Lesson 6: Introducing Double Number Line Diagrams

2-6: Learning Goals

- Let’s use number lines to represent equivalent ratios.

2-6-1: Adjusting Another Factor

Find the value of each product mentally.

$(4.5) \cdot 4$

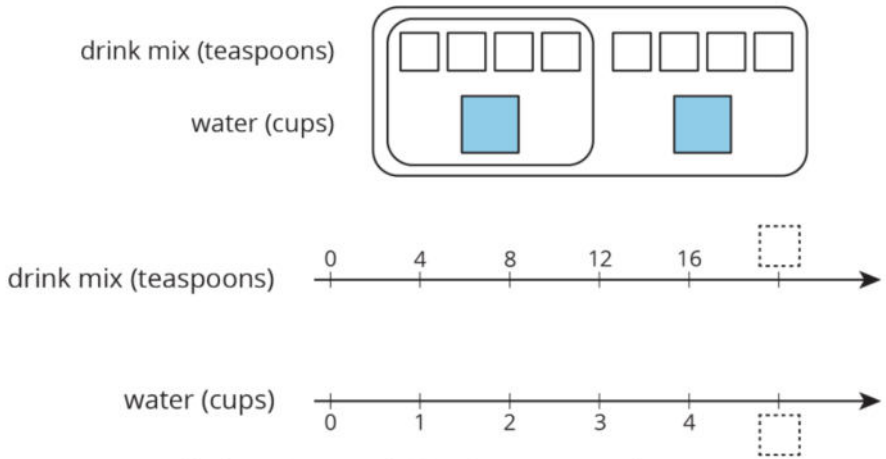
$(4.5) \cdot 8$

$\frac{1}{10} \cdot 65$

$\frac{2}{10} \cdot 65$

2-6-2: Drink Mix on a Double Number Line

The other day, we made drink mixtures by mixing 4 teaspoons of powdered drink mix for every cup of water. Here are two ways to represent multiple batches of this recipe:



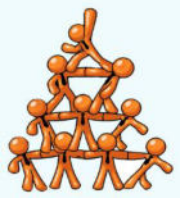
1. How can we tell that $4 : 1$ and $12 : 3$ are equivalent ratios?

2. How are these representations the same? How are these representations different?

3. How many teaspoons of drink mix should be used with 3 cups of water?

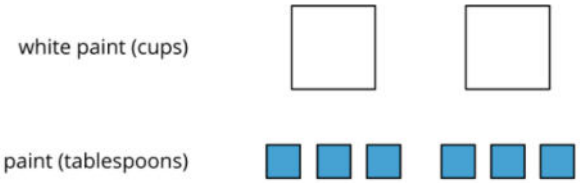
4. How many cups of water should be used with 16 teaspoons of drink mix?

5. What numbers should go in the empty boxes on the **double number line diagram**? What do these numbers mean?

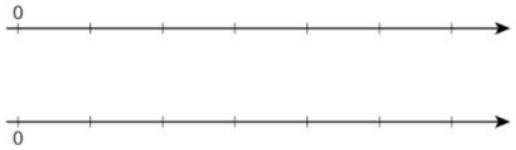


2-6-3: Blue Paint on a Double Number Line

Here is a diagram showing Elena's recipe for light blue paint.



1. Complete the double number line diagram to show the amounts of white paint and blue paint in different-sized batches of light blue paint.



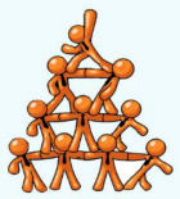
2. Compare your double number line diagram with your partner. Discuss your thinking. If needed, revise your diagram.

3. How many cups of white paint should Elena mix with 12 tablespoons of blue paint? How many batches would this make?

4. How many tablespoons of blue paint should Elena mix with 6 cups of white paint? How many batches would this make?

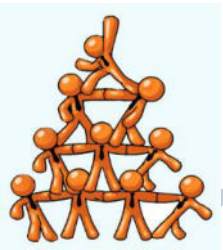
5. Use your double number line diagram to find another amount of white paint and blue paint that would make the same shade of light blue paint.

6. How do you know these mixtures would make the same shade of light blue paint?



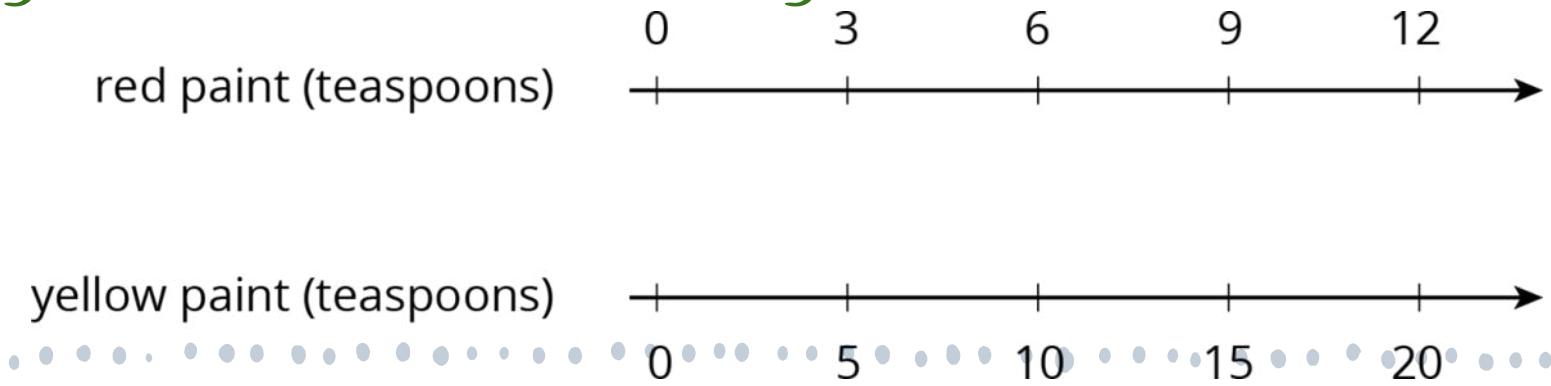
2-6: Lesson Synthesis

- Double number lines easily display equivalent ratios, with the numbers in each equivalent ratio lining up vertically.
- A double number line diagram can be used when a discrete diagram would be cumbersome or even impossible.



2-6: double number line diagram

A double number line diagram is a pair of parallel number lines with the numbers 0 aligned. Each number line is marked in equal increments and numbered. The tick marks are aligned, but the numbers on the two lines are often different. A pair of aligned numbers on the diagram represents a ratio that is equivalent to every other pair of aligned numbers on the diagram.



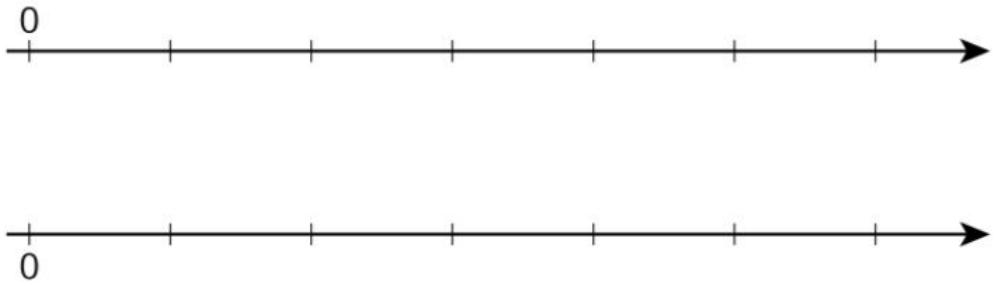
2-6: Learning Targets

- When I have a double number line that represents a situation, I can explain what it means.
- I can label a double number line diagram to represent batches of a recipe or color mixture.

2-6-4: Cool Down

A recipe for one batch of cookies uses 5 cups of flour and 2 teaspoons of vanilla.

1. Complete the double number line diagram to show the amount of flour and vanilla needed for 1, 2, 3, 4, and 5 batches of cookies.



