

Lessons 4.1 and 4.2

Triangle Sum Properties & Properties of Isosceles Triangles

- *Classify triangles and find measures of their angles.*
- *Discover the properties of Isosceles Triangles.*

HOMEWORK:
Lesson 4.1/1-9 and
4.2/1-10

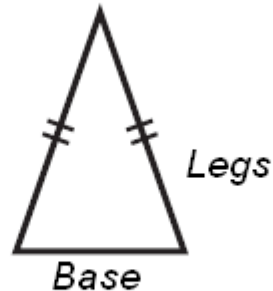
Classification By Sides

Equilateral



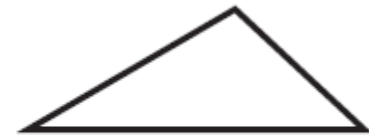
3 congruent sides

Isosceles



At least 2 congruent sides

Scalene



No congruent sides

Classification By Angles

Acute



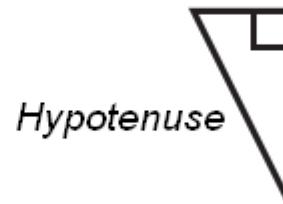
3 acute angles

Equiangular



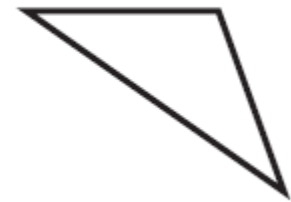
3 congruent

Right



1 right angle

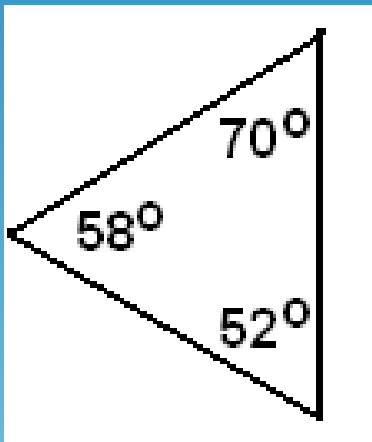
Obtuse



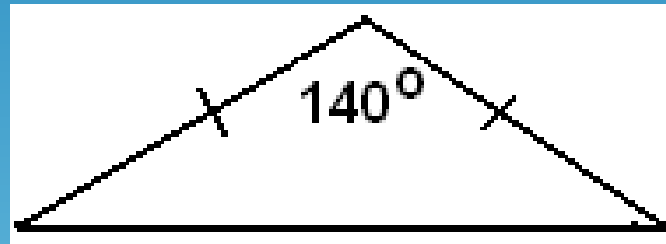
1 obtuse angle

Classifying Triangles

In classifying triangles, be as specific as possible.



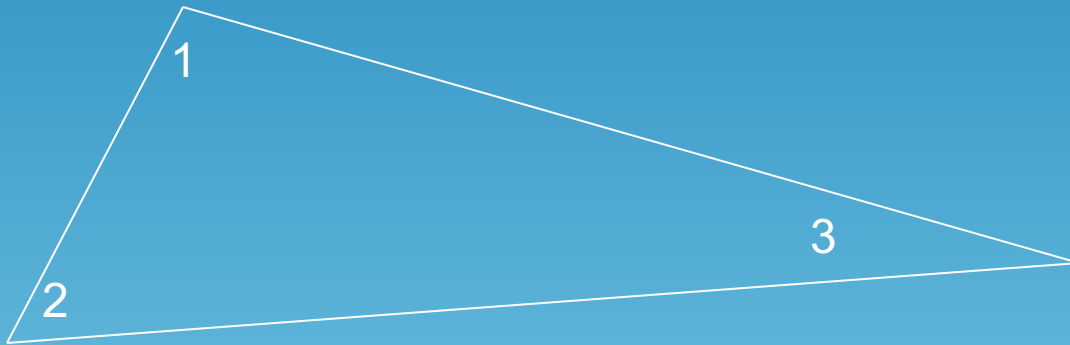
Acute,
Scalene



Obtuse,
Isosceles

Triangle Sum Theorem *****NEW**

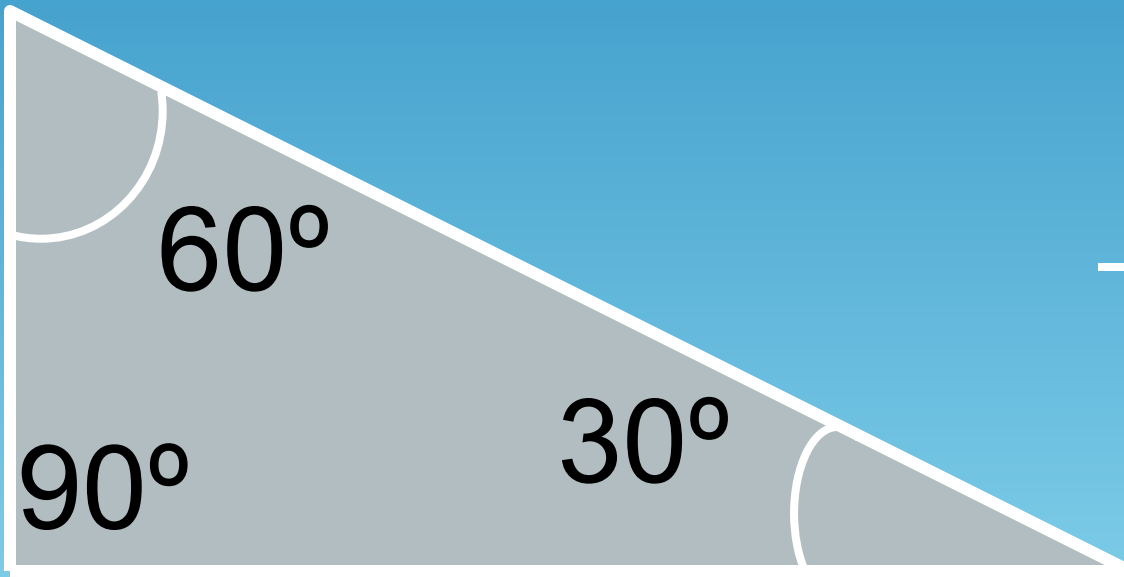
The sum of the measures of the interior angles of a triangle is 180° .



$$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$$

Property of triangles

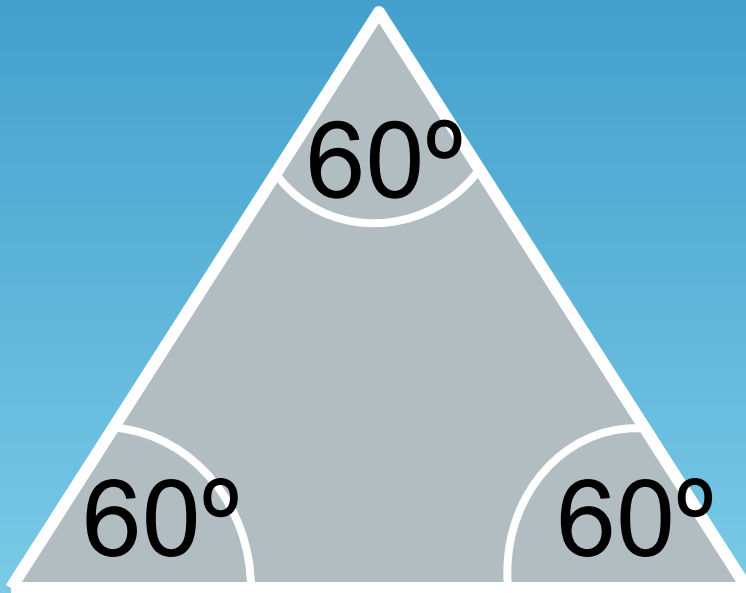
The sum of all the angles equals 180° degrees.



$$\begin{array}{r} 60^\circ \\ 90^\circ \\ + 30^\circ \\ \hline 180^\circ \end{array}$$

