

Teaching for Mathematical
Understanding
Grades 3 - 5

First District RESA

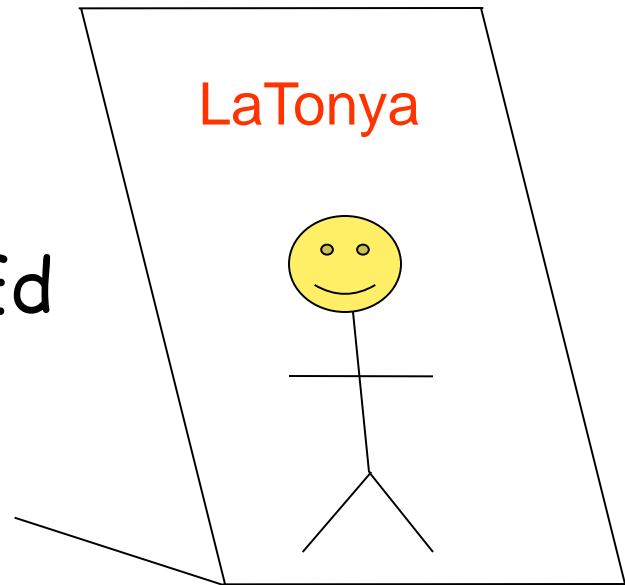
Fractions! Fractions! Fractions!

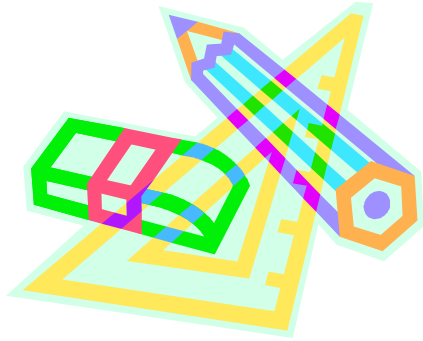


Welcome!

Please sign in on your school's sign in sheet, and make your name card

- Fold it
- With a marker, write your name on it
- Include:
 - Grades taught
 - Regular or Special Ed





Welcome!

Activity One
Can You Find Your Partner?

Please take a card before
having a seat.



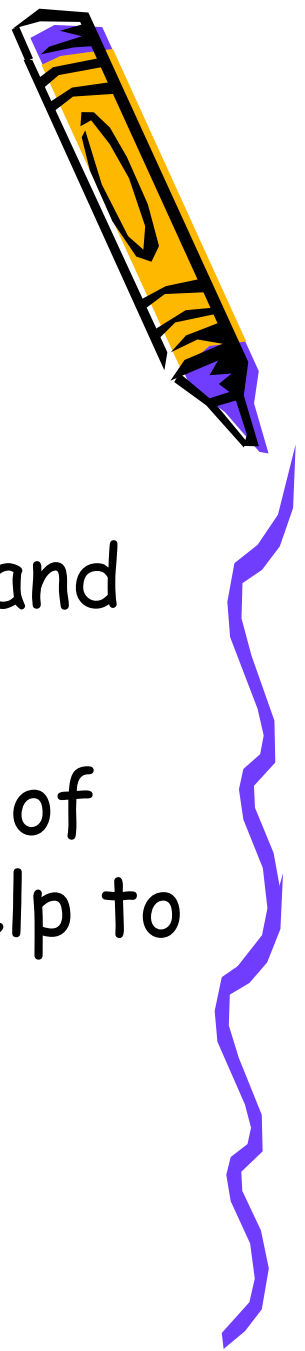
Housekeeping:

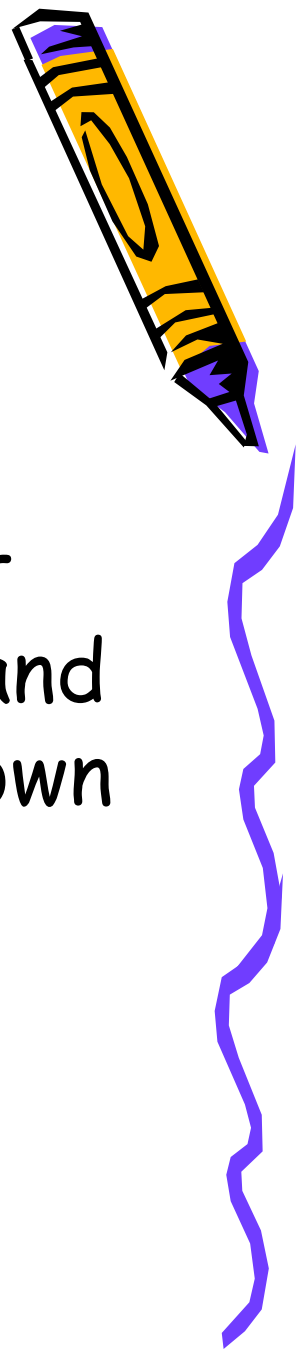
- Time
- Breaks
- Lunch
- Cell phones
- Signal for Quiet



Essential Questions

- Why are the differences between **knowing/doing** and **understanding** important in mathematics teaching and learning?
- How does effective implementation of **Georgia Standards of Excellence** help to facilitate student understanding?





Enduring Understandings

The **quality** and **quantity** of student **experiences** increase the number and strength of connections among known information, leading to greater **understanding**.

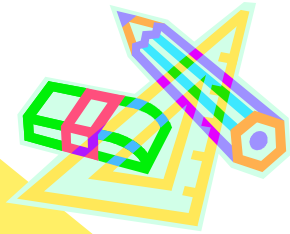


Enduring Understandings

Experiences include opportunities to

- memorize **facts**,
- maintain **old learning** as a foundation for **new learning**,
- **reason** and problem solve in **rigorous** and **relevant** contexts,
- **receive**, **discover**, or **develop** information,
- **engage** with that information **immediately** and **after** time has passed.

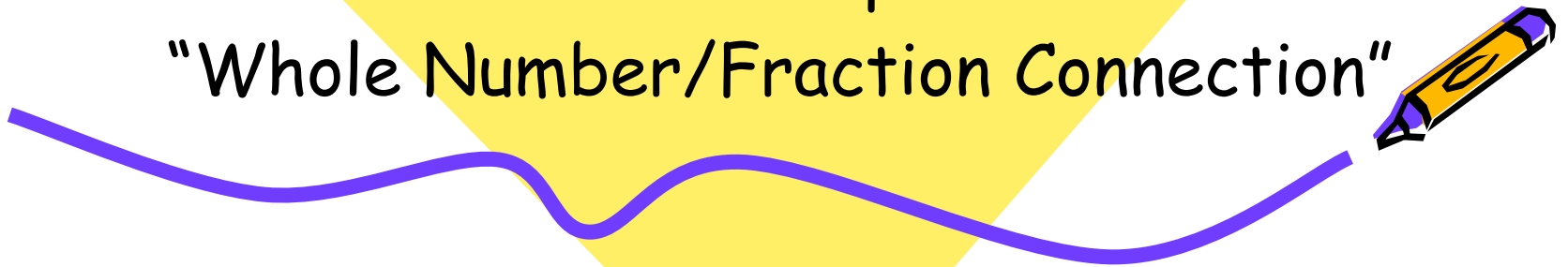




Teaching for Mathematical Understanding

Number and Operations

"Whole Number/Fraction Connection"



Now we will...

- Focus attention on experiences that create connections among whole numbers and fractions.
- Develop concept-based approaches to more difficult solutions.



Progression of Georgia Standards of Excellence



- 1st Grade (in Geometry)

- Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

- 2nd Grade (in Geometry)

- Partition circles and rectangles into two, three and four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.



Progression of Georgia Standards of Excellence



- 3rd Grade

- Develop understanding of fractions as numbers.