2. If you use 20 cups of flour, how many teaspoons of vanilla should you use?

3. If you use 6 teaspoons of vanilla, how many cups of flour should you use?

Lesson 7: Creating Double Number Line Diagrams

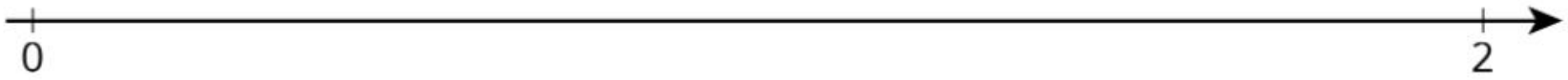
2-7: Learning Goals

- Let's draw double number line diagrams like a pro.

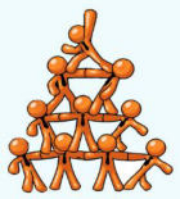
2-7-1: Ordering on a Number Line

1. Locate and label the following numbers on the number line:

$\frac{1}{2}$ $\frac{1}{4}$ $1\frac{3}{4}$ 1.5 1.75

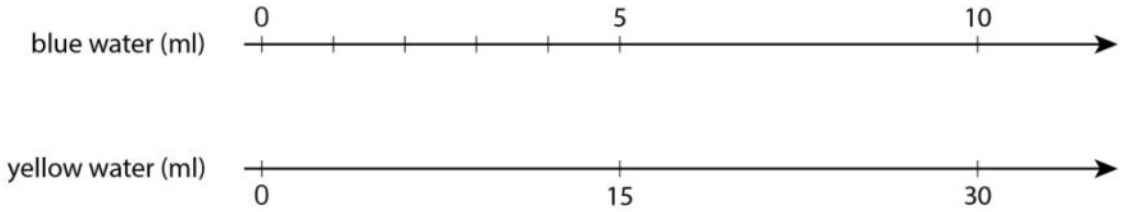


2. Based on where you placed the numbers, locate and label four more fractions or decimals on the number line.



2-7-2: Just a Little Green

The other day, we made green water by mixing 5 ml of blue water with 15 ml of yellow water. We want to make a very small batch of the same shade of green water. We need to know how much yellow water to mix with only 1 ml of blue water.



1. On the number line for blue water, label the four tick marks shown.

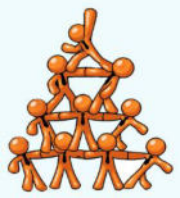
2. On the number line for yellow water, draw and label tick marks to show the amount of yellow water needed for each amount of blue water.

3. How much yellow water should be used for 1 ml of blue water? Circle where you can see this on the double number line.

4. How much yellow water should be used for 11 ml of blue water?

5. How much yellow water should be used for 8 ml of blue water?

6. Why is it useful to know how much yellow water should be used with 1 ml of blue water?



2-7-3: Art Paste on a Double Number Line

A recipe for art paste says "For every 2 pints of water, mix in 8 cups of flour."

1. Follow the instructions to draw a double number line diagram representing the recipe for art paste.

    a. Use a ruler to draw two parallel lines.
    b. Label the first line "pints of water." Label the second line "cups of flour."
    c. Draw at least 6 equally-spaced tick marks that line up on both lines.
    d. Along the water line, label the tick marks with the amount of water in 0, 1, 2, 3, 4, and 5 batches of art paste.
    e. Along the flour line, label the tick marks with the amount of flour in 0, 1, 2, 3, 4, and 5 batches of art paste.

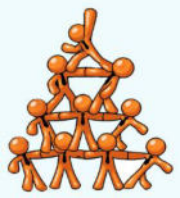
2. Compare your double number line diagram with your partner's. Discuss your thinking. If needed, revise your diagram.

3. Next, use your double number line to answer these questions:

    a. How much flour should be used with 10 pints of water?

    b. How much water should be used with 24 cups of flour?

    c. How much flour **per** pint of water does this recipe use?



2-7-4: Revisiting Tuna Casserole

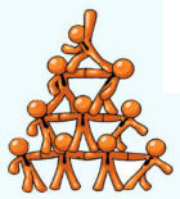
The other day, we looked at a recipe for tuna casserole that called for 10 ounces of cream of chicken soup for every 3 cups of elbow-shaped pasta.

1. Draw a double number line diagram that represents the amounts of soup and pasta in different-sized batches of this recipe.

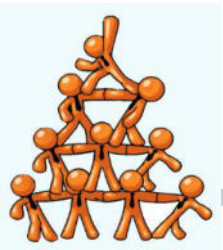
2. If you made a large amount of tuna casserole by mixing 40 ounces of soup with 15 cups of pasta, would it taste the same as the original recipe? Explain or show your reasoning.

3. The original recipe asked for 6 ounces of tuna for every 3 cups of pasta. Add a line to your diagram to represent the amount of tuna in different batches of casserole.

4. How many ounces of soup should you mix with 30 ounces of tuna to make a casserole that tastes the same as the original recipe?



2-7: Lesson Synthesis



2-7: per

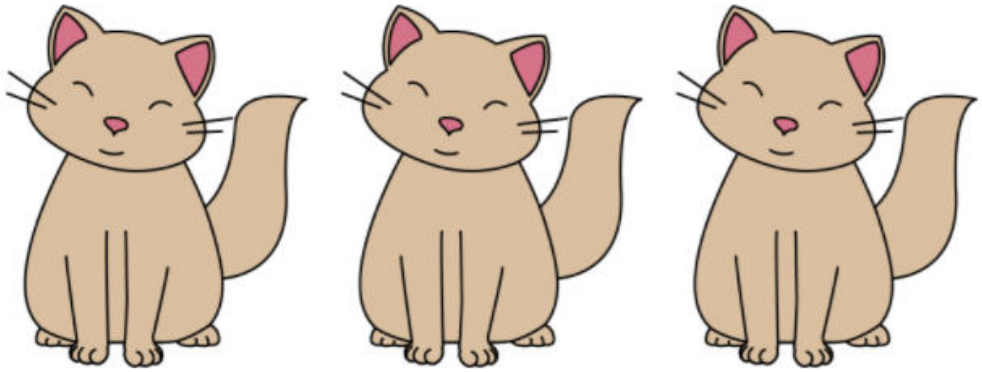
The word per means "for each." For example: he paid $5 for each ticket, so the cost was $5 per ticket.

2-7: Learning Targets

- I can create a double number line diagram and correctly place and label tick marks to represent equivalent ratios.
- I can explain what the word per means.

2-7-5: Cool Down

Each of these cats has 2 ears, 4 paws, and 1 tail.



1. Draw a double number line diagram that represents a ratio in the situation.

2. Write a sentence that describes this situation and that uses the word *per*.

Lesson 8: How Much for One?

2-8: Learning Goals

- Let’s use ratios to describe how much things cost.

2-8-1: Remainders in Division

Find the quotient mentally.

246÷12

2-8-2: Grocery Shopping

Answer each question and explain or show your reasoning. If you get stuck, consider drawing a double number line diagram.

1. Eight avocados cost $4.

   a. How much do 16 avocados cost?

   b. How much do 20 avocados cost?

   c. How much do 9 avocados cost?

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2. Twelve large bottles of water cost $9.

   a. How many bottles can you buy for $3?

   b. What is the cost per bottle of water?

   c. How much would 7 bottles of water cost?

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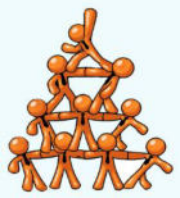
3. A 10-pound sack of flour costs $8.

   a. How much does 40 pounds of flour cost?

   b. What is the cost per pound of flour?



2-8-3: More Shopping

1. Four bags of chips cost $6.
    a. What is the cost per bag?
    b. At this rate, how much will 7 bags of chips cost?
2. At a used book sale, 5 books cost $15.
    a. What is the cost per book?
    b. At this rate, how many books can you buy for $21?
3. Neon bracelets cost $1 for 4.
    a. What is the cost per bracelet?
    b. At this rate, how much will 11 neon bracelets cost?

