

Triangle Midsegments

Warm Up

Use the points $A(2, 2)$, $B(12, 2)$ and $C(4, 8)$ for Exercises 1–5.

1. Find X and Y , the midpoints of AC and CB . $(3, 5), (8, 5)$
2. Find XY . 5
3. Find AB . 10
4. Find the slope of AB . 0
5. Find the slope of XY . 0
6. What is the slope of a line parallel to $3x + 2y = 12$? $-\frac{3}{2}$

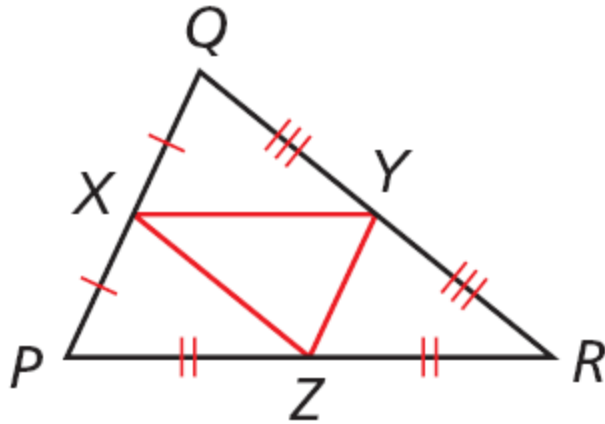
Objective

Prove and use properties of triangle midsegments.

Vocabulary

midsegment of a triangle

A **midsegment of a triangle** is a segment that joins the midpoints of two sides of the triangle. Every triangle has three midsegments, which form the *midsegment triangle*.



Midsegments: $\overline{XY}$, $\overline{YZ}$, $\overline{ZX}$

Midsegment triangle: $\triangle XYZ$

Example 1: Examining Midsegments in the Coordinate Plane

The vertices of $\triangle XYZ$ are $X(-1, 8)$, $Y(9, 2)$, and $Z(3, -4)$. M and N are the midpoints of $\overline{XZ}$ and $\overline{YZ}$. Show that $\overline{MN} \parallel \overline{XY}$ and $MN = \frac{1}{2}XY$.

Step 1 Find the coordinates of M and N .

$$\text{mdpt. of } \overline{XZ} = \left(\frac{-1+3}{2}, \frac{8+(-4)}{2} \right) = (1, 2)$$

$$\text{mdpt. of } \overline{YZ} = \left(\frac{9+3}{2}, \frac{2+(-4)}{2} \right) = (6, -1)$$

Example 1 Continued

Step 2 Compare the slopes of $\overline{MN}$ and $\overline{XY}$.

$$\text{slope of } \overline{MN} = \frac{-1-2}{6-1} = -\frac{3}{5}$$

$$\text{slope of } \overline{XY} = \frac{2-8}{9-(-1)} = -\frac{3}{5}$$

Since the slopes are the same, $\overline{MN} \parallel \overline{XY}$.

Example 1 Continued

Step 3 Compare the heights of $\overline{MN}$ and $\overline{XY}$.

$$MN = \sqrt{(6-1)^2 + (-1-2)^2} = \sqrt{34}$$

$$XY = \sqrt{[9 - (-1)]^2 + (2-8)^2} = 2\sqrt{34}$$

$$\text{Since } \sqrt{34} = \frac{1}{2}(2\sqrt{34}), MN = \frac{1}{2}XY.$$

Check It Out! Example 1

The vertices of $\triangle RST$ are $R(-7, 0)$, $S(-3, 6)$, and $T(9, 2)$. M is the midpoint of $\overline{RT}$, and N is the midpoint of $\overline{ST}$. Show that $\overline{MN} \parallel \overline{RS}$ and

$$MN = \frac{1}{2}RS.$$

Step 1 Find the coordinates of M and N .

$$\text{mdpt. of } \overline{RT} = \left(\frac{-7+9}{2}, \frac{0+2}{2} \right) = (1, 1)$$

$$\text{mdpt. of } \overline{ST} = \left(\frac{-3+9}{2}, \frac{6+2}{2} \right) = (3, 4)$$

Check It Out! Example 1 Continued

Step 2 Compare the slopes of $\overline{MN}$ and $\overline{RS}$.

$$\text{slope of } \overline{MN} = \frac{3}{2}$$

$$\text{slope of } \overline{RS} = \frac{3}{2}$$

Since the slopes are equal $\overline{MN} \parallel \overline{RS}$.

