6.3 Exponential Functions

In this section, we will study the following topics:

- Evaluating exponential functions with base *b*
- Graphing exponential functions with base *b*

DEFINITION

Exponential Functions

$$f(x) = b^x$$

where b is the base and the independent variable is in the exponent.

So, in an exponential function, **the variable is in the exponent.**

Exponential Functions

Which of the following are exponential functions?

$$f(x) = x^3$$

$$f(x) = 3^x$$

$$f(x) = 5^\pi$$

$$f(x) = 1^x$$

Graphs of Exponential Functions

Just as the graphs of all quadratic functions have the same basic shape, the graphs of exponential functions have the same basic characteristics.

They can be broken into two categories—

- *exponential growth*
- *exponential decay (decline)*

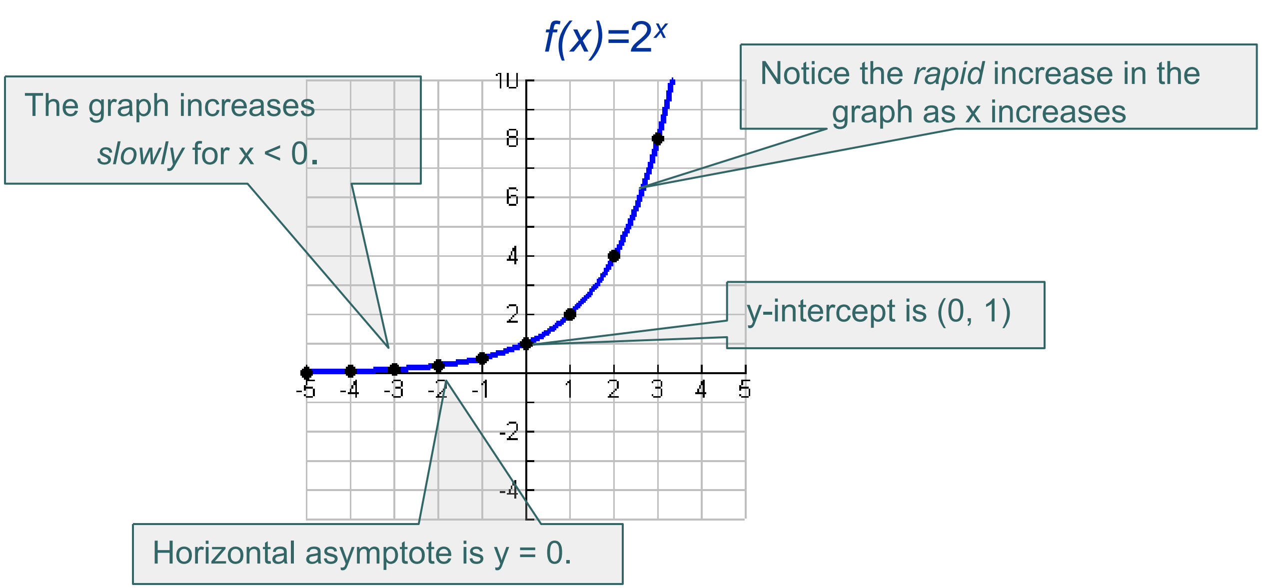
The Graph of an Exponential Growth Function

We will look at the graph of an exponential function that increases as x increases, known as the **exponential growth function**.