    Pause here so you teacher can review your work.

4. Your teacher will assign you one of the problems. Create a visual display that shows your solution to the problem. Be prepared to share your solution with the class.

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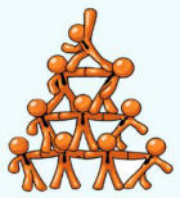
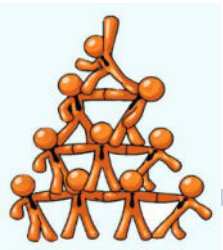


2-8: Lesson Synthesis

How can you find a unit price?



2-8: Unit price

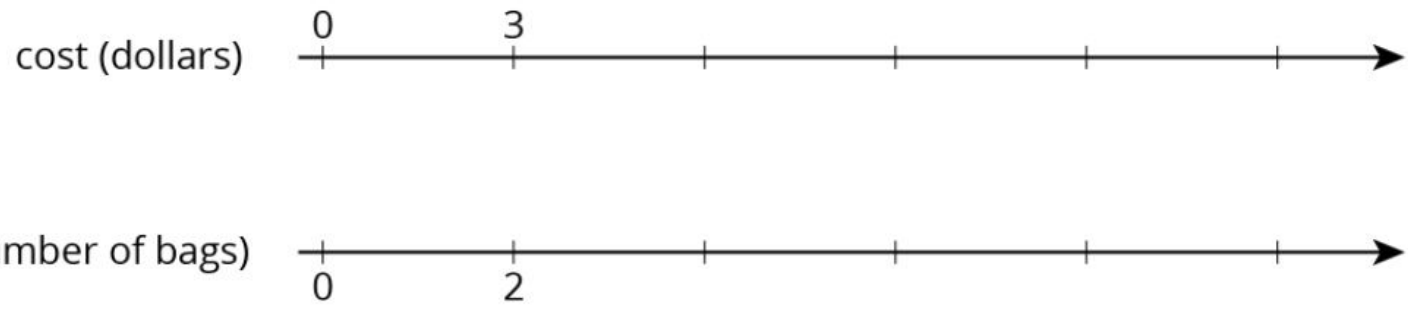
The unit price is the cost for one item or one unit of measure.

2-8: Learning Targets

- If I know the price of multiple things, I can find the price per thing.
- I can explain what the phrase “at this rate” means, using prices as an example.
- I can choose and create diagrams to help me reason about prices.

2-8-4: Cool Down

Here is a double number line showing that it costs $3 to buy 2 bags of rice:



1. At this rate, how many bags of rice can you buy with $12?

2. Find the cost per bag.

3. How much do 20 bags of rice cost?

Lesson 9: Constant Speed

2-9: Learning Goals

- Let’s use ratios to work with how fast things move.

2-9-1: Dividing by Powers of 10

Find the quotient mentally.

30÷10

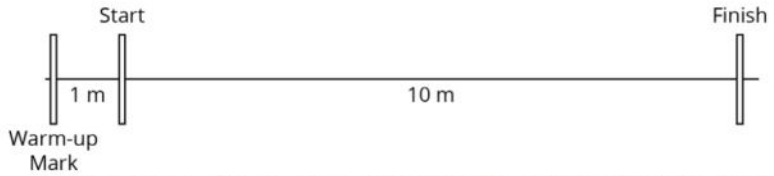
34÷10

3.4÷10

34÷100

2-9-2: Moving 10 Meters

Your teacher will set up a straight path with a 1-meter warm-up zone and a 10-meter measuring zone. Follow the following instructions to collect the data.



1. a. The person with the stopwatch (the "timer") stands at the finish line. The person being timed (the "mover") stands at the warm-up line.

   b. On the first round, the mover starts moving *at a slow, steady speed* along the path. When the mover reaches the start line, they say, "Start!" and the timer starts the stopwatch.

   c. The mover keeps moving steadily along the path. When they reach the finish line, the timer stops the stopwatch and records the time, rounded to the nearest second, in the table.

   d. On the second round, the mover follows the same instructions, but this time, moving *at a quick, steady speed*. The timer records the time the same way.

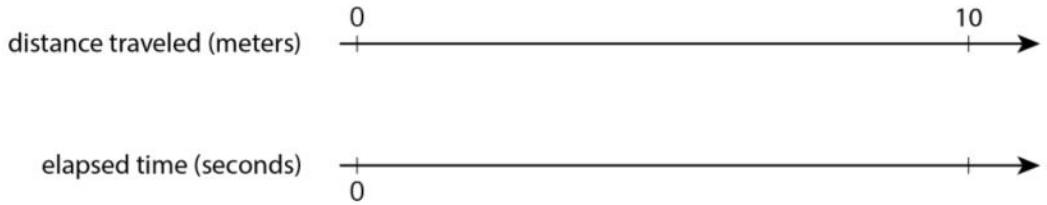
   e. Repeat these steps until each person in the group has gone twice: once at a slow, steady speed, and once at a quick, steady speed.

your slow moving time (seconds)	your fast moving time (seconds)

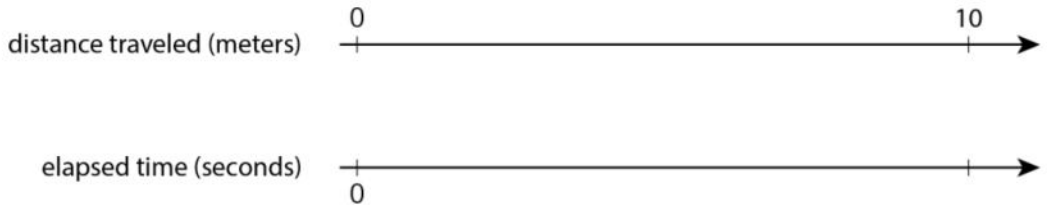
2-9-2: Moving 10 Meters

2. After you finish collecting the data, use the double number line diagrams to answer the questions. Use the times your partner collected while you were moving.

Moving slowly:



Moving quickly:



a. Estimate the distance in meters you traveled in 1 second when moving slowly.

b. Estimate the distance in meters you traveled in 1 second when moving quickly.

c. Trade diagrams with someone who is not your partner. How is the diagram representing someone moving slowly different from the diagram representing someone moving quickly?

2-9-3: Moving for 10 Seconds

Lin and Diego both ran for 10 seconds, each at their own constant speed. Lin ran 40 meters and Diego ran 55 meters.

1. Who was moving faster? Explain your reasoning.

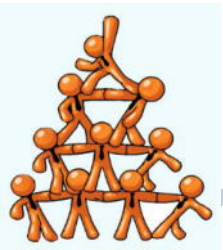
2. How far did each person move in 1 second? If you get stuck, consider drawing double number line diagrams to represent the situations.

3. Use your data from the previous activity to find how far *you* could travel in 10 seconds at your quicker speed.

4. Han ran 100 meters in 20 seconds at a constant speed. Is this speed faster, slower, or the same as Lin's? Diego's? Yours?

2-9: Lesson Synthesis

- meters per second
- dollars per item



2-9: meters per second

A unit to measure speed that tells you how many meters an object travels in one second.

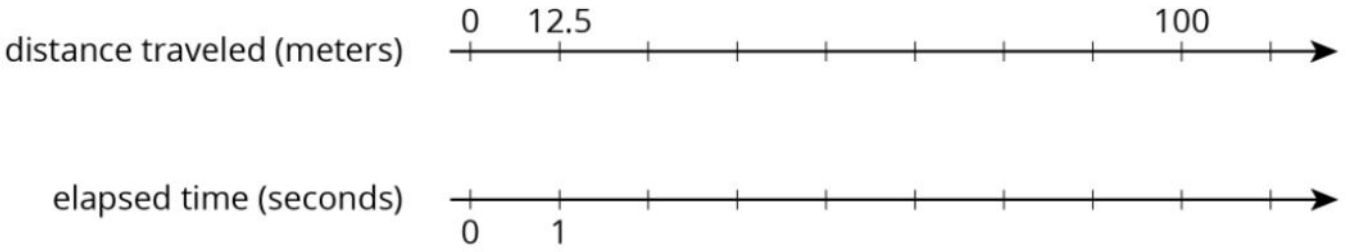
2-9: Learning Targets

- If I know an object is moving at a constant speed, and I know two of these things: the distance it travels, the amount of time it takes, and its speed, I can find the other thing.
- I can choose and create diagrams to help me reason about constant speed.

