Trigonometric ratios

Warm Up

Write each fraction as a decimal rounded to the nearest hundredth.

1. $\frac{2}{3}$ 0.67

2. $\frac{7}{24}$ 0.29

Solve each equation.

3. $0.8 = \frac{5.8}{x}$ $x = 7.25$

4. $0.94 = \frac{x}{8.5}$ $x = 7.99$

Objectives

Find the sine, cosine, and tangent of an acute angle.

Use trigonometric ratios to find side lengths in right triangles and to solve real-world problems.

Vocabulary

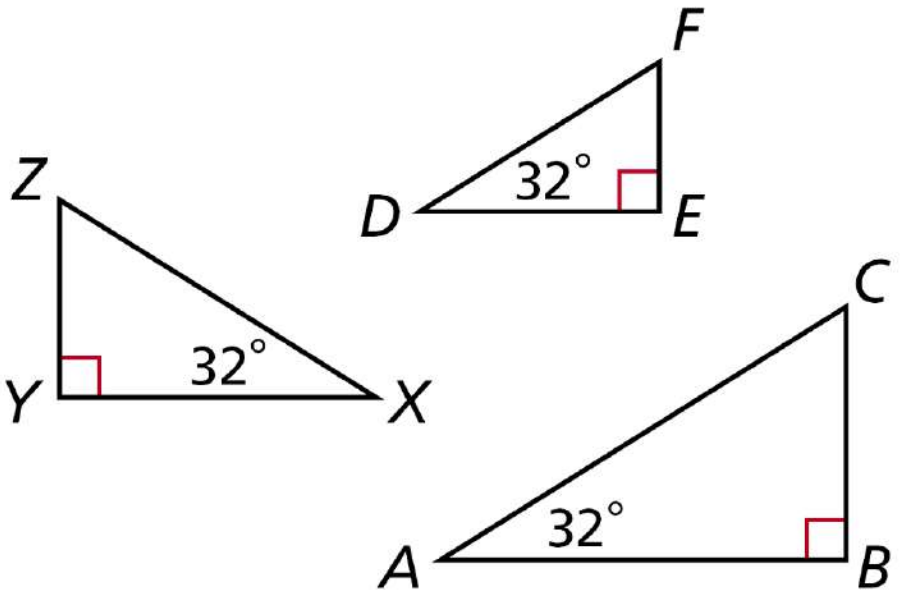
trigonometric ratio

sine

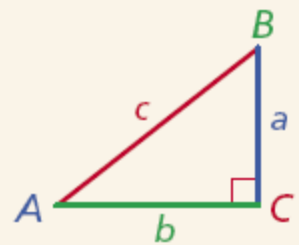
cosine

tangent

By the AA Similarity Postulate, a right triangle with a given acute angle is similar to every other right triangle with that same acute angle measure. So $\Delta ABC \sim \Delta DEF \sim \Delta XYZ$, and $\frac{BC}{AC} = \frac{EF}{DF} = \frac{YZ}{XZ}$. These are *trigonometric ratios*. A **trigonometric ratio** is a ratio of two sides of a right triangle.



Trigonometric Ratios

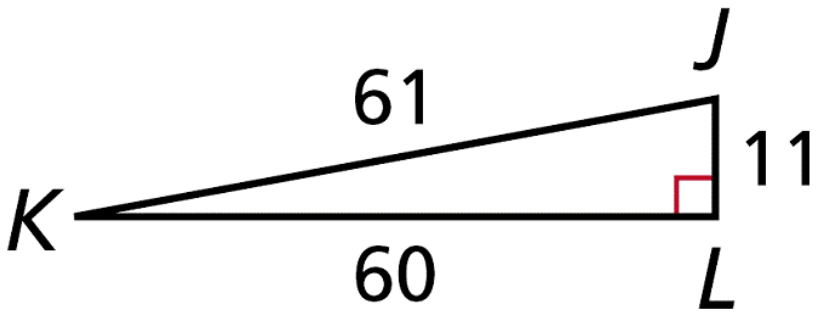
DEFINITION	SYMBOLS	DIAGRAM
The **sine** of an angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.	$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{a}{c}$ $\sin B = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{b}{c}$	
The **cosine** of an angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse.	$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{b}{c}$ $\cos B = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{a}{c}$	
The **tangent** of an angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.	$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{a}{b}$ $\tan B = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{b}{a}$	

Writing Math

In trigonometry, the letter of the vertex of the angle is often used to represent the measure of that angle. For example, the sine of $\angle A$ is written as sin A.

Example 1A: Finding Trigonometric Ratios

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



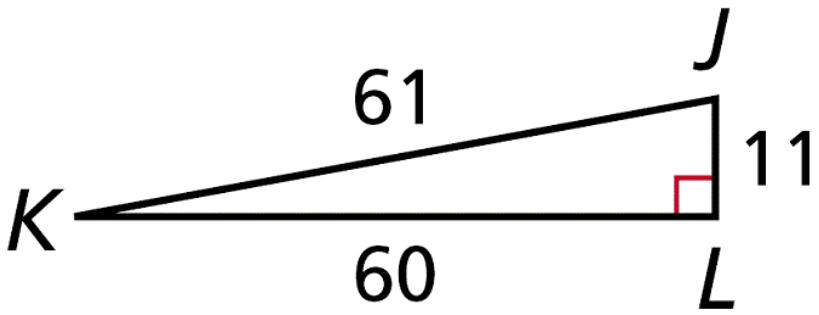
sin *J*

$$\sin J = \frac{60}{61} \approx 0.98$$

The sine of an ∠ is $\frac{\textit{opp. leg}}{\textit{hyp.}}$.