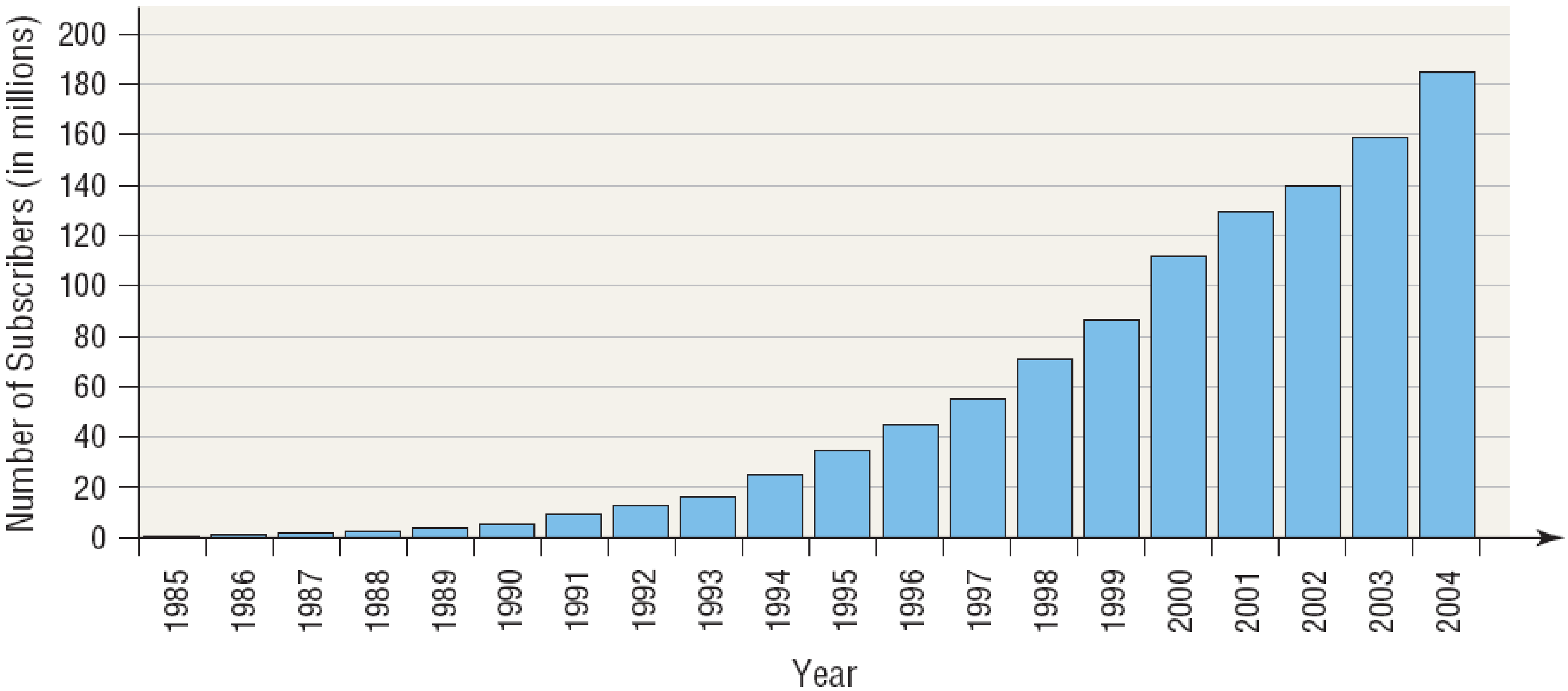
It has the form $f(x) = a^x$ where a > 1.

Example: $f(x) = 2^x$

x	$f(x)$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	



Number of Cellular Phone Subscribers at Year End



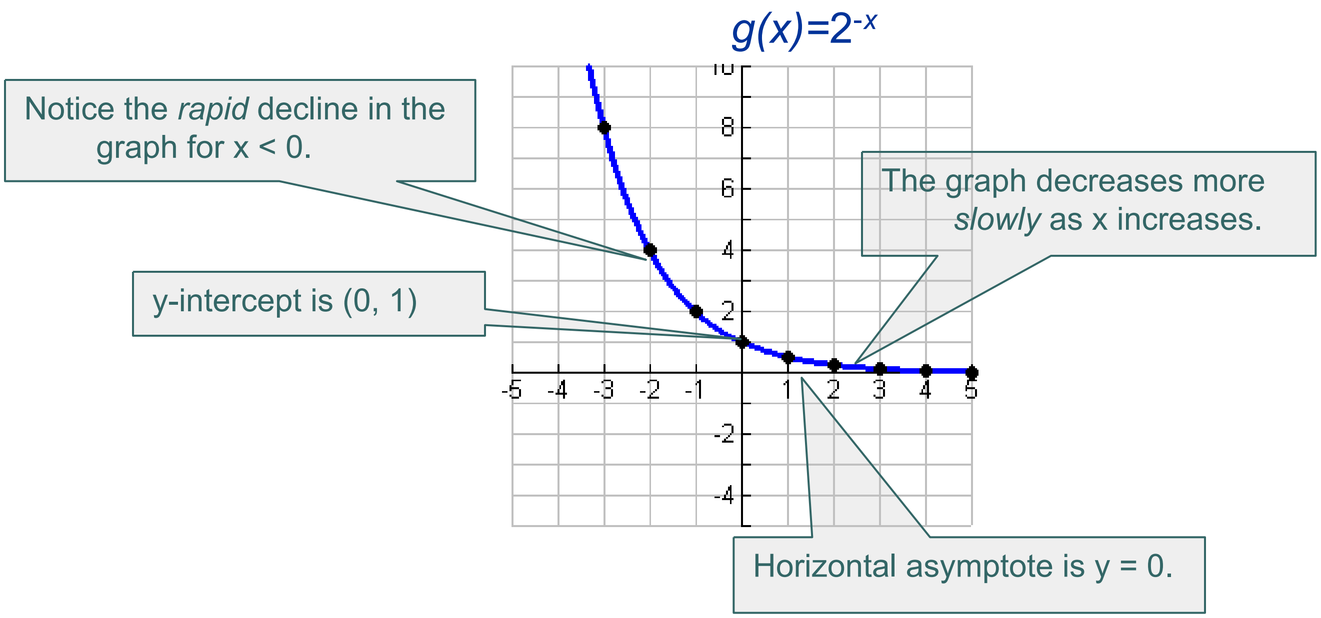
The Graph of an Exponential Decay (Decline) Function

