Circles

SHS Analytic Geometry

Unit 5

Objectives/Assignment

Week 1: G.GPE.1; G.GPE.4

- *Students should be able to derive the formula for a circle given the Pythagorean Theorem*
- *Students should be able to derive the equation of a circle given its center and radius.*
- *Students should be able to complete the square to find the center and radius of a circle given by an equation.*
- *Students should prove properties involving circles on the coordinate plane.*
- *Students should be able to solve circle Application Problems.*

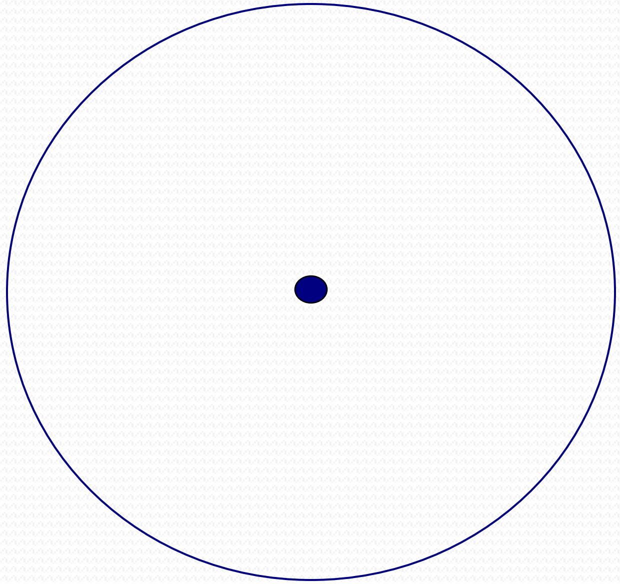
What is a circle?

A circle is the set of all points in a plane equidistant from a fixed point.

equi

fixed point

"Equi" means same, so equidistant means the same distance.

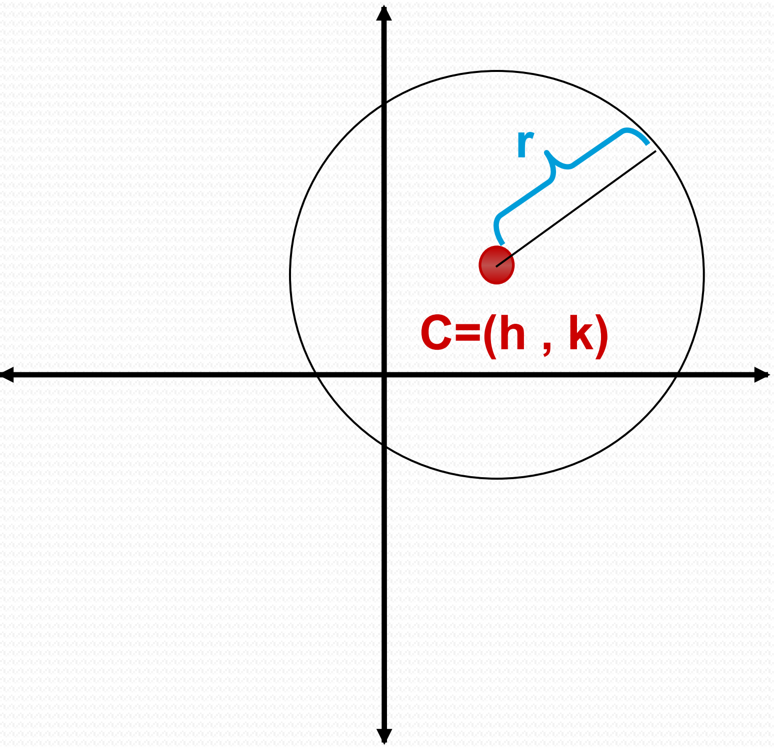


The fixed point is called the center.

EQ: What does a triangle have to do with a circle?
http://learni.st/learnings/28476-the-equation-of-a-basic-circle?board_id=3075

CIRCLE TERMS

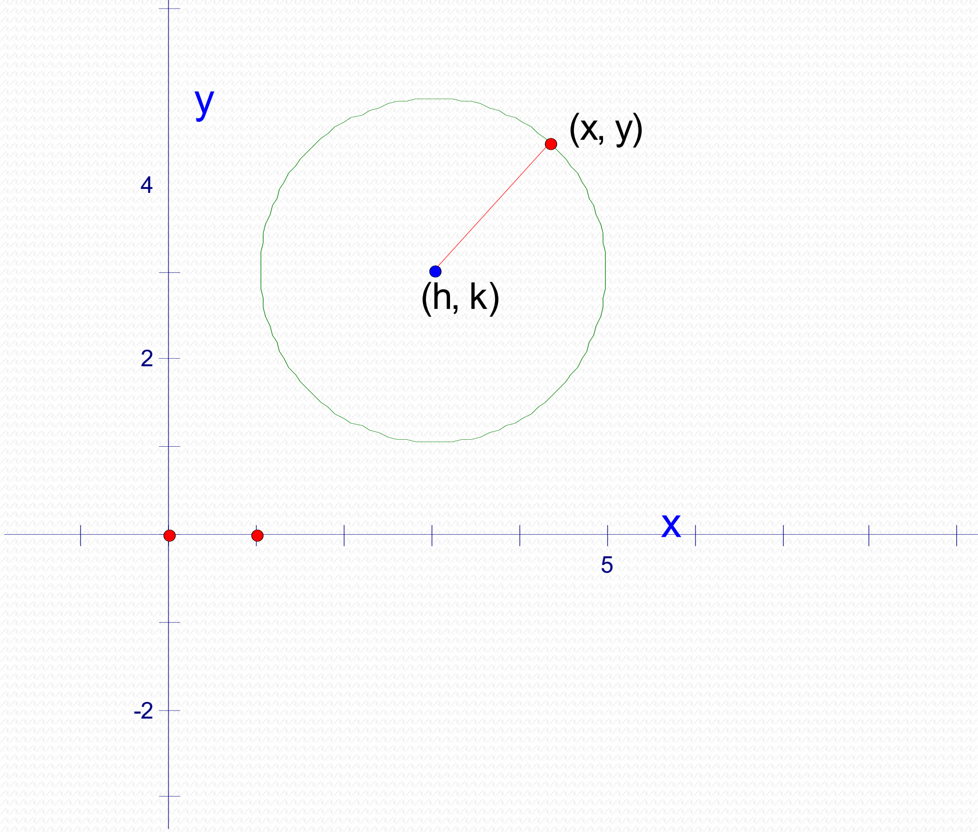
Definition: A circle is an infinite number of points a set distance away from a center



EQUATION FORM	$(x - h)^2 + (y - k)^2 = r^2$
CENTER	(h, k)
RADIUS	r
MIDPOINT FORMULA	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
DISTANCE FORMULA	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

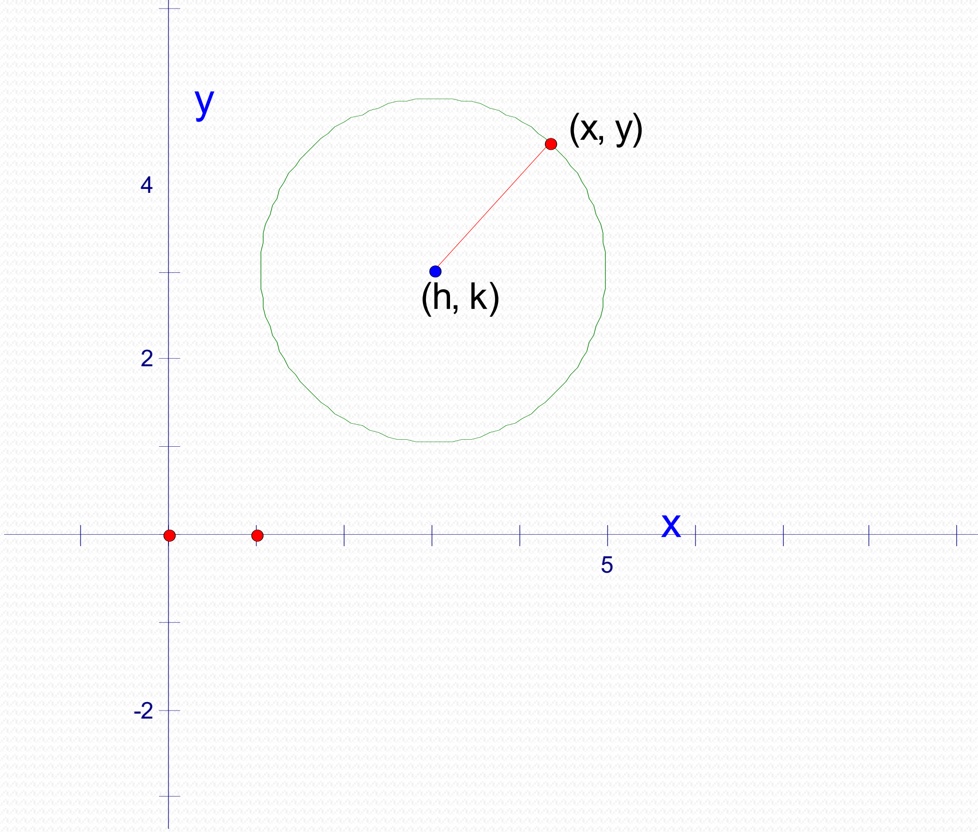
Finding Equations of Circles

- You can write an equation of a circle in a coordinate plane if you know its radius and the coordinates of its center.



