Grade 6 Unit Two

OVERVIEW In this unit, students will:

- gain a deeper understanding of proportional reasoning through instruction and practice
- develop and use multiplicative thinking
- develop a sense of proportional reasoning
- develop the understanding that ratio is a comparison of two numbers or quantities
- find percents using the same processes for solving rates and proportions
- solve real-life problems involving measurement units that need to be converted

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students understand the problem context in order to translate them into ratios/rates.

2. Reason abstractly and quantitatively. Students understand the relationship between two quantities in order to express them mathematically. They use ratio and rate notation as well as visual models and contexts to demonstrate reasoning.

3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding appropriateness of representations given ratio and rate contexts. For example, does a tape diagram adequately represent a given ratio scenario.

4. Model with mathematics. Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.

5. Use appropriate tools strategically. Students choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.

6. Attend to precision. Students use and interpret mathematical language to make sense of ratios and rates.

7. Look for and make use of structure. The structure of a ratio is unique and can be used across a wide variety of problem-solving situations. For instance, students recognize patterns that exist in ratio tables, including both the additive and multiplicative properties. In addition, students use their knowledge of the structures of word problems to make sense of real-world problems.

8. Look for and express regularity in repeated reasoning. Students utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts. Students can generalize the relationship between representations, understanding that all formats represent the same ratio or rate.**Georgia Department of Education** Georgia Standards of Excellence Framework *GSE Grade 6*

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students.

Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary
http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.
http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

• **Percent:** A fraction or ratio in which the denominator is 100. A number compared to 100.

• **Proportion:** An equation which states that two ratios are equal.

• **Rate:** A comparison of two quantities that have different units of measure

• **Ratio:** compares quantities that share a fixed, multiplicative relationship.

• **Rational number:** A number that can be written as a/b where a and b are integers, but b is not equal to 0.

• **Tape diagram:** A thinking tool used to visually represent a mathematical problem and transform the words into an appropriate numerical operation. Tape diagrams are linear drawings that look like a segment of tape, used to illustrate number relationships. Also known as Singapore Strips, strip diagrams, bar models or graphs, fraction strips, or length models.• **Unit Ratio:** are ratios written as some number to 1.

• **Quantity:** is an amount that can be counted or measured.

Grade 7 Unit 3 STANDARDS FOR MATHEMATICAL CONTENT

Understand ratio concepts and use ratio reasoning to solve problems.

MGSE6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MGSE6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MGSE6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

MGSE6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

MGSE6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. ***For example, given 1 in. = 2.54 cm, how many centimeters are in 6 inches?***

BIG IDEAS

• A ratio is a number that relates two quantities or measures within a given situation in a multiplicative relationship (in contrast to a difference or additive relationship).The relationships and rules that govern whole numbers, govern all rational numbers.

• Making explicit the type of relationships that exist between two values will minimize confusion between multiplicative and additive situations.

• Ratios can express comparisons of a part to whole, (a/b with $b \neq 0$), for example, the ratio of the number of boys in a class to the number of students in the class.

• The ratio of the length to the width of a rectangle is a part-to-part relationship.

• Understand that fractions are also part-whole ratios, meaning fractions are also ratios. Percentages are ratios and are sometimes used to express ratios.

- Both part-to-whole and part-to-part ratios compare two measures of the same type of thing. A ratio can also be a rate.
- A rate is a comparison of the measures of two different things or quantities; the measuring unit is different for each value. For example if 4 similar vans carry 36 passengers, then the comparison of 4 vans to 36 passengers is a ratio.
- All rates of speed are ratios that compare distance to time, such as driving at 45 miles per hour or jogging at 7 minutes per mile.
- Ratios use division to represent relations between two quantities.

ESSENTIAL QUESTIONS

- What kinds of problems can I solve by using ratios?
- How can I tell if a relationship is multiplicative?
- What is the difference between a multiplicative and an additive relationship?
- What are equivalent ratios?
- What are rates?
- How are unit rates helpful in solving real-world problems?
- How are ratios and rates similar and different?
- What are percentages?
- What information do I get when I compare two numbers using a ratio?