- 4th Grade

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.



Understand decimal notation for fractions, and compare decimal fractions.



Progression of Georgia Standards of Excellence



- 5th Grade

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- 6th Grade

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.



3rd Grade Overview



In this unit, students will:

- Develop an understanding of fractions, beginning with unit fractions.
- View fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole.
- Understand that the size of a fractional part is relative to the size of the whole. For example, $\frac{1}{2}$ of the paint in a small bucket could be less paint than $\frac{1}{3}$ of the paint in a larger bucket, but $\frac{1}{3}$ of a ribbon is longer than $\frac{1}{5}$ of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one.
- Solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.



3rd Grade Overview

In this unit, students will:

- Recognize that the numerator is the top number (term) of a fraction and that it represents the number of equal-sized parts of a set or whole; recognize that the denominator is the bottom number (term) of a fraction and that it represents the total number of equal-sized parts or the total number of objects of the set
- Explain the concept that the larger the denominator, the smaller the size of the piece
- Compare common fractions with like denominators and tell why one fraction is greater than, less than, or equal to the other
- Represent halves, thirds, fourths, sixths, and eighths using various fraction models.



4th Grade Overview



In this unit students will:

- understand representations of simple equivalent fractions
- compare fractions with different numerators and different denominators

In this unit students will:

- Identify visual and written representations of fractions
- Understand representations of simple equivalent fractions
- Understand the concept of mixed numbers with common denominators to 12
- Add and subtract fractions with common denominators
- Add and subtract mixed numbers with common denominators
- Convert mixed numbers to improper fractions and improper fractions to mixed fractions
- Understand a fraction a/b as a multiple of $1/b$. (for example: model the product of $\frac{3}{4}$ as $3 \times 1/4$).
- Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
- Multiply a whole number by a fraction



4th Grade Overview

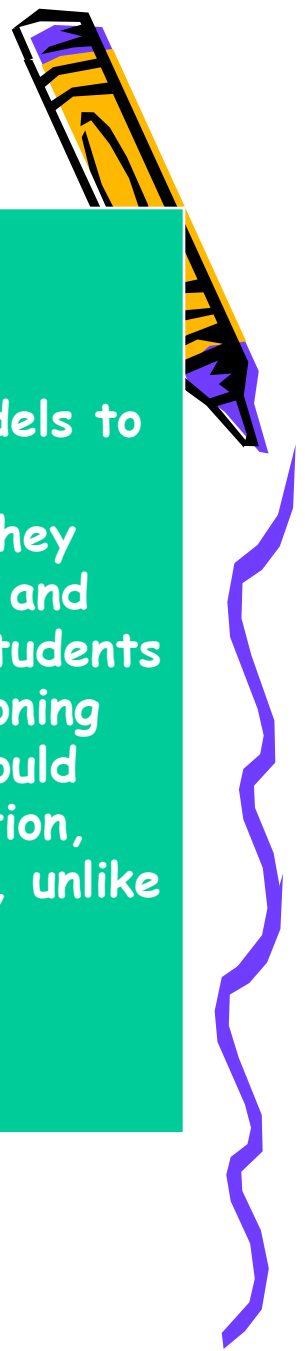


In this unit students will:

- express fractions with denominators of 10 and 100 as decimals
- understand the relationship between decimals and the base ten system
- understand decimal notation for fractions
- use fractions with denominators of 10 and 100 interchangeably with decimals
- express a fraction with a denominator 10 as an equivalent fraction with a denominator 100
- add fractions with denominators of 10 and 100 (including adding tenths and hundredths)
- compare decimals to hundredths by reasoning their size
- understand that comparison of decimals is only valid when the two decimals refer to the same whole
- justify decimals comparisons using visual models