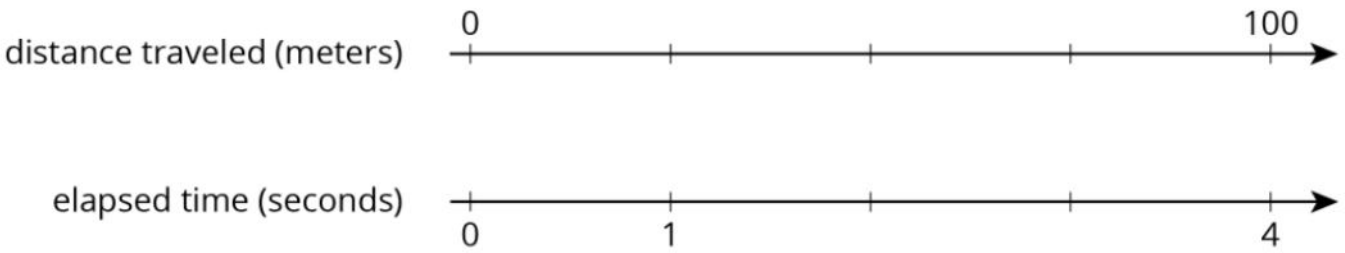
2-9-4: Cool Down

Two trains are traveling at constant speeds on different tracks.

Train A:



Train B:



Which train is traveling faster? Explain your reasoning.

Lesson 10: Comparing Situations by Examining Ratios

2-10: Learning Goals

- Let’s use ratios to compare situations.

2-10-1: Treadmills

2-10-1: Treadmills

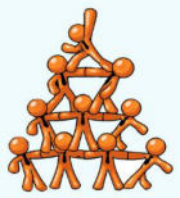
Mai and Jada each ran on a treadmill. The treadmill display shows the distance, in miles, each person ran and the amount of time it took them, in minutes and seconds.

Here is Mai's treadmill display:

Here is Jada's treadmill display:

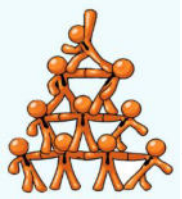
1. What is the same about their workouts? What is different about their workouts?

2. If each person ran at a constant speed the entire time, who was running faster? Explain your reasoning.



2-10-2: Concert Tickets

Diego paid $47 for 3 tickets to a concert. Andre paid $141 for 9 tickets to a concert. Did they pay at the **same rate**? Explain your reasoning.

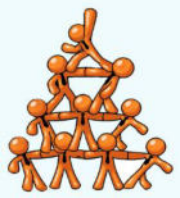


2-10-3: Sparkling Orange Juice

Lin and Noah each have their own recipe for making sparkling orange juice.

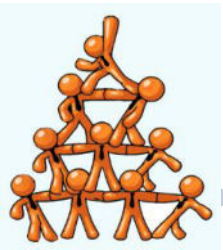
- Lin mixes 3 liters of orange juice with 4 liters of soda water.
- Noah mixes 4 liters of orange juice with 5 liters of soda water.

How do the two mixtures compare in taste? Explain your reasoning.



2-10: Lesson Synthesis

- How did we know that the people on the treadmill were not going the same speed?
- How did we know the people paid the same rate for the concert tickets?
- How did we know that the sparkling orange juice recipes did not taste the same?
- How were all these problems alike?



2-10: Same rate

In two situations involving ratios of the same two quantities, if the ratio of the quantities in one situation is equivalent to the ratio of the quantities in the other situation then we say the two situations involve the same rate.

2-10: Learning Targets

- I can decide whether or not two situations are happening at the same rate.
- I can explain what it means when two situations happen at the same rate.
- I know some examples of situations where things can happen at the same rate.

2-10-4: Cool Down

Andre ran 2 kilometers in 15 minutes, and Jada ran 3 kilometers in 20 minutes. Both ran at a constant speed.

Did they run at the *same* speed? Explain your reasoning.

Lesson 11: Representing Ratios with Tables

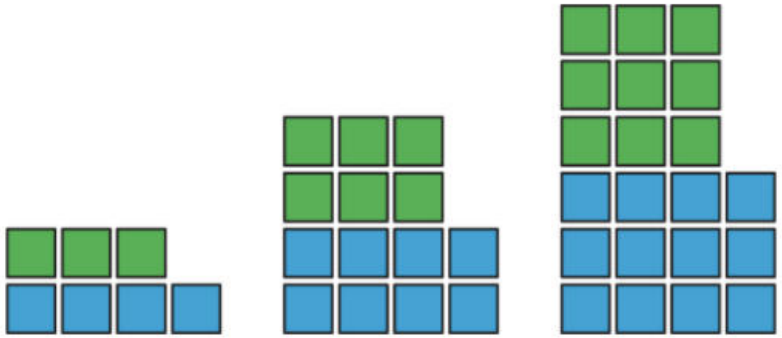
2-11: Learning Goals

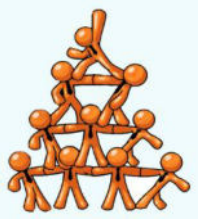
- Let’s use tables to represent equivalent ratios.

2-11-1: How Is It Growing?

Look for a pattern in the figures.

1. How many total tiles will be in:
   a. the 4th figure?
   b. the 5th figure?
   c. the 10th figure?
2. How do you see it growing?





2-11-2: A Huge Amount of Sparkling Orange Juice

Noah's recipe for one batch of sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

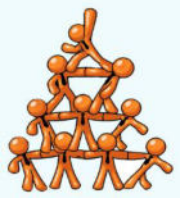
1. Use the double number line to show how many liters of each ingredient to use for different-sized batches of sparkling orange juice.

2. If someone mixes 36 liters of orange juice and 45 liters of soda water, how many batches would they make?

3. If someone uses 400 liters of orange juice, how much soda water would they need?

4. If someone uses 455 liters of soda water, how much orange juice would they need?

5. Explain the trouble with using a double number line diagram to answer the last two questions.



2-11-3: Batches of Trail Mix

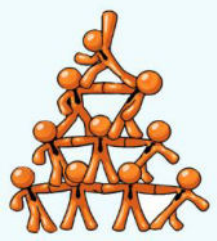
A recipe for trail mix says: "Mix 7 ounces of almonds with 5 ounces of raisins." Here is a **table** that has been started to show how many ounces of almonds and raisins would be in different-sized batches of this trail mix.

almonds (oz)	raisins (oz)
7	5
28	
	10
3.5	
	250
56	

1. Complete the table so that ratios represented by each row are equivalent.

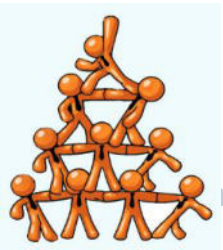
2. What methods did you use to fill in the table?

3. How do you know that each row shows a ratio that is equivalent to $7 : 5$? Explain your reasoning.



2-11: Lesson Synthesis

sugar	flour
2	5
8	20
26	65
240	600



2-11: table

A table is a way to organize information. Each rectangle in the table is called a cell. Each horizontal set of entries is called a row, and each vertical set of entries is called a column. The first row in a table often contains headers to explain what information is in each column.

This table shows the tail-lengths of three different pets. It has four rows and two columns.

The first cell in each column tells you what kind of information is in that column.

tail length (inches)
22
12
2

2-11: Learning Targets

- When I see a table representing a set of equivalent ratios, I can come up with numbers to make a new row.
- If I am looking at a table of values, I know where the rows are and where the columns are.
- When I see a table representing a set of equivalent ratios, I can explain what the numbers mean.