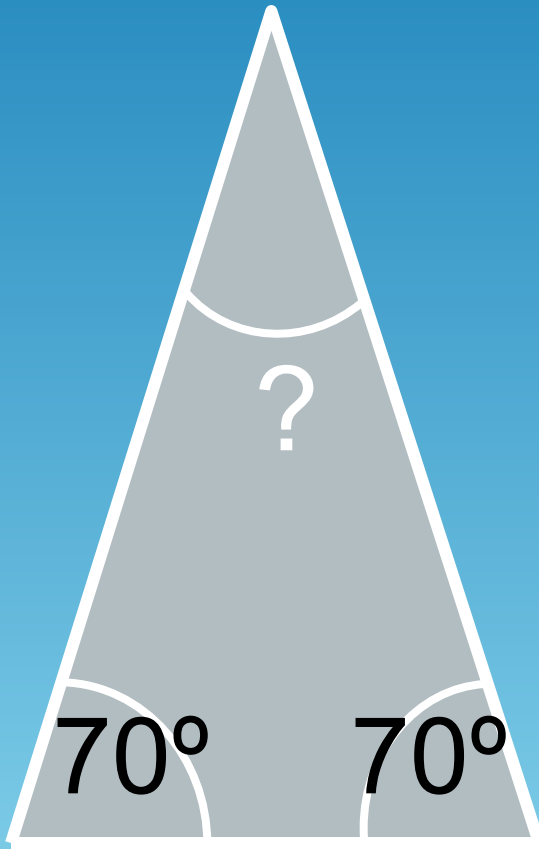
Property of triangles

The sum of all the angles equals 180° degrees.



$$\begin{array}{r} 60^\circ \\ 60^\circ \\ + 60^\circ \\ \hline 180^\circ \end{array}$$

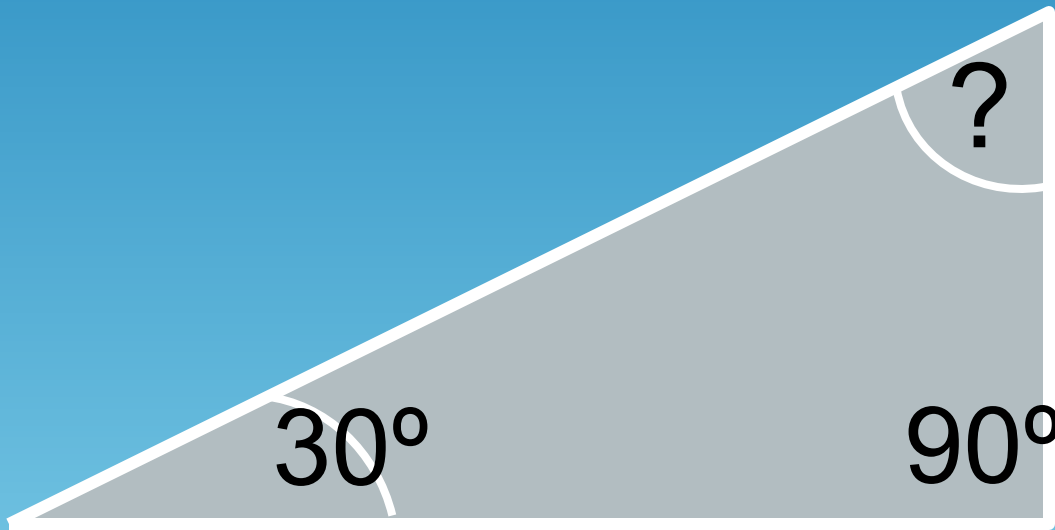
What is the missing angle?



$$\begin{array}{r} 70^\circ \\ 70^\circ \\ + \quad ? \\ \hline 180^\circ \end{array}$$

$$180 - 140 = 40^\circ$$

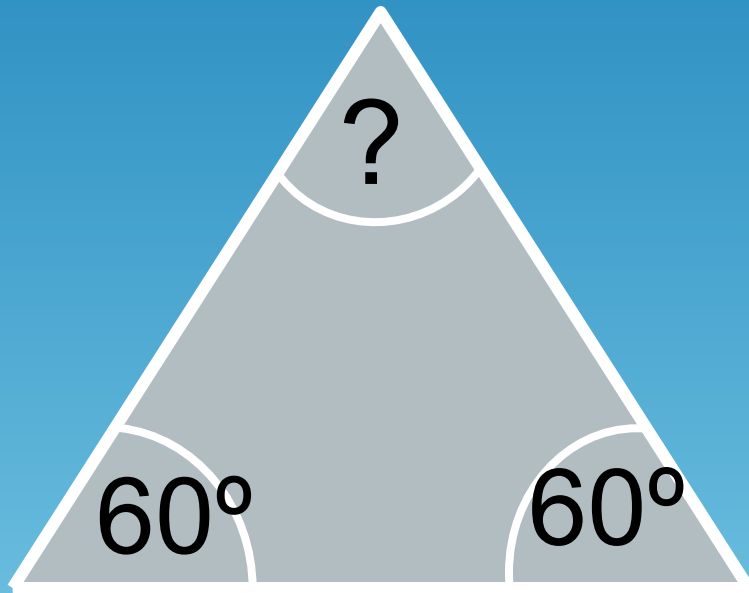
What is the missing angle?



$$\begin{array}{r} 90^\circ \\ 30^\circ \\ + \quad ? \\ \hline 180^\circ \end{array}$$

$$180 - 120 = 60^\circ$$

What is the missing angle?



$$\begin{array}{r} 60^\circ \\ 60^\circ \\ + \quad ? \\ \hline 180^\circ \end{array}$$

$$180 - 120 = 60^\circ$$

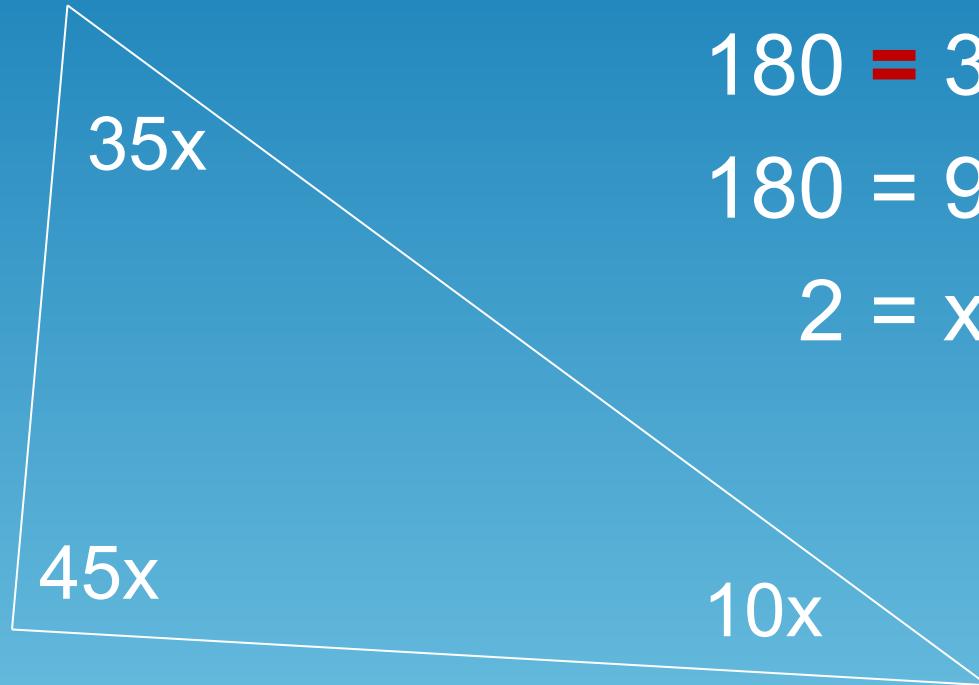
What is the missing angle?



$$\begin{array}{r} 30^\circ \\ 78^\circ \\ + \quad ? \\ \hline 180^\circ \end{array}$$