Finding Equations of Circles

- Suppose the radius is r and the center is (h, k). Let (x, y) be any point on the circle. The distance between (x, y) and (h, k) is r, so you can use the Distance Formula. (Told you it wasn't going away).



Finding Equations of Circles

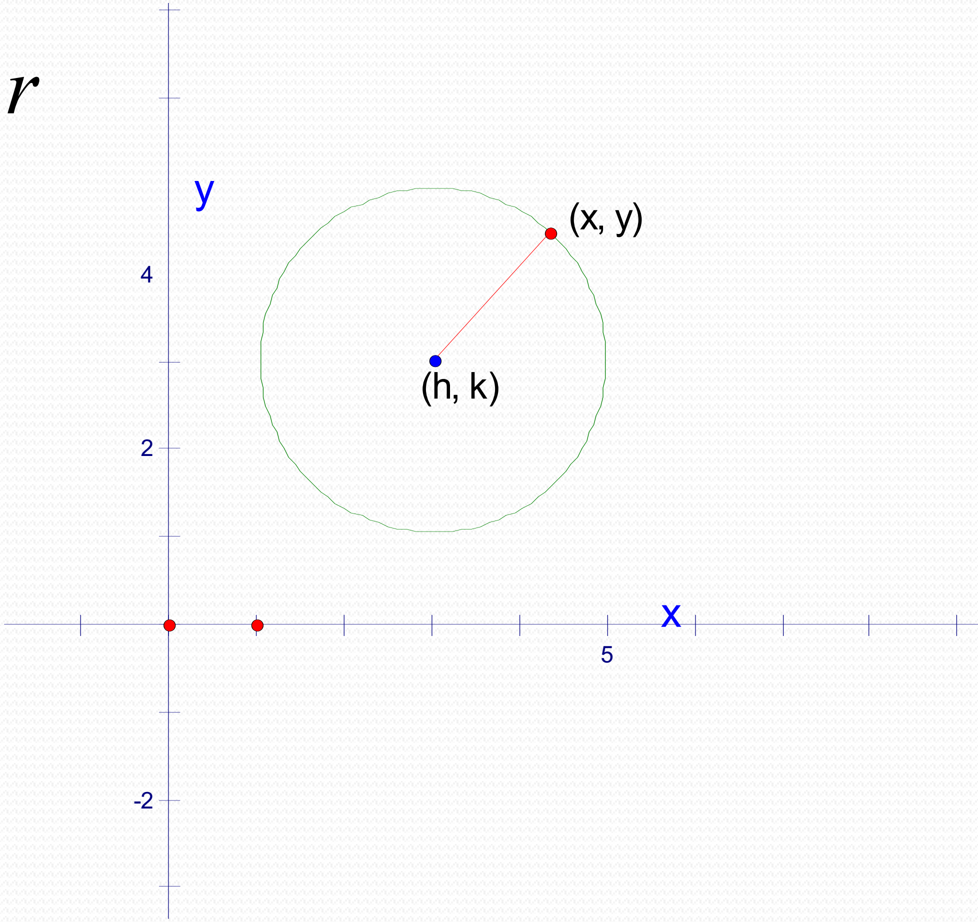
$$\sqrt{(x-h)^2+(y-k)^2}=r$$

- Square both sides to find the standard equation of a circle with radius r and center (h, k).

$(x - h)^2 + (y - k)^2 = r^2$

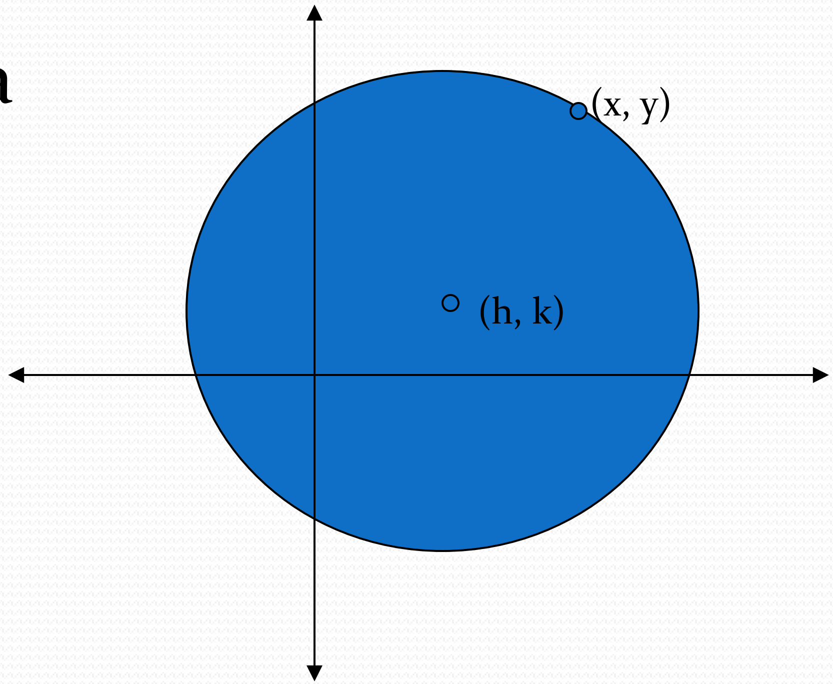
If the center is at the origin, then the standard equation is

$x^2 + y^2 = r^2$.



Equation of a circle

Use the distance formula to determine the equation of a circle



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Using (h, k) and (x, y), we get

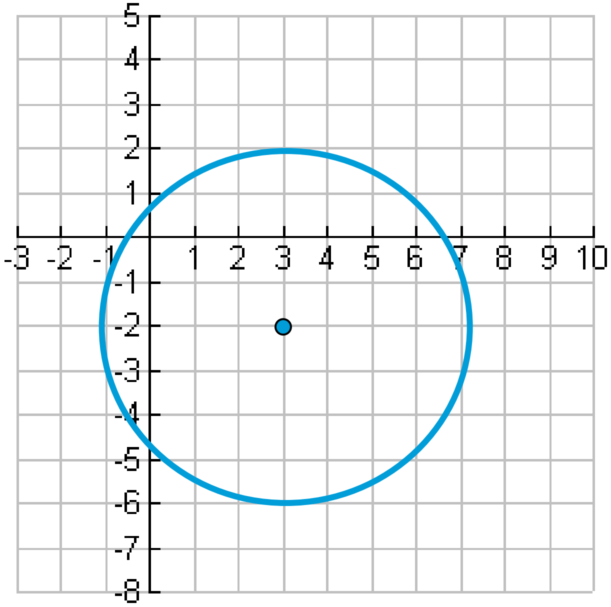
$$d = \sqrt{(x-h)^2 + (y-k)^2}$$

The distance d is actually equal to the radius of the circle, so we get

$$r = \sqrt{(x-h)^2 + (y-k)^2} \quad \text{or} \quad r^2 = (x-h)^2 + (y-k)^2.$$

EX 1 Write an equation of a circle with center (3, -2) and a radius of 4.

h k r



$$(x-h)^2+(y-k)^2=r^2$$

$$(x-3)^2+(y-(-2))^2=4^2$$

$$(x-3)^2+(y+2)^2=16$$

EX 2 Write an equation of a circle with center (-4, 0) and a *diameter* of 10.

h k 2r

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-(-4))^2+(y-0)^2=5^2$$

$$(x+4)^2+y^2=25$$

EX 3 Write an equation of a circle with center (2, -9) and a *radius of* $\sqrt{11}$.

h k r

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-2)^2+(y-(-9))^2=\sqrt{11}^2$$

$$(x-2)^2+(y+9)^2=11$$

EX 4 Find the coordinates of the center and the measure of the radius.

