Theoretical Probability

Warm Up

Solve each proportion.

1. Which represents a greater amount 0.04 or 3.9 percent?

0.04

2. A bag contains 9 lettered tiles. There are 5 Es, 3 Ts, and 1 X. What letter would you be most likely to draw?

An E

Theoretical Probability

Essential ?

How can you describe the likelihood of an event?

Standard

MCC7.SP.6

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

Theoretical Probability

Theoretical Probability

How can you describe the likelihood of an event?

MCC7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

MCC7.SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

Vocabulary

theoretical probability

equally likely

fair

Going Deeper

Essential question: *How can you find the theoretical probability of an event?*

video tutor

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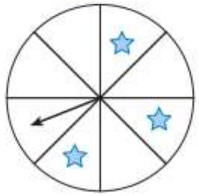
1 EXPLORE Finding Theoretical Probability

At a school fair, you have a choice of spinning Spinner A or Spinner B. You win a MP3 player if the spinner lands on a section with a star in it. Which spinner should you choose if you want the best chance of winning?

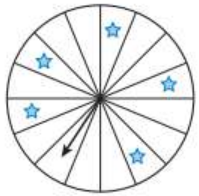
Complete the table.

	Spinner A	Spinner B
Total Number of Outcomes		
Number of Sections with Stars		
P(winning MP3) = $\frac{\text{Number of sections with stars}}{\text{Total number of outcomes}}$	___	___

Spinner A



Spinner B



Compare the ratios for Spinner A and Spinner B.

The ratio for Spinner _____ is greater than the ratio for Spinner _____.

I should choose _____ for the best chance of winning.

REFLECT

1a. *Theoretical probability* is a way to describe how you found the chance of winning a MP3 player in the scenario above. Using the spinner example to help you, explain in your own words how to find the theoretical probability of an event.

_____

_____

_____

1b. Suppose you choose Spinner A. What is the probability that you will not win? Show your work below.

Theoretical Probability

Theoretical probability is used to find the probability of an event when all the outcomes are equally likely.

Equally likely outcomes have the same probability.

THEORETICAL PROBABILITY

$$\text{probability} = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

If each possible outcome of an experiment is equally likely, then the experiment is said to be **fair**. Experiments involving number cubes and coins are usually assumed to be fair.

Theoretical Probability

A.

Andy has 20 marbles in a bag. Of these, 9 are clear and 11 are blue. Find the probability of drawing a clear marble from the bag? Write your answer as a fraction, as a decimal, and as a percent.

$$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

$$P(\text{clear}) = \frac{\text{number of clear marbles}}{\text{total number of marbles}}$$ *Write the ratio.*

$$= \frac{9}{20}$$ *Substitute.*

$$= 0.45 = 45\%$$ *Write as a decimal and write as a percent.*

The theoretical probability of drawing a clear marble is $\frac{9}{20}$, 0.45, or 45%.

Theoretical Probability

B.

Jane has 20 marbles in a bag. Of these 8 are green. Find the probability of drawing a green marble from the bag? Write your answer as a fraction, as a decimal, and as a percent.

$$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

$$P(\text{green}) = \frac{\text{number of green marbles}}{\text{total number of marbles}}$$ *Write the ratio.*

$$= \frac{8}{20}$$ *Substitute.*

$$= 0.4 = 40\%$$ *Write as a decimal and write as a percent.*

The theoretical probability of drawing a green marble is $\frac{8}{20}$, 0.4, or 40%.

Theoretical Probability

c.

Find the probability of rolling a number more than 4 on a fair number cube.

For a fair number cube, each of the six possible outcomes is equally likely. There are 2 ways to roll a number greater than 4: 5 or 6.

$$P = \frac{\text{number of ways the event can occur}}{\text{total number of equally likely outcomes}}$$

$$P(\text{number more than 4}) = \frac{\text{2 numbers more than 4}}{\text{6 possible outcomes}}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

$$\approx 0.33 \approx 33\%$$

The theoretical probability of rolling a number more than 4 is $\frac{1}{3}$, 0.33, or 33%.

Theoretical Probability

D.

There are 13 boys and 10 girls on the track team. The name of each of the team members is written on an index card. A card is drawn at random to choose a student to run a sprint and the card is replaced in the stack.

