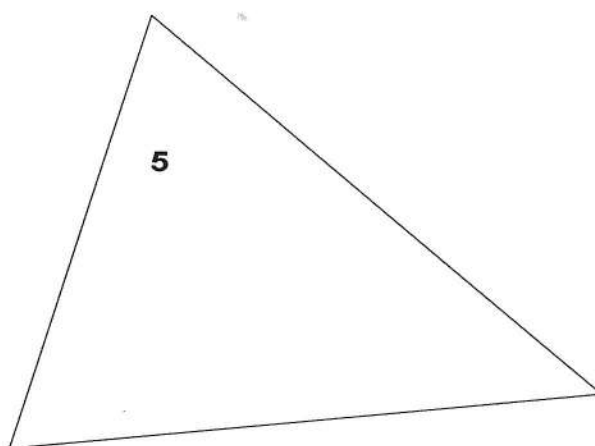
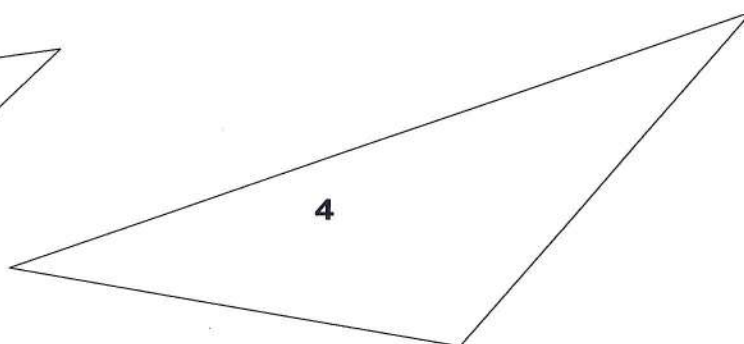
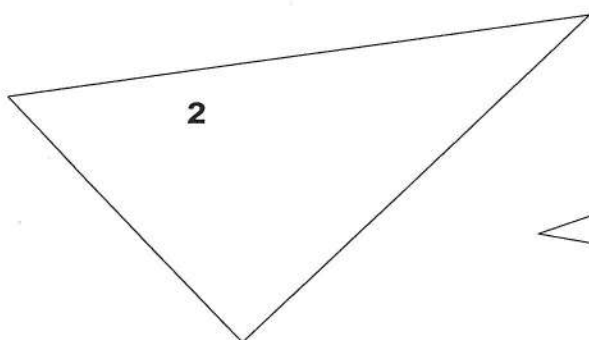
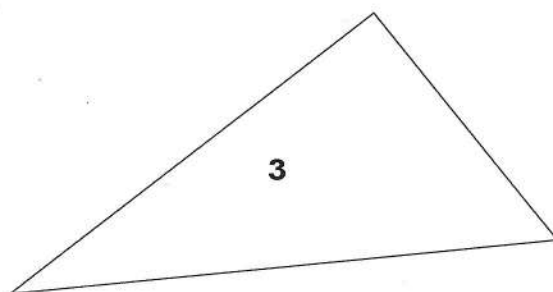
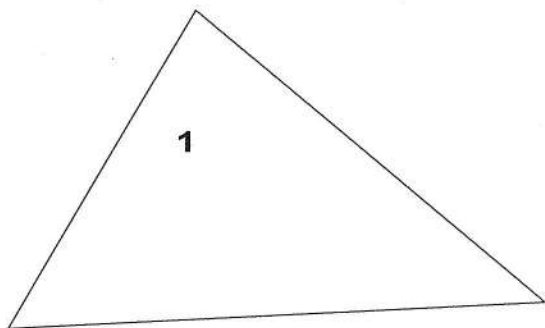
These combinations of straws will not form a triangle.

Length of Shortest Straw (cm)	Length of Middle Straw (cm)	Length of Longest Straw (cm)

Conclusion: _____

Activator #2: Triangle Inequality

Measure the lengths of the three sides of each of the following triangles. Compare the sum of any two of those sides of one triangle to the third side in the same triangle. What do you notice is ALWAYS true?