$$180 - 108 = 72^\circ$$

Find all the angle measures



$$180 = 35x + 45x + 10x$$

$$180 = 90x$$

$$2 = x$$

$90^\circ, 70^\circ, 20^\circ$

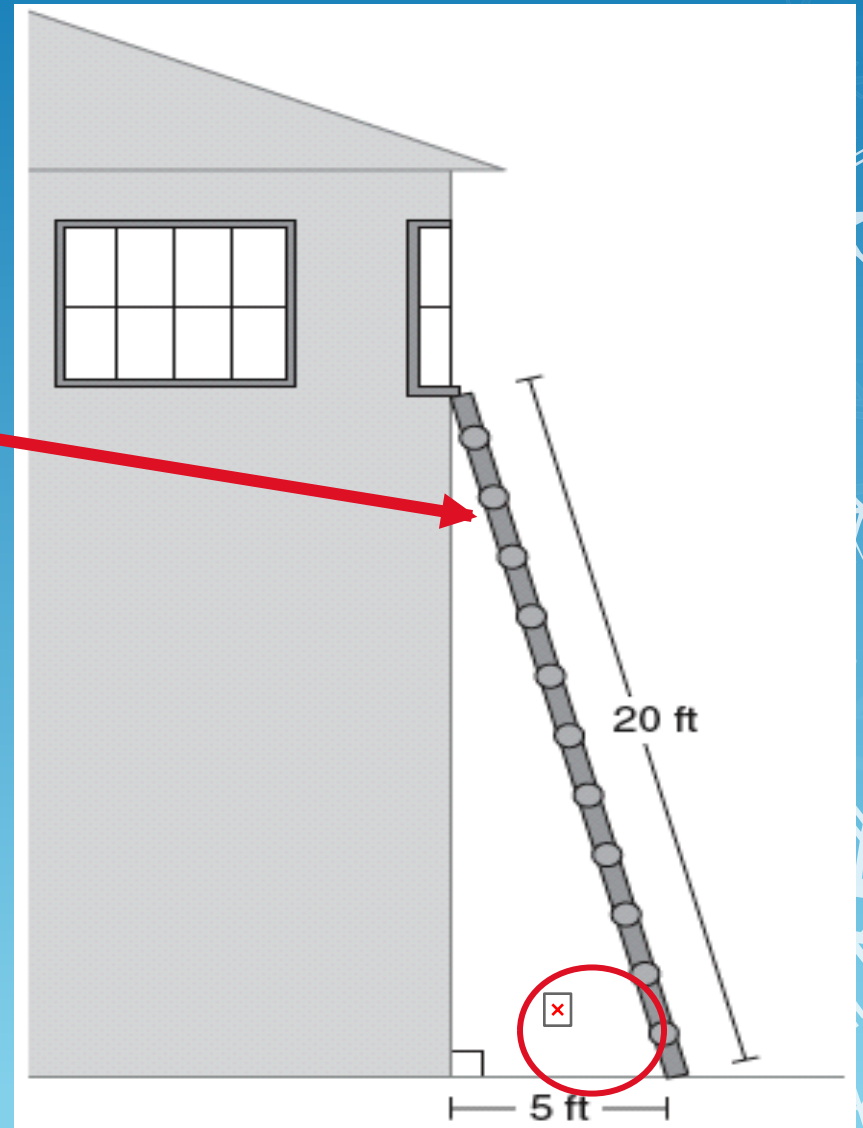
What can we find out?

The ladder is leaning on the ground at a 75° angle. At what angle is the top of the ladder touching the building?

$$180 = 75 + 90 + x$$

$$180 = 165 + x$$

$$15^\circ = x$$

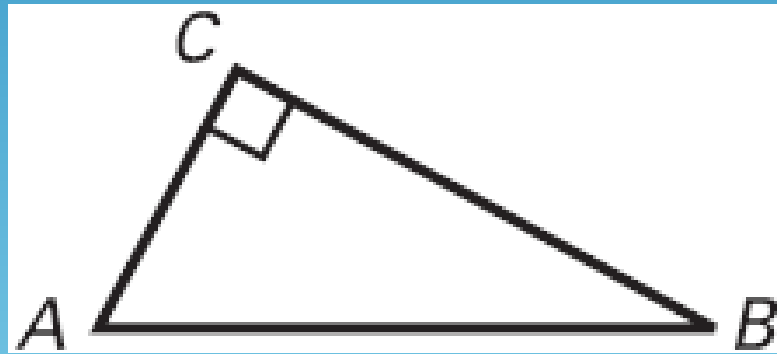


Corollary to Triangle Sum Theorem

A *corollary* is a statement that readily follows from a theorem.

The *acute* angles of a right triangle are *complementary*.


$$m\angle A + m\angle B = 90^\circ$$



Find the missing angles.

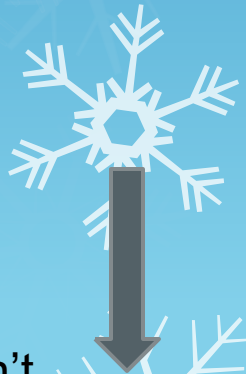
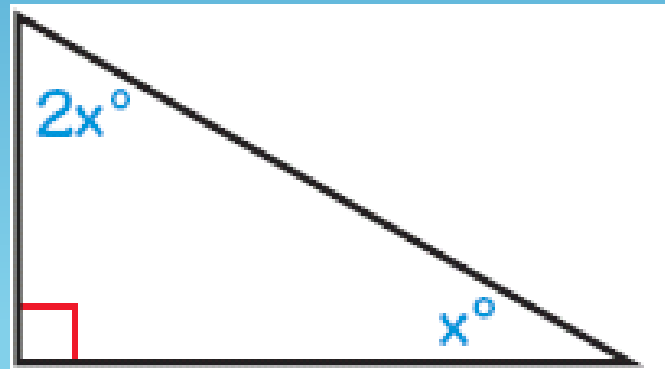


The tiled staircase shown below forms a right triangle.

The measure of one acute angle in the triangle is twice the measure of the other angle.



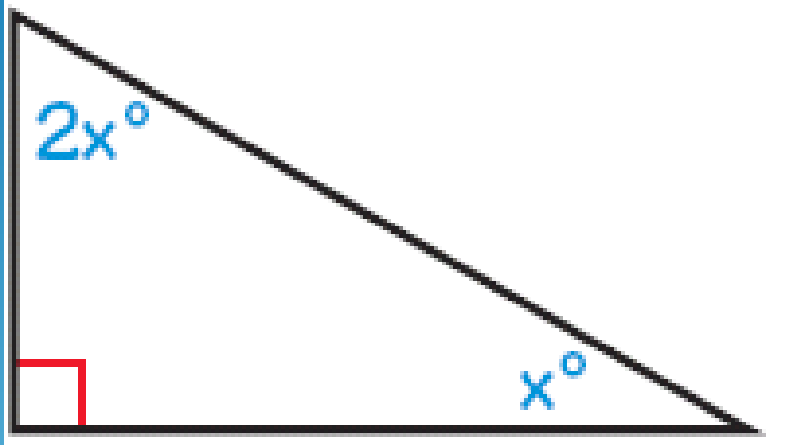
Find the measure of each acute angle.



Con't

Find the missing angles.

SOLUTION:



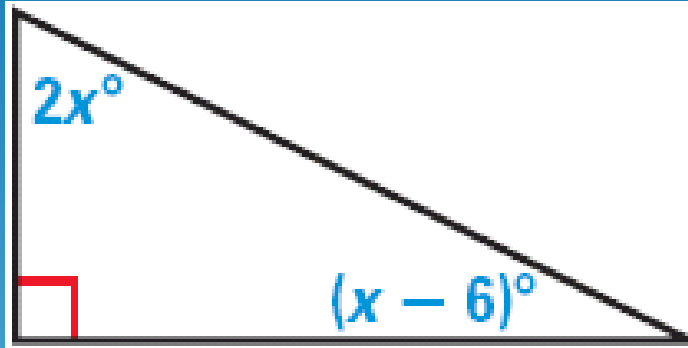
$$2x + x = 90$$

$$3x = 90$$

$$x = 30^\circ$$

$$2x = 60^\circ$$

Find the missing angles.



$$2x + (x - 6) = 90^\circ$$

$$3x - 6 = 90$$

$$3x = 96$$

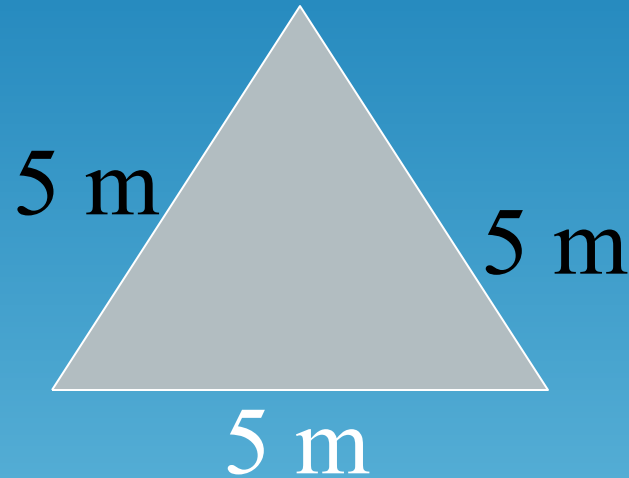
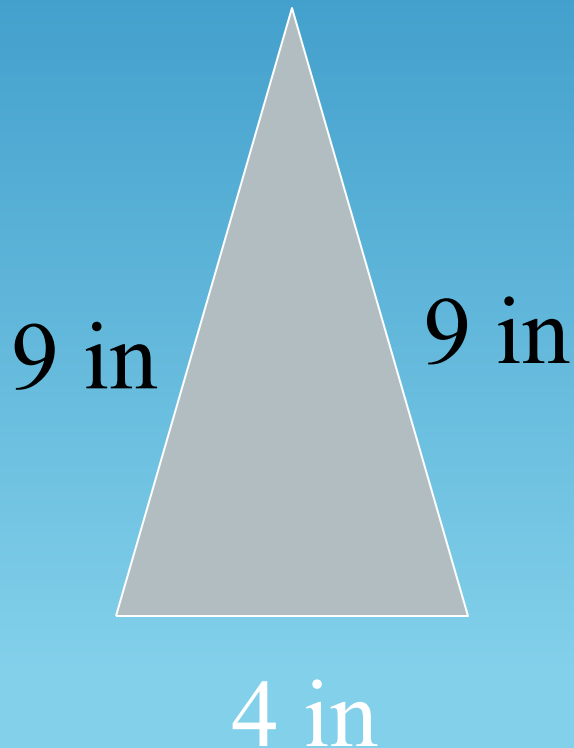
$$x = 32$$

