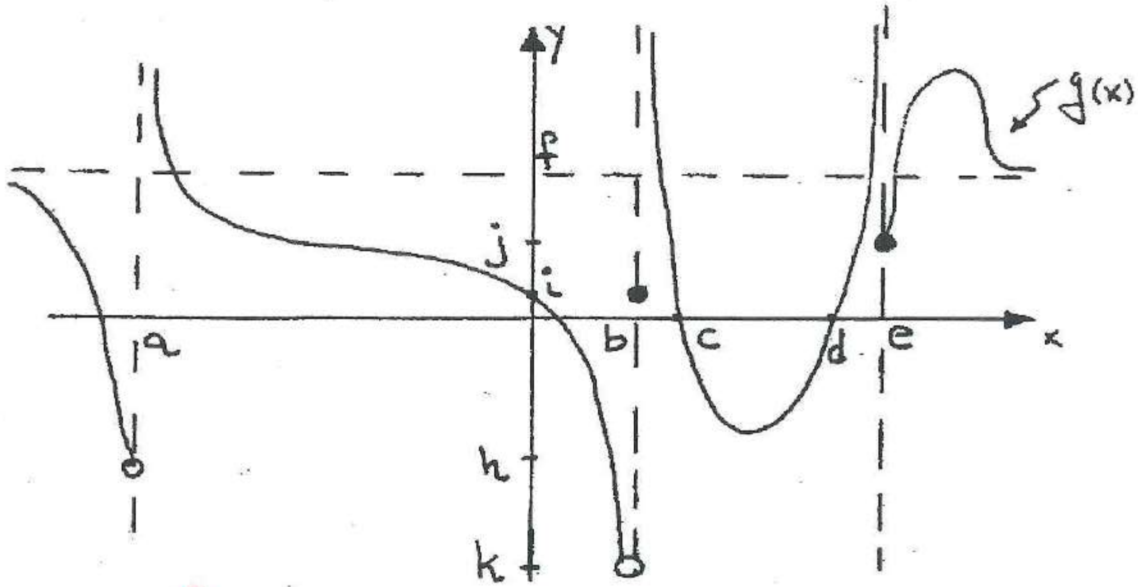
AP Calculus LIMITS TEST – Part 1 (non-calculator)

Name: Key Date: ________ Period: ____

Use the graph of G(x) below to answer the following questions. If a limit does not exist, justify your answer.

** SHOW ALL WORK ON A SEPERATE SHEET AND ATTACH. **

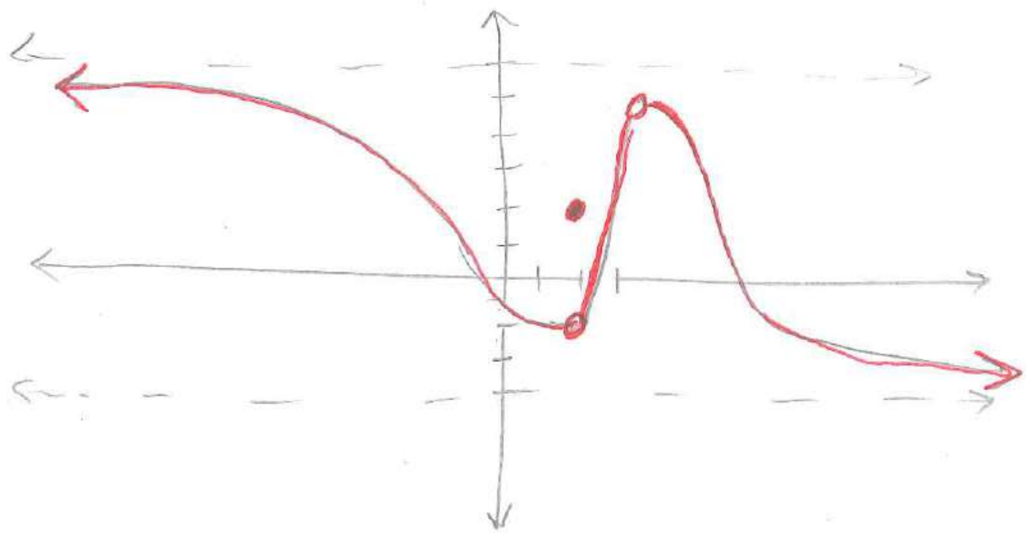


1. $\lim_{x\to\infty} g(x) =$ f
2. $\lim_{x\to-\infty} g(x) =$ f
3. $\lim_{x\to a+} g(x) =$ ∞
4. $\lim_{x\to a-} g(x) =$ h
5. $\lim_{x\to a} g(x) =$ DNE since $\lim_{x\to a^-} g(x) = h \neq \lim_{x\to a^+} g(x) = \infty$
6. $\lim_{x\to b-} g(x) =$ K
7. $\lim_{x\to 0} g(x) =$ i
8. $\lim_{x\to b} g(x) =$ DNE since $\lim_{x\to b^-} g(x) = k \neq \lim_{x\to b^+} g(x) = \infty$
9. $g(e) =$ j
10. $g(b) =$ i
11. g(a) = undefined
12. Define the horizontal asymptote in g(x) using limit notation: $\lim_{x\to\pm\infty} g(x) = f$, so $y = f$ is a H.A.
13. Determine whether the following exists in g(x). If yes, define using limit notation:
    a) Removable Discontinuity No
    b) Jump Discontinuity No
    c) Infinite Discontinuity yes, $\lim_{x\to a^+} g(x) = \infty$ $\lim_{x\to b^+} g(x) = \infty$ $\lim_{x\to e^-} g(x) = e$

14.) Graph (and label) a possible graph for the function $k(x)$ satisfying the following conditions:

$\lim_{x\to\infty} k(x) = -3$ $\quad$ $\lim_{x\to -\infty} k(x) = 6$

$\lim_{x\to 3^-} k(x) = 5$ $\quad$ $\lim_{x\to 2} k(x) = -1$ $\quad$ and $\quad k(2) = 2$ $\quad$ VLT



For #15-24, find each limit analytically or show that it does not exist.