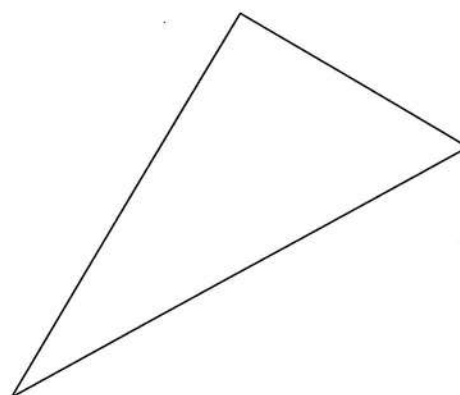
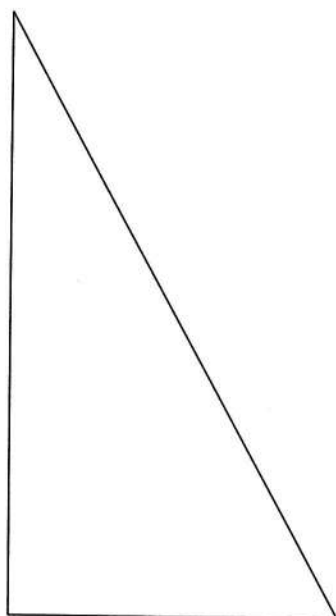
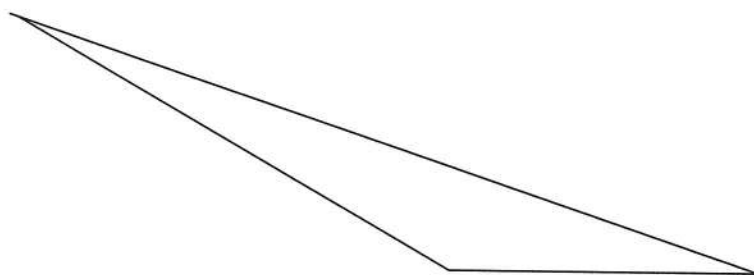
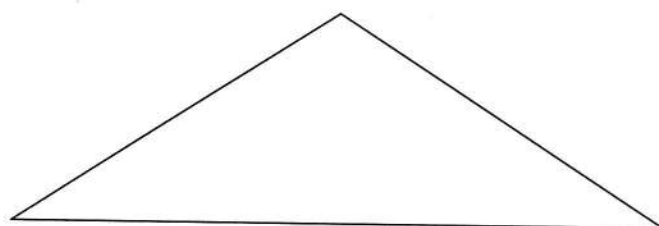
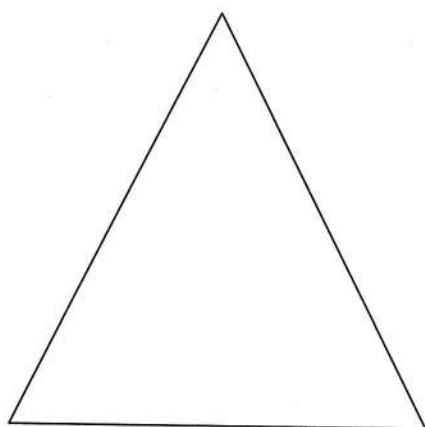


Triangle Measurement Activating Strategy #3

Measure the angles of each triangle with a protractor. Put the measure (to the nearest degree) inside the angle.

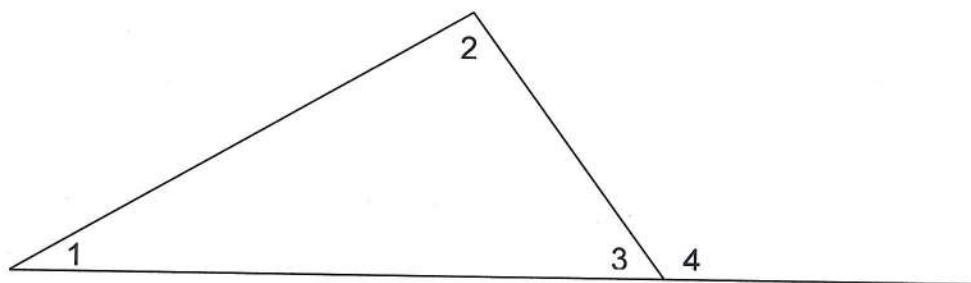
Measure the lengths of the sides to the nearest millimeter. Put that measurement beside the line segment.