5th Grade Overview



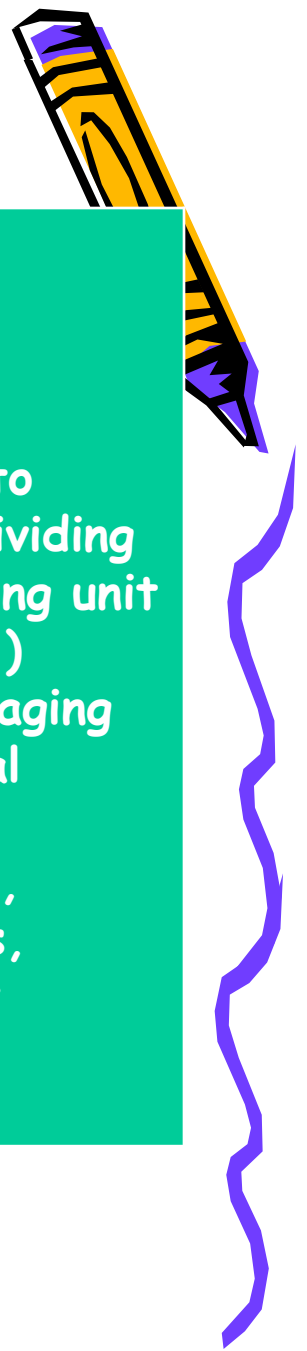
In this unit students will:

- Use equivalent fractions as a strategy to add and subtract fractions.

Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: fraction, equivalent, addition/ add, sum, subtraction/subtract, difference, unlike denominator, numerator, benchmark fraction, estimate, reasonableness, and mixed numbers.



5th Grade Overview



In this unit students will:

- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: fraction, numerator, denominator, operations, multiplication/multiply, division/divide, mixed numbers, product, quotient, partition, equal parts, equivalent, factor, unit fraction, area, side lengths, fractional side lengths, scaling, comparing.



6th Grade Overview



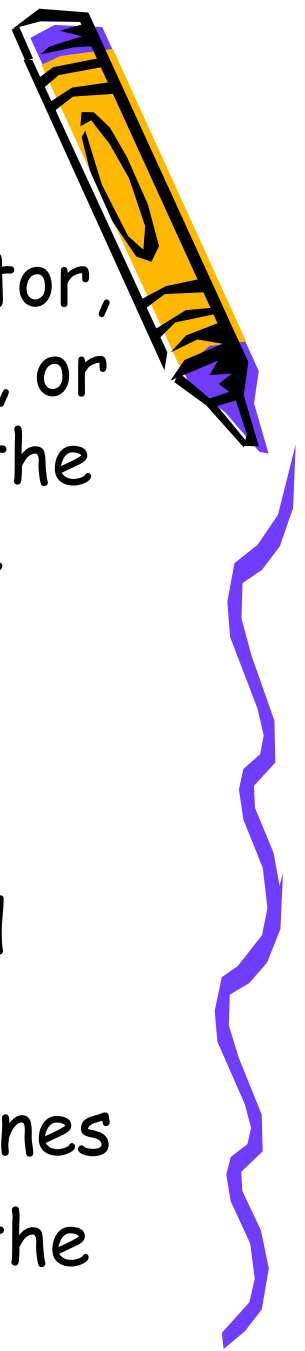
In this unit students will:

- Unit 1: Extending students' experience with whole number computation in elementary grades, division of fractions by fractions and all four operations on decimals are a focus in the first unit. Tasks utilize hands-on activities as a means to building understanding, rather than rote memorization of algorithms.
- Students also find common factors and multiples and deepen and extend their understanding of the distributive property to work with fractions.



