

Ted's Quest for a Tablet

Name: _____

Ted has had his eye on a tablet computer for several months, and he's trying to figure out a way to save enough money so that he can buy one using cash instead of credit. Ted is trying every possible method to build his computer fund – he's looking for change in the couch, he's drawn up a savings plan, he's budgeting in other areas of his life, and so on. But Ted is growing impatient, and he's afraid that he will resort to using a credit card so that he can get one quickly, but of course that means Ted will have to pay interest on his purchase.

Let's first look at a few ways that Ted has attempted to build his fund.

1. Change under the Sofa Cushion

Ted first tries the easiest way to find money – he looks under the cushions of his sofa! And he finds, to his amazement, enough coins to equal \$7.75! If Ted has

- found only nickels, dimes, and quarters,
- a total of 65 coins,
- 5 more nickels than the total number of dimes and quarters together,

which of the following can be used to find the number of nickels, n , the number of dimes, d , and the number of quarters, q , that Ted has?

$$(a) \begin{cases} n + d + q = 7.75 \\ 65n + 65d + 65q = 7.75 \\ n - 5 = d + q \end{cases}$$

$$(b) \begin{cases} n + d + q = 65 \\ 0.05n + 0.1d + 0.25q = 65 \\ n - (d + q) = 5 \end{cases}$$

$$(c) \begin{cases} n + d + q = 65 \\ 0.05n + 0.1d + 0.25q = 7.75 \\ n - d - q = 5 \end{cases}$$

$$(d) \begin{cases} n + d + q = 7.75 \\ 0.05n + 0.1d + 0.25q = 7.75 \\ (d + q) + 5 = n \end{cases}$$

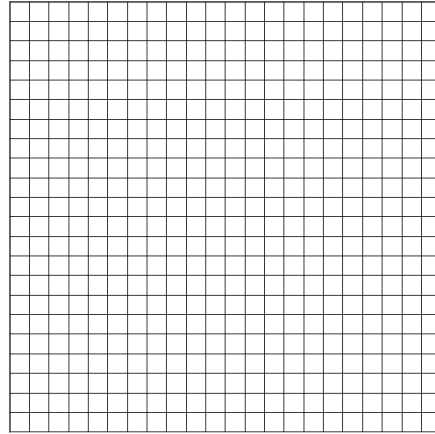
Explain your choice.

2. Money in the Bank

Ted's grandmother, Miss Tedrina Mae, really likes to spoil her grandson and knows how much he wants a tablet computer. She sets up a checking account (with no interest) for Ted with an initial amount of \$200 and will she add \$15 each month afterward.

(a) Write an equation whose solution is the number of months, m , it takes for the account balance to reach \$395.

(b) Make a plot of the balance after m months for $m = 1, 5, 9, 13, 17$ and indicate on the plot the solution to your equation in part (a).



Ted loves his grandmother and gladly accepts her \$200 gift, but he doesn't want her to open up a checking account and put extra money in it (she's done too much already!) – he believes he has a better idea, anyway. Ted remembered that he took some notes in a finance course he was enrolled in at the local community college regarding interest-bearing accounts:

Compound Interest (2 Types)

1. n compounding periods: $A = P \left(1 + \frac{r}{n} \right)^{nt}$

2. Continuous compounding: $A = Pe^{rt}$

Values of n :

Annually = Once per year

Semiannually = Twice per year

Quarterly = Four times per year

Monthly = Twelve times per year

Weekly = Fifty-two times per year

A = Final amount

P = Principal (starting or initial amount)

r = interest rate (decimal form)

n = compounding periods per year

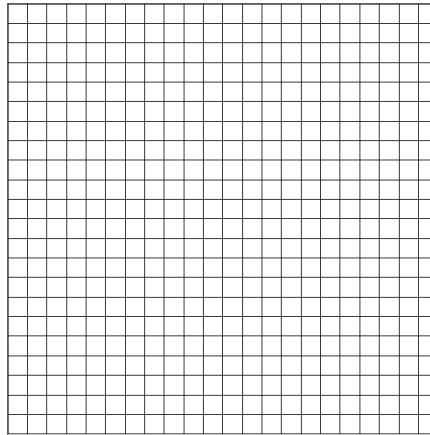
t = time in years

There are two banks that Ted will decide between to open a savings account (which is one type of interest-bearing account) – Bernoulli Bank or Euler Federal Bank. Bernoulli Bank offers a 6.75% quarterly-compounded interest rate, and Euler Federal offers a 6.75% continuously-compounded interest rate.