Find the theoretical probability of drawing a boy's name.

$$P(\text{boy}) = \frac{\text{number of boys on the team}}{\text{number of members on the team}}$$

$$P(\text{boy}) = \frac{13}{23}$$ *Substitute.*

Theoretical Probability

Remember!

The sum of the probabilities of an event and its complement is 1.

Not listed…just watch

Theoretical Probability

Additional Example 2B: *School Application*

There are 13 boys and 10 girls on the track team. The name of each of the team members is written on an index card. A card is drawn at random to choose a student to run a sprint and the card is replaced in the stack.

Find the theoretical probability of drawing a girl's name.

$P(\text{boy}) + P(\text{girl}) = 1$

$\frac{13}{23} + P(\text{girl}) = 1$ *Substitute* $\frac{13}{23}$ *for P(boy).*

$-\frac{13}{23} \quad = -\frac{13}{23}$ *Subtract* $\frac{13}{23}$ *from both sides*

$P(\text{girl}) = \frac{10}{23}$ *Simplify.*

Check It Out: Example 2A

A teacher has written the name of each student on a piece of paper and placed the names in a box. She randomly draws a paper from the box to determine which student will present the answer to the problem of the day.

If there are 15 boys and 12 girls in the class, what is the theoretical probability that a girl's name will be drawn?

$$P(\text{girl}) = \frac{\text{number of girls in the class}}{\text{number of students in the class}}$$

Find the theoretical probability.

$$= \frac{12}{27}$$

Substitute.

Theoretical Probability

Check It Out: Example 2B

What is the theoretical probability that a boy's name will be drawn?

$P(\text{girl}) + P(\text{boy}) = 1$

$\frac{12}{27} + P(\text{boy}) = 1$ *Substitute* $\frac{12}{27}$ *for P(girl).*

$-\frac{12}{27} = -\frac{12}{27}$ *Subtract* $\frac{12}{27}$ *from both sides*

$P(\text{boy}) = \frac{15}{27}$ *Simplify.*

MCC7.SP.7a

2 EXAMPLE Calculating Theoretical Probability

A **A bag contains 8 red marbles and 12 green marbles. You choose one marble out of the bag without looking. What is the probability that you choose a red marble? Write your answer in simplest form.**

$$P(\text{red marble}) = \frac{\text{number of red marbles}}{\text{total number of marbles in the bag}} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

B **You roll a number cube one time. What is the probability that you roll a 3 or 4? Write your answer in simplest form.**

$$P(\text{rolling a 3 or 4}) = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

TRY THIS!

Find the probability of each event. Write your answer in simplest form.

2a. A spinner has 3 equally-sized sections—one is green, one is yellow, and one is blue. You spin the spinner once. What is the probability that the spinner lands on blue?

_____

2b. You pick at random one card from a standard deck of playing cards. What is the probability that you pick a black card?

_____

REFLECT

2c. Describe a situation that has a theoretical probability of $\frac{1}{4}$?

_____

_____

Making Predictions

Theoretical Probability

Learn to use probability to predict events.

Vocabulary

prediction

Theoretical Probability

A **prediction** is something you can reasonably expect to happen in the future. Weather forecasters use several different methods of forecasting to make predictions about the weather.

One way to make a prediction is to use probability.

Day	Made	Sold	Left Over
1	36	18	18
2	30	19	11
3	30	17	13
4	30	19	11
5	24	22	2
6	24	21	3
7	24	19	5
8	24	18	6

A Find the experimental probability that the bakery will sell at least 20 lemon muffins in one day.

There were _____ days on which the bakery sold at least 20 lemon muffins.

There are _____ days in the table in all.

$P(\geq 20) = \frac{2}{8} = \frac{\square}{\square}$

B Predict how many days in a month of 30 days that the bakery will sell out if it makes 20 lemon muffins each day.

$\frac{\square}{\square} \cdot 30 = \frac{\square}{\square} =$ _____

Multiply the experimental probability of selling at least 20 muffins by the number of days. Then round to the nearest whole number.

The bakery will sell out of lemon muffins on about _____ days if 20 muffins are made each day.

C Predict how many days in a month of 30 days that the bakery will sell out if it makes 22 lemon muffins each day.