Example 1B: Finding Trigonometric Ratios

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



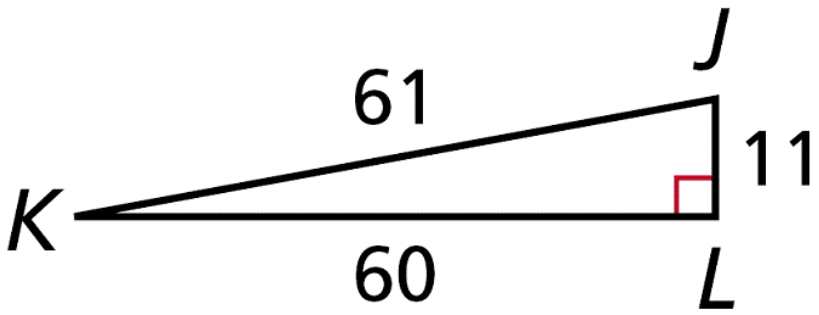
cos *J*

$$\cos J = \frac{11}{61} \approx 0.18$$

The cosine of an ∠ is $\frac{\text{adj. leg}}{\text{hyp.}}$.

Example 1C: Finding Trigonometric Ratios

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



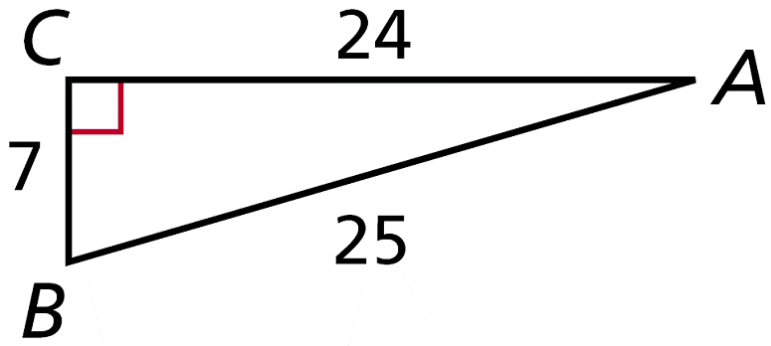
tan *K*

$$\tan K = \frac{11}{60} \approx 0.18$$

The tangent of an ∠ is $\frac{\text{opp. leg}}{\text{adj. leg}}$.

Check It Out! Example 1a

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



cos *A*

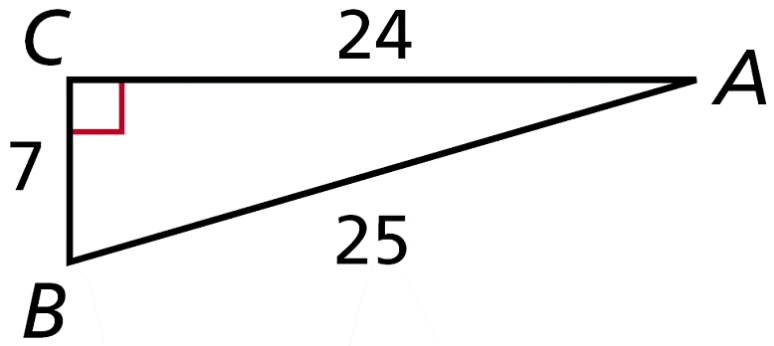
$$\cos A = \frac{24}{25} \approx 0.96$$

The cosine of an ∠ is $\frac{\textit{adj. leg}}{\textit{hyp.}}$.

Check It Out! Example 1b

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.

tan *B*

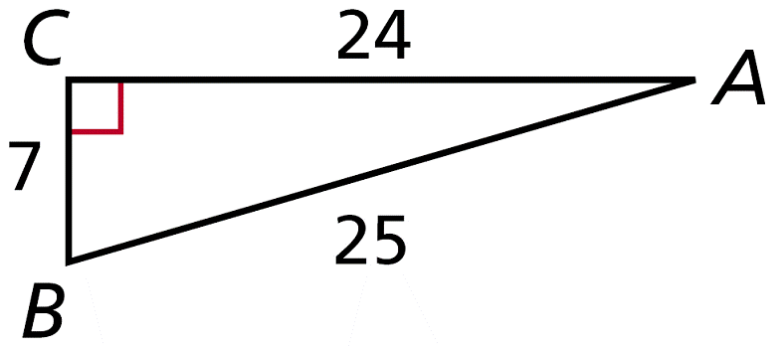


$$\tan B = \frac{24}{7} \approx 3.43$$

The tangent of an ∠ is $\frac{\text{opp. leg}}{\text{adj. leg}}$.

Check It Out! Example 1c

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



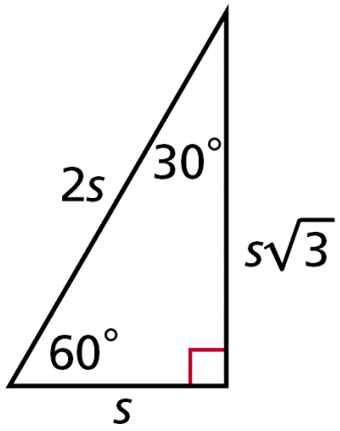
sin *B*

$$\sin B = \frac{24}{25} \approx 0.96$$

The sine of an ∠ is $\frac{\text{opp. leg}}{\text{hyp.}}$.