$$2x = 2(32) = \mathbf{64^\circ}$$

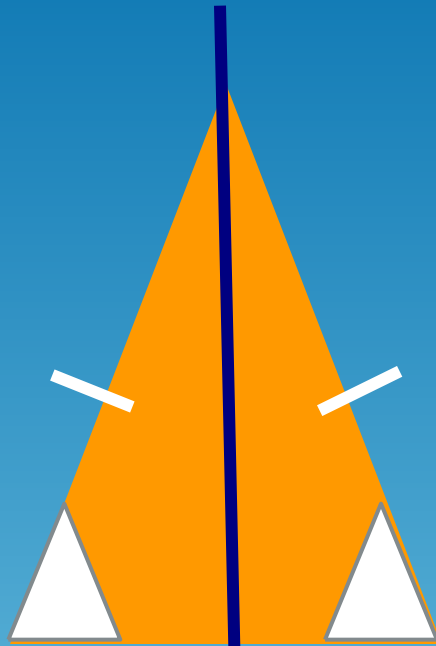
$$(x - 6) = 32 - 6 = \mathbf{26^\circ}$$

Isosceles Triangle

at least two sides have the same length



Properties of an Isosceles Triangle

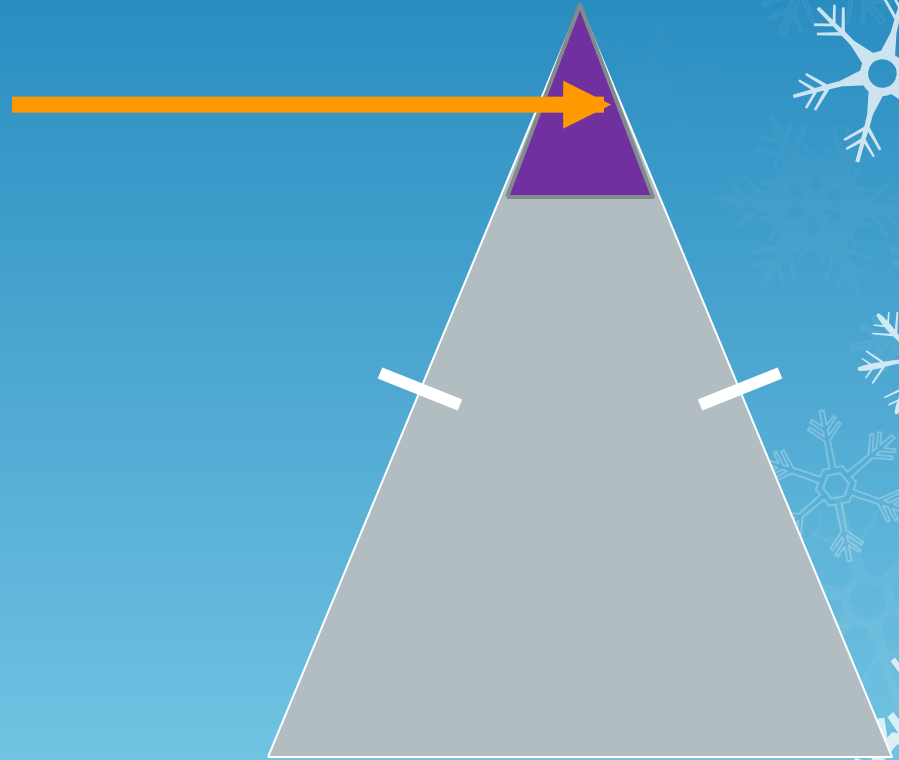


- Has at least 2 equal sides
- Has 2 equal angles
- Has 1 line of symmetry



Parts of an Isosceles Triangle:

The *vertex angle* is the angle **between** two congruent sides



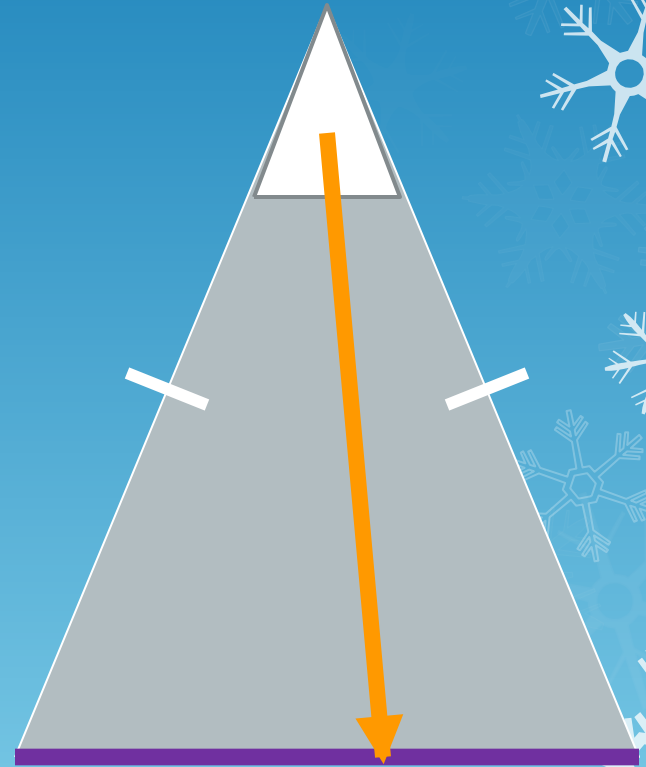
Parts of an Isosceles Triangle:

The *base angles*
are the angles
opposite the
congruent sides



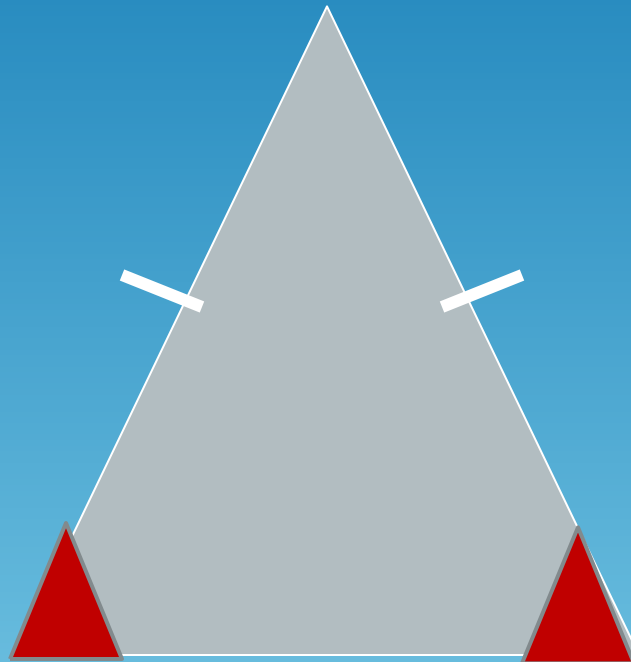
Parts of an Isosceles Triangle:

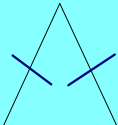
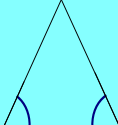
The ***base*** is
the side
opposite the
vertex angle



Isosceles Triangle Conjecture

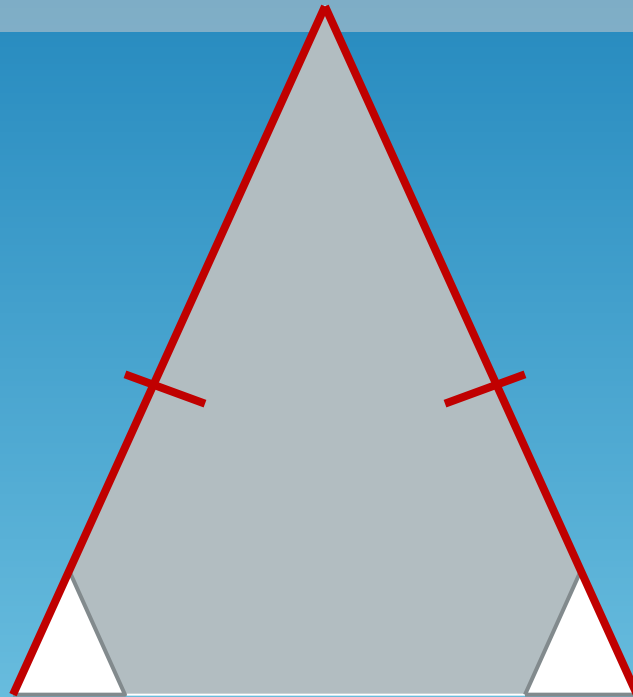
*If a triangle is isosceles, then **base** angles are congruent.*

