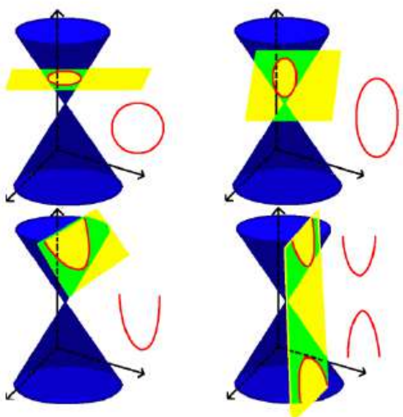
Pre-Calculus Unit 1 Task 1: Our Only Focus: Circles & Parabolas Review

Name: ____ **Date:** _______________ **Period:** ____

For most students, you last learned about conic sections in Analytic Geometry, which was a while ago. Before we begin looking back over the first two types of conic sections that you have already discovered, let's take a look at the geometric meaning of a conic section. First, why "conic"? Conic sections can be defined several ways, and what we'll focus on in this unit is deriving the formulas for the last two types of conic sections from special points called foci. But for the purpose of (re)introduction, the geometric meaning of "conic" comes into focus.



The reason we call these graphs conic sections is that they represent different slices of a double-napped cone. The diagram above shows how the different graphs can be "sliced off" of the figure. You should notice right away that the vast majority of these graphs are relations, not functions – in fact, only one case of parabolas (those featured in your past learning about quadratic functions) are actually functions. That doesn't limit the usefulness of these special planar graphs, however.

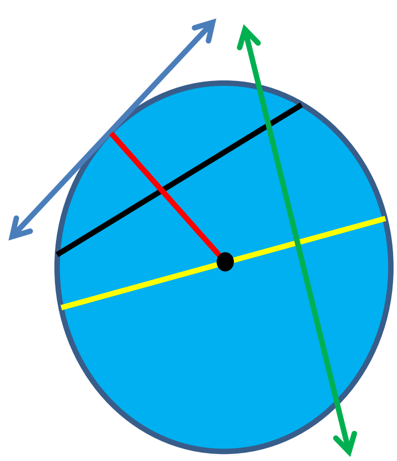
The General Form of a Quadratic Relation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \ \ where \ \ A, B, C \ cannot \ all \ be \ zero$$

The graphs of all conic sections follow this same equation. It should be noted that for our more basic purposes, the B coefficient will always be zero.

Consider…the Circle

Circles should be old news to you, but just as a quick review let's see if you can remember their important parts.



For the circle to the left, label the following features on the diagram:

Center
Diameter
Radius
Chord
Secant Line
Tangent Line
Point of Tangency

That the locus of points making up a circle are equidistant from the circle's center. This leads to an important idea about the center – it serves as the focus of the circle. The points making up the circle are all entirely dependent upon the location of that important focal point.

Now let's review the standard form of the equation describing a circle.

Standard Form of a Circle:

$$(x-h)^2 + (y-k)^2 = r^2 \text{ with center at } (h,k) \text{ and radius } r$$

(Re)Presenting the…Parabola

The conic parabolas you learned about in Analytic Geometry were either functions (opened either up or down) or relations (opened either left or right). Let's do a little parabola review and see what you remember about these.

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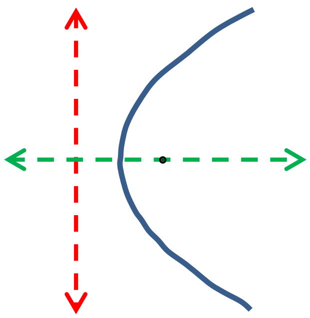

Label the following features on the sketch to the left.

Vertex
Focus
Axis of Symmetry
Directrix
The directed distance p (label 2 different places)

Now let's see if you can answer some basic questions.

(a) What relationship does the locus of points forming a parabola have with the focus and directrix?

(b) What relationship does the vertex have with the focus and the directrix?

(c) What relationship does the directed distance p have with the focus and the directrix?

Just as with circles, the most useable form for parabolas is standard form. Therefore, we need to know the following:

Standard Form of a Parabola and Related Information

With vertex (h,k) and directed distance from the vertex to the focus p:

$$\text{Vertical Axis of Symmetry}: (x-h)^2 = 4p(y-k)$$

If p is positive, the parabola opens up; if p is negative, the parabola opens down.

$$\text{Horizontal Axis of Symmetry}: (y-k)^2 = 4p(x-h)$$

If p is positive, the parabola opens to the right; if p is negative, the parabola opens to the left.

Just as with circles, often you will be given either an equation for a parabola that is not in standard form and you'll need to convert the equation to standard form.

Consider the following equation of a parabola:

$$5y^2 - 6x + 10y - 7 = 0$$

This parabola has been written in general form. Using what we know about the coefficients from general form, we have $C = 5, D = -6, E = 10$, and $F = -7$. It's easy to see that the y term is squared, so either the parabola will open left or right, but beyond this, it's difficult to tell anything else about the relation. Therefore, once again, we will have to convert to standard form by manipulating terms and completing the square:

$$5y^2 - 6x + 10y - 7 = 0$$

$$5y^2 + 10y = 6x + 7$$

$$5\left(y^2 + 2y + \left(\frac{2}{2}\right)^2\right) = 6x + 7 + 5\left(\frac{2}{2}\right)^2$$

Completing the Square!!

$$5(y^2 + 2y + 1) = 6x + 12$$

$$5(y+1)^2 = 6(x+2)$$

$$(y+1)^2 = \frac{6}{5}(x+2)$$

So what do we now know? Well, we know the vertex of the parabola is at. $(-2, -1)$. We know the parabola opens to the right because p is positive. How do we know it's positive? Let's see…

Standard Form: $(y-k)^2 = 4p(x-h)$

so $$4p = \frac{6}{5} \; so \; p = \frac{6}{20} = \frac{3}{10}$$

Therefore the focus is at $\left(-2 + \frac{3}{10}, -1\right) = \left(\frac{-17}{10}, -1\right)$ and the directrix would be at $x = -2 - \frac{3}{10}$ which simplifies to. $x = -\frac{23}{10}$

Convert the following equations of parabolas into standard form.

1. $x^2 + x - y = 5$

2. $2y^2 + 16y = -x - 27$

3. $x = -y^2 + 6y - 5$

Two more things before we go…

Circles and parabolas are from the past – they're not our focus now. But the next two conic sections are built upon your knowledge of these simplest of conics. Therefore, think about (and answer!) these two questions.

1. We know that the general form of a quadratic relation is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

   What relationship do the coefficients A and C have for a circle? For a parabola?

2. Why was this activity named "our only focus"?