What do you see?



Activator #4 – Exterior and Remote Interior Angles

Use this figure to help visualize the following questions.



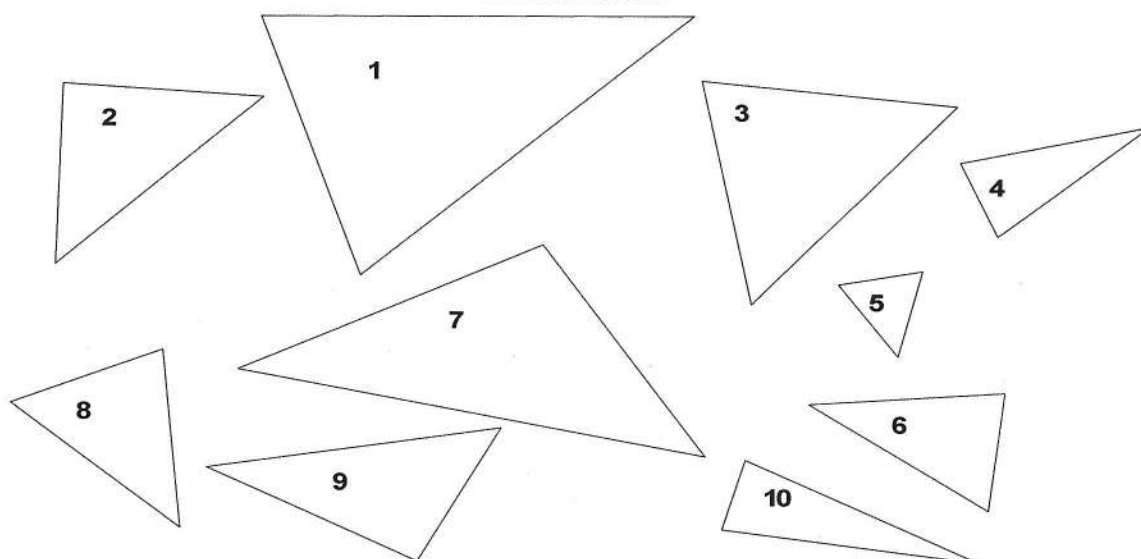
Measure the four angles to the nearest degree with a protractor.

Measure of $\angle 1$	Measure of $\angle 2$	Measure of $\angle 3$	Measure of $\angle 4$

1. What kind of angles are $\angle 3$ and $\angle 4$? _____
2. What should the sum of $\angle 3$ and $\angle 4$ be? _____
3. What is $\angle 3 + \angle 4$ from your measurements? _____
4. What is $\angle 1 + \angle 2$? _____
5. $\angle 4$ is called an exterior angle. Angles 1 and 2 are called the remote interior angles. Why? _____
5. Can you make any conjecture about $\angle 4$ and the sum of $\angle 1$ and $\angle 2$?
_____ What would it be? _____

Triangle Inequality Theorem Task 1

For each triangle, measure each of the sides. Then, fill in the chart with the needed information.



Triangle #	Shortest Side	Middle Side	Short + Middle	Inequality < or >	Longest Side
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Triangle Inequality Learning Task 2 (adapted from the Illuminations website)

During this activity, you will compare the sum of the measures of any two sides of a triangle with the measure of the third side.

1. Break a piece of linguini into three pieces or cut straws into three pieces, and use the pieces to form a triangle. Measure each side length to the nearest tenth of a centimeter. In the table below, record the measures of each side of the triangle from smallest to largest; then, find the sum of the measures of the small and medium sides. Repeat this activity twice, with two other triangles, to complete the chart.