Ted plans on investing his \$200 gift – plus an additional \$50 bill that he found in the sofa cushion when he was looking for change – in a savings account for one year.

(c) Give the simplified form of the equation representing the amount earned at any given month at Bernoulli Bank.

(d) Graph the function from part (c). What type of function is this?



(e) Fill in the table below and explain which bank you would choose.

Year	Bernoulli Bank	Euler Federal Bank
1		
2		
3		
4		
5		
6		
t		

(f) If Ted chooses the savings account with Euler Federal, how long will it take for Ted to save the same amount that he would have had in six months with his grandmother’s original plan?

3. Ted’s Totally Terrific Guitars

Ted just happens to be a really talented craftsman, and is known throughout his hometown as being a very good guitar builder. Ted hopes to make some money from this business for his tablet computer purchase. Ted builds three types of guitars: archtops, electrics, and acoustics. Ted builds 1 archtop guitar per month, 2 acoustic guitars per month, and 3 electric guitars per month. Suppose that it takes Ted x hours to build an archtop guitar, y hours to build an electric guitar, and z hours to build an acoustic guitar.

(a) Write an equation relating x , y , and z if Ted spends a total of 134 hours per month building guitars.

(b) If Ted charges \$90 per hour for an archtop guitar, \$45 per hour for an electric guitar, and \$65 per hour for an acoustic guitar, write an equation relating x , y , and z if Ted builds \$9240 worth of guitars.

Ted buys large blocks of specific varieties of expensive woods to build his guitars. Ted has found that the best varieties of tone woods for instruments are spruce and mahogany. Ted buys w blocks of spruce for y dollars each, and x blocks of mahogany for z dollars each. In a given month, Ted spends a total of C dollars, where $C = wy + xz$. For the following, write an equation whose solution is the given quantity.

(c) The number of blocks of mahogany that Ted can afford to buy if he wishes to spend a total of \$5000 this month, mahogany costs \$450 per block, and he has already bought 7 blocks of spruce at \$200 each.

(d) The price of spruce blocks if Ted bought 9 of them, in addition to the 10 blocks of mahogany that Ted bought at \$425 a block, for a total of \$6275.

(e) The price of a block of mahogany, given that a block of spruce costs $\frac{5}{9}$ as much as a block of mahogany, and Ted has bought 12 blocks of spruce and 15 blocks of mahogany for a total cost of \$9750.

4. Ted's Toss

Ted is still trying to find ways to make money, and since the county fair is in town, he's decided to try his skill in the ball-throwing competition (this is a fairly simple county fair!) with a \$50 grand prize. The winner is the thrower that produces the longest time in the air for the ball. When Ted throws this particular type of ball, it moves vertically upward at a speed of v feet/second and rises a distance of d feet in t seconds, given by

$$d = 4 + vt - 10t^2$$

Write an equation whose solution is

(a) the time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet;

(b) the speed with which the ball must be thrown to rise 20 feet in 2 seconds.

Before we move on to the second part of “Ted’s Toss”, let’s look at a little quadratic review.

Quick Quadratic Review

Graph these equations on your graphing calculator at the same time. What happens? Why?

$$Y1: (x - 3)(x + 1)$$

$$Y2: x^2 - 2x - 3$$

$$Y3: (x - 1)^2 - 4$$

$$Y4: x^2 - 2x + 1$$

Below are the first three equations from the previous problem.

$$Y1: (x - 3)(x + 1)$$

$$Y2: x^2 - 2x - 3$$

$$Y3: (x - 1)^2 - 4$$

These three equations all describe the same function. What are the coordinates of the following points on the graph of the function? From which equation is each point most easily determined? Explain.

Vertex: _____

y-intercept: _____

x-intercept(s): _____

Make up an equation for a quadratic function whose graph satisfies the given condition. Use whatever form is most convenient.

(a) Has a vertex at $(-2, -5)$.