We will look at the graph of an exponential function that decreases as x increases, known as the **exponential decay function**.

It has the form $f(x) = a^{-x}$ where a > 1.

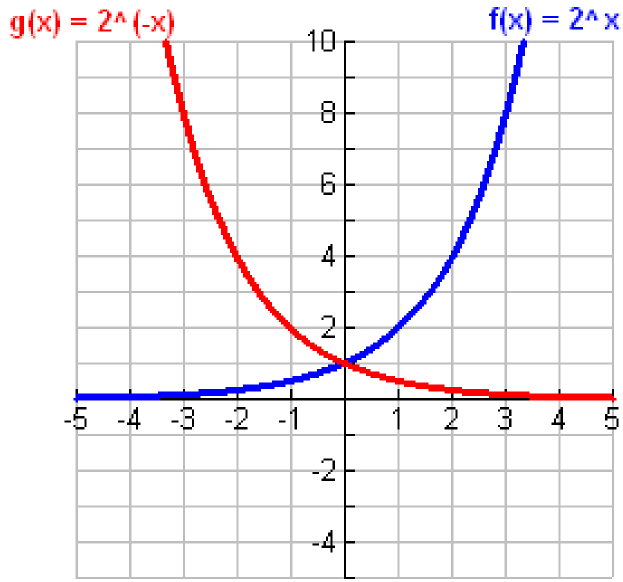
Example: $g(x) = 2^{-x}$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



Graphs of Exponential Functions

Notice that *f(x)* = 2^x and *g(x)* = 2^{-x} are *reflections of one another about the y-axis.*



Both graphs have: y-intercept (____,____) and
horizontal asymptote *y* = ___________.

The domain of *f(x) and g(x)* is __________; the range is ________.

Graphs of Exponential Functions

Also, note that $g(x) = 2^{-x} = \left(\frac{1}{2}\right)^{x}$, using the properties of exponents.

So an exponential function is a **DECAY** function if

- The base *a* is greater than one and the function is written as $f(x) = b^{-x}$