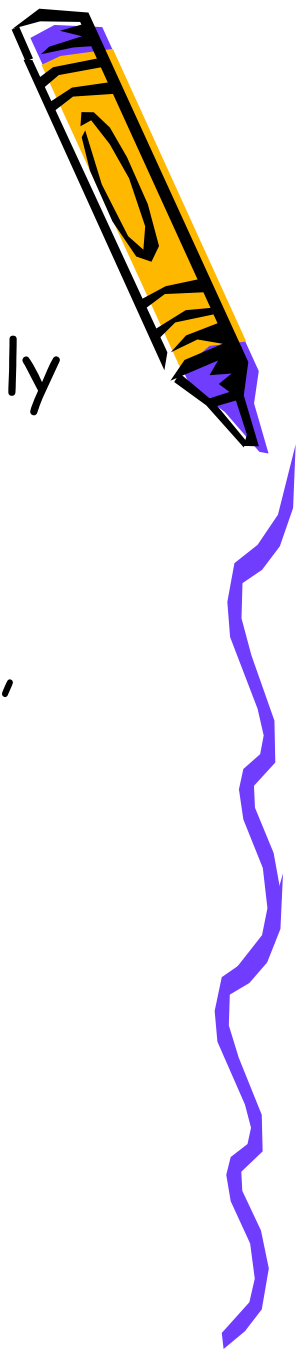
Common Misconceptions

- The idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is.
- Students think all shapes can be divided the same way.
- Student count "tick marks" on number lines (linear/length model) rather than the distance or region partitioned.



Common Misconceptions

- Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator.
- Students often mix models when adding, subtracting, or comparing fractions.



Vocabulary Issues/ Vocabulary Ideas for Usage



- Issues

- Please make sure students have a clear understanding of "numerator" and "denominator".

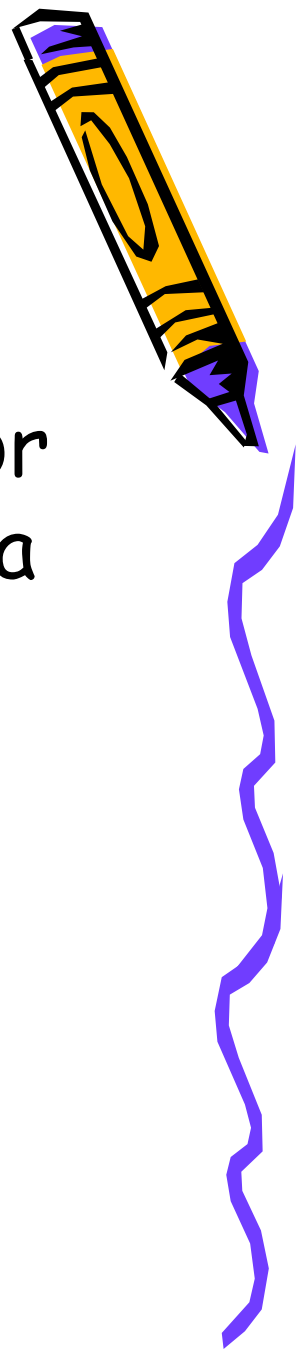
- Suggestions

- How to say the fractions - $\frac{1}{2}$ means "one out of two equal parts/pieces".



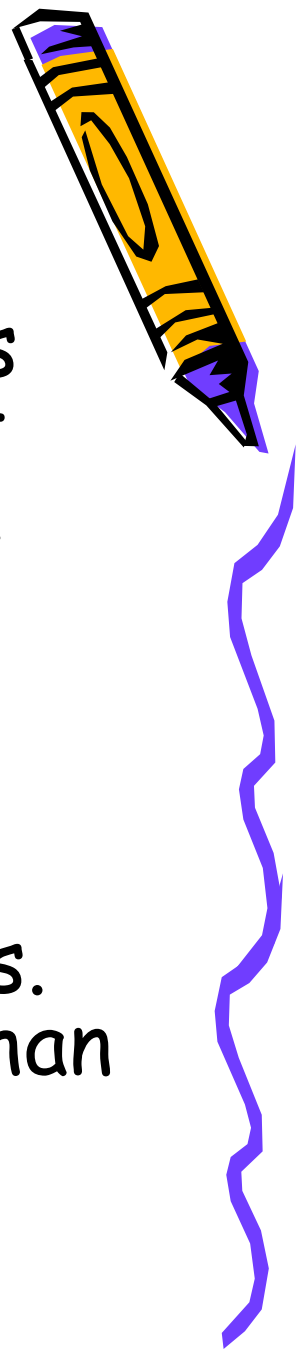
Fractional Parts

- Fractional parts are equal shares or equal-sized portions of a whole or a unit.
- The whole or unit must remain standard in order to make the fractional parts accurate.