(b) Has a y-intercept at $(0, 6)$

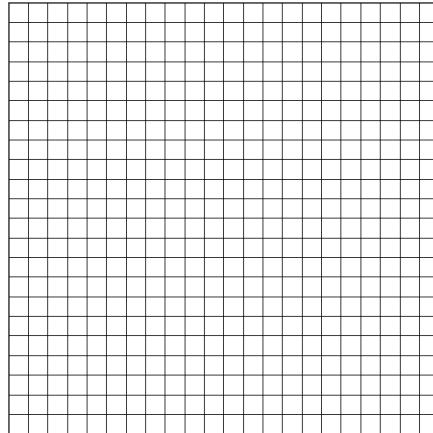
(c) Has x-intercepts at $(3, 0)$ and $(5, 0)$

(d) Has x-intercepts at the origin and $(4, 0)$

(e) Goes through the points $(4, -10)$ and $(1, 2)$

Now let’s say that Ted has a pretty good throwing arm (he has entered a throwing competition, after all!). In fact, on his particular throw for this competition, Ted throws the ball at a speed of 90 ft/sec. Using $d = 4 + vt - 10t^2$, answer the following.

(c) Write the equation that will be used to model Ted's throw, and then graph his throw, from the time it leaves his hand to the time it falls back to the ground. Label all axes and units carefully. At what height does the ball leave Ted's hand?



(d) For Ted's throw, what is the maximum height of the ball? Show how you would find this using the equation you created in part (c), and then check your answer with your graph.

Ted was the last person to throw in the competition. Unfortunately, up to the time of Ted's throw, his arch-nemesis Billy Bob Bigglesby (who already owns the most expensive tablet computer on the market) had the longest throw with the ball having stayed in the air for 8.8 seconds. The model for Billy Bob's throw is the same as for Ted [$d = 4 + vt - 10t^2$]. Obviously, they are the same height and have roughly the same throwing position.

(e) Who won the \$50 prize, Ted or Billy Bob? How must the speed of Billy Bob's throw compare to Ted's?

(f) What was the speed of Billy Bob's throw if the maximum height for his throw was 197.6 feet?

5. Ted's Test

Ted also happens to be a student at the local college, where he is taking a physics course. Ted has a special scholarship arrangement – instead of having his tuition paid, the foundation that pays for his scholarship gives him \$250 for every course in which he makes an A. Ted is taking a physics final exam, and he is being asked to re-arrange important equations in order to solve for a desired variable.

Help Ted out by using inverse operations to solve the equations for the unknown variable, or for the designated variable if there is more than one.

(a) $-3 = \frac{x}{-27}$

(b) $16z = \frac{1}{4}$

(c) $\frac{1}{3}w + 7 = \frac{9}{5}$

(d) $a^2 + b^2 = c^2$ for b

(e) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for c

(f) $F = qE$ for E

(g) $E_k = \frac{1}{2}mv^2$ for v

(h) $ax + c = R$ for x

(i) $x = x_0 + v_0t + \frac{1}{2}at^2$ for a

(j) $F = \frac{GMm}{r^2}$

6. Ted's Tablet

Ted finally made enough money (by checking sofa cushions, building guitars, using his savings account, throwing balls in competitions, and doing well in college) to buy himself the tablet computer of his dreams! Ted bought the computer for exactly \$817.53, which is the list price plus sales tax. Find the list price of the tablet if Ted bought the computer in

(a) Vidalia, where the sales tax is 5%;

(b) Marietta, where the sales tax is 7.55%;

(c) a city in Georgia where the sales tax is r .

Solve the following:

1. A vintage Wheat Penny had a value of \$1.17 in 2005. Its value has been increasing at 9% per year. What is the total value after now?
2. Gina deposited \$1600 in an account that pays 1.5% interest compounded quarterly. What will the balance be in 2 years?
3. The Garcias have \$5,000 in a savings account. The bank pays 2.5% interest on savings account, compounded monthly. Find the total balance after 15 years.
4. Determine the amount of **interest** earned on a \$100,000 investment if it is invested at 5.2% annual interest compounded quarterly for 12 years.
5. The Yellowing Cab Company has a savings plan for employees. If an employee makes an initial deposit of \$1000, the company pays 8% interest compounded weekly. If an employee withdraws the money after five years, how much is withdrawn?
6. Actinium-226 has a half-life of 29 hours. If 100 mg of actinium-226 disintegrates over a period of 63 hours, how many mg of actinium-226 will remain?
7. The half-life of radon-222 is 3.8 days. How much of a 100 g sample is left after 15.2 days?
8. In 1985, there were 285 cell phone subscribers in the small town of Seaverville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?
9. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of twelve hours?
10. You want to start saving for your retirement. You are not sure which type of account to use. If you deposit \$1500.00 at 2.75% interest which account will pay the highest return after 35 years? Show your work.
 - a. Compounded monthly
 - b. Compounded quarterly
 - c. Compounded yearly
 - d. Compounded continuously