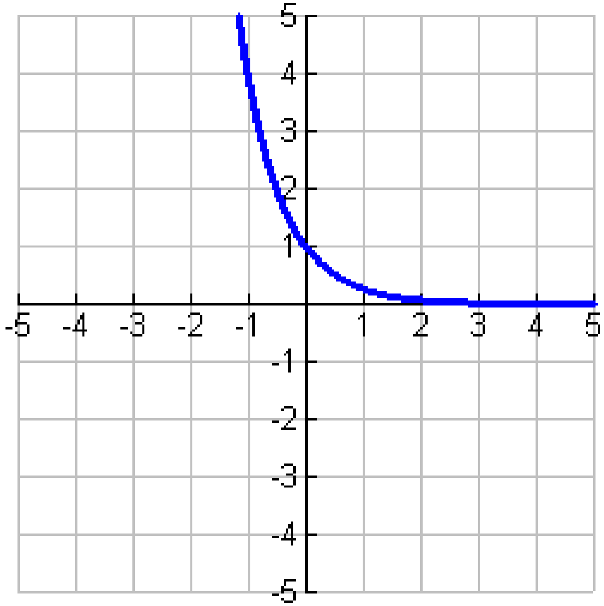
-OR-

- The base *a* is between 0 and 1 and the function is written as $f(x) = b^{x}$

Graphs of Exponential Functions

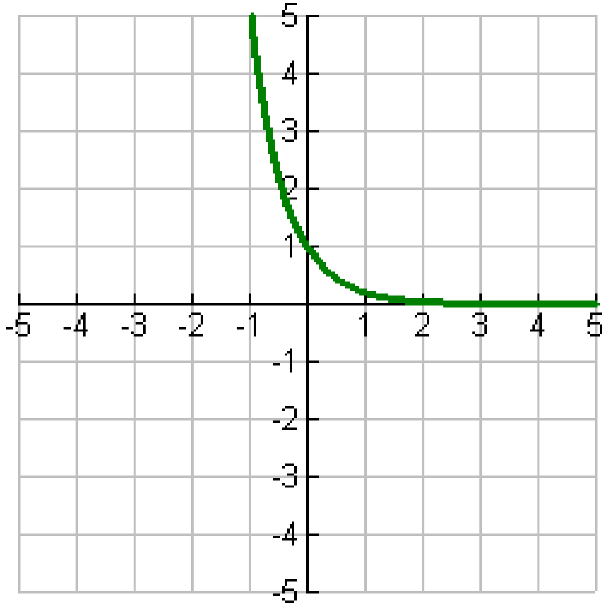
Examples:

$$f(x) = 0.25^x$$



In this case, a = 0.25 ($0 < a < 1$).