Example 2: Finding Trigonometric Ratios in Special Right Triangles

Use a special right triangle to write cos 30° as a fraction.



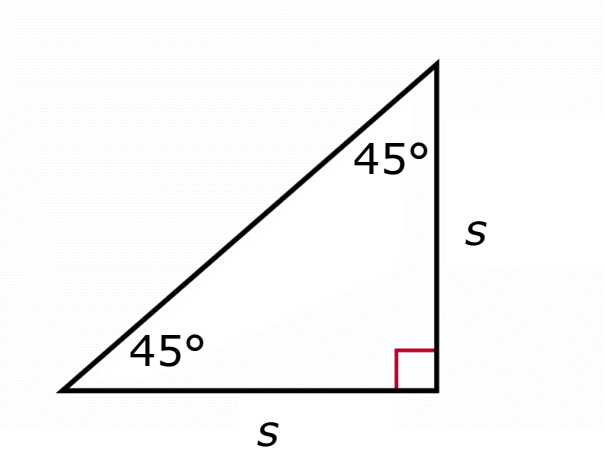
Draw and label a 30º-60º-90º Δ.

$$\cos 30° = \frac{s\sqrt{3}}{2s} = \frac{\sqrt{3}}{2}$$

The cosine of an ∠ is $\frac{\text{adj. leg}}{\text{hyp.}}$.

Check It Out! Example 2

Use a special right triangle to write tan 45° as a fraction.



Draw and label a 45º-45º-90º Δ.

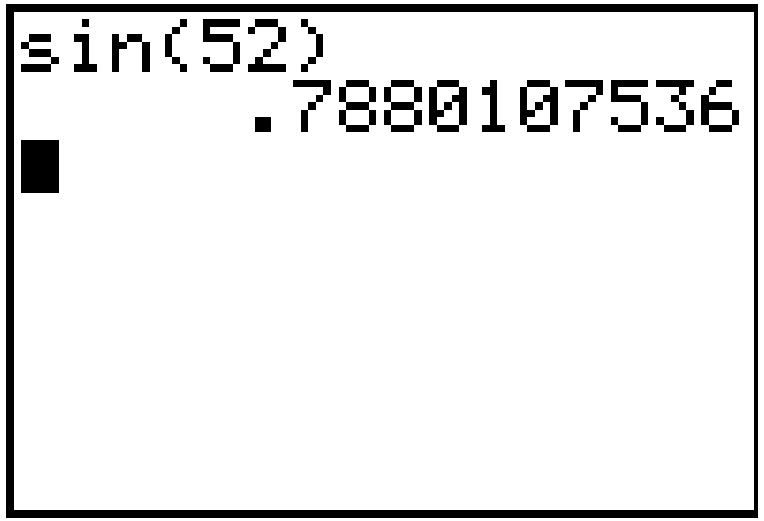
$$\tan 45^\circ = \frac{s}{s} = 1$$

The tangent of an ∠ is $\frac{\text{opp. leg}}{\text{adj. leg}}$.

Example 3A: Calculating Trigonometric Ratios

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

sin 52°



sin 52° ≈ 0.79

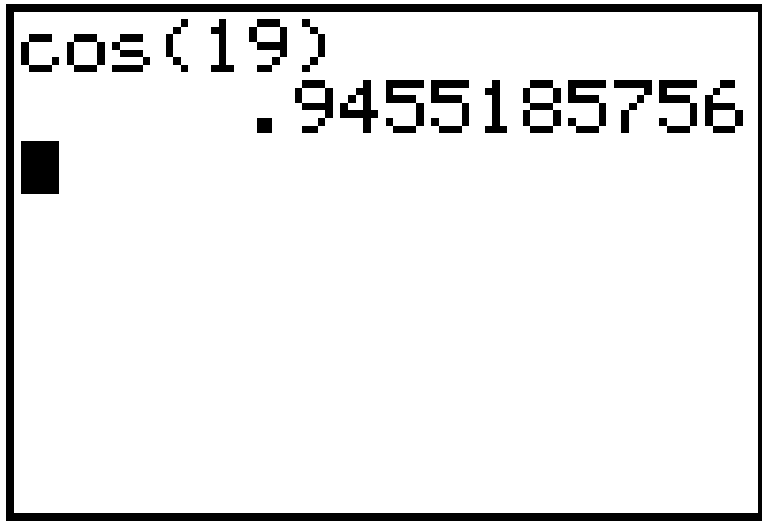
Caution!

Be sure your calculator is in degree mode, not radian mode.