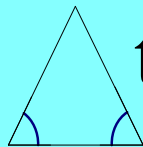
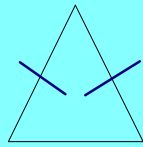


If  then 

Converse of Isosceles Triangle Conjecture

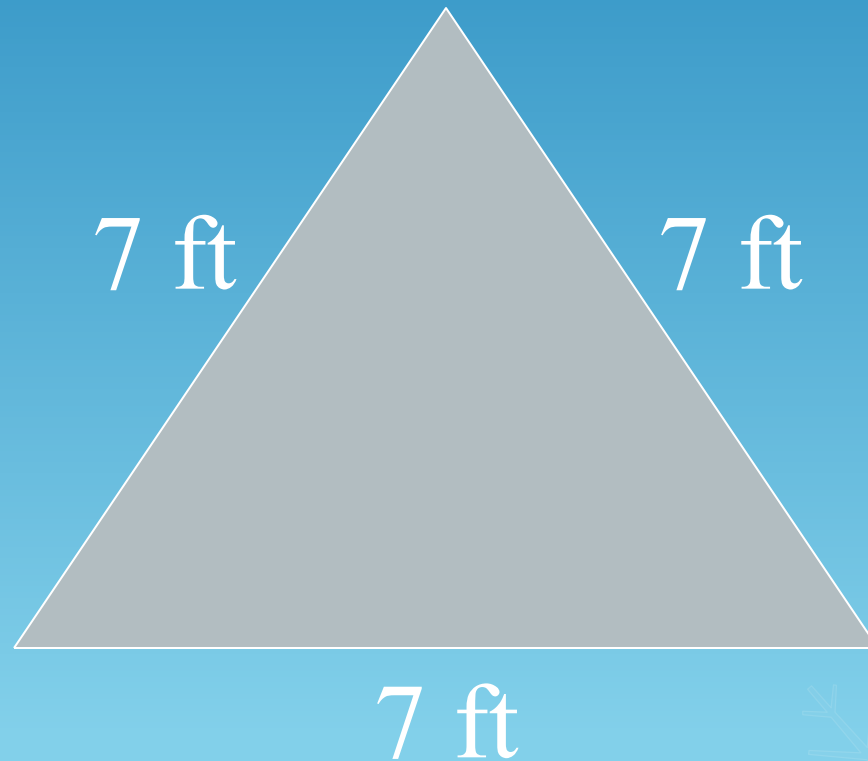
*If a triangle has two congruent angles,
then it is an isosceles triangle.*



If  then 

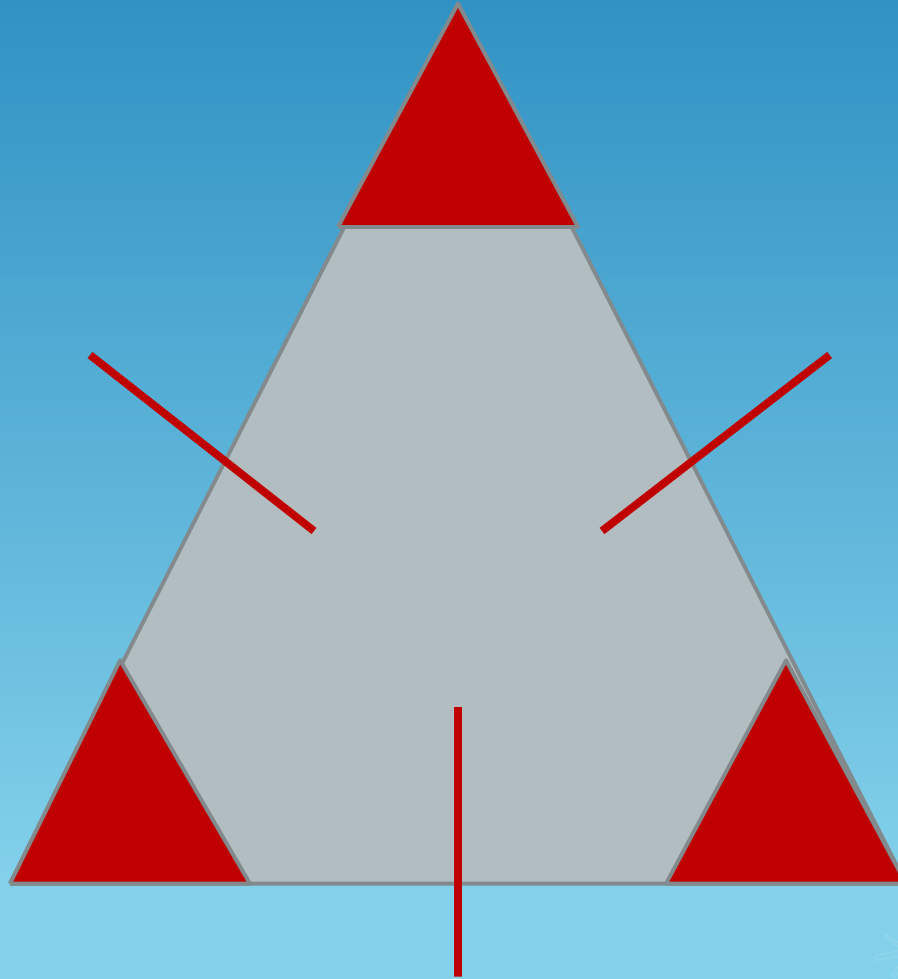
Equilateral Triangle

Triangle with all three sides are congruent



Equilateral Triangle Conjecture

An *equilateral* triangle is *equiangular*, and
an equiangular triangle is equilateral.



Find the missing angle measures.

$\angle 68^\circ$ and $\angle a$ are base angles $\rightarrow$
 $\therefore$ they are congruent

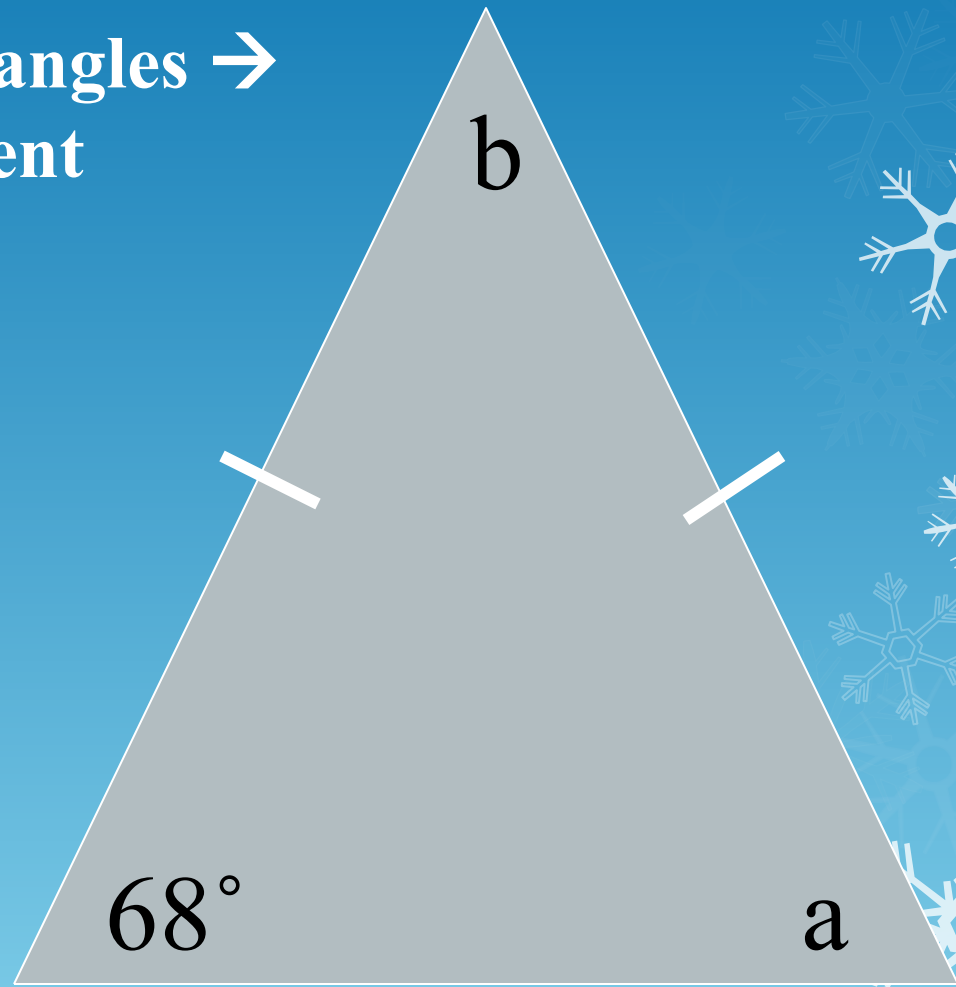
$$m\angle a = 68^\circ$$

Triangle sum to find $\angle b$

$$m\angle b = 180 - 68 - 68$$

$$m\angle b = 180 - 136$$

$$m\angle b = 44^\circ$$



Find the missing angle measures.