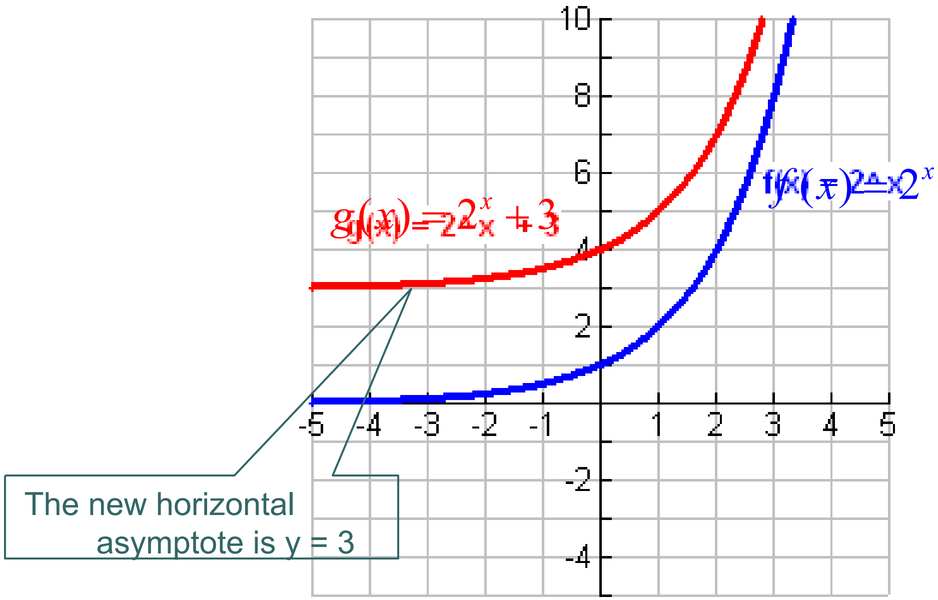
$$f(x) = 5.6^{-x}$$



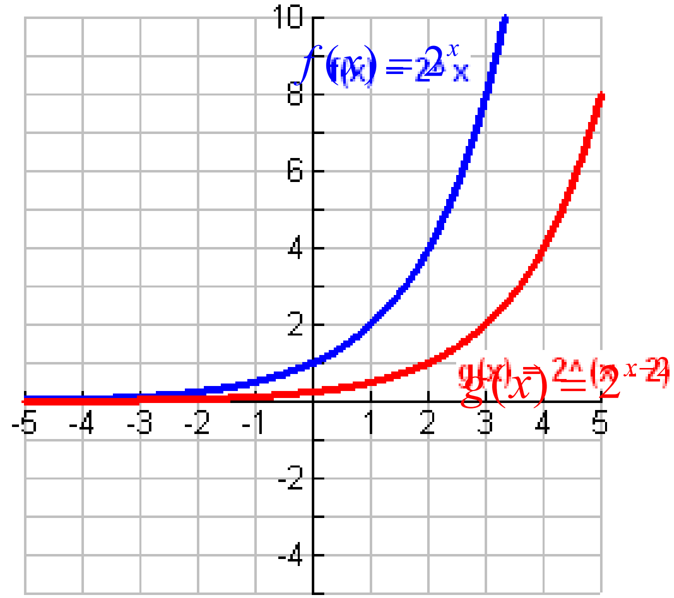
In this case, a = 5.6 ($a > 1$).

Transformations of Graphs of Exponential Functions

Look at the following shifts and reflections of the graph of $f(x) = 2^x$.

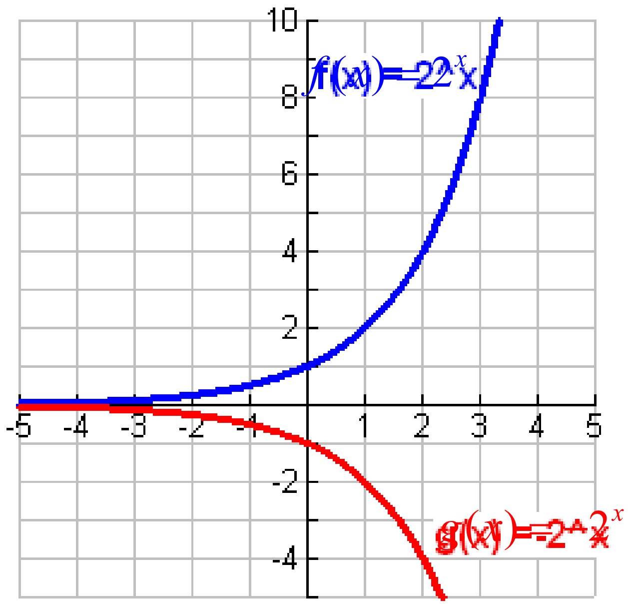


Transformation: Vertical shift of 3 units upward

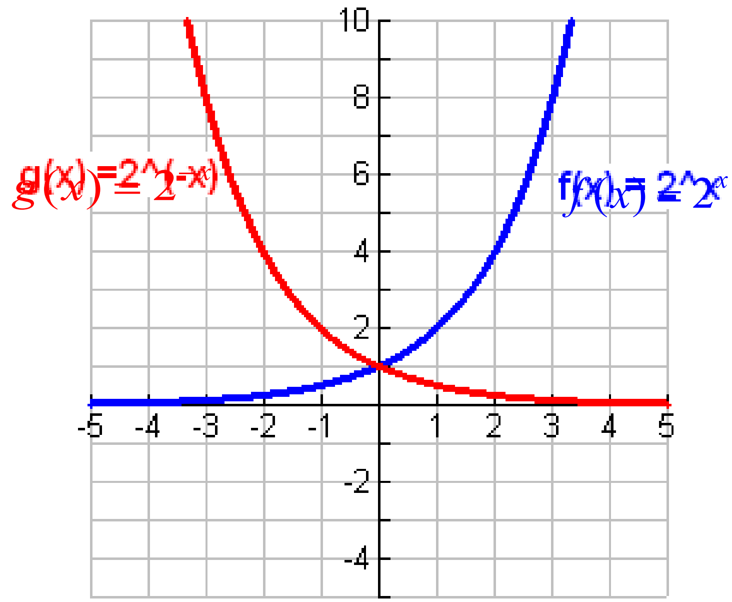


Horizontal shift of 2 units to the right.

Transformations of Graphs of Exponential Functions



Reflection in x-axis.



Reflection in y-axis.

Transformations of Graphs of Exponential Functions

Describe the **transformation(s)** that the graph of $f(x) = 2^x$ must undergo in order to obtain the graph of each of the following functions.