Small	Medium	Large	Small + Medium

2. Break a piece of linguini or cut straws into three pieces so that it is impossible to form a triangle. Measure each side of the non-triangle to the nearest tenth of a centimeter. In the table below, record the measures of each side of the non-triangle from smallest to largest; then, find the sum of the measures of the small and medium sides. Repeat this activity twice, with two other non-triangles, to complete the chart.

Small	Medium	Large	Small + Medium

3. Compare the sum of the measures of the small and medium sides to the measure of the large side for each triangle you created. Describe what you notice.
4. Compare the sum of the measures of the small and medium sides to the measure of the large side for each non-triangle you created. Describe what you notice.
5. Based on your observations, write a conjecture about the relationship between the sum of the measures of the small and medium sides of a triangle and the measure of the large side of the triangle. Provide a reason for your conjecture.

6. Using a partner's measurements, test your conjecture. If your conjecture holds for your partner's measurements, provide a convincing reason why your conjecture would hold for any triangle. If your conjecture does not hold for your partner's measurements, revise your conjecture.
7. Is it possible to have a triangle such that the sum of the measures of the small and medium sides is equal to the measure of the large side? Provide a convincing reason for your answer. (You may use linguini, if you like.)
8. If the sum of the measure of the small and medium sides of the triangle is greater than the measure of the large side of the triangle, we can conclude that the sum of the measures of any other pair of sides of the triangle will be greater than the measure of the remaining side. Explain why this conclusion is possible.
9. In the box below, write three inequalities that are always true for a triangle with side lengths s , m , and l . (These inequalities should be based on your conclusion from Question 8.)

Triangle Inequality Theorem

In a triangle with side lengths s , m , and l

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} > \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} > \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} > \underline{\hspace{1cm}}$$

10. Consider the situation where you know that two sides of a triangle are 4 and 7. Would each of the following values be possibilities for the third side of the triangle?

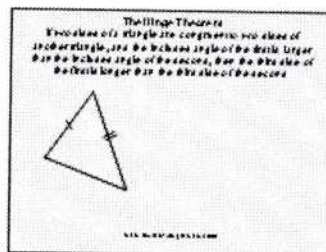
Possible length of 3 rd side	Yes or No?
3	
3.5	
4	
5	
6	
7	
8	
9	
10	
10.5	
11	

11. Using the information from number 10, what are all the possible values for the third side? Write your answer as an inequality.

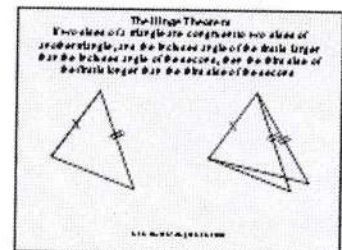
Powerpoint Presentation on the Hinge Theorem