Check It Out! Example 1 Continued

Step 3 Compare the heights of $\overline{MN}$ and $\overline{RS}$.

$$MN = \sqrt{(4-1)^2 + (3-1)^2} = \sqrt{13}$$

$$RS = \sqrt{(6-0)^2 + [-3-(-7)]^2} = \sqrt{52} = 2\sqrt{13}$$

The length of $\overline{MN}$ is half the length of $\overline{RS}$.

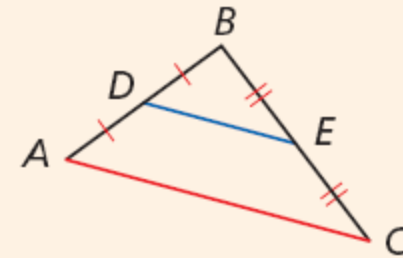
The relationship shown in Example 1 is true for the three midsegments of every triangle.

Theorem 5-4-1

Triangle Midsegment Theorem

A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.

$$\overline{DE} \parallel \overline{AC}, DE = \frac{1}{2}\overline{AC}$$



Example 2A: Using the Triangle Midsegment Theorem

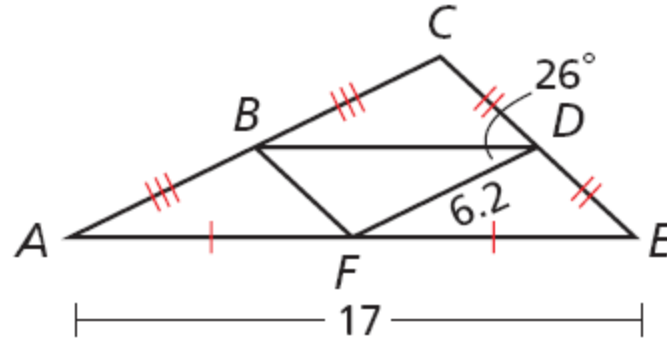
Find each measure.

BD

$$BD = \frac{1}{2}AE \quad \Delta \text{ Midsegment Thm.}$$

$$BD = \frac{1}{2}(17) \quad \text{Substitute 17 for AE.}$$

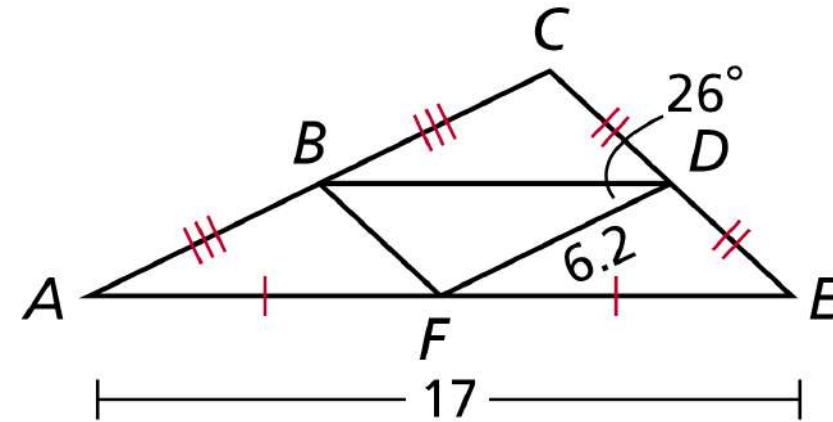
$$BD = 8.5 \quad \text{Simplify.}$$



Example 2B: Using the Triangle Midsegment Theorem

Find each measure.

$m\angle CBD$



$$\overline{DF} \parallel \overline{CA}$$

△ Midsegment Thm.

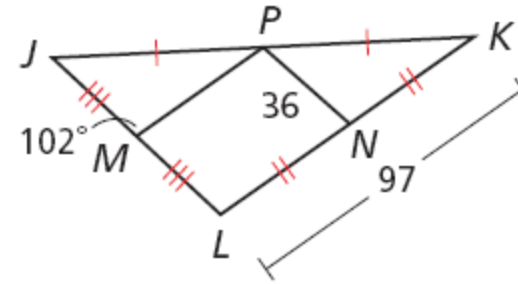
$$m\angle CBD = m\angle BDF \quad \text{Alt. Int. } \angle\text{s Thm.}$$

$$m\angle CBD = 26^\circ \quad \text{Substitute } 26^\circ \text{ for } m\angle BDF.$$

Check It Out! Example 2a

Find each measure.