2-11-4: Cool Down

In previous lessons, we worked with a diagram and a double number line that represent this cookie recipe. Here is a table that represents the same situation.

flour (cups)	vanilla (teaspoons)
5	2
15	6
$2\frac{1}{2}$	1

1. Write a sentence that describes a ratio shown in the table.

2. What does the second row of numbers represent?

3. Complete the last row for a different batch size that hasn't been used so far in the table. Explain or show your reasoning.

Lesson 12: Navigating a Table of Equivalent Ratios

2-12: Learning Goals

- Let’s use a table of equivalent ratios like a pro.

2-12-1: Multiplying by a Unit Fraction

Find the product mentally.

$\frac{1}{3} \cdot 21$

$\frac{1}{6} \cdot 21$

$(5.6) \cdot \frac{1}{8}$

$\frac{1}{4} \cdot (5.6)$

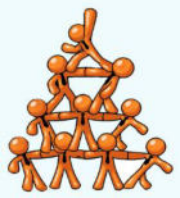
2-12-2: Comparing Taco Prices

number of tacos	price in dollars

Use the table to help you solve these problems. Explain or show your reasoning.

1. Noah bought 4 tacos and paid $6. At this rate, how many tacos could he buy for $15?

2. Jada's family bought 50 tacos for a party and paid $72. Were Jada's tacos the same price as Noah's tacos?



2-12-3: Hourly Wages

Lin is paid $90 for 5 hours of work. She used the following table to calculate how much she would be paid at this rate for 8 hours of work.

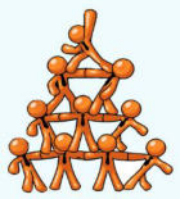
amount earned ($)	time worked (hours)
90	5
18	1
144	8

1. What is the meaning of the 18 that appears in the table?

2. Why was the number $\frac{1}{5}$ used as a multiplier?

3. Explain how Lin used this table to solve the problem.

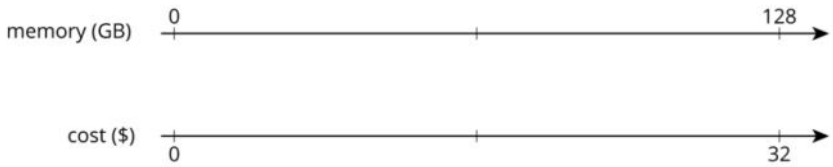
4. At this rate, how much would Lin be paid for 3 hours of work? For 2.1 hours of work?



2-12-4: Zeno's Memory Card

In 2016, 128 gigabytes (GB) of portable computer memory cost $32.

1. Here is a double number line that represents the situation:



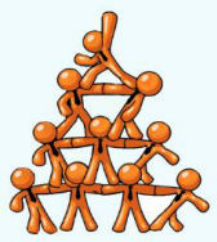
One set of tick marks has already been drawn to show the result of multiplying 128 and 32 each by $\frac{1}{2}$. Label the amount of memory and the cost for these tick marks.

Next, keep multiplying by $\frac{1}{2}$ and drawing and labeling new tick marks, until you can no longer clearly label each new tick mark with a number.

2. Here is a table that represents the situation. Find the cost of 1 gigabyte. You can use as many rows as you need.

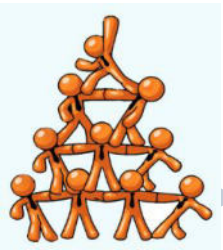
memory (gigabytes)	cost (dollars)
128	32

3. Did you prefer the double number line or the table for solving this problem? Why?



2-12: Lesson Synthesis

- In problems with equivalent ratios, finding an equivalent ratio containing a “1” is often a good strategy.
- To create a new row in a table of equivalent ratios, take an existing row and multiply both values by the same number.
- Remember that we can multiply whole numbers by unit fractions to get smaller numbers.



2-12: Learning Targets

- I can solve problems about situations happening at the same rate by using a table and finding a "1" row.
- I can use a table of equivalent ratios to solve problems about unit price.

2-12-5: Cool Down

A shop sells bagels for $5 per dozen. You can use the table to answer the questions. Explain your reasoning.

number of bagels	price in dollars
12	5

1. At this rate, how much would 6 bagels cost?

2. How many bagels can you buy for $50?

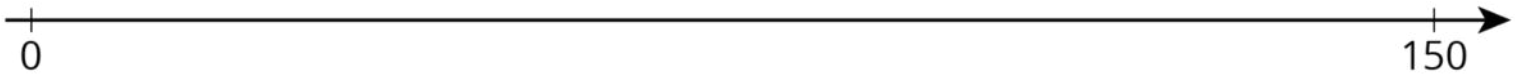
Lesson 13: Tables and Double Number Line Diagrams

2-13: Learning Goals

- Let’s contrast double number lines and tables.

2-13-1: Constant Dividend

1. Find the quotients mentally.
   - 150÷2
   - 150÷4
   - 150÷8
2. Locate and label the quotients on the number line.



2-13-2: Moving 3,000 Meters

The other day, we saw that Han can run 100 meters in 20 seconds.

Han wonders how long it would take him to run 3,000 meters at this rate. He made a table of equivalent ratios.

20	100
10	50
1	5
3,000	

1. Do you agree that this table represents the situation? Explain your reasoning.

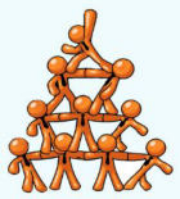
2. Complete the last row with the missing number.

3. What question about the situation does this number answer?

4. What could Han do to improve his table?

5. Priya can bike 150 meters in 20 seconds. At this rate, how long would it take her to bike 3,000 meters?

6. Priya's neighbor has a dirt bike that can go 360 meters in 15 seconds. At this rate, how long would it take them to ride 3,000 meters?



2-13-3: The International Space Station

- Can you name two things in our town (or city) that are about 1 kilometer apart?
- How long do you think it would take you to walk 1 kilometer?
- What might be a typical speed limit on a highway, in kilometers per hour?

2-13-3: The International Space Station

The International Space Station orbits around the Earth at a constant speed. Your teacher will give you either a double number line or a table that represents this situation. Your partner will get the other representation.

1. Complete the parts of your representation that you can figure out for sure.

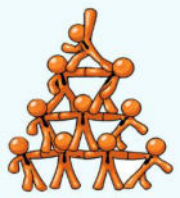
2. Share information with your partner, and use the information that your partner shares to complete your representation.

International Space Station Copyright Owner: NASA License: Public Domain Via: NASA

3. What is the speed of the International Space Station?

4. Place the two completed representations side by side. Discuss with your partner some ways in which they are the same and some ways in which they are different.