Example 3B: Calculating Trigonometric Ratios

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

cos 19°

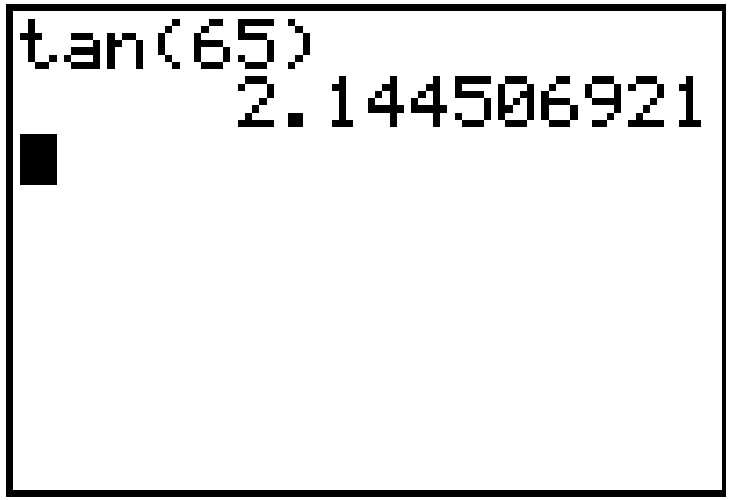


cos 19° ≈ 0.95

Example 3C: Calculating Trigonometric Ratios

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

tan 65°

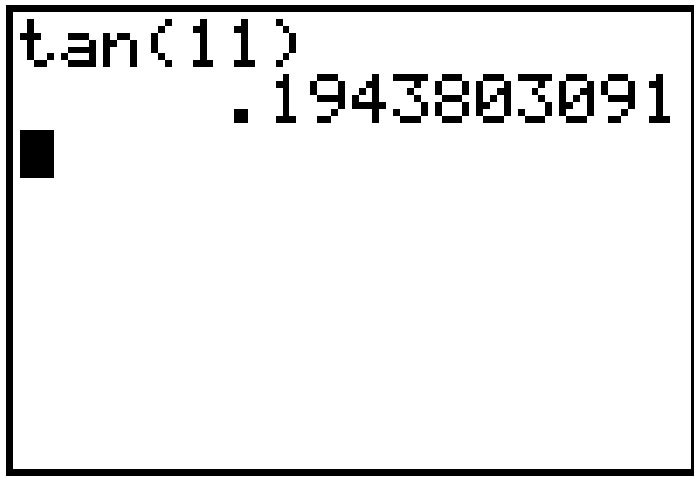


tan 65° ≈ 2.14

Check It Out! Example 3a

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

tan 11°

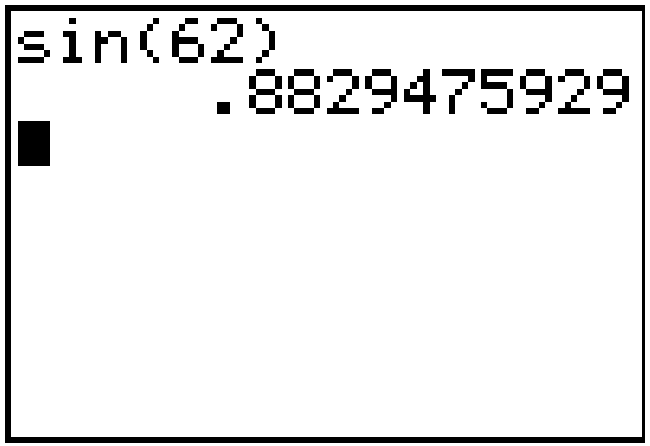


tan 11° ≈ 0.19

Check It Out! Example 3b

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

sin 62°



sin 62° ≈ 0.88

Check It Out! Example 3c

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

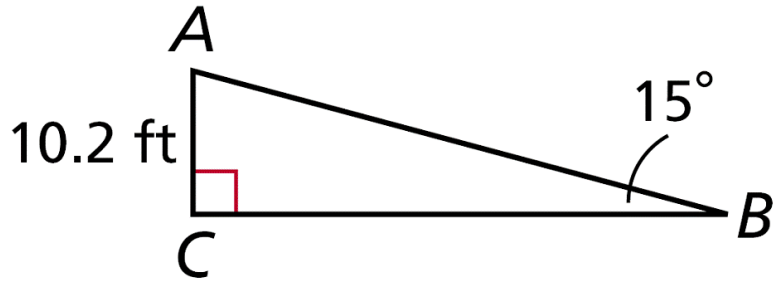
cos 30°

$\cos 30° \approx 0.87$

The hypotenuse is always the longest side of a right triangle. So the denominator of a sine or cosine ratio is always greater than the numerator. Therefore the sine and cosine of an acute angle are always positive numbers less than 1. Since the tangent of an acute angle is the ratio of the lengths of the legs, it can have any value greater than 0.

Example 4A: Using Trigonometric Ratios to Find Lengths

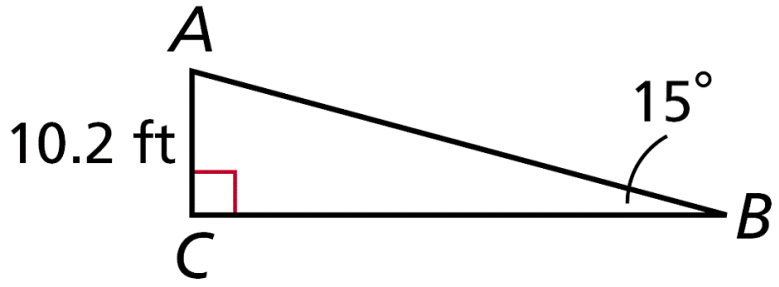
Find the length. Round to the nearest hundredth.



BC

$\overline{BC}$ is adjacent to the given angle, $\angle B$. You are given AC, which is opposite $\angle B$. Since the adjacent and opposite legs are involved, use a tangent ratio.