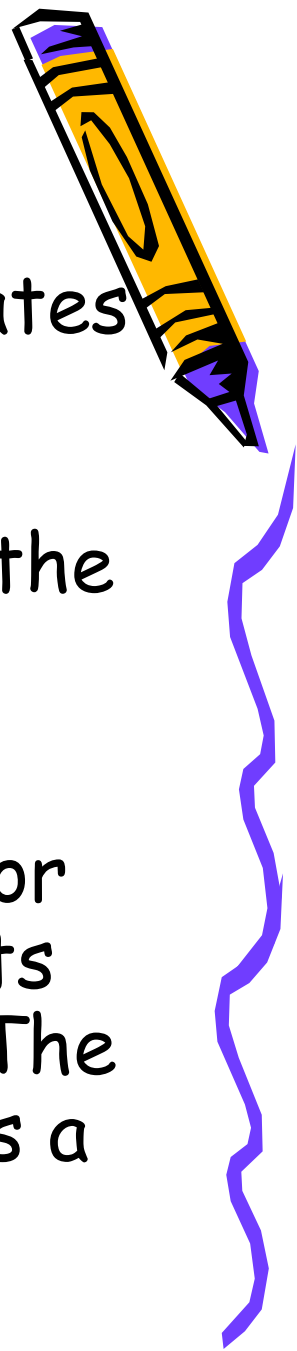


Fractional Parts

- Fractional parts have special names that tell us how many parts of that size are needed to make the whole. For example, thirds require three equal parts to make a whole.
- The more fractional parts used to make a whole, the smaller the parts. For example, eighths are smaller than fifths.



Fractional Parts

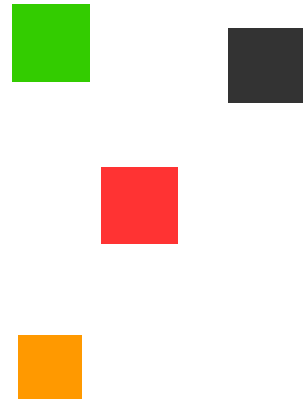


- The denominator of a fraction indicates the number of equal size pieces the whole has been divided into. The denominator is a divisor that names the fractional part of the whole under consideration.
- The numerator of a fraction counts or tells how many of the fractional parts of a whole are under consideration. The numerator is a multiplier; it indicates a multiple of a fractional part.



Coasting Along

How can you use unit squares to connect region models to set models?





Coasting Along, Part 1



- Choose any 16 unit squares. Arrange them on the grid to form a square coaster design. You may change colors and designs until you have a design you like.
- Use the crayons to make two copies of your design.
- On the back of your top design, list the colors you used. Next to each, write the fraction that stands for the part of the whole design that the color represents.
- Exchange recordings with a partner. (Don't peek at the backs!) Write a fraction for each color in your partner's design. Then check the back of that design to see if you both wrote the same fractions. Discuss the results.
- Be ready to tell how to figure out the fractional parts of a coaster design.

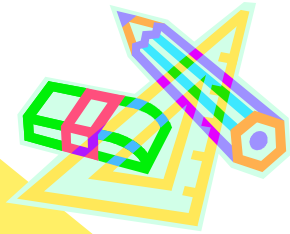




Coasting Along, Part 2

- The coaster you made represents one whole or unit.
- On the back of your top design, you listed the colors along with the fraction that stands for the part of the whole design that each color represents.
- Cut your bottom coaster design into unit squares.
- The group of 16 unit squares represents a set.
- Do the fractions identified when the unit squares formed the coaster (region) remain the same when the cut apart unit squares form the set?
- How are parts of a whole and parts of a set similar? Different?