$P(\geq 22) = \frac{\square}{\square}$

Find the experimental probability that the bakery will sell at least 22 lemon muffins in one day.

$\frac{\square}{\square} \cdot 30 = \frac{\square}{\square} =$ _____

Multiply the experimental probability of selling at least 22 muffins by the number of days. Then round to the nearest whole number.

Rounded to the nearest day, the bakery will sell out of lemon muffins on about _____ days if 22 muffins are made each day.

D Use the predictions to make a decision.

Would it be better for the bakery to make 20 lemon muffins or 22 lemon muffins each day next month? Justify your answer.

TRY THIS!

1a. Why might it be a good idea for the bakery to make 30 lemon muffins each day? Use data from the table to support your answer.

1b. Why might it be a bad idea for the bakery to make 30 lemon muffins each day? Use data from the table to support your answer.

Additional Example 1: Using Experimental Probability to Make Predictions

Lawrence finds the experimental probability of his reaching first base is 40%. Out of 350 at-bats, how many times can he expect to reach first base?

Method 1: Set up an equation.

$\frac{4}{10} \cdot 350 = x$ *Multiply the probability by the number of at bats.*

$140 = x$

Additional Example 1 Continued

Method 2: Set up a proportion.

$\frac{4}{10} = \frac{x}{350}$ *Think: 4 out of 10 is how many out of 350.*

$4 \cdot 350 = 10 \cdot x$ *The cross products are equal.*

$\frac{1400}{10} = \frac{10x}{10}$ *Multiply.*

Divide each side by 10 to isolate the variable.

$140 = x$

Lawrence can predict that he will reach first base about 140 of 350 times.

Check It Out: Example 1

Malia finds the experimental probability of her scoring a goal is 20%. Out of 225 attempts, how many times can she expect to score a goal?

Method 1: Set up an equation.

$\frac{2}{10} \cdot 225 = x$ *Multiply the probability by the number of attempts.*

$45 = x$

Check It Out: Example 1 Continued

Method 2: Set up a proportion.

$$\frac{2}{10} = \frac{x}{225}$$

Think: 2 out of 10 is how many out of 225.

$$2 \cdot 225 = 10 \cdot x$$

The cross products are equal.

$$\frac{450}{10} = \frac{10x}{10}$$

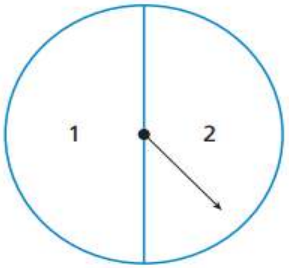
Multiply.

Divide each side by 10 to isolate the variable.

$$45 = x$$

Malia can predict that she will score about 45 goals of 225 attempts.

Sara and Sheldon are playing a game with the spinner shown. On each turn, a player spins the pointer of the spinner twice.



A Sara gets a point if the sum of the numbers she spins is odd. Sheldon gets a point if the sum of the numbers he spins is even. Is this game fair?

Complete the table. For each outcome, tell who would get a point.

Outcome	Point Winner
1, then 1	
1, then 2	
2, then 1	
2, then 2	

For each turn, the probability that Sara will get a point is ____ = ____.

For each turn, the probability that Sheldon will get a point is ____ = ____.

The game is / is not fair because _____

_____

The game is / is not fair because _____

_____

B Now Sara and Sheldon are playing a different game. Sara gets a point if the product of the numbers she spins is odd. Sheldon gets a point if the product of the numbers he spins is even. Is this game fair?

Complete the table. For each outcome, tell who would get a point.

Outcome	Point Winner
1, then 1	
1, then 2	
2, then 1	
2, then 2	

For each turn, the probability that Sara will get a point is ——.

For each turn, the probability that Sheldon will get a point is ——.

The game is / is not fair because _____

_____

TRY THIS!

2a. Alfonso and Leif are playing a game in which they take turns rolling a number cube. Alfonso gets a point if the number he rolls is less than 3. Leif gets a point if the number he rolls is 3 or greater. Is this game fair? Explain.

_____

_____

_____

REFLECT

2b. What If...? Alfonso and Leif each take 20 turns. Predict their scores, and explain how you determined your answers.

_____

Practice Worksheet Both sides