Example 4A Continued



$$\tan B = \frac{\text{opp. leg}}{\text{adj. leg}} = \frac{AC}{BC}$$ *Write a trigonometric ratio.*

$$\tan 15° = \frac{10.2}{BC}$$ *Substitute the given values.*

$$BC = \frac{10.2}{\tan 15°}$$ *Multiply both sides by BC and divide by tan 15°.*

$BC \approx 38.07$ ft *Simplify the expression.*

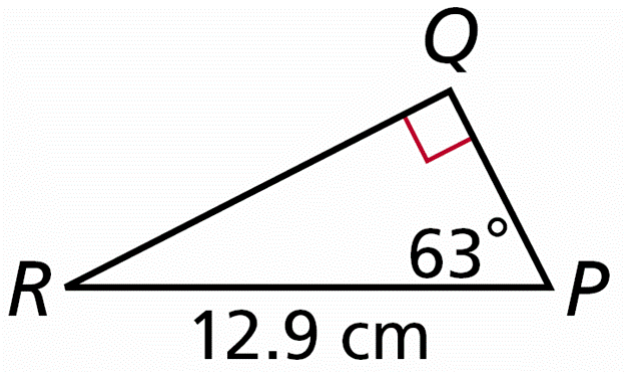
Caution!

Do not round until the final step of your answer. Use the values of the trigonometric ratios provided by your calculator.

Example 4B: Using Trigonometric Ratios to Find Lengths

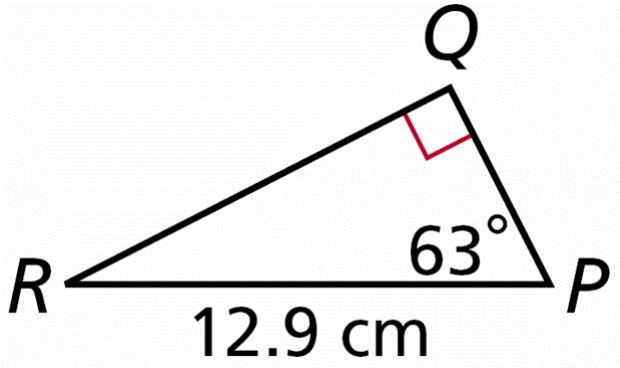
Find the length. Round to the nearest hundredth.

QR



$\overline{QR}$ is opposite to the given angle, $\angle P$. You are given PR, which is the hypotenuse. Since the opposite side and hypotenuse are involved, use a sine ratio.

Example 4B Continued



$$\sin P = \frac{\text{opp. leg}}{\text{hyp.}} = \frac{QR}{PR}$$ *Write a trigonometric ratio.*

$$\sin 63° = \frac{QR}{12.9}$$ *Substitute the given values.*

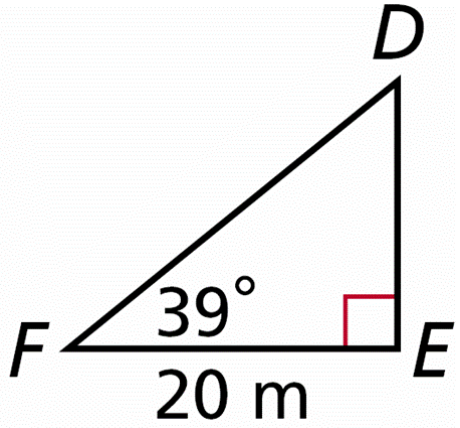
$12.9(\sin 63°) = QR$ *Multiply both sides by 12.9.*

$11.49 \text{ cm} \approx QR$ *Simplify the expression.*

Example 4C: Using Trigonometric Ratios to Find Lengths

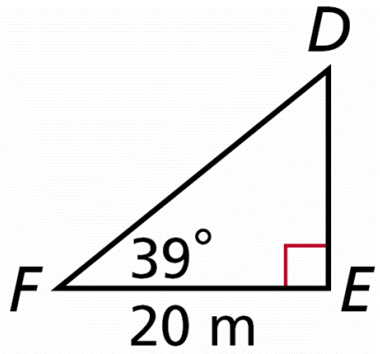
Find the length. Round to the nearest hundredth.

FD



$\overline{FD}$ is the hypotenuse. You are given *EF*, which is adjacent to the given angle, $\angle F$. Since the adjacent side and hypotenuse are involved, use a cosine ratio.

Example 4C Continued



$$\cos F = \frac{\text{adj. leg}}{\text{hyp}} = \frac{EF}{FD}$$ *Write a trigonometric ratio.*

$$\cos 39° = \frac{20}{FD}$$ *Substitute the given values.*

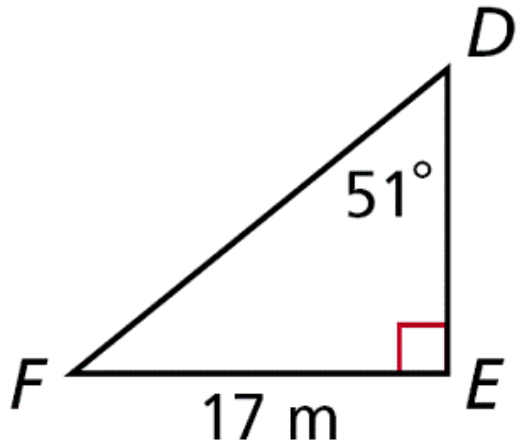
$$FD = \frac{20}{\cos 39°}$$ *Multiply both sides by FD and divide by cos 39°.*

$FD \approx 25.74$ m *Simplify the expression.*

Check It Out! Example 4a

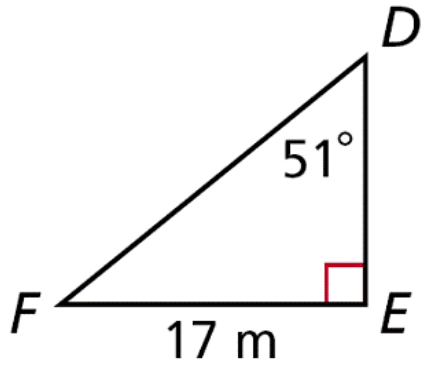
Find the length. Round to the nearest hundredth.