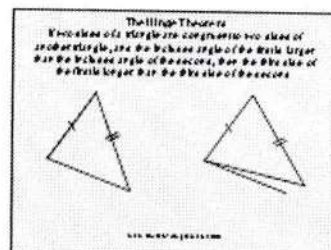
1



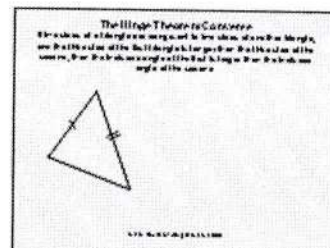
2



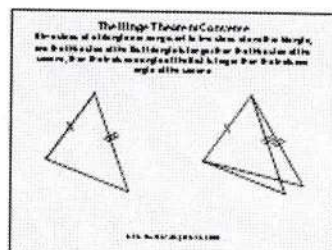
3



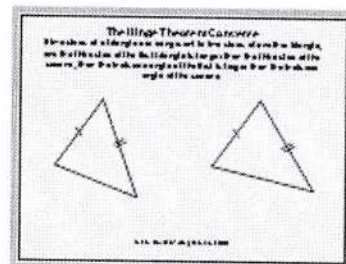
4



5



6

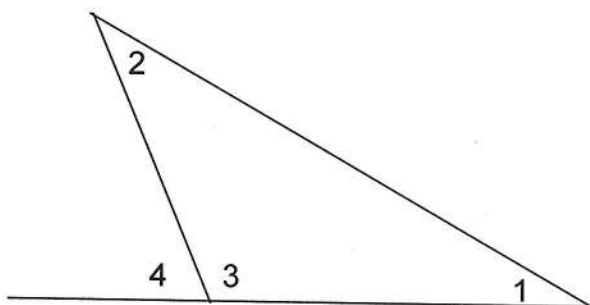


7

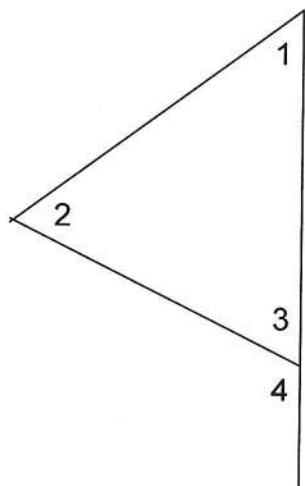
Identifying Exterior and Interior Angles Graphic Organizer #1

Identify the interior and exterior angles of each triangle. Measure with a protractor the two remote interior angles. Then calculate the measure of the remote interior angle and the third interior angle.