$\angle c$ & $\angle d$ are base angles and are congruent

Triangle sum = 180°

$$\therefore 180 = 119 + c + d$$

$$180 - 119 = c + d$$

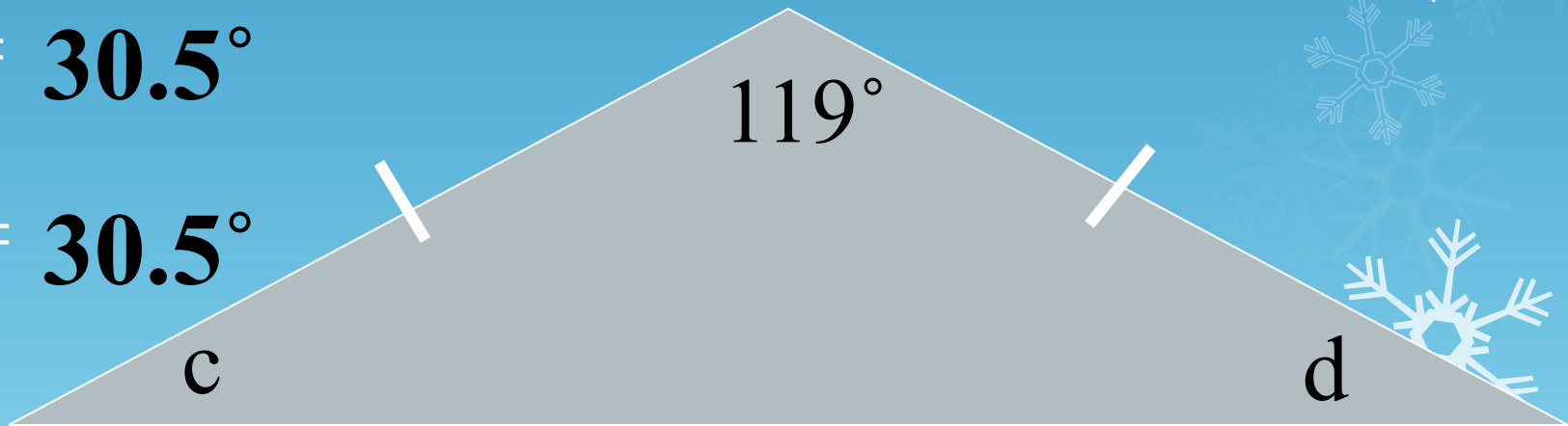
$$61 = c + d$$

$$\angle c = \frac{1}{2} (61) = 30.5$$

$$\angle d = \frac{1}{2} (61) = 30.5$$

$$m\angle c = 30.5^\circ$$

$$m\angle d = 30.5^\circ$$



Find the missing angle measures.

EFG is an equilateral triangle

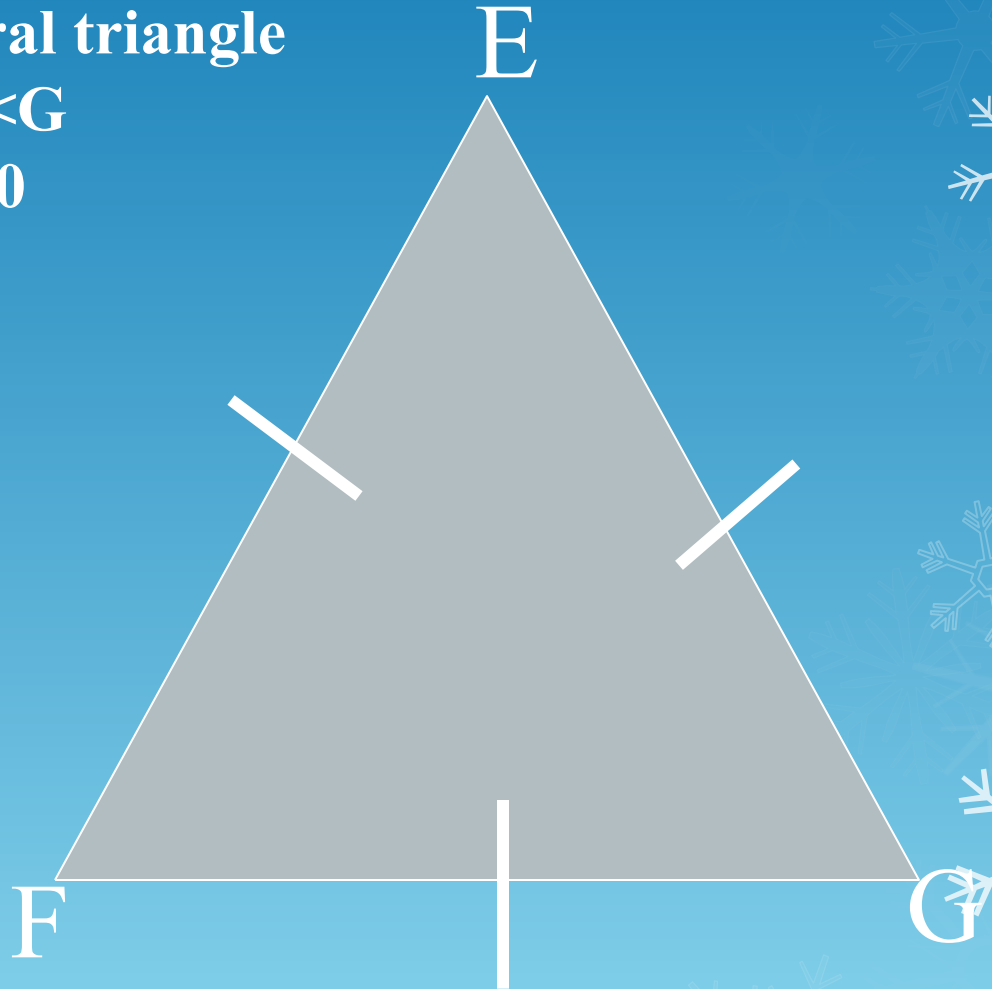
$$\therefore \angle E = \angle F = \angle G$$

$$180 / 3 = 60$$

$$m\angle E = 60^\circ$$

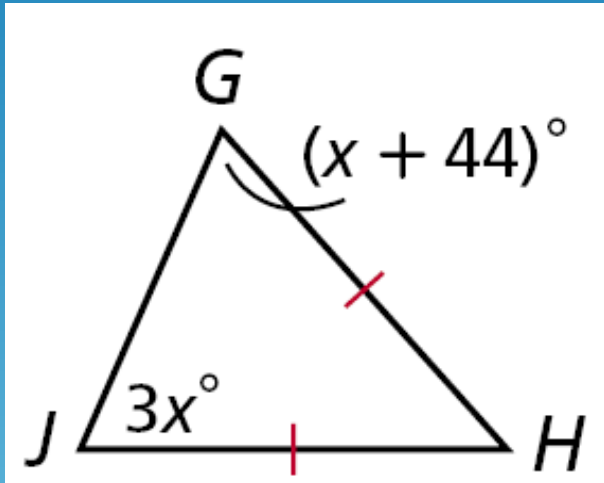
$$m\angle F = 60^\circ$$

$$m\angle G = 60^\circ$$



Find the missing angle measures.

Find $m\angle G$.