Opposite signs!

$$(x-6)^2+(y+3)^2=25^2$$

(6, -3)

Radius 5

Take the square root!

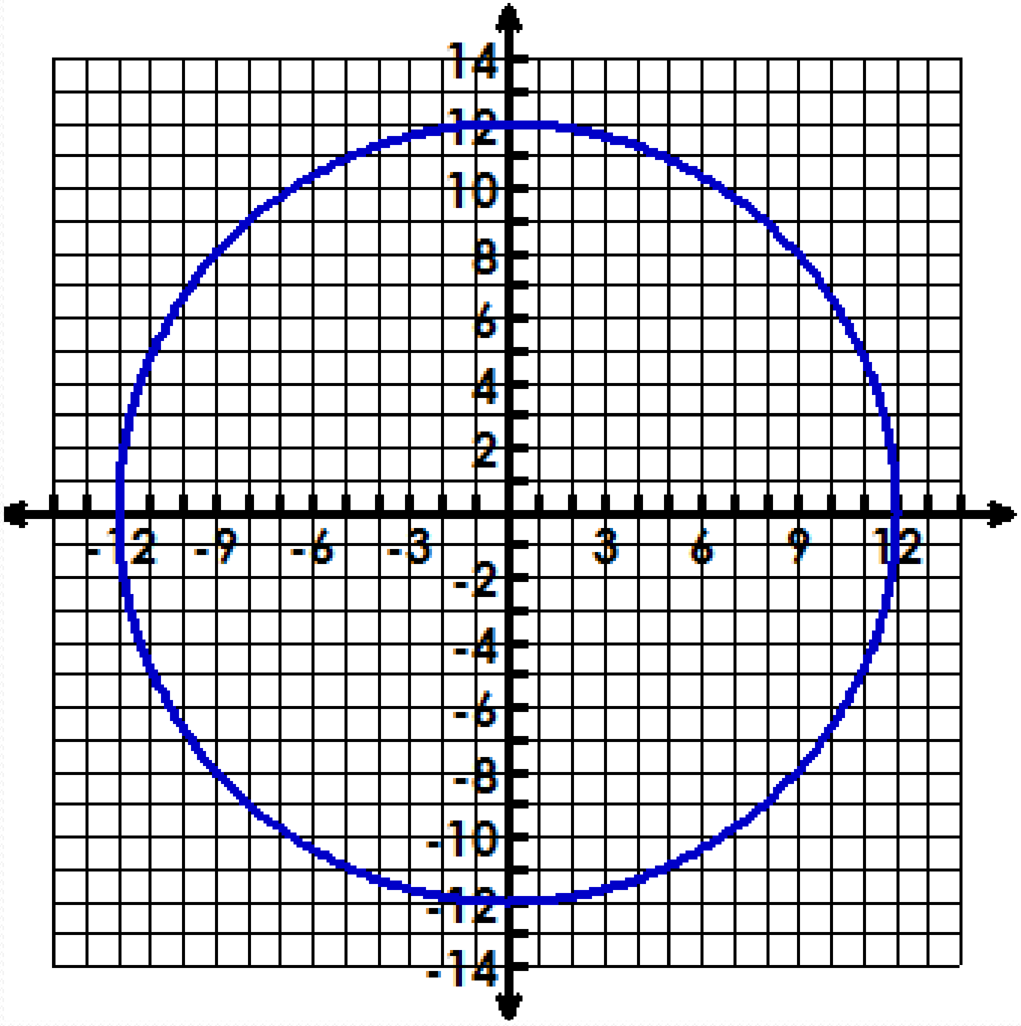
5. Find the center, radius, & equation of the circle.

The center is (0, 0)

The radius is 12

The equation is

$$x^2 + y^2 = 144$$



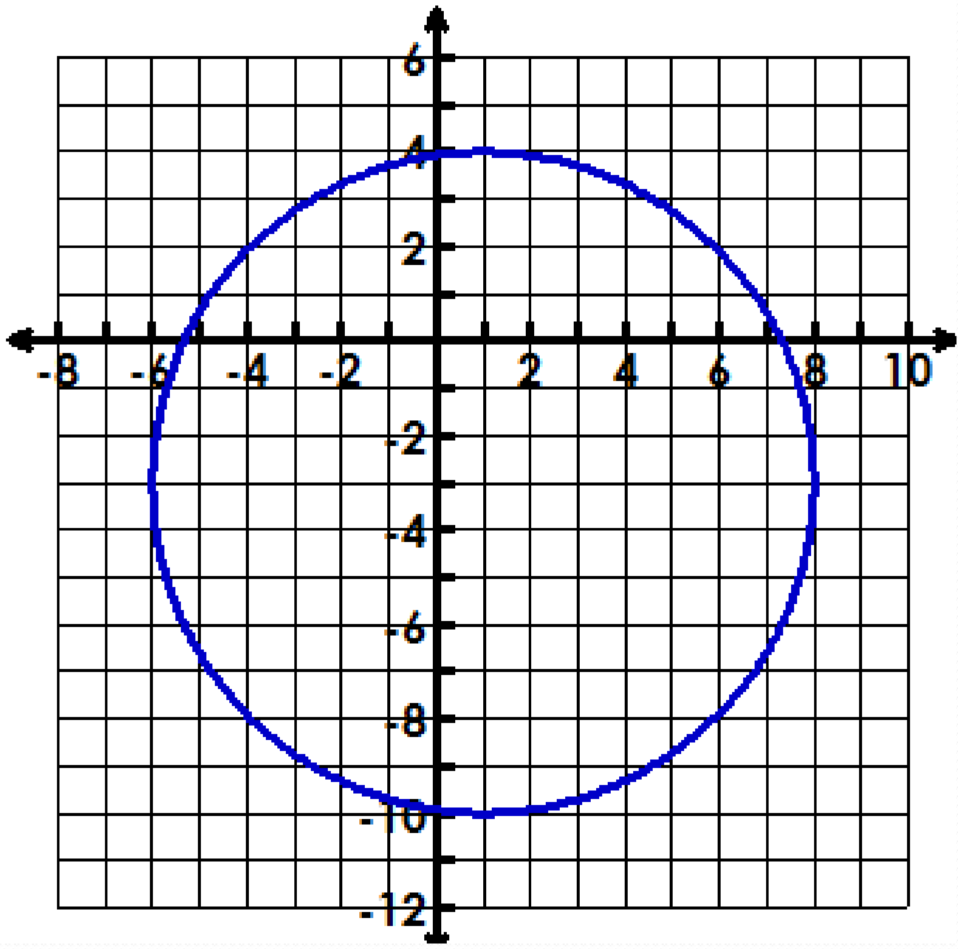
6. Find the center, radius, & equation of the circle.

