Two way Frequency table

Warm Up

A bag contains 4 red and 2 yellow marbles. A marble is selected, kept out of the bag, and another marble is selected. Find each conditional probability of selecting the second marble.

1. $P(\text{red} \mid \text{red})$ *0.6*

2. $P(\text{red} \mid \text{yellow})$ *0.8*

3. $P(\text{yellow} \mid \text{yellow})$ *0.2*

4. $P(\text{yellow} \mid \text{red})$ *0.4*

Continued : Warm Up

5. *A bag contains 4 red and 2 yellow marbles. A marble is selected, kept out of the bag, and another marble is selected. Find P(two red marbles).*

0.4

Objectives

Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified.

Vocabulary

joint relative frequency

marginal relative frequency

conditional relative frequency

A *two-way table* is a useful way to organize data that can be categorized by two variables. Suppose you asked 20 children and adults whether they liked broccoli. The table shows one way to arrange the data.

The **joint relative frequencies** are the values in each category divided by the total number of values, shown by the shaded cells in the table. Each value is divided by 20, the total number of individuals.

	Yes	No
Children	3	8
Adults	7	2

The **marginal relative frequencies** are found by adding the joint relative frequencies in each row and column.

	Yes	No	Total
Children	0.15	0.4	0.55
Adults	0.35	0.1	0.45
Total	0.5	0.5	1

To find a **conditional relative frequency** , divide the joint relative frequency by the marginal relative frequency. Conditional relative frequencies can be used to find conditional probabilities.

Example 1: Finding Joint and Marginal Relative Frequencies

The table shows the results of randomly selected car insurance quotes for 125 cars made by an insurance company in one week. Make a table of the joint and marginal relative frequencies.

	Teen	Adult
0 accidents	15	53
1 accident	4	32
2+ accidents	9	12

Example 1: Continued

Divide each value by the total of 125 to find the joint relative frequencies, and add each row and column to find the marginal relative frequencies.

	Teen	Adult	Total
0 acc.	0.12	0.424	0.544
1 acc.	0.032	0.256	0.288
2 + acc.	0.072	0.096	0.168
Total	0.224	0.776	1

Check It Out! Example 1

The table shows the number of books sold at a library sale. Make a table of the joint and marginal relative frequencies.

	Fiction	Nonfiction
Hardcover	28	52
Paperback	94	36

Check It Out! Example 1 Continued

Divide each value by the total of 210 to find the joint relative frequencies, and add each row and column to find the marginal relative frequencies.

	Fiction	Nonfiction	Total
Hardcover	0.133	0.248	0.381
Paperback	0.448	0.171	0.619
Total	0.581	0.419	1

Example 2: Using Conditional Relative Frequency to Find Probability

A reporter asked 150 voters if they plan to vote in favor of a new library and a new arena. The table shows the results.

		Library	
		Yes	No
Arena	Yes	21	30
	No	57	42

Example 2A Continued

A. Make a table of the joint and marginal relative frequencies.

		Library		
		Yes	No	Total
Arena	Yes	0.14	0.2	0.34
	No	0.38	0.28	0.66
	Total	0.52	0.48	1

Example 2B Continued

B. If you are given that a voter plans to vote *no* to the new library, what is the probability the voter also plans to say no to the new arena?

$$\frac{0.28}{0.48} \approx 0.58$$

Check It Out! Example 2

The classes at a dance academy include ballet and tap dancing. Enrollment in these classes is shown in the table.

		Ballet: Yes	Ballet: No
Tap	Yes	38	52
	No	86	24

2a. Copy and complete the table of the joint relative frequencies and marginal relative frequencies.

		Ballet: Yes	Ballet: No	Total
Tap	Yes			
	No			
	Total			1

Check It Out! Example 2 continued

Ballet

Tap	Yes	No	Total
Yes	0.19	0.26	0.45
No	0.43	0.12	0.55
Total	0.62	0.38	1

2b. If you are given that a student is taking ballet, what is the probability that the student is not taking tap?

$$\frac{0.43}{0.62} \approx 0.69 \text{ or } 69\%$$

Example 3: Comparing Conditional Probabilities

A company sells items in a store, online, and through a catalog. A manager recorded whether or not the 50 sales made one day were paid for with a gift card.

	Gift Card	Another Method
Store	𝍸 I	𝍸 IIII
Online	𝍸 IIII	𝍸 𝍸 III
Catalog	𝍸	𝍸 III

Use conditional probabilities to determine for which method a customer is most likely to pay with a gift card.

Example 3 Continued

	Gift Card	Another Method	TOTAL
Store	**0.12**	**0.18**	**0.30**
Online	**0.18**	**0.26**	**0.44**
Catalog	**0.10**	**0.16**	**0.26**
TOTAL	**0.40**	**0.60**	**1**

P(gift card if in store) = 0.4
P(gift card if online) ≈ 0.41
P(gift card if by catalog) ≈ 0.38
so most likely if buying online.

A customer is most likely to pay with a gift card if buying online.

Check It Out! Example 3

Francine is evaluating three driving schools. She asked 50 people who attended the schools whether they passed their driving tests on the first try. Use conditional probabilities to determine which is the best school.

Use conditional probabilities to determine which is the best school.

	Pass	Fail
Al's Driving	14	8
Drive Time	11	7
Crash Course	5	5

Check It Out! Example 3 Continued

	Pass	Fail	TOTAL
Al's Driving	0.28	0.16	0.44
Drive Time	0.22	0.14	0.36
Crash Course	0.10	0.10	0.20
TOTAL	0.60	0.40	1

Al's Driving has the best pass rate, about 64%, versus 61% for Drive Time and 50% for Crash Course.

Lesson Quiz: Part I

1. At a juice-bottling factory, quality-control technicians randomly select bottles and mark them *pass* or *fail*. The manager randomly selects the results of 50 tests and organizes the data by shift and result. The table below shows these results.

	Pass	Fail
Morning	14	5
Afternoon	10	4
Evening	11	6

Lesson Quiz: Part I continued

1. Make a table of the joint and marginal relative frequencies.

	Pass	Fail	Total
Morn.	0.28	0.1	0.38
After.	0.2	0.08	0.28
Eve.	0.22	0.12	0.34
Total	0.7	0.3	1

Lesson Quiz: Part 2

2. Find the probability that a bottle was inspected in the afternoon given that it failed the inspection.

$$\frac{0.08}{0.3} \approx 0.27$$

Lesson Quiz: Part 3

3. Use conditional probabilities to determine on which shift a bottle is most likely to pass inspection.

P(pass if in morning) ≈ 0.74,
P(pass if in afternoon) ≈ 0.71,
P(pass if in evening) ≈ 0.65,
so most likely to pass in the morning