State the **domain**, **range** and the **horizontal asymptote** for each.

1. $f(x) = 2^x - 5$

Transformations of Graphs of Exponential Functions

Describe the **transformation(s)** that the graph of $f(x) = 2^x$ must undergo in order to obtain the graph of each of the following functions.

State the **domain**, **range** and the **horizontal asymptote** for each.

2. $f(x) = 2^{x+1}$

Transformations of Graphs of Exponential Functions

Describe the **transformation(s)** that the graph of $f(x) = 2^x$ must undergo in order to obtain the graph of each of the following functions.

State the **domain**, **range** and the **horizontal asymptote** for each.

3. $f(x) = 2^{-x} + 4$

Transformations of Graphs of Exponential Functions

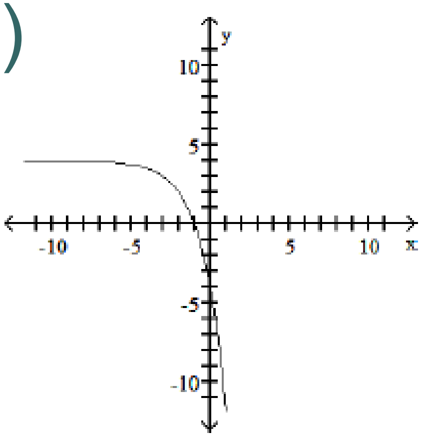
Describe the **transformation(s)** that the graph of $f(x) = 2^x$ must undergo in order to obtain the graph of each of the following functions.

State the **domain**, **range** and the **horizontal asymptote** for each.

4. $f(x) = -2^{x-3}$

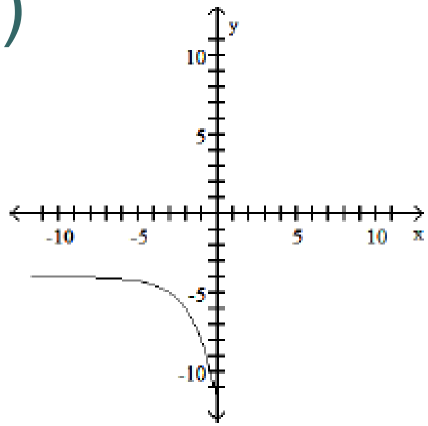
Graph using transformations and determine the domain, range and horizontal asymptote. $f(x) = -2^{x+3} + 4$

A)



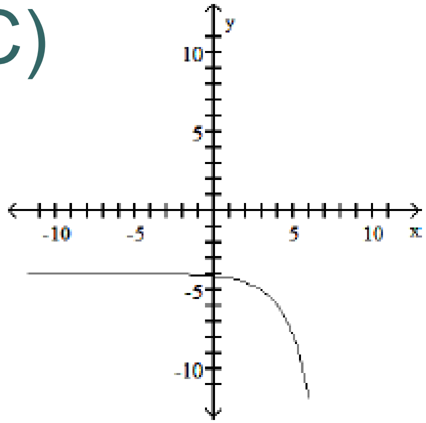
domain of f: $(-\infty, \infty)$; range of f: $(-\infty, 4)$;
horizontal asymptote: $y = 4$

B)



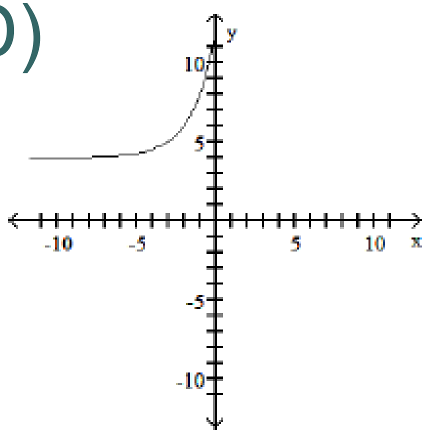
domain of f: $(-\infty, \infty)$; range of f: $(-\infty, -4)$;
horizontal asymptote: $y = -4$

C)



domain of f: $(-\infty, \infty)$; range of f: $(-\infty, -4)$;
horizontal asymptote: $y = -4$

D)



domain of f: $(-\infty, \infty)$; range of f: $(-4, \infty)$;
horizontal asymptote: $y = 4$