JL



$$PN = \frac{1}{2}JL \quad \Delta \text{ Midsegment Thm.}$$

$$2(36) = JL \quad \text{Substitute 36 for PN and multiply both sides by 2.}$$

$$72 = JL \quad \text{Simplify.}$$

Check It Out! Example 2b

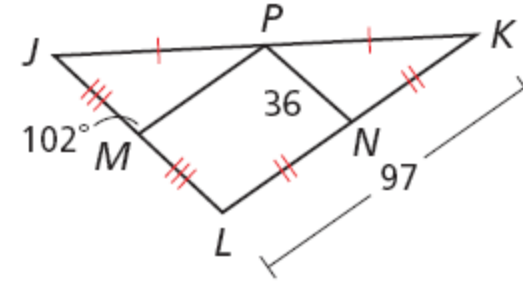
Find each measure.

PM

$$PM = \frac{1}{2}LK \quad \Delta \text{ Midsegment Thm.}$$

$$PM = \frac{1}{2}(97) \quad \text{Substitute 97 for } LK.$$

$$PM = 48.5 \quad \text{Simplify.}$$



Check It Out! Example 2c

Find each measure.

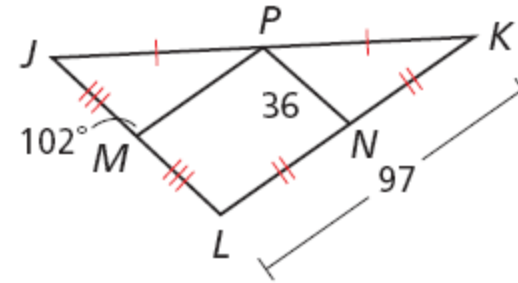
$m\angle MLK$

$$\overline{MP} \parallel \overline{LK}$$

Δ Midsegment Thm.

$$m\angle MLK = m\angle JMP \quad \text{Similar triangles}$$

$$m\angle MLK = 102^\circ \quad \text{Substitute.}$$



Example 3: Indirect Measurement Application

In an A-frame support, the distance PQ is 46 inches. What is the length of the support $\overline{ST}$ if S and T are at the midpoints of the sides?



$$ST = \frac{1}{2}PQ \quad \Delta \text{ Midsegment Thm.}$$

$$ST = \frac{1}{2}(46) \quad \text{Substitute 46 for } PQ.$$

$$ST = 23 \quad \text{Simplify.}$$

The length of the support ST is 23 inches.

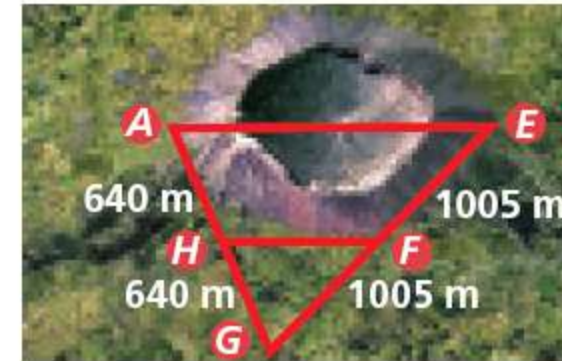
Check It Out! Example 3

What if...? Suppose Anna's result in Example 3 (p. 323) is correct. To check it, she measures a second triangle. How many meters will she measure between H and F ?

$$HF = \frac{1}{2}AE \quad \Delta \text{ Midsegment Thm.}$$

$$HF = \frac{1}{2}(1550) \quad \text{Substitute 1550 for } AE.$$

$$HF = 775 \text{ m} \quad \text{Simplify.}$$



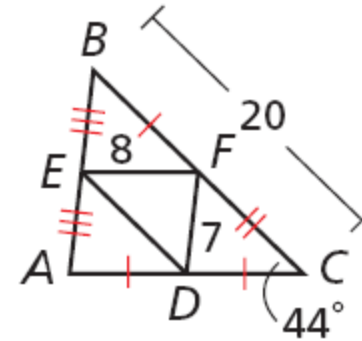
Lesson Quiz: Part I

Use the diagram for Items 1–3. Find each measure.

1. ED 10

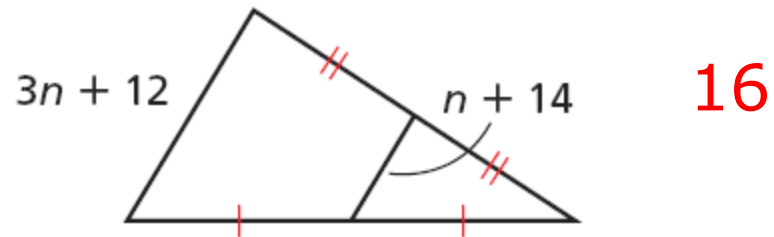
2. AB 14

3. $m\angle BFE$ 44°



Lesson Quiz: Part II

4. Find the value of n .



5. $\triangle XYZ$ is the midsegment triangle of $\triangle WUV$.
What is the perimeter of $\triangle XYZ$?