15) $\lim_{x\to -2} \frac{x^3+2x^2}{x+2}$ $\quad \frac{x^2(\cancel{x+2})}{\cancel{x+2}}$ $\quad$ **4**

16) $\lim_{x\to 0} \frac{\sqrt{3}-\sqrt{x^2+3}}{x^2} \cdot \frac{(\sqrt{3}+\sqrt{x^2+3})}{(\sqrt{3}+\sqrt{x^2+3})} = \frac{3-(x^2+3)}{x^2(\sqrt{3}+\sqrt{x^2+3})} = \frac{-1}{\sqrt{3}+\sqrt{x^2+3}}$ $\quad$ **$\frac{-1}{2\sqrt{3}}$ OR $-\frac{\sqrt{3}}{6}$**

17) $\lim_{x\to\infty} \frac{x^{16}+2x^8-1}{3+4x^{16}}$ $\quad$ **$\frac{1}{4}$**

18.) $\lim_{x\to 1} \frac{x^3-1}{x-1} = \frac{(\cancel{x-1})(x^2+x+1)}{\cancel{x-1}}$ $\quad$ **3**

19.) $\lim_{x\to -2} \frac{x}{(1-x)(x+2)^2}$ $\quad$ **$-\infty$**

20.) $\lim_{x\to 3^+} \frac{2x-6}{x-3} = \lim_{x\to 3^+} 2$ $\quad$ **2**

21.) $\lim_{x\to 3} \pi^4$ $\quad$ **π^4**

22.) $\lim_{x\to\infty} \frac{3x+2}{1-8x^2} =$ $\quad$ **0**

23.) $\lim_{x\to\infty} \frac{4-3x^3}{2x^3+3x-1} =$ $\quad$ **$\frac{-3}{2}$**

24.) $\lim_{x\to\infty} \frac{3x+2}{1-8x^2} =$ $\quad$ **0**

25.) Find and justify the existence of any horizontal or vertical asymptote of the function $f(x) = \frac{x^2-x-12}{2x^2-8x}$

$f(x) = \frac{(\cancel{x-4})(x+3)}{2x(\cancel{x-4})} = \frac{x+3}{2x}$ $\quad$ VA $x \neq 0$ $\quad$ hole $x \neq 4$

VA: $x=0$ since $\lim_{x\to 0^-} f(x) = -\infty$ and $\lim_{x\to 0^+} f(x) = \infty$

HA: $y=\frac{1}{2}$ since $\lim_{x\to\pm\infty} f(x) = \frac{1}{2}$

AP Calculus: LIMITS RE-TEST – Part 2 (with calculator)

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26) Suppose f(x) is continuous and $\lim_{x\to 4} f(x) = 12$. Which of the following must be true? (circle all that apply)

a) 4 is in the domain of f $f(4)=12$
b) $f(4) = 12$
c) $\lim_{x\to 4} f(x) = 12$

A B C

27) Find the value of the "c" such that f is everywhere continuous.

$$f(x) = \begin{cases} x^2, & x \le 3 \\ \frac{c}{x}, & x > 3 \end{cases}$$

$(3)^2 = (3)^2 = \frac{c}{3}$

$27 = c$

$c = 27$

28) Use limit notation to describe the information in the table.

x	h(x)
0.498	1.1978
0.4991	1.1983
0.4998	1.1991
0.4999	1.1999

$\lim_{x\to \frac{1}{2}^-} h(x) = 1.2$

29) Identify all asymptotes of the function and use calculus justifications to support your answers.

$$p(x) = \frac{x^2 - 4}{2x^2 + 3x - 2}$$

$\frac{(x-2)\cancel{(x+2)}}{(2x-1)\cancel{(x+2)}}$ VA: $x = \frac{1}{2}$

HA: $y = \frac{1}{2}$ since $\lim_{x\to\pm\infty} p(x) = \frac{1}{2}$ / VA: $x = \frac{1}{2}$ since $\lim_{x\to\frac{1}{2}^-} p(x) = \infty$, $\lim_{x\to\frac{1}{2}^+} p(x) = -\infty$

30) $\lim_{x\to \frac{3\pi}{2}} \frac{\cos^2 x}{1+\sin x} =$ 2

$\frac{0}{0} \Rightarrow \lim_{x\to\frac{3\pi}{2}} \frac{(1-\sin x)\cancel{(1+\sin x)}}{\cancel{1+\sin x}} = 1 - -1$

For #31-33, determine if the statement is true or false. Support all of your answers.

31.) If $f(x)$ is a polynomial, then the function given by $r(x) = \frac{f(x)}{2x+2}$ has a hole at $x = -1$.

False, only a hole if f(x) is divisible by 2x+2. May be a V.A.

32.) A rational function can have infinitely many horizontal asymptotes.

False, at most 2 HAs.

33.) If $\lim_{z\to c} q(z) = L$, and $q(c) =$ is defined, then $q(c)$ is continuous at $z = c$.

2-sided limit function

False, only if $g(c) = L$.