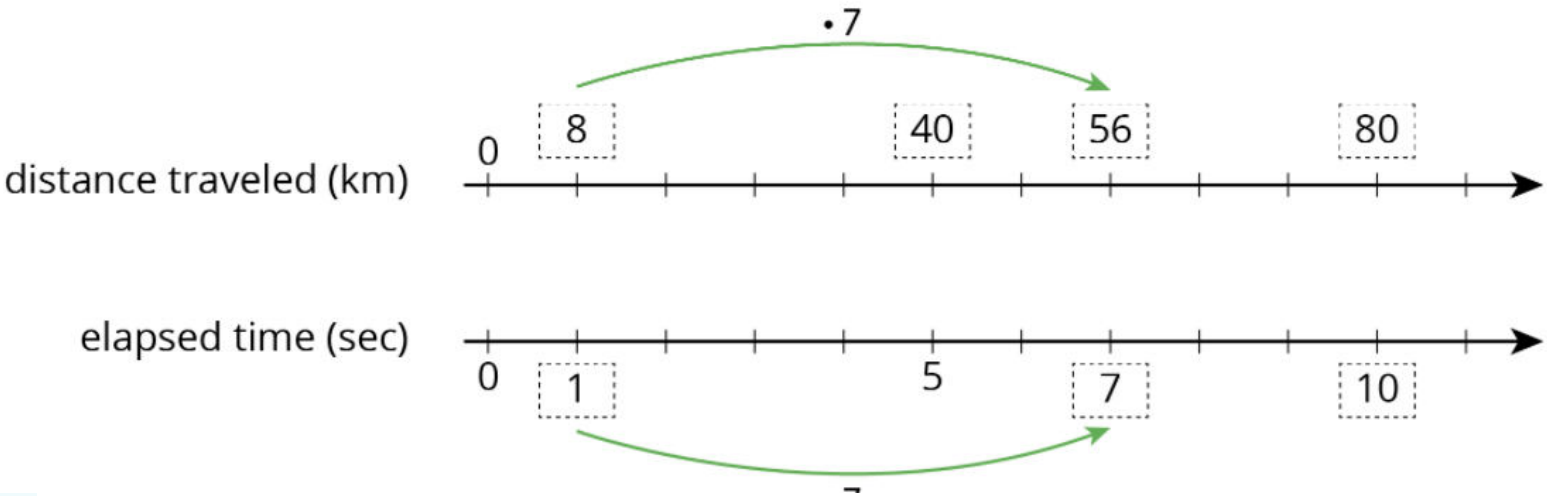
5. Record at at least one way that they are the same and one way they are different.

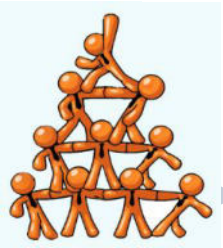


2-13: Lesson Synthesis

distance traveled (km)	elapsed time (sec)
0	0
80	10
8	1
40	5
56	7

•7 •7





2-13: Learning Targets

- I can create a table that represents a set of equivalent ratios.
- I include column labels when I create a table, so that the meaning of the numbers is clear.
- I can explain why sometimes a table is easier to use than a double number line to solve problems involving equivalent ratios.

2-13-4: Cool Down

In a sprint to the finish, a professional cyclist travels 380 meters in 20 seconds. At that rate, how far does the cyclist travel in 3 seconds?

Lesson 14: Solving Equivalent Ratio Problems

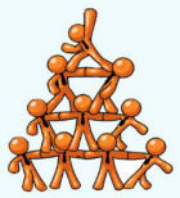
2-14: Learning Goals

- Let's practice getting information from our partner.

2-14-1: What Do You Want to Know?

Here is a problem: A red car and a blue car enter the highway at the same time and travel at a constant speed. How far apart are they after 4 hours?

What information would you need to solve the problem?



2-14-2: Hot Chocolate and Potatoes

Problem Card

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. Solve the problem and explain your reasoning to your partner.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

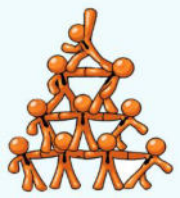
Data Card

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to *ask* for information. *Only* give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask "Why do you need that information?"
4. After your partner solves the problem, ask them to explain their reasoning and listen to

2-14-3: Comparing Reading Rates

- Lin read the first 54 pages from a 270-page book in the last 3 days.
- Diego read the first 100 pages from a 320-page book in the last 4 days.
- Elena read the first 160 pages from a 480-page book in the last 5 days.

If they continue to read every day at these rates, who will finish first, second, and third? Explain or show your reasoning.



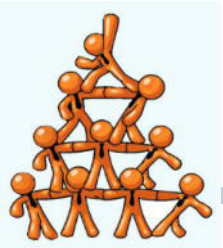
2-14: Lesson Synthesis

- If you eat 12 strawberries in 3 minutes, how long will it take to eat 8 strawberries at that rate?

number of strawberries	number of minutes
12	3
1	$\frac{1}{4}$
8	2

- If you jump 8 times in 10 seconds, how many jumps can you make in 45 seconds at that rate?

number of jumps	number of seconds
8	10



2-14: Learning Targets

- I can decide what information I need to know to be able to solve problems about situations happening at the same rate.
- I can explain my reasoning using diagrams that I choose.

2-14-4: Cool Down

Jada wants to know how fast the water comes out of her faucet. What information would she need to know to be able to determine that?

Lesson 15: Part-Part-Whole Ratios

2-15: Learning Goals

- Let's look at situations where you can add the quantities in a ratio together.

2-15-1: True or False: Multiplying by a Unit Fraction

True or false?

$\frac{1}{5} \cdot 45 = \frac{45}{5}$

$\frac{1}{5} \cdot 20 = \frac{1}{4} \cdot 24$

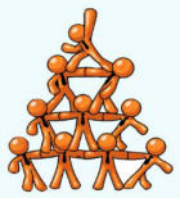
$42 \cdot \frac{1}{6} = \frac{1}{6} \cdot 42$

$486 \cdot \frac{1}{12} = \frac{480}{12} + \frac{6}{12}$

2-15-2: Cubes of Paint

To make a particular green paint, we need to mix 1 ml of blue paint to 3 ml of yellow.

- How much green paint will this recipe yield?
- If each cube represents 2 ml instead of 1 ml, how much of blue and yellow do the snap cubes represent? How many ml of green paint will we have?
- Is there another way to represent 2 ml of blue and 6 ml of yellow using snap cubes?
- How do we refer to 2 ml of blue and 6 ml of yellow in terms of ‘batches’?



2-15-2: Cubes of Paint

A recipe for maroon paint says, "Mix 5 ml of red paint with 3 ml of blue paint."

1. Use snap cubes to represent the amounts of red and blue paint in the recipe. Then, draw a sketch of your snap-cube representation of the maroon paint.

   a. What amount does each cube represent?

   b. How many milliliters of maroon paint will there be?

2. a. Suppose each cube represents 2 ml. How much of each color paint is there?

   Red: _____ ml Blue: _____ ml Maroon: _____ ml

   b. Suppose each cube represents 5 ml. How much of each color paint is there?

   Red: _____ ml Blue: _____ ml Maroon: _____ ml

3. a. Suppose you need 80 ml of maroon paint. How much red and blue paint would you mix? Be prepared to explain your reasoning.

   Red: _____ ml Blue: _____ ml Maroon: 80 ml

   b. If the original recipe is for one batch of maroon paint, how many batches are in 80 ml of maroon paint?

2-15-3: Sneakers, Chicken, and Fruit Juice

Solve each of the following problems and show your thinking. If you get stuck, consider drawing a **tape diagram** to represent the situation.

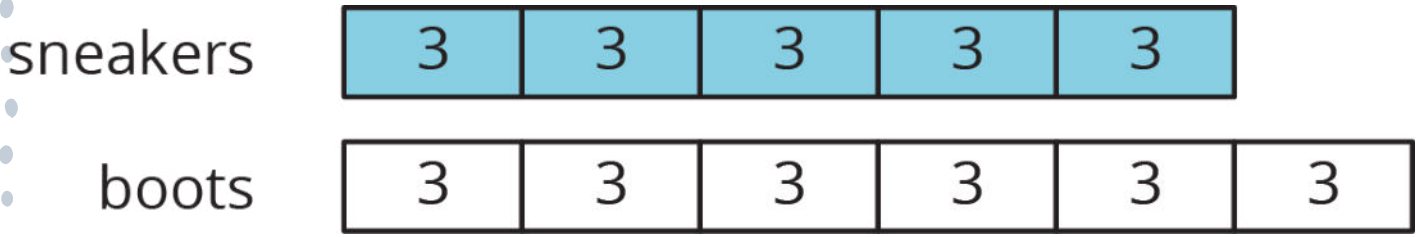
1. The ratio of students wearing sneakers to those wearing boots is 5 to 6. If there are 33 students in the class, and all of them are wearing either sneakers or boots, how many of them are wearing sneakers?
2. A recipe for chicken marinade says, “Mix 3 parts oil with 2 parts soy sauce and 1 part orange juice.” If you need 42 cups of marinade in all, how much of each ingredient should you use?
3. Elena makes fruit punch by mixing 4 parts cranberry juice to 3 parts apple juice to 2 parts grape juice. If one batch of fruit punch includes 30 cups of apple juice, how large is this batch of fruit punch?