DF



$\overline{DF}$ is the hypotenuse. You are given EF, which is opposite to the given angle, $\angle D$. Since the opposite side and hypotenuse are involved, use a sine ratio.

Check It Out! Example 4a Continued



$$\sin D = \frac{\text{opp. leg}}{\text{hyp}} = \frac{EF}{DF}$$ *Write a trigonometric ratio.*

$$\sin 51° = \frac{17}{DF}$$ *Substitute the given values.*

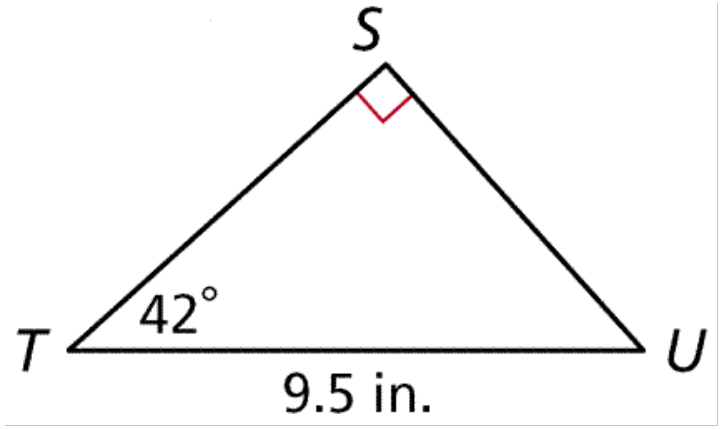
$$DF = \frac{17}{\sin 51°}$$ *Multiply both sides by DF and divide by sin 51°.*

$DF \approx 21.87$ cm *Simplify the expression.*

Check It Out! Example 4b

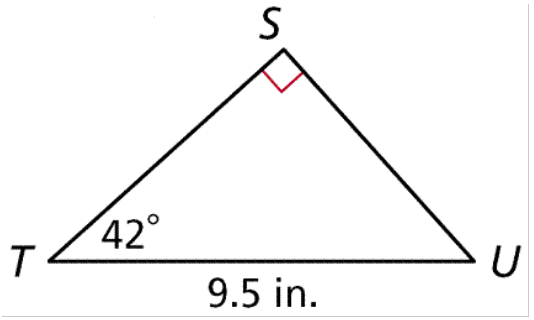
Find the length. Round to the nearest hundredth.

ST



$\overline{ST}$ is a leg. You are given *TU*, which is the hypotenuse. Since the adjacent side and hypotenuse are involved, use a cosine ratio.

Check It Out! Example 4b Continued



$$\cos T = \frac{\text{adj. leg}}{\text{hyp}} = \frac{ST}{TU}$$ *Write a trigonometric ratio.*

$$\cos 42° = \frac{ST}{9.5}$$ *Substitute the given values.*

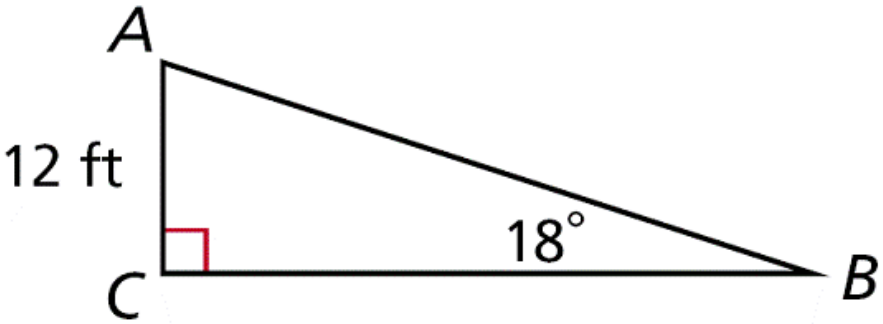
$ST = 9.5(\cos 42°)$ *Multiply both sides by 9.5.*

$ST \approx 7.06$ in. *Simplify the expression.*

Check It Out! Example 4c

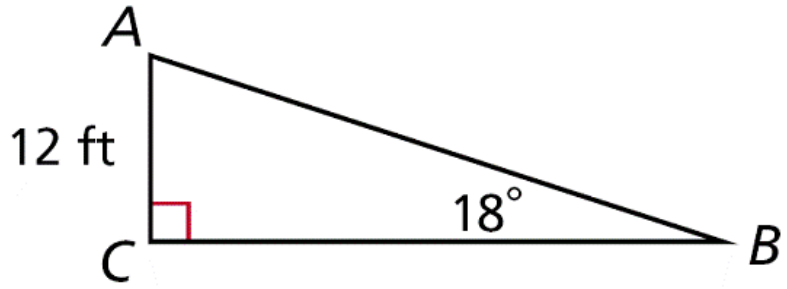
Find the length. Round to the nearest hundredth.