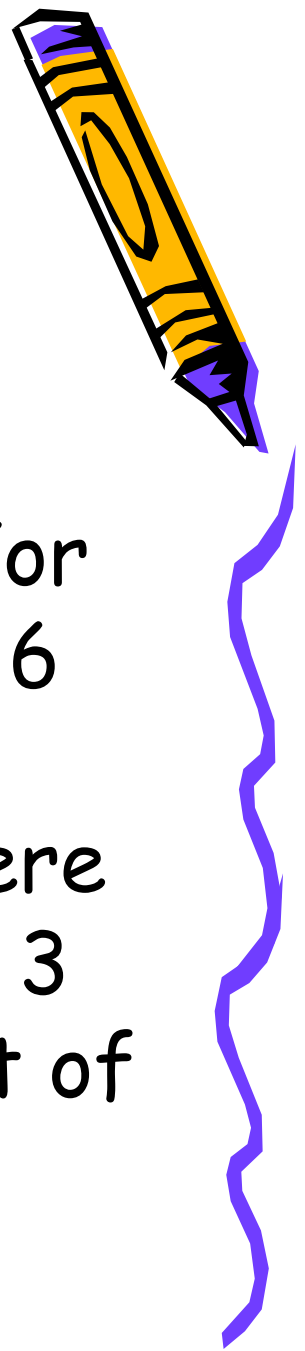
Teaching for Mathematical Understanding

Number and Operations
"Equivalent Fractions"

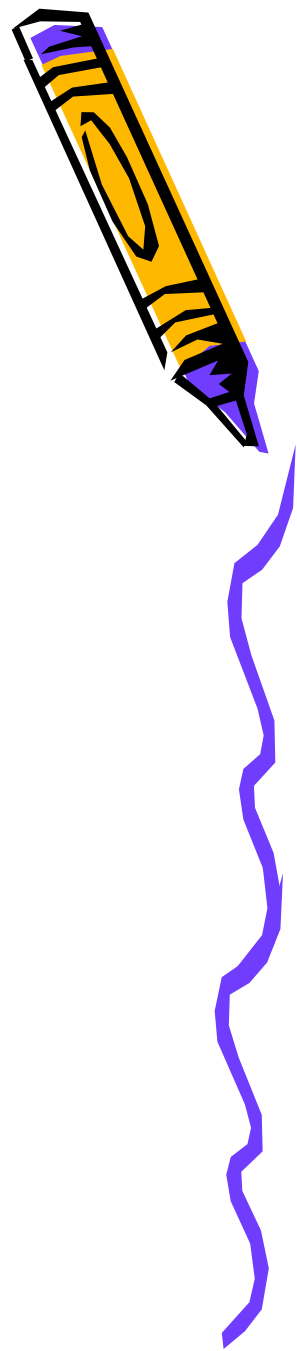


Equivalent Fractions

- Equivalent fractions describe the same part of the whole, but have different sized fractional parts. For example, the fraction $\frac{6}{8}$ describes 6 of eight equal pieces. If the eight parts are grouped in twos, then there are four equal size parts. It takes 3 of those parts to describe the part of the whole named by $\frac{6}{8}$.



Simplified Fractions

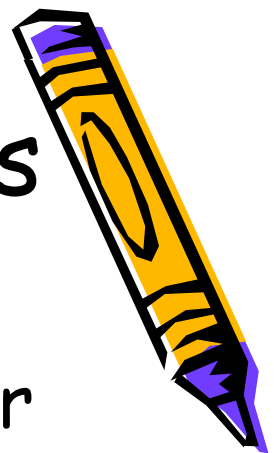


- A simplified fraction describes an equivalent fraction with fewer larger pieces.
- So $\frac{3}{4}$ is the simplified form of $\frac