$\triangle GHJ$ is isosceles

$$\therefore \angle G = \angle J$$

$$x + 44 = 3x$$

$$44 = 2x$$

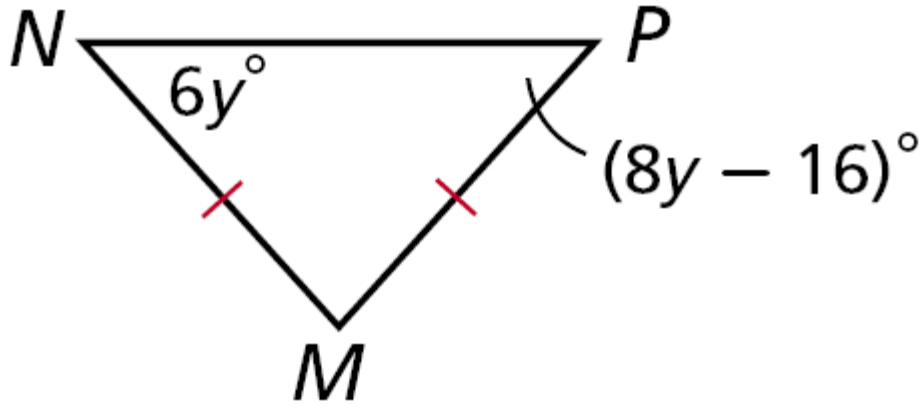
$$x = \mathbf{22}$$

$$\text{Thus } m\angle G = 22 + 44 = \mathbf{66^\circ}$$

$$\text{And } m\angle J = 3(22) = \mathbf{66^\circ}$$

Find the missing angle measures.

Find $m\angle N$



Base angles are =

$$6y = 8y - 16$$

$$-2y = -16$$

$$y = 8$$

Thus $m\angle N = 6(8) = \mathbf{48^\circ}$.

$$m\angle P = 8(8) - 16 = \mathbf{48^\circ}$$

Find the missing angle measures.

Using Properties of Equilateral Triangles

Find the value of x .

$\triangle LKM$ is equilateral

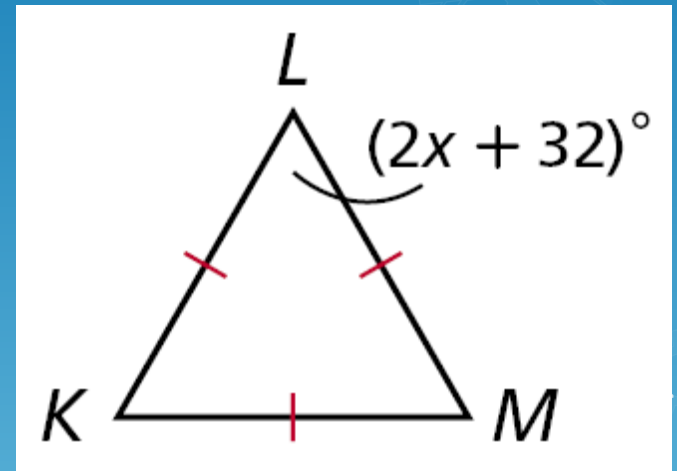
$$\therefore m\angle K = m\angle L = m\angle M$$

$$180/3 = 60^\circ$$

$$2x + 32 = 60$$

$$2x = 37$$

$$x = 18.5^\circ$$



Find the missing side measures.

Using Properties of Equilateral Triangles

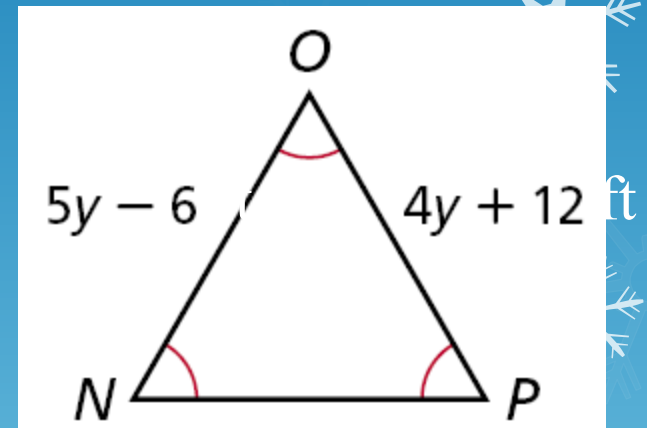
Find the value of y .

$\triangle NPO$ is equiangular $\therefore$
 $\triangle NPO$ is also equilateral.

$$5y - 6 = 4y + 12$$

$$y - 6 = 12$$

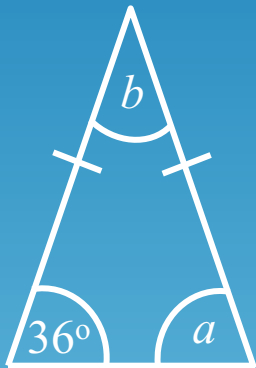
$$y = \mathbf{18}$$



$$\text{Side NO} = 5(18) - 6 = \mathbf{90ft}$$

Find the missing angle measures.

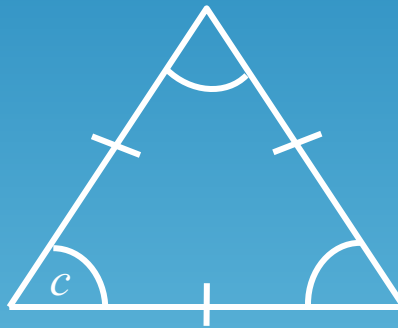
Using the symbols describing shapes answer the following questions:



Isosceles triangle
Two angles are equal

$$a = 36^\circ$$

$$\begin{aligned} b &= 180 - (2 \times 36) \\ &= 108^\circ \end{aligned}$$



Equilateral triangle
all angles are equal

$$c = 180 \div 3 = 60^\circ$$

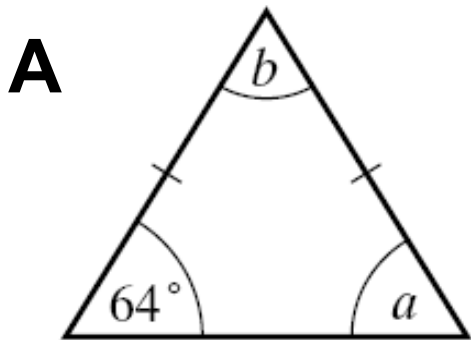


Right-angled triangle

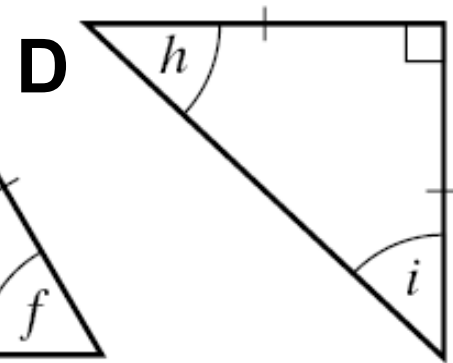
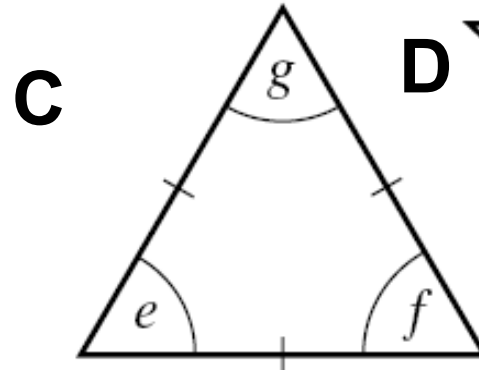
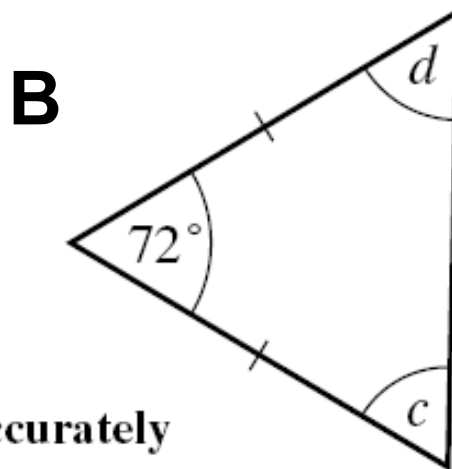
$$\begin{aligned} d &= 180 - (45 + 90) \\ &= 45^\circ \end{aligned}$$

Find the missing angle measures.