BC



$\overline{BC}$ is a leg. You are given AC, which is the opposite side to given angle, $\angle B$. Since the opposite side and adjacent side are involved, use a tangent ratio.

Check It Out! Example 4c Continued



$$\tan B = \frac{\text{opp. leg}}{\text{adj. leg}} = \frac{AC}{BC}$$ *Write a trigonometric ratio.*

$$\tan 18° = \frac{12}{BC}$$ *Substitute the given values.*

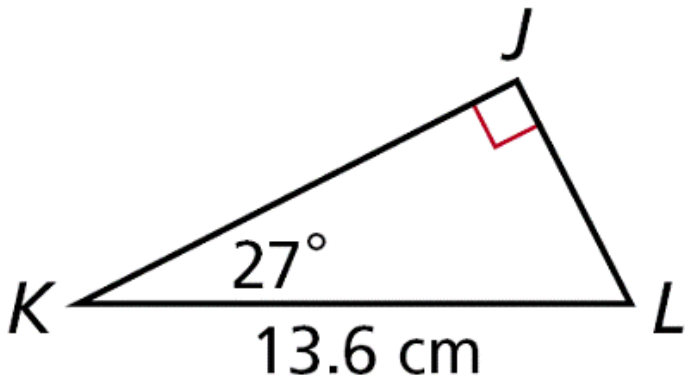
$$BC = \frac{12}{\tan 18°}$$ *Multiply both sides by BC and divide by tan 18°.*

$BC \approx 36.93$ ft *Simplify the expression.*

Check It Out! Example 4d

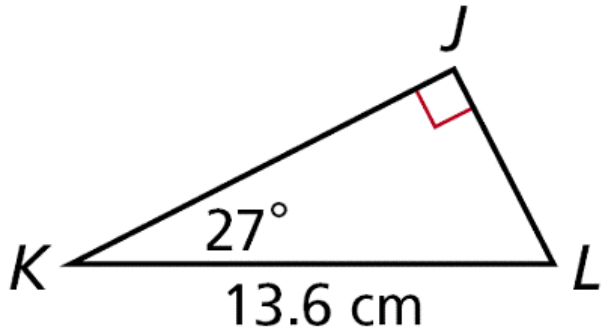
Find the length. Round to the nearest hundredth.

JL



$\overline{JL}$ is the opposite side to the given angle, $\angle K$. You are given KL, which is the hypotenuse. Since the opposite side and hypotenuse are involved, use a sine ratio.

Check It Out! Example 4d Continued



$$\sin K = \frac{\text{opp. leg}}{\text{hyp}} = \frac{JL}{KL}$$ *Write a trigonometric ratio.*

$$\sin 27° = \frac{JL}{13.6}$$ *Substitute the given values.*

$JL = 13.6(\sin 27°)$ *Multiply both sides by 13.6.*

$JL \approx 6.17$ cm *Simplify the expression.*

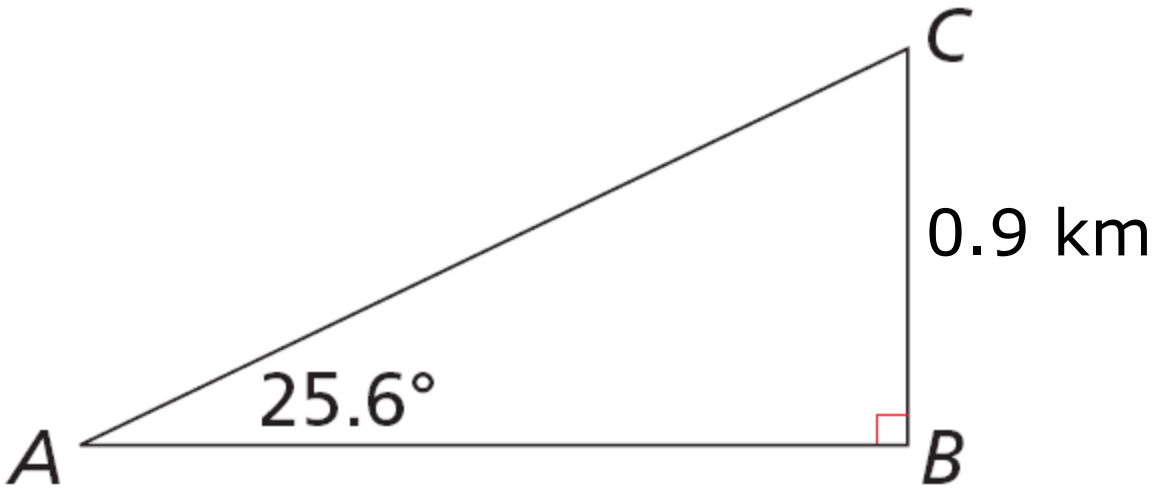
Example 5: *Problem-Solving Application*

The Pilatusbahn in Switzerland is the world's steepest cog railway. Its steepest section makes an angle of about 25.6º with the horizontal and rises about 0.9 km. To the nearest hundredth of a kilometer, how long is this section of the railway track?

Example 5 Continued

1 Understand the Problem

Make a sketch. The **answer** is *AC*.



Example 5 Continued

2 Make a Plan

$\overline{CA}$ is the hypotenuse. You are given BC, which is the leg opposite $\angle A$. Since the opposite and hypotenuse are involved, write an equation using the sine ratio.