The center is **(1, -3)**

The radius is **7**

The equation is

$$(x - 1)^2 + (y + 3)^2 = 49$$

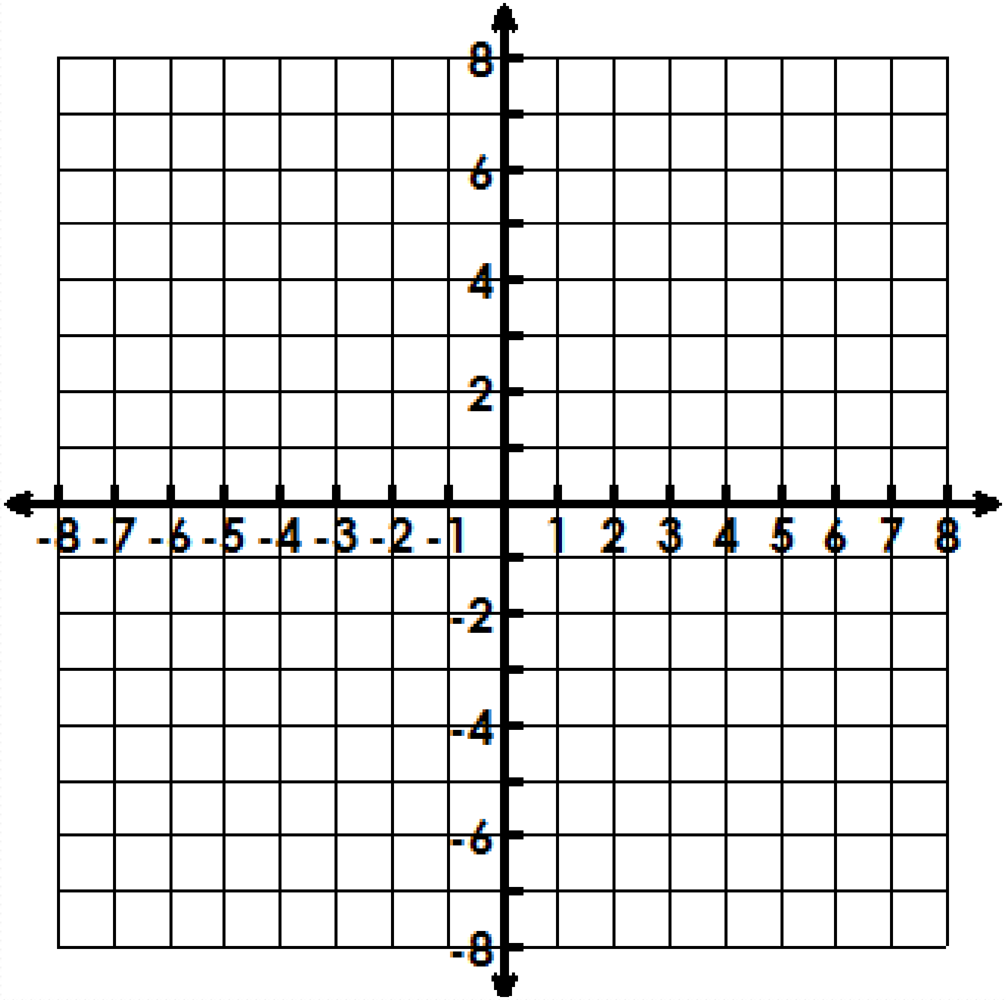


7. Graph the circle, identify the center & radius.

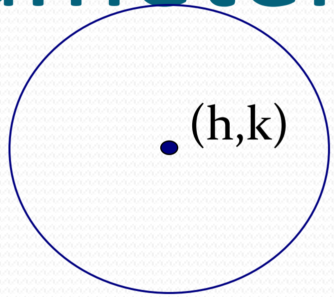
$(x-3)^2 + (y-2)^2 = 9$

Center **(3, 2)**

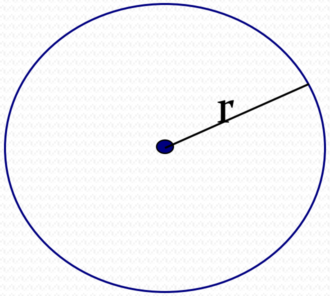
Radius of **3**



Parameters of circle



- Center: (h,k)
  - The fixed point described in the definition of a circle



- Radius: r
  - The distance from the center of the circle to any point on the circle

Ex. 1: Writing a Standard Equation of a Circle

- Write the standard equation of the circle with a center at (-4, 0) and radius 7.1

$(x - h)^2 + (y - k)^2 = r^2$	Standard equation of a circle.
$[(x - (-4)]^2 + (y - 0)^2 = 7.1^2$	Substitute values.
$(x + 4)^2 + (y - 0)^2 = 50.41$	Simplify.

Ex. 2: Writing a Standard Equation of a Circle

- The point (1, 2) is on a circle whose center is (5, -1). Write a standard equation of the circle.

$(x - h)^2 + (y - k)^2 = r^2$ Standard equation of a circle.

$[(x - 5)]^2 + [y - (-1)]^2 = 5^2$ Substitute values.

$(x - 5)^2 + (y + 1)^2 = 25$ Simplify.

Graphing Circles

- If you know the equation of a circle, you can graph the circle by identifying its center and radius.

Ex. 3: Graphing a circle

- The equation of a circle is

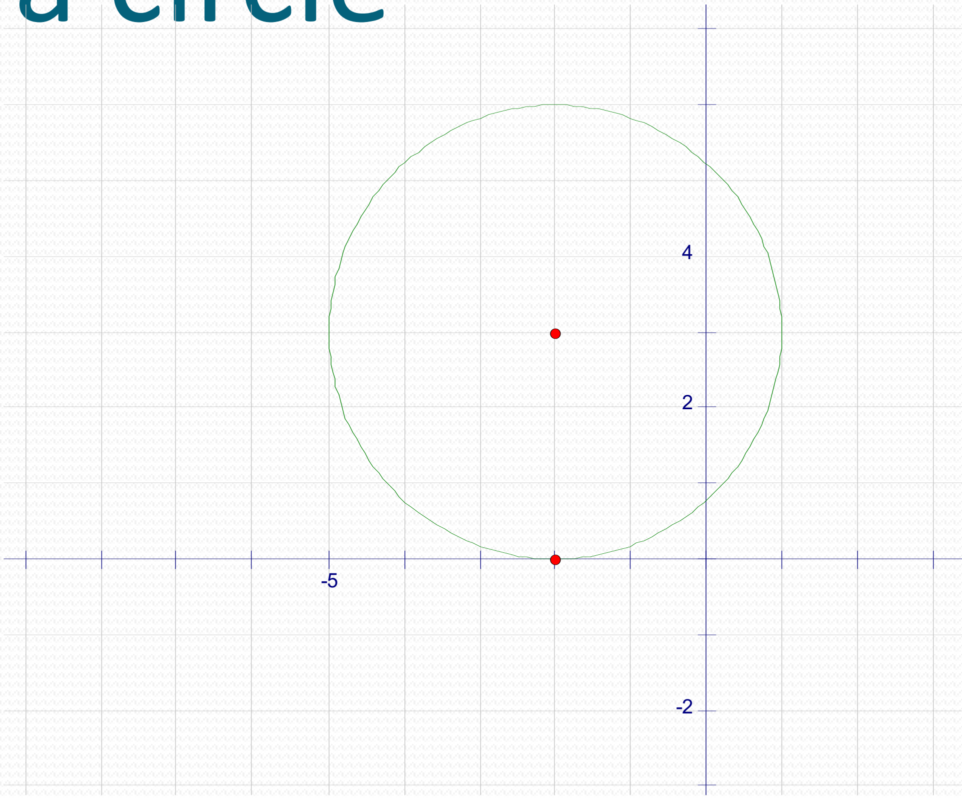
$(x+2)^2 + (y-3)^2 = 9$. Graph the circle.

First rewrite the equation to find the center and its radius.

- $(x+2)^2 + (y-3)^2 = 9$
- $[x - (-2)]^2 + (y - 3)^2 = 3^2$
- The center is (-2, 3) and the radius is 3.

Ex. 3: Graphing a circle

- To graph the circle, place the point of a compass at (-2, 3), set the radius at 3 units, and swing the compass to draw a full circle.



Ex. 2: Writing a Standard Equation of a Circle

- The point (1, 2) is on a circle whose center is (5, -1). Write a standard equation of the circle.

r = Use the Distance Formula

r = Substitute values.

r = Simplify.

r = Simplify.

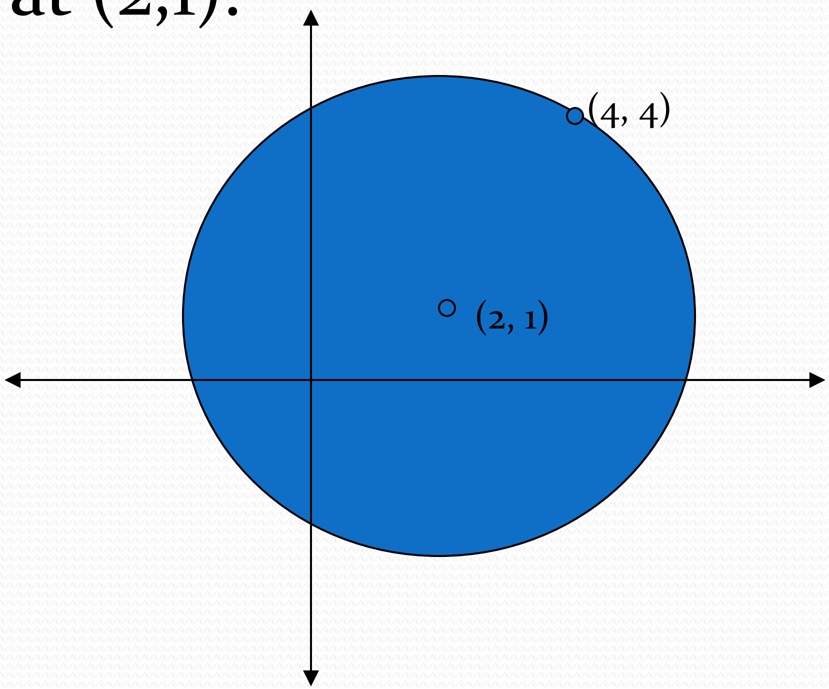
r = Addition Property

r = 5 Square root the result.

Given the graph of a circle, state its equation

- From our graph we see that the center is at (2,1).
- To write the equation of a circle you must know the center and the radius.

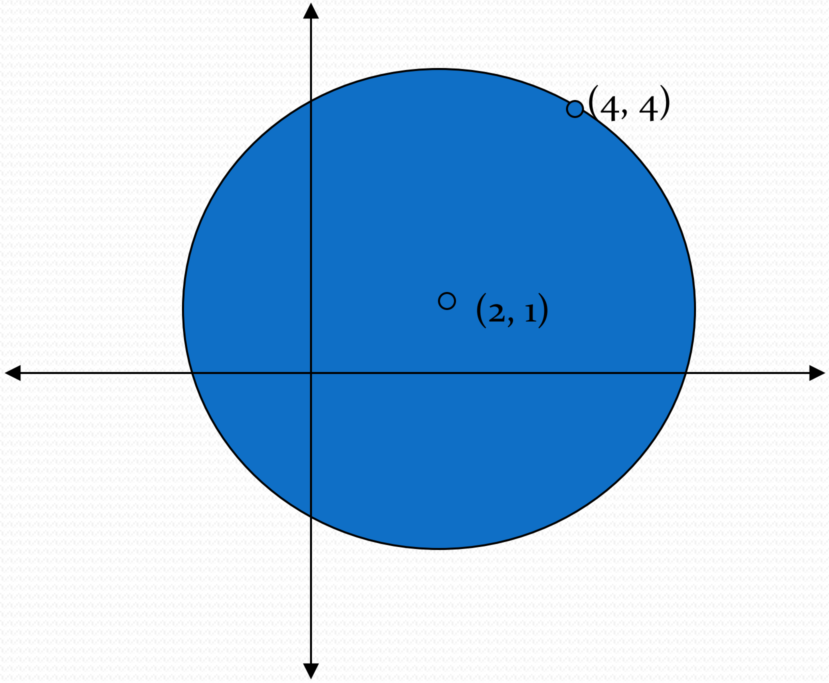
$$r = \sqrt{(4-2)^2 + (4-1)^2}$$
$$r = \sqrt{2^2 + 3^2}$$
$$r = \sqrt{4+9}$$
$$r = \sqrt{13}$$



- Use the center and the point (4,4) to find the radius.

Given the graph of a circle, state its equation

Center (2,1) $r = \sqrt{13}$



The equation of the circle is

$$r^2 = (x - h)^2 + (y - k)^2$$

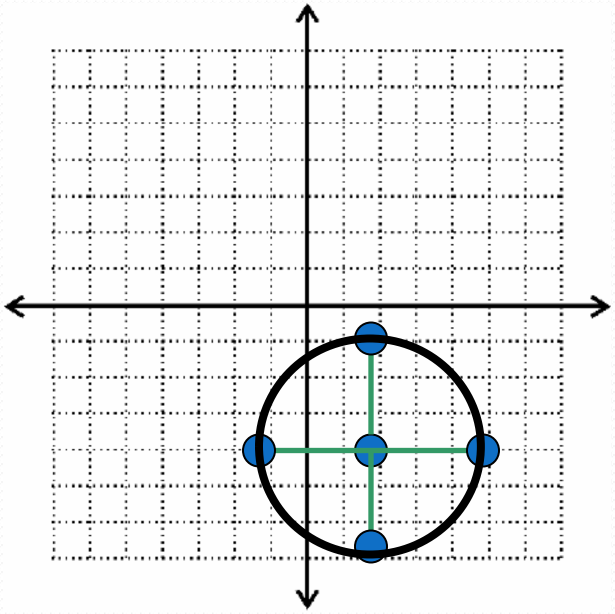
$$\left(\sqrt{13}\right)^2 = (x - 2)^2 + (y - 1)^2$$

$$13 = (x - 2)^2 + (y - 1)^2$$

WRITE and GRAPH

- A) write the equation of the circle in standard form
- $x^2 + y^2 - 4x + 8y + 11 = 0$
- Group the x and y terms
- $x^2 - 4x + y^2 + 8y + 11 = 0$
- Complete the square for x/y
- $x^2 - 4x + 4 + y^2 + 8y + 16 = -11 + 4 + 16$
- $(x - 2)^2 + (y + 4)^2 = 9$
- YAY! Standard Form!

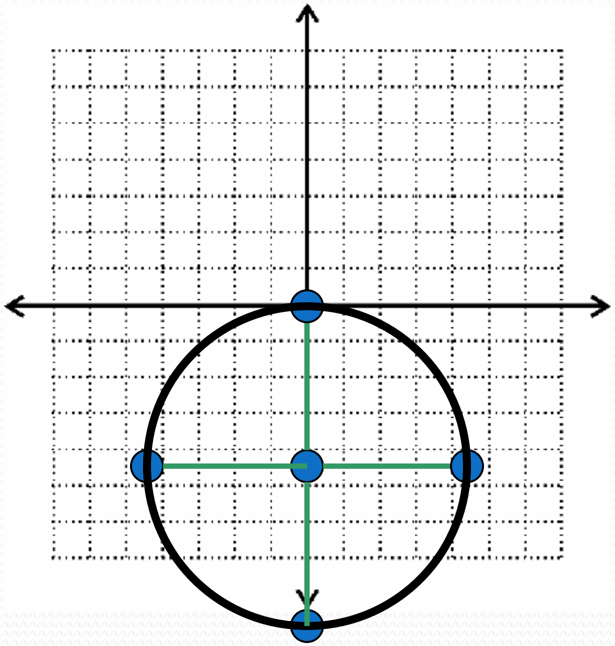
- B) GRAPH
- Plot Center (2,-4)
- Radius = 3



WRITE and GRAPH

- A) write the equation of the circle in standard form
- $4x^2 + 4y^2 + 36y + 5 = 0$
- Group the x and y terms
- $4x^2 + 4y^2 + 36y + 5 = 0$
- Complete the square for x/y
- $4x^2 + 4(y^2 + 9y) = -5$
- $4x^2 + 4(y^2 + 9y + 81/4) = -5 + 81$
- $4x^2 + 4(y + 9/2)^2 = 76$
- $x^2 + (y + 9/2)^2 = 19$
- YAY! Standard Form!

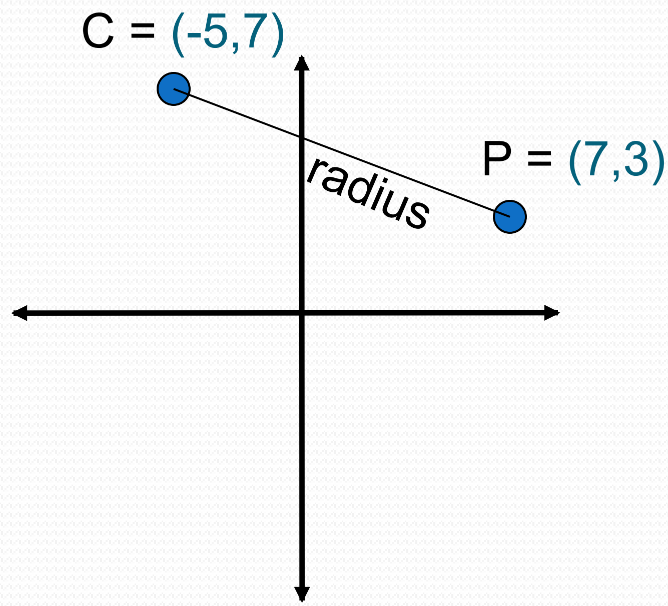
- B) GRAPH
- Plot Center (0 , -9/2)
- Radius = √19 = 4.5



WRITING EQUATIONS

Write the EQ of a circle that has a center of (-5,7) and passes through (7,3)

- Plot your info
- Need to find values for h, k, and r
- $(h , k) = (-5 , 7)$
- How do we find r?
- Use distance formula with C and P.



- Plug into formula
- $(x - h)^2 + (y - k)^2 = r^2$
- $(x + 5)^2 + (y - 7)^2 = (4\sqrt{10})^2$
- $(x + 5)^2 + (y - 7)^2 = 160$

Let's Try One

Write the EQ of a circle that has endpoints of the diameter at (-4,2) and passes through (4,-6)

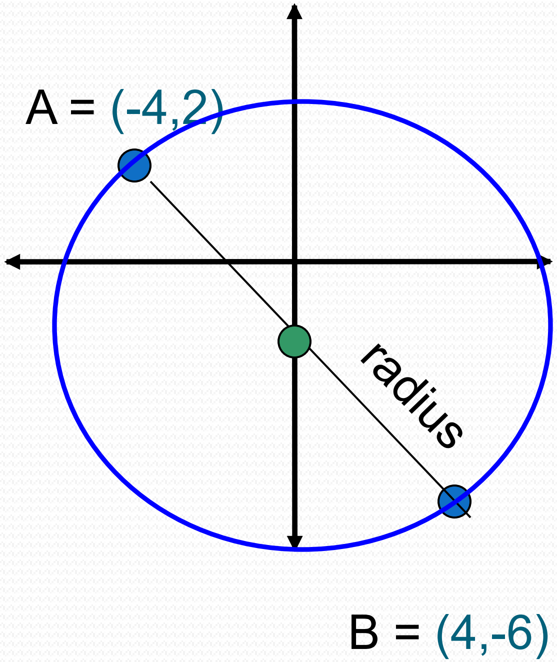
- Plot your info
- Need to find values for h, k, and r
- How do we find (h,k)?
- Use midpoint formula

$$C = \left(\frac{-4+4}{2}, \frac{2+-6}{2} \right)$$

- (h , k) = (0 , -2)
- How do we find r?
- Use dist form with C and B.

$$\text{Dist} = \sqrt{32} = 4\sqrt{2}$$

- Plug into formula
- $(x - h)^2 + (y - k)^2 = r^2$
- $(x)^2 + (y + 2)^2 = 32$



Hint: Where is the center? How do you find it?

Suppose the equation of a circle is $(x-5)^2 + (y+2)^2 = 9$

- Write the equation of the new circle given that:

A) The center of the circle moved up 4 spots and left 5:

- $(x - 0)^2 + (y - 2)^2 = 9$

Center moved from (5,-2) → (0,2)

B) The center of the circle moved down 3 spots and right 6:

- $(x - 11)^2 + (y + 5)^2 = 9$

Center moved from (5,-2) → (11,-5)

Let's Try One

Find the center and radius of the circle with equation $(x + 4)^2 + (y - 2)^2 = 36$.

$(x - h)^2 + (y - k)^2 = r^2$ Use the standard form.

$(x + 4)^2 + (y - 2)^2 = 36$ Write the equation.

$(x - (-4))^2 + (y - 2)^2 = 6^2$ Rewrite the equation in standard form.

$h = -4 \quad k = 2 \quad r = 6$ Find h, k, and r.

The center of the circle is (–4, 2). The radius is 6.

Let's Try One

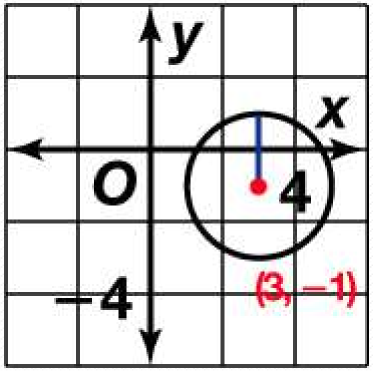
Graph $(x - 3)^2 + (y + 1)^2 = 4$.

$(x - h)^2 + (y - k)^2 = r^2$

Find the center and radius of the circle.

$(x - 3)^2 + (y - (-1))^2 = 4$

$h = 3$ $k = -1$ $r^2 = 4$, or $r = 2$



Draw the center (3, –1) and radius 2.
Draw a smooth curve.

Equation of a Circle

$$(x-h)^2+(y-k)^2=r^2$$

$\underline{\text{Center}}:(h,k)$

$\underline{\text{Radius}}:\ r$

Writing the Equation of a Circle

1. Group x terms together, y-terms together, and move constants to the other side
2. Complete the square for the x-terms
   - *Remember that whatever you do to one side, you must also do to the other*
3. Complete the square for the y-terms
   - *Remember that whatever you do to one side, you must also do to the other*

Example: Write the equation and find the center and radius length of :

$$x^2 + y^2 - 10x + 8y - 8 = 0$$

$$(x^2 - 10x) + (y^2 + 8y) = 8$$ **Group terms**

$$(x^2 - 10x + \_) + (y^2 + 8y + \_) = 8 + \_ + \_$$ **Complete the square**

$$(x^2 - 10x + 25) + (y^2 + 8y + 16) = 8 + 25 + 16$$

$$(x-5)^2 + (y+4)^2 = 49$$

$$(x-5)^2 + (y-(-4))^2 = (7)^2$$

Center : $(5, -4)$ Radius length: 7

You try!!: Write the equation and find the center and radius length of :

$$x^2 + y^2 + 6x - 12y + 20 = 0$$

$$(x^2 + 6x) + (y^2 - 12y) = -20$$

$$(x^2 + 6x + 9) + (y^2 - 12y + 36) = -20 + 9 + 36$$

$$(x + 3)^2 + (y - 6)^2 = 25$$

$$(x - (-3))^2 + (y - 6)^2 = (5)^2$$

Center : $(-3, 6)$ Radius length: 5

THINK ABOUT IT

Find the center, the length of the radius, and write the equation of the circle if the endpoints of a diameter are (-8,2) and (2,0).

Center: Use midpoint formula!

$$\left(\frac{-8+2}{2},\frac{2+0}{2}\right)=\left(-3,1\right)$$

Length: use distance formula with radius and an endpoint

$$\sqrt{(2-(-3))^2+(0-1)^2}=\sqrt{26}$$

Equation: *Put it all together*

$$(x-(-3))^2+(y-1)^2=\left(\sqrt{26}\right)^2 \quad or \quad (x+3)^2+(y-1)^2=26$$

Ex. 4: Applying Graphs of Circles

- A bank of lights is arranged over a stage. Each light illuminates a circular area on the stage. A coordinate plane is used to arrange the lights, using the corner of the stage as the origin. The equation $(x - 13)^2 + (y - 4)^2 = 16$ represents one of the disks of light.

  A. Graph the disk of light.

  B. Three actors are located as follows: Henry is at (11, 4), Jolene is at (8, 5), and Martin is at (15, 5). Which actors are in the disk of light?

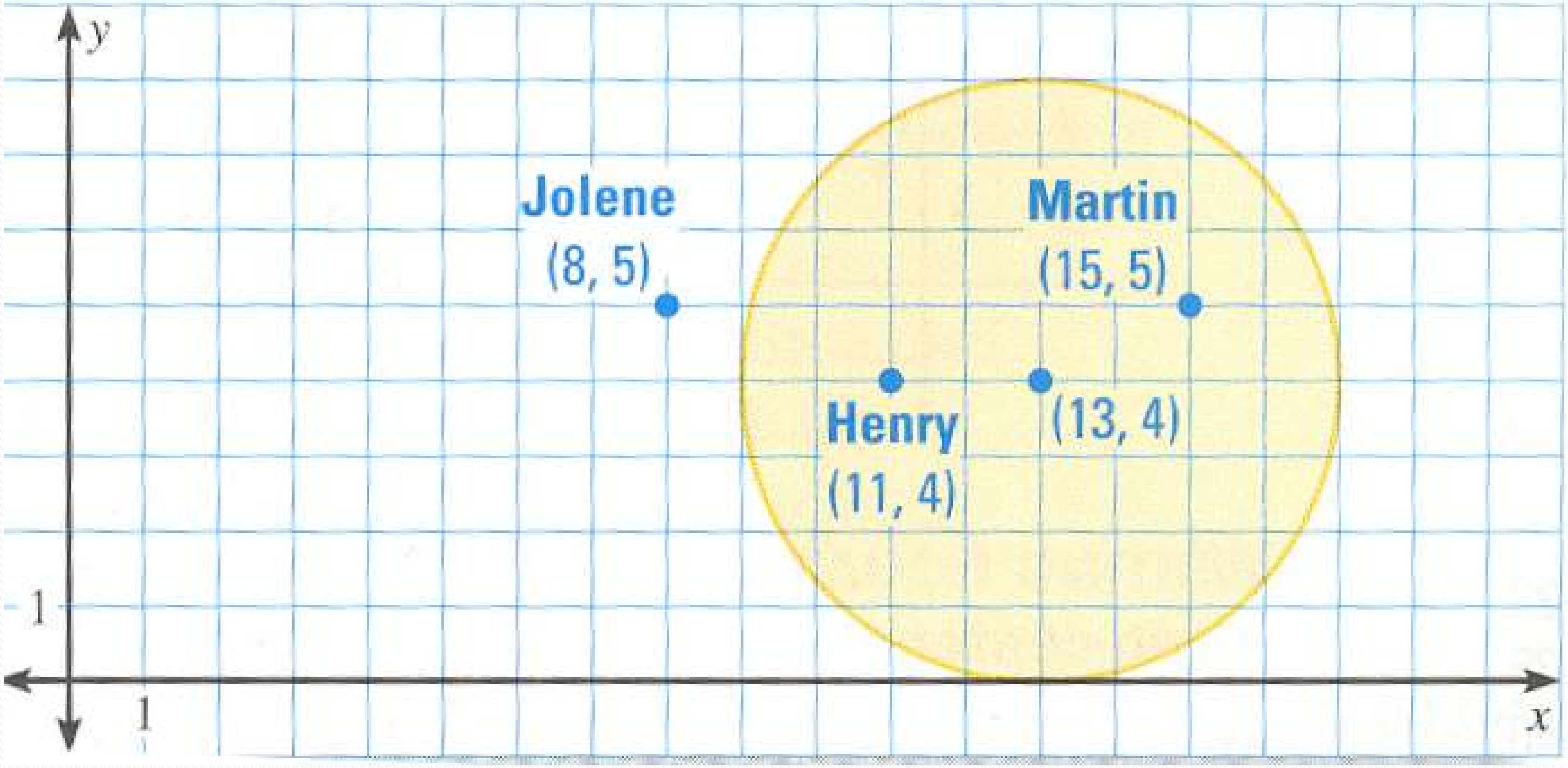
Ex. 4: Applying Graphs of Circles

1. Rewrite the equation to find the center and radius.

- $(x – h)^2 + (y – k)^2 = r^2$
- $(x - 13)^2 + (y - 4)^2 = 16$
- $(x – 13)^2 + (y – 4)^2 = 4^2$
- The center is at (13, 4) and the radius is 4. The circle is shown on the next slide.

Ex. 4: Applying Graphs of Circles

1. Graph the disk of light



The graph shows that Henry and Martin are both in the disk of light.

Ex. 4: Applying Graphs of Circles

- A bank of lights is arranged over a stage. Each light

Use the Distance Formula

r =

Substitute values.

r =

Simplify.

r =

Simplify.

r =

Addition Property

r = 5

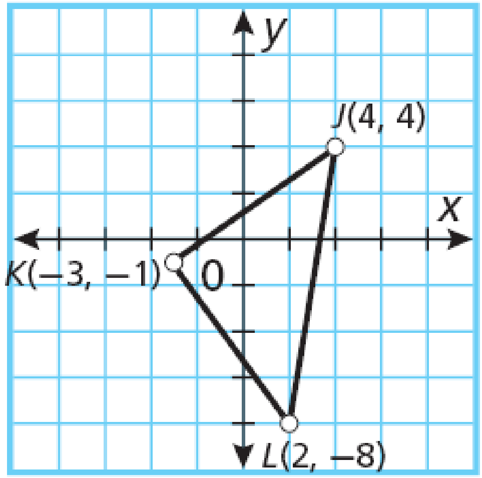
Square root the result.

Example 3: Radio Application

An amateur radio operator wants to build a radio antenna near his home without using his house as a bracing point. He uses three poles to brace the antenna. The poles are to be inserted in the ground at three points equidistant from the antenna located at *J*(4, 4), *K*(–3, –1), and *L*(2, –8). What are the coordinates of the base of the antenna?

Step 1 Plot the three given points.

Step 2 Connect *J*, *K*, and *L* to form a triangle.

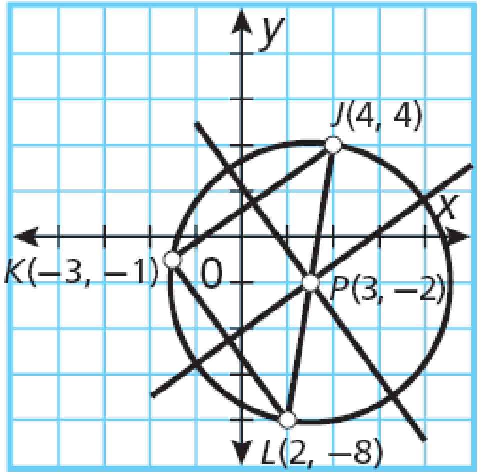


Example 3 Continued

Step 3 Find a point that is equidistant from the three points by constructing the perpendicular bisectors of two of the sides of Δ*JKL.*

The perpendicular bisectors of the sides of Δ*JKL* intersect at a point that is equidistant from *J*, *K*, and *L*.

The intersection of the perpendicular bisectors is *P* (3, –2). *P* is the center of the circle that passes through *J*, *K*, and *L*.



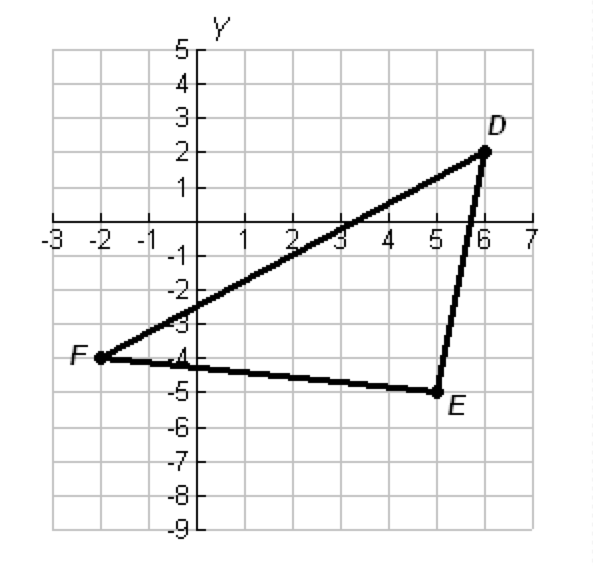
The base of the antenna is at *P* (3, –2).

Check It Out! Example 3

What if…? Suppose the coordinates of the three cities in Example 3 (p. 801) are *D*(6, 2) , *E*(5, –5), and *F*(-2, -4). What would be the location of the weather station?

Step 1 Plot the three given points.

Step 2 Connect *D*, *E*, and *F* to form a triangle.

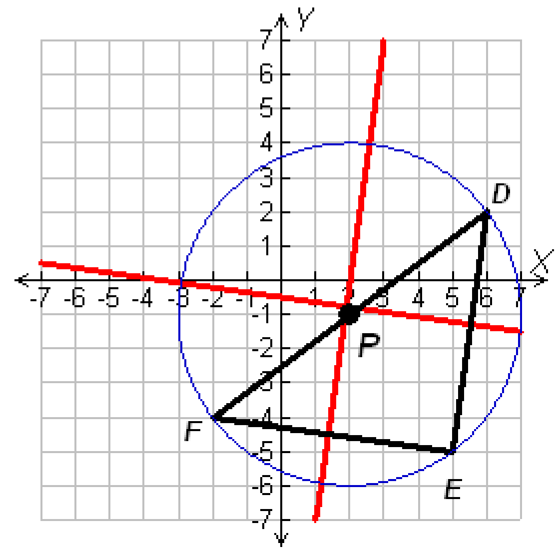


Check It Out! Example 3 Continued

Step 3 Find a point that is equidistant from the three points by constructing the perpendicular bisectors of two of the sides of Δ*DEF.*

The perpendicular bisectors of the sides of Δ*DEF* intersect at a point that is equidistant from *D*, *E*, and *F*.

The intersection of the perpendicular bisectors is *P*(2, –1). *P* is the center of the circle that passes through *D*, *E*, and *F*.



The base of the antenna is at *P*(2, –1).

Lesson Quiz: Part III

5. A carpenter is planning to build a circular gazebo that requires the center of the structure to be equidistant from three support columns located at $E(-2, -4)$, $F(-2, 6)$, and $G(10, 2)$.

What are the coordinates for the location of the center of the gazebo?

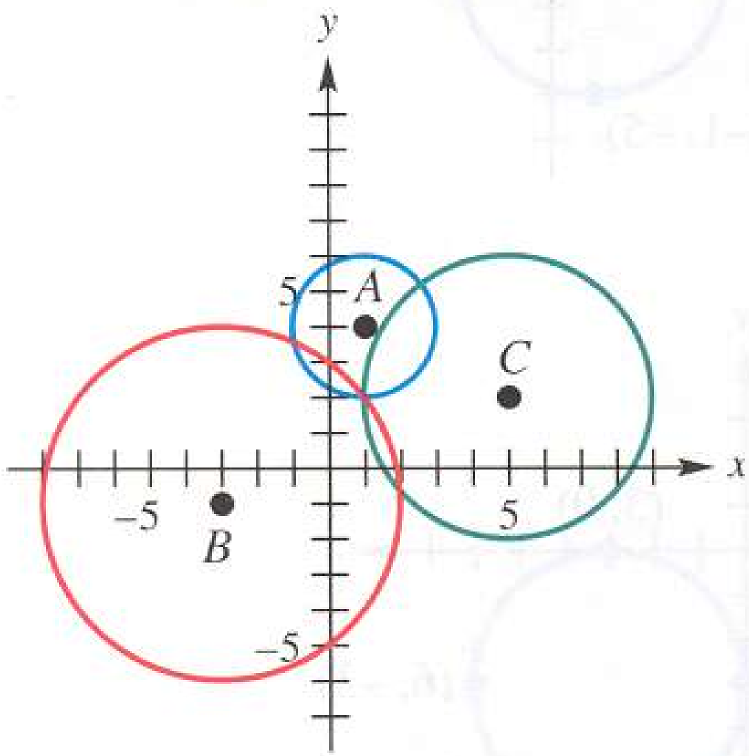
$(3, 1)$

Example 6 LOCATING THE EPICENTER OF AN EARTHQUAKE

Three receiving stations are located on a coordinate plane at points (1, 4), (–3, –1), and (5, 2). The distance from the earthquake epicenter to each station should be 2 units, 5 units, and 4 units respectively.

Solution Graph the three circles. From the graph it appears that the epicenter is located at (1, 2). To check this algebraically, determine the equation for each circle and substitute $x = 1$ and $y = 2$.

Example 6 LOCATING THE EPICENTER OF AN EARTHQUAKE



Station *A*:

$$(x-1)^2+(y-4)^2=4$$

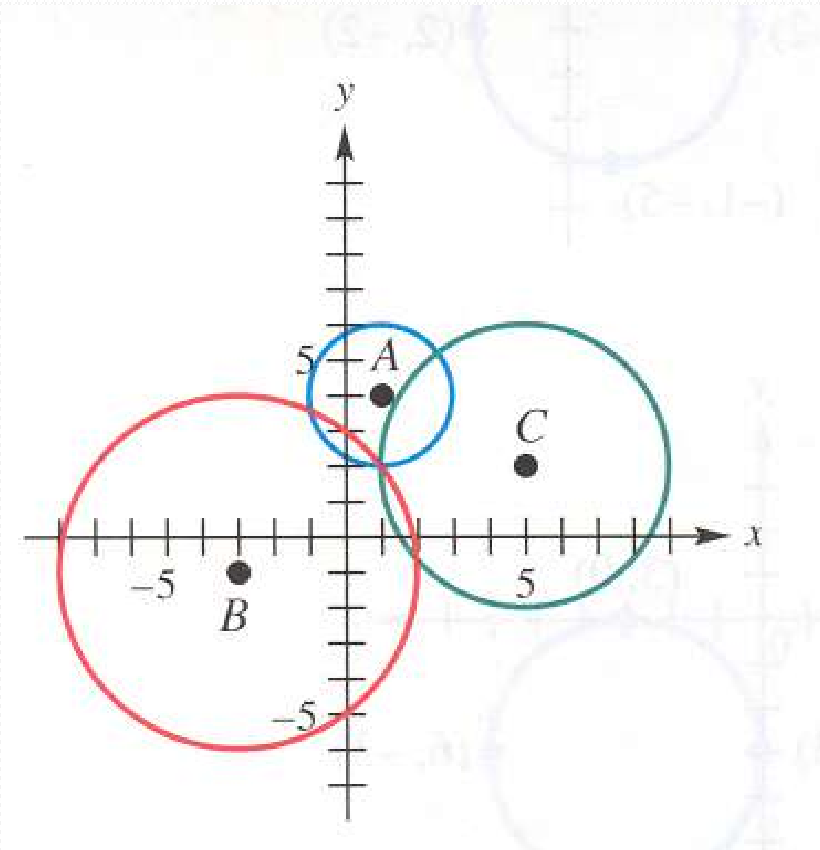
$$(1-1)^2+(2-4)^2=4$$

$$0+4=4$$

$$4=4$$

Example 6

LOCATING THE EPICENTER OF AN EARTHQUAKE



Station *B*:

$$(x+3)^2+(y+1)^2=25$$

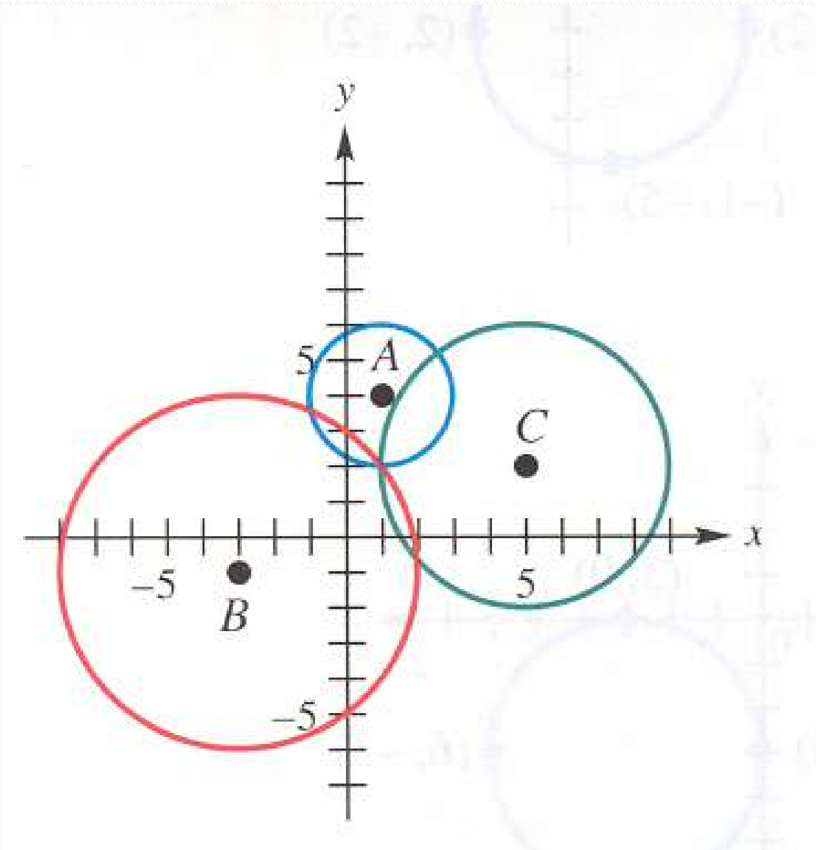
$$(1+3)^2+(2+1)^2=25$$

$$16+9=25$$

$$25=25$$

Example 6

LOCATING THE EPICENTER OF AN EARTHQUAKE



Station *C*:

$$(x-5)^2+(y-2)^2=16$$

$$(1-5)^2+(2-2)^2=16$$

$$16+0=16$$

$$16=16$$

Example 6

LOCATING THE EPICENTER OF AN EARTHQUAKE

Three receiving stations are located on a coordinate plane at points (1, 4), (–3, –1), and (5, 2). The distance from the earthquake epicenter to each station should be 2 units, 5 units, and 4 units respectively.

The point (1, 2) does lie on all three graphs; thus, we can conclude that the epicenter is at (1, 2).