1 Work out the angles marked by letters.



Not drawn accurately



A

$$\begin{aligned} a &= 64^\circ \\ b &= 180 - (2 \times 64^\circ) \\ &= 52^\circ \end{aligned}$$

B

$$\begin{aligned} c &= d \\ c + d &= 180 - 72 \\ c + d &= 108 \\ c = d &= 54^\circ \end{aligned}$$

C Equilateral triangle

$$e = f = g = 60^\circ$$

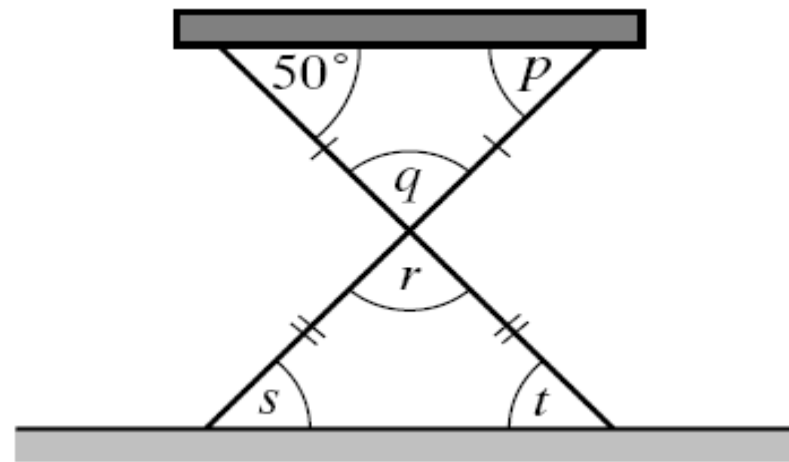
D

$$\begin{aligned} h &= i \\ h + i &= 180 - 90 \\ h + i &= 90 \\ h = i &= 45^\circ \end{aligned}$$

Get Real!

The diagram shows a cross-legged stool.

Calculate the angles marked by letters.



$$p = 50^\circ$$

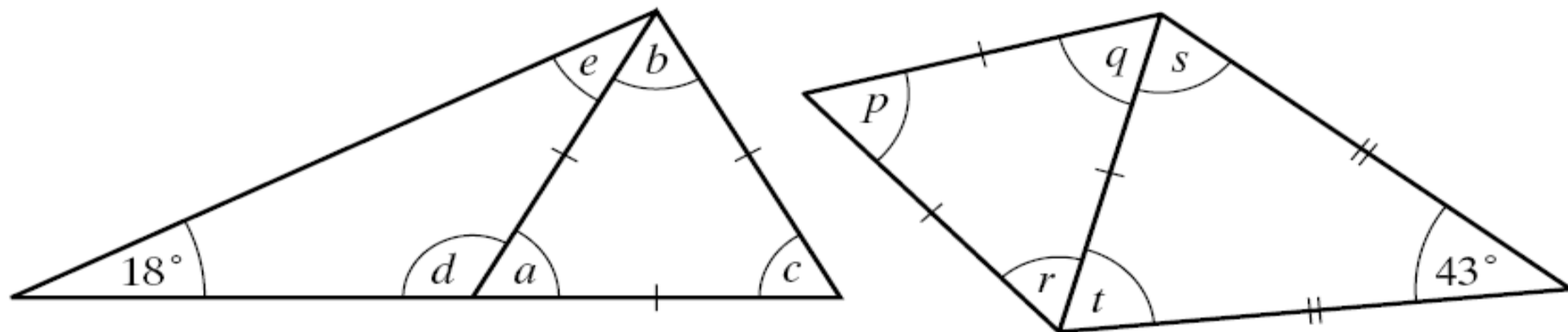
$$q = 180 - (2 \times 50^\circ) = 80^\circ$$

$$r = q = 80^\circ \quad \text{vertical angles are equal}$$

$$\text{Therefore : } s = t = p = 50^\circ$$

Find the missing angle measures.

Calculate the angles marked by letters.



Not drawn accurately

$$a = b = c = 60^\circ$$

$$d = 180 - 60 = 120^\circ$$

$$e + 18 = a = 60$$

exterior angle = sum of remote interior angles

$$e = 60 - 18 = 42^\circ$$

$$p = q = r = 60^\circ$$

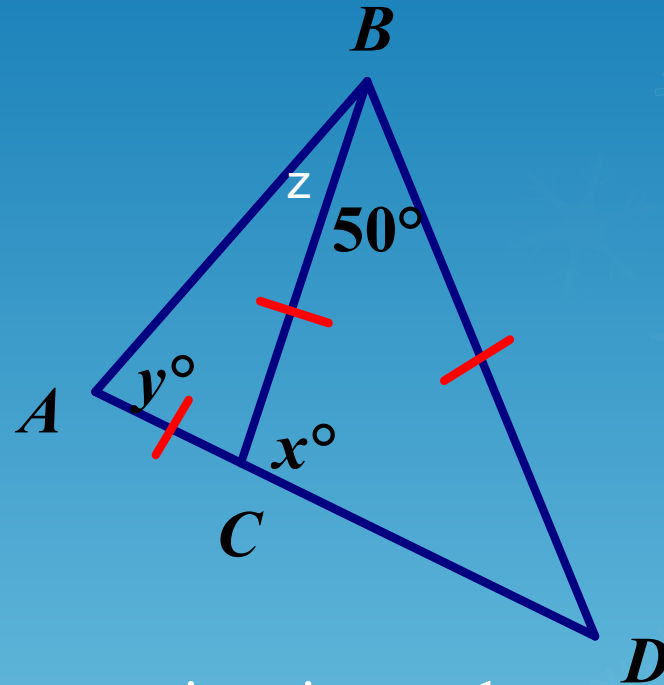
$$s = t = \frac{180 - 43}{2} = 68.5^\circ$$

Find the missing angle measures.

1) Find the value of x

2) Find the value of y

1) x is a base angle

$$180 = x + x + 50$$
$$130 = 2x$$
$$x = 65^\circ$$


2) y & z are remote interior angles
and base angles of an isosceles
triangle

Therefore: $y + z = x$ *and* $y = z$

$$y + z = 80^\circ$$

$$y = 40^\circ$$

Find the missing angle measures.

- 1) Find the value of x
- 2) Find the value of y

1) $\triangle CDE$ is equilateral

All angles = 60°

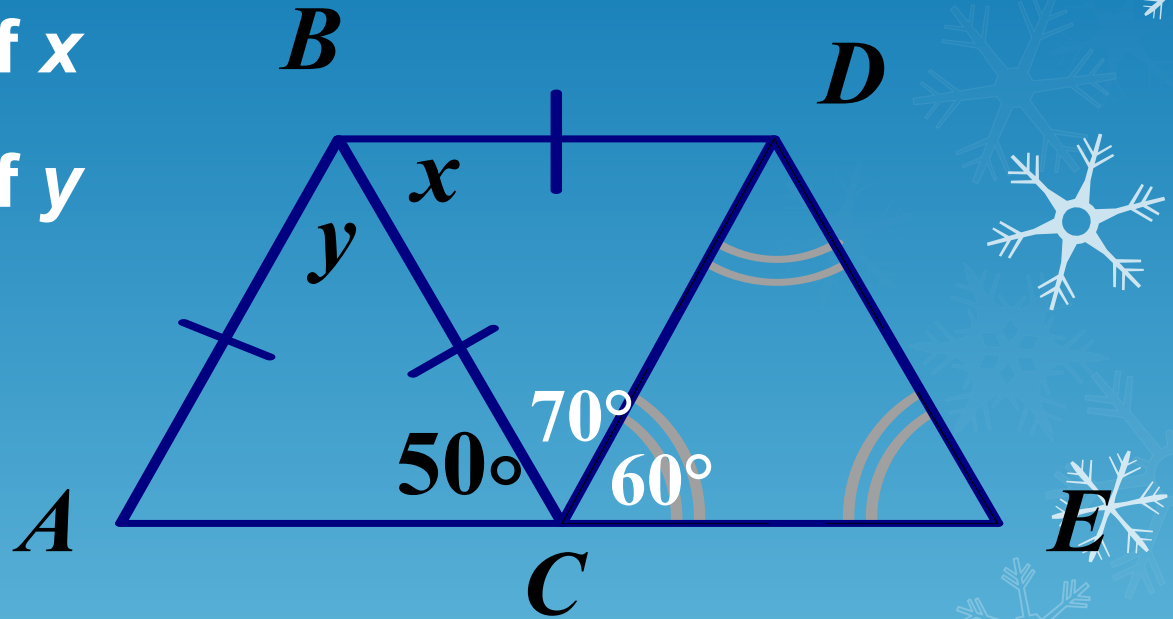
Using Linear Pair

$$\angle BCD = 70^\circ$$

x is the vertex angle

$$x = 180 - 70 - 70$$

$$x = 40^\circ$$



2) y is the vertex angle

$$y = 180 - 100$$

$$y = 80^\circ$$

Homework

In your textbook:

Lesson 4.1 / 1-9; 4.2 / 1-10