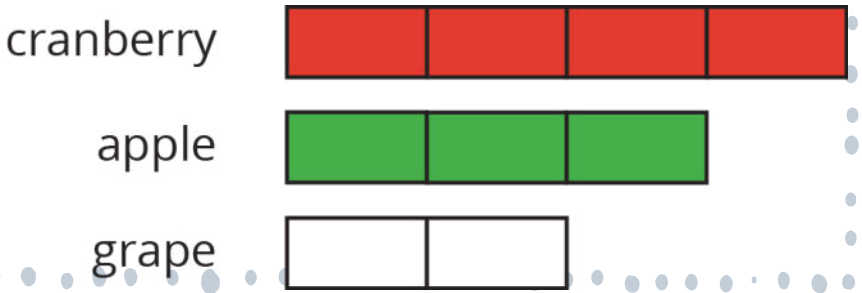
2-15-4: Invent Your Own Ratio Problem

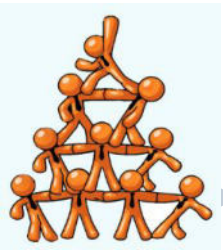
1. Invent another ratio problem that can be solved with a tape diagram and solve it. If you get stuck, consider looking back at the problems you solved in the earlier activity.
2. Create a visual display that includes:
    - The new problem that you wrote, without the solution.
    - Enough work space for someone to show a solution.
3. Trade your display with another group, and solve each other's problem. Include a tape diagram as part of your solution. Be prepared to share the solution with the class.
4. When the solution to the problem you invented is being shared by another group, check their answer for accuracy.

2-15: Lesson Synthesis

- Can anyone identify what made these problems different?
- How can a **tape diagram** represent these types of situations?
- How does changing the value of each part of the tape affect the total amount?



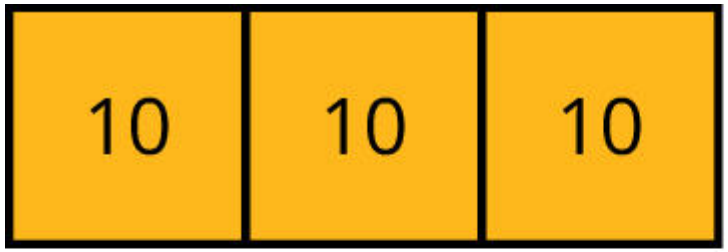


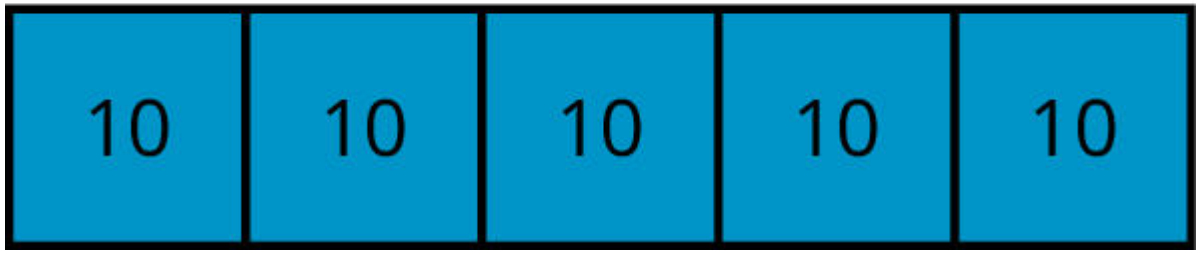


2-15: tape diagram

A tape diagram can be used to represent a ratio between two quantities measured in the same units.

The tape diagram shows a ratio of 30 gallons of yellow paint to 50 gallons of blue paint.





2-15: Learning Targets

- I can solve problems when I know a ratio and a total amount.
- I can create tape diagrams to help me reason about problems involving a ratio and a total amount.

2-15-5: Cool Down

The first floor of a house consists of a kitchen, playroom, and dining room. The areas of the kitchen, playroom, and dining room are in the ratio 4:3:2. The combined area of these three rooms is 189 square feet. What is the area of each room?

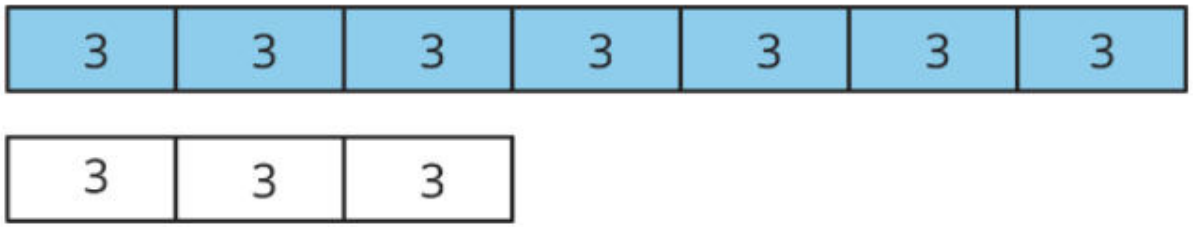
Lesson 16: Solving More Ratio Problems

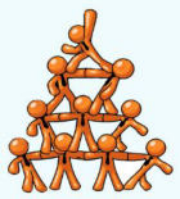
2-16: Learning Goals

- Let’s compare all our strategies for solving ratio problems.

2-16-1: You Tell the Story

Describe a situation with two quantities that this tape diagram could represent.



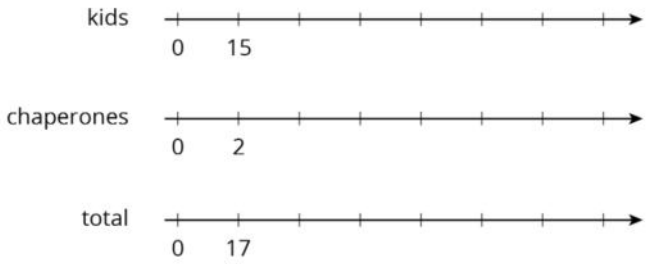


2-16-2: A Trip to the Aquarium

Consider the problem: A teacher is planning a class trip to the aquarium. The aquarium requires 2 chaperones for every 15 students. The teacher plans accordingly and orders a total of 85 tickets. How many tickets are for chaperones, and how many are for students?

1. Solve this problem in *one* of three ways:

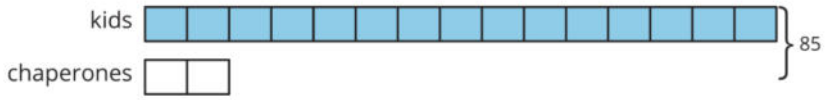
   a. Use a triple number line.



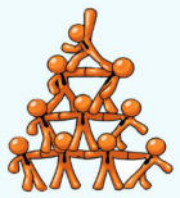
   b. Use a table. (Fill rows as needed.)

kids	chaperones	total
15	2	17

   c. Use a tape diagram.