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics.

OVERVIEW

The units in this instructional framework emphasize key standards that assist students to develop a deeper understanding of numbers. They learn to express different representations of rational numbers (e.g., fractions, decimals, and percent's), discover how to identify and explain the constant of proportionality, and represent proportional relationships and scale drawings within real-world contexts. The Big Ideas that are expressed in this unit are integrated with such routine topics as estimation, mental and basic computation. All of these concepts need to be reviewed throughout the year.

Take what you need from the tasks and modify as required. These tasks are suggestions, something that you can use as a resource for your classroom.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of ratio and unit rates in real-world contexts. They persevere by selecting and using appropriate representations for the given contexts.

2. Reason abstractly and quantitatively. Students will reason about the value of the rational number in relation the models that are created to represent them.

3. Construct viable arguments and critique the reasoning of others. Students use arguments to justify their reasoning when creating and solving proportions used in real-world contexts.

4. Model with mathematics. Students create models using tape diagrams, double number lines, manipulatives, tables and graphs to represent real-world and mathematical situations involving ratios

and proportions. For example, students will examine the relationships between slopes of lines and ratio tables in the context of given situations.
5. Use appropriate tools strategically. Students use visual representations such as the coordinate plane to show the constant of proportionality.
6. Attend to precision. Students attend to the ratio and rate language studied in grade 6 to represent and solve problems involving rates and ratios.

7. Look for and make use of structure. Students look for patterns that exist in ratio tables in order to make connections between the constant of proportionality in a table with the slope of a graph.

8. Look for and express regularity in repeated reasoning. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

STANDARDS FOR MATHEMATICAL CONTENT

Analyze proportional relationships and use them to solve real-world and mathematical problems.

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
MGSE7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.
MGSE7.RP.2d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1,r)$ where r is the unit rate.
MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.
MGSE7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

BIG IDEAS

- Fractions, decimals, and percents can be used interchangeably.
- Ratios and rates use multiplication/division to represent relationships between two quantities.
- The constant of proportionality is also considered to be the unit rate.

ESSENTIAL QUESTIONS

- How can you compute ratios of length in like or different units?
- How can you compute unit rates involving rational numbers, fractions and complex fractions?
- How do I interpret a unit rate (using words and mathematically)?
- What strategies can be used to compare ratios?
- How do I verify if two quantities are directly proportional?

- How can I use tables, graphs or equations to determine whether a relationship is proportional?
- How do I interpret a distance time graph and determine a point of intersection?
- How can models be used to solve percent problems?
- How do I apply mental math strategies to solve percent problems?
- How are distances and measurements translated into a map or scale drawing?
- How do I determine the an appropriate scale for the area (such as my yard or school) that I am measuring and mapping?
- How is the unit rate represented in tables, graphs, equations and diagrams?
- How is unit rate computed in real-world problems?

- How are ratios and their relationships used to solve real world problems?

- How do I solve and interpret solutions of real-world percent problems?

- How do I utilize percent of increase and decrease as an aspect of multiplication?

- How does my understanding of unit rate save me money?

- How can I determine the unit rate for a product that I might purchase?

CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and decimals, including application of order of operations

- knowledge of equivalent fractions

- addition and subtraction of common fractions with like denominators

- measuring length and finding perimeter and area of rectangles and squares

- characteristics of 2-D and 3-D shapes

***FLUENCY**

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems

within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students: flexibly use a combination of deep understanding, number sense, and memorization. are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them. are able to articulate their reasoning. find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

SELECTED TERMS AND SYMBOLS

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- **Constant of Proportionality**
- **Equivalent Fractions**
- **Fraction**
- **Multiplicative inverse**
- **Percent rate of change**
- **Proportion**
- **Ratio**
- **Similar Figures**
- **Unit Rate**
- **Scale factor**