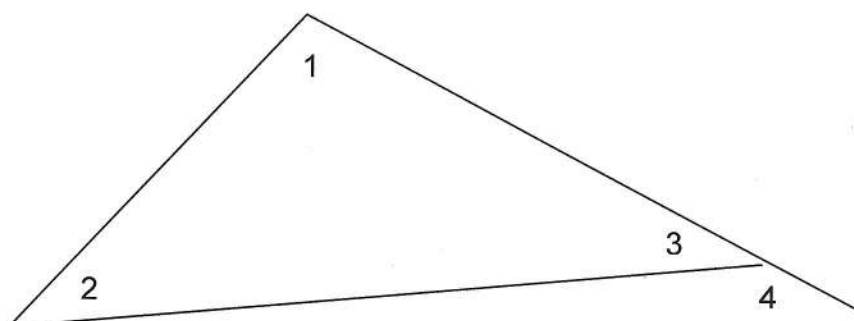
Triangle 1



Triangle 2

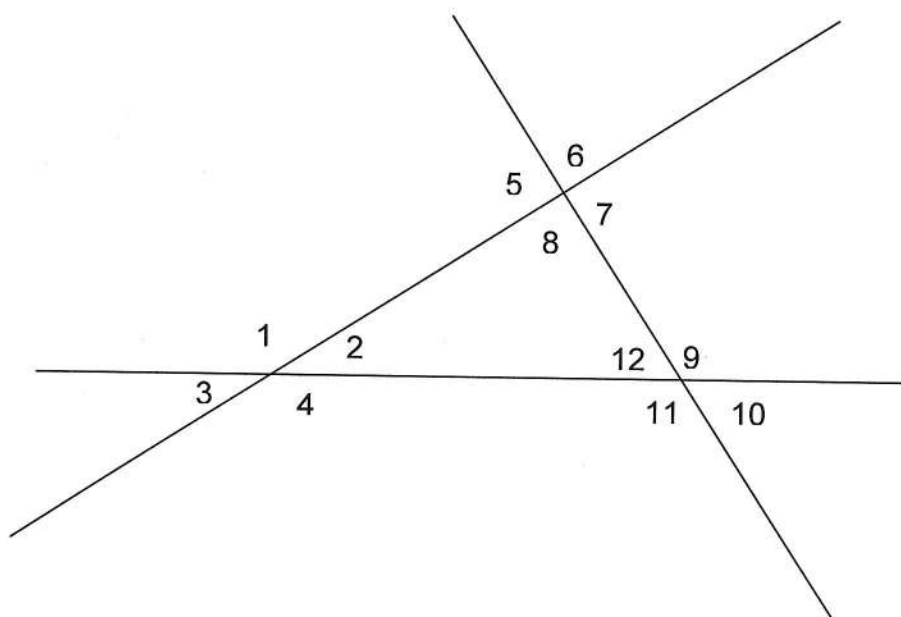


Triangle 3



Interior and Exterior Angles Continued

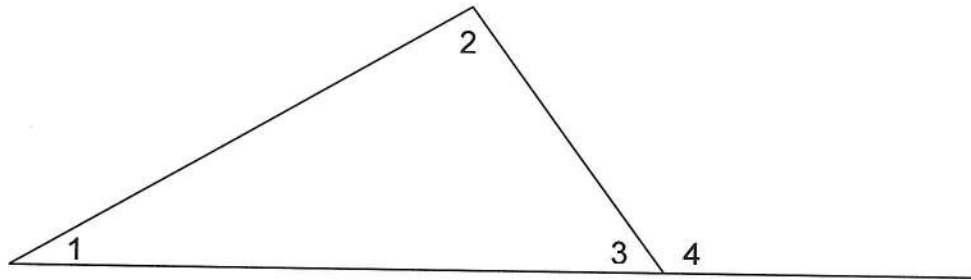
Answer the following questions using the following figure.



Consider the triangle in the center of the drawing:

1. What are the interior angles?
2. What are the exterior angles?
3. With reference to angle 9, what are the two remote interior angles?
4. Given remote interior angles 8 and 12, what is the exterior angle?
5. With reference to angle 4, what are the two remote interior angles?
6. Given remote interior angles 2 and 12, what is the exterior angle?

Exterior Angle Theorem Practice



1. Given $m\angle 1 = 30$ and $m\angle 2 = 65$, find the $m\angle 4$. _____
2. Given $m\angle 1 = 43$ and $m\angle 2 = 87$, find the $m\angle 4$. _____
3. Given $m\angle 1 = 56$ and $m\angle 2 = 64$, find the $m\angle 4$. _____
4. Given $m\angle 1 = 35$ and $m\angle 2 = 68$, find the $m\angle 4$. _____
5. Given $m\angle 1 = 48$ and $m\angle 2 = 73$, find the $m\angle 4$. _____

Comparisons

Put $<$ or $>$ in the blanks to form a true statement.

$$m\angle 1 \quad \underline{\hspace{1cm}} \quad m\angle 4$$

$$m\angle 2 \quad \underline{\hspace{1cm}} \quad m\angle 4$$

Frayer Models

Definition	Illustration
<div>Triangle Inequality Theorem</div>	
Examples that work	Examples that do not work

Definition	Illustration
<div>Side-Angle Inequality Theorem</div>	
Example	Another Example

Frayer Models

Definition	Illustration
<div>Side-Angle-Side Inequality (Hinge) Theorem</div>	
Example	Another Example

Definition	Illustration
<div>Exterior Angle Inequality Theorem</div>	
Example	Another Example

Ticket Out the Door

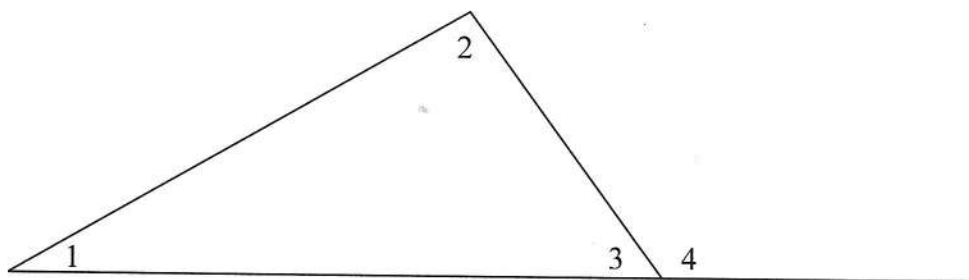
Name: _____

The measures of two sides of a triangle are 6 inches and 10 inches. What measures would work for the third side? Be sure to give all possible measures that would form a triangle with the 6 and 10 inch sides.

Ticket Out the Door

Name: _____

Find the measures of all the angles of the drawing if $m\angle 1 = x$, $m\angle 2$ is $2x$, and $m\angle 3 = 30^\circ$.



Poor Captain Robot Learning Task

Captain Robot's positronic brain is misfiring and he will only take instructions to move three distances. He will no longer acknowledge angles in directions and chooses all of them himself. He will not travel the same path twice and refuses to move at all if he cannot end up back at his starting point.

1. Captain Robot was given the following sets of instructions. Determine which instructions Captain Robot will use and which sets he will ignore. Be ready to defend your choices.

Instruction Set #1: 10 feet, 5 feet, 8 feet

Instruction Set #2: 7 feet, 4.4 feet, 8 feet

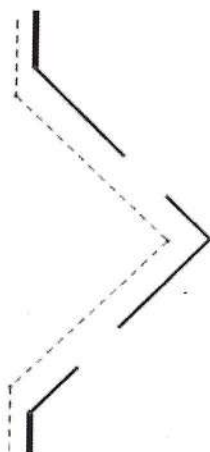
Instruction Set #3: 10 feet, 2 feet, 8 feet

Instruction Set #4: 5 feet, 5 feet, 2.8 feet

Instruction Set #5: 7 feet, 5.1 feet, 1 foot

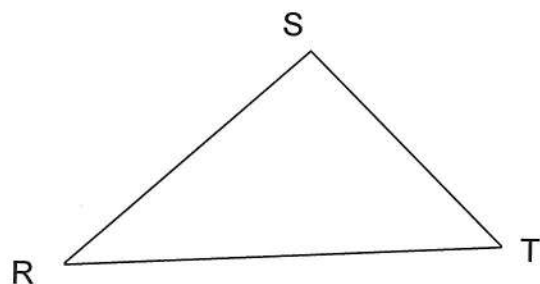
2. Determine a method that will always predict whether or not Captain Robot will move.
3. Captain Robot traveled from point A to point B to point C. His largest turn occurred at point C and his smallest turn occurred at point A. Order the sides of the triangle from largest to smallest.
4. Is there a relationship between the lengths of the side and the measures of the angles of a triangle? Explain why or why not.

The museum has decided Captain Robot needs to patrol two access doors off the side of the museum. Captain Robot's addition to his route is shown below.



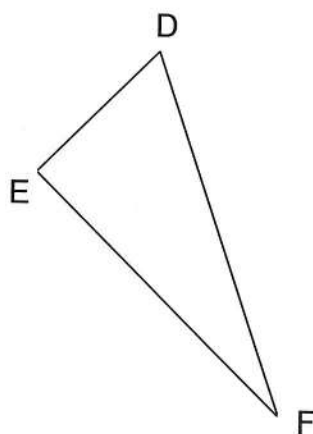
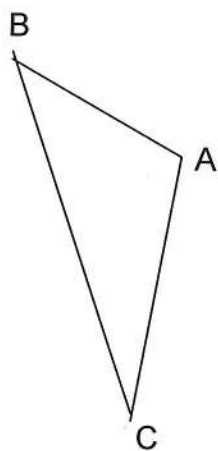
5. Determine the angles Captain Robot will need to turn.
6. Look carefully at the angles you chose for the robot's route. What do you notice?
7. Draw several triangles. Measure the interior angles and one exterior angle. What pattern do you notice? Make a generalization relating the interior and exterior angle.

Activator – Review of Corresponding Sides and Included Angles



1. What is the included angle of \overline{RS} and \overline{TR} ?

$$\triangle ABC \cong \triangle DEF$$



2. Which angle corresponds to $\angle A$?
3. Which segment corresponds to \overline{DE} ?

Given $\triangle DEF \cong \triangle RST$

4. Name the 3 corresponding pairs of segments.
5. Name the 3 corresponding pairs of angles.

VOCABULARY

