Unit 3 Study Guide

Convert between exponents and radicals:

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
5^x (using exponents)	5^0	$5^{\frac{1}{3}}$	$5^{\frac{2}{3}}$	5^1	$5^{\frac{4}{3}}$	$5^{\frac{5}{3}}$	5^2
5^x (equivalent expression)	1	$\sqrt[3]{5}$	$\sqrt[3]{5^2}$ or $\sqrt[3]{25}$	5	$\sqrt[3]{5^4}$ or $\sqrt[3]{625}$	$\sqrt[3]{5^5}$ or $\sqrt[3]{3125}$	25

Ex. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

Ex. $81^{\frac{3}{4}} = \sqrt[4]{81^3} = (\sqrt[4]{81})^3 = 3^3 = 27$

Use the Quadratic Formula to solve a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the Discriminant to determine the number and type of solutions:

- $b^2 - 4ac > 0$ 2 real solutions
- $b^2 - 4ac = 0$ 1 real solution
- $b^2 - 4ac < 0$ 2 non-real solutions

Solve Exponential Equations:

Real Numbers

$(x+1)^2 = 81$

$\sqrt{(x+1)^2} = \sqrt{81}$

$x + 1 = \pm 9$

$x = -1 \pm 9$

$x = -1 + 9 = 8$

or $x = -1 - 9 = -10$

Real Numbers

$(x+1)^2 = -81$

No solution ∅

positive ≠negative

Complex Numbers

$(x+1)^2 = -81$

$\sqrt{(x+1)^2} = \sqrt{-81}$

$x + 1 = \pm 9i$

$x = -1 \pm 9i$

Solve Rational Equations:

$\sqrt{x-2} - 3 = -1$

$\sqrt{x-2} = -1 + 3$

$\sqrt{x-2} = 2$

$(\sqrt{x-2})^2 = 2^2$

$x - 2 = 4$

$x = 6$

$\sqrt{x-2} + 3 = -1$

$\sqrt{x-2} = -1 - 3$

$\sqrt{x-2} = -4$

Not possible, positive can't equal negative (in the real numbers)

$\sqrt[3]{x-7} = -3$

$(\sqrt[3]{x-7})^3 = (-3)^3$

$x - 7 = -27$

$x = -20$

$\sqrt[3]{x-7} = 3$

$(\sqrt[3]{x-7})^3 = 3^3$

$x - 7 = 27$

$x = 34$

Operations with Complex Numbers

Add (real + real, imaginary + imaginary)	Subtract (real - real, imaginary - imaginary)	Multiply (FOIL & $i^2 = -1$)
$(a + bi) + (c + di)$	$(a + bi) - (c + di)$	$(a + bi)(c + di)$
$(a + c) + (bi + di)$	$(a - c) + (bi - di)$	$ac + adi + bci + bdi^2$
		$(ac - bd) + (ad + bc)i$

The Imaginary Number

$$i = \sqrt{-1}$$

$$i^2 = (\sqrt{-1})^2 = -1$$

Number of Solutions

$\sqrt{x} = c \Rightarrow$ 1 solution

$\sqrt{x} = -c \Rightarrow$ 0 solutions in the real numbers (positive ≠negative)

$x^2 = c \Rightarrow$ 2 solutions

$\sqrt[3]{x} = c \; or \; \sqrt[3]{x} = -c \Rightarrow$ 1 solution (same sign as *c*)

$x^3 = c \; or \; x^3 = -c \Rightarrow$ 1 solution (same sign as *c*)