2. After your class discusses all three strategies, which do you prefer for this problem and why?

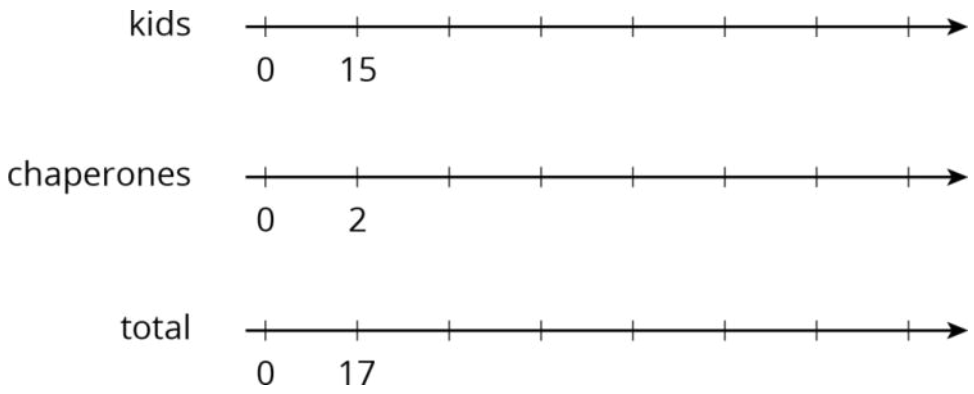


2-16-3: Salad Dressing and Moving Boxes

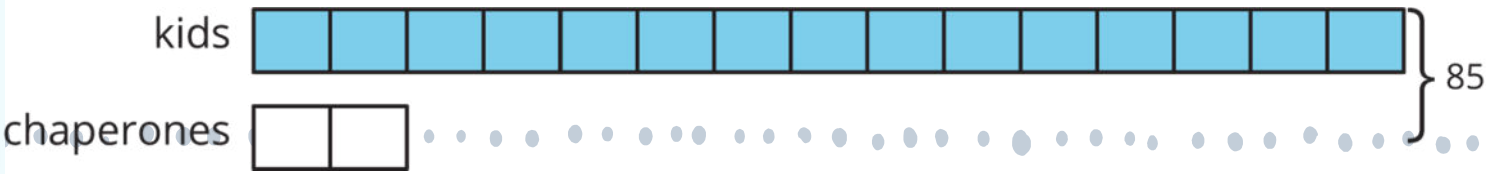
Solve each problem, and show your thinking. Organize it so it can be followed by others. If you get stuck, consider drawing a double number line, table, or tape diagram.

1. A recipe for salad dressing calls for 4 parts oil for every 3 parts vinegar. How much oil should you use to make a total of 28 teaspoons of dressing?
2. Andre and Han are moving boxes. Andre can move 4 boxes every half hour. Han can move 5 boxes every half hour. How long will it take Andre and Han to move all 72 boxes?

2-16: Lesson Synthesis



kids	chaperones	total
15	2	17



2-16: Learning Targets

- I can use diagrams to help someone else understand why my solution makes sense.
- I can choose and create diagrams to help think through my solution.
- I can solve all kinds of problems about equivalent ratios.

2-16-4: Cool Down

You are having a pizza-making party. You will need 6 ounces of dough and 4 ounces of sauce for each person at the party (including yourself, the host). Once you have a total count of guests, you buy exactly the needed amount of all the ingredients. The dough and sauce that you buy weigh 130 ounces all together.

1. How many ounces of dough did you buy?
2. How many ounces of sauce did you buy?
3. How many guests are coming to the party?

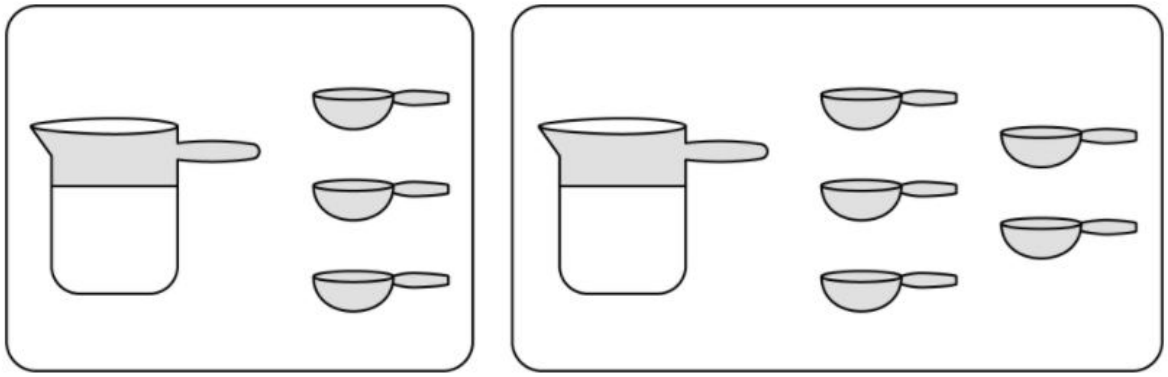
Lesson 17: A Fermi Problem

2-17: Learning Goals

- Let’s solve a Fermi problem.

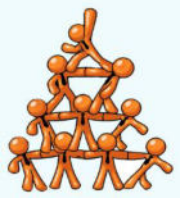
2-17-1: Fix It!

Andre likes a hot cocoa recipe with 1 cup of milk and 3 tablespoons of cocoa. He poured 1 cup of milk but accidentally added 5 tablespoons of cocoa.



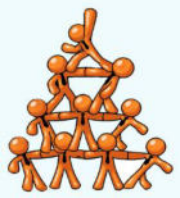
1. How can you fix Andre's mistake and make his hot cocoa taste like the recipe?

2. Explain how you know your adjustment will make Andre's hot cocoa taste the same as the one in the recipe.



2-17-2: Who Was Fermi?

1. Record the Fermi question that your class will explore together.
2. Make an estimate of the answer. If making an estimate is too hard, consider writing down a number that would definitely be too low and another number that would definitely be too high.
3. What are some smaller sub-questions we would need to figure out to reasonably answer our bigger question?
4. Think about how the smaller questions above should be organized to answer the big question. Label each smaller question with a number to show the order in which they should be answered. If you notice a gap in the set of sub-questions (i.e., there is an unlisted question that would need to be answered before the next one could be tackled), write another question to fill the gap.

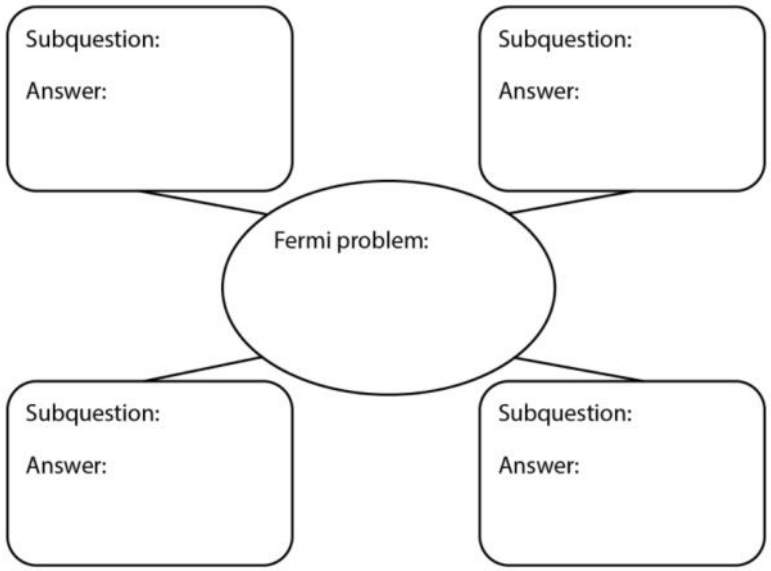


2-17-3: Researching Your Own Fermi Problem

1. Brainstorm at least five Fermi problems that you want to research and solve. If you get stuck, consider starting with "How much would it cost to . . .?" or "How long would it take to . . .?"

2. Pause here so your teacher can review your questions and approve one of them.

3. Use the graphic organizer to break your problem down into sub-questions.

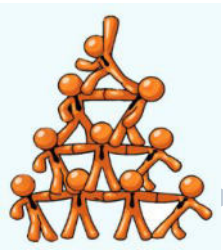


4. Find the information you need to get closer to answering your question. Measure, make estimates, and perform any necessary calculations. If you get stuck, consider using tables or double number line diagrams.

5. Create a visual display that includes your Fermi problem and your solution. Organize your thinking so it can be followed by others.

2-17: Lesson Synthesis

- Where did you make an estimate?
- When did you figure out what additional information you would need?
- What simplifying assumptions did you make?



2-17: Learning Targets

- I can decide what information I need to know to be able to solve a real-world problem about ratios and rates.
- I can apply what I have learned about ratios and rates to solve a more complicated problem.