Example 5 Continued

3 **Solve**

$$\sin A = \frac{\text{opp. leg}}{\text{hyp.}} = \frac{BC}{CA}$$ *Write a trigonometric ratio.*

$$\sin 25.6^\circ = \frac{0.9}{CA}$$ *Substitute the given values.*

$$CA = \frac{0.9}{\sin 25.6^\circ}$$ *Multiply both sides by CA and divide by sin 25.6°.*

$CA \approx 2.0829$ km *Simplify the expression.*

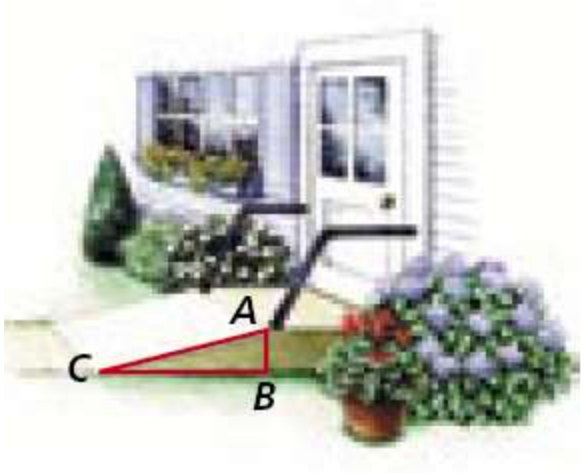
Example 5 Continued

Look Back

The problem asks for *CA* rounded to the nearest hundredth, so round the length to 2.08. The section of track is 2.08 km.

Check It Out! Example 5

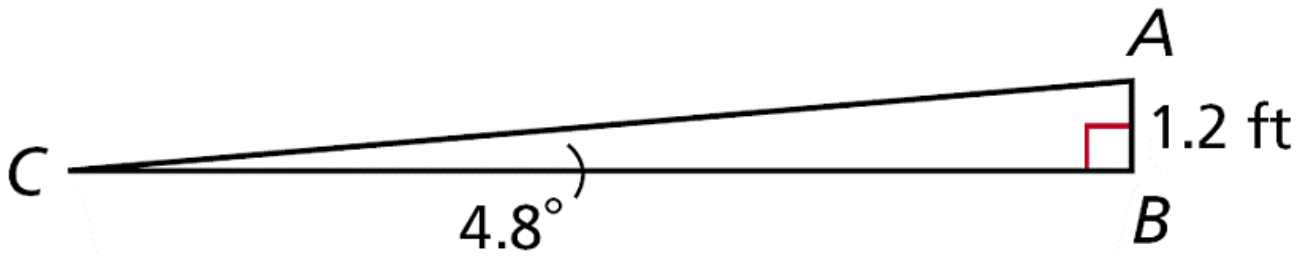
Find *AC*, the length of the ramp, to the nearest hundredth of a foot.



Check It Out! Example 5 Continued

1 Understand the Problem

Make a sketch. The **answer** is *AC*.



Check It Out! Example 5 Continued

Make a Plan

$\overline{AC}$ is the hypotenuse to $\angle C$. You are given AB, which is the leg opposite $\angle C$. Since the opposite leg and hypotenuse are involved, write an equation using the sine ratio.

Check It Out! Example 5 Continued

3 Solve

$$\sin C = \frac{\text{opp. leg}}{\text{hyp.}} = \frac{AB}{AC}$$ *Write a trigonometric ratio.*

$$\sin 4.8° = \frac{1.2}{AC}$$ *Substitute the given values.*

$$AC = \frac{1.2}{\sin 4.8°}$$ *Multiply both sides by AC and divide by sin 4.8°.*

$AC \approx 14.3407$ ft *Simplify the expression.*

Check It Out! Example 5 Continued

Look Back

The problem asks for *AC* rounded to the nearest hundredth, so round the length to 14.34. The length of ramp covers a distance of 14.34 ft.

Lesson Quiz: Part I

Use a special right triangle to write each trigonometric ratio as a fraction.

1. sin 60° $\frac{\sqrt{3}}{2}$

2. cos 45° $\frac{\sqrt{2}}{2}$

Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

3. tan 84° 9.51

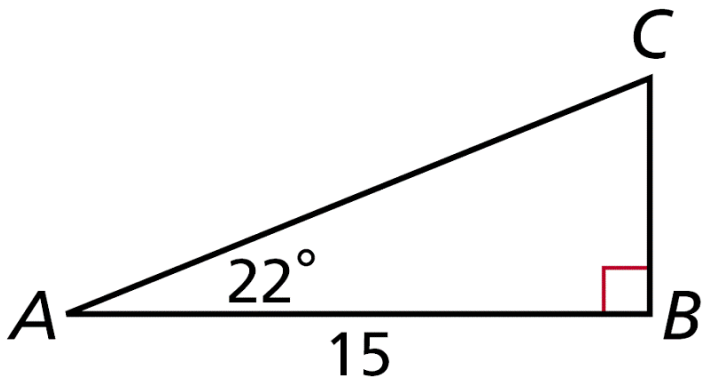
4. cos 13° 0.97

Lesson Quiz: Part II

Find each length. Round to the nearest tenth.

5. *CB* 6.1

6. *AC* 16.2



Use your answers from Items 5 and 6 to write each trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.

7. sin *A* $\frac{6.1}{16.2} \approx 0.38$ **8.** cos *A* $\frac{15}{16.2} \approx 0.93$ **9.** tan *A* $\frac{6.1}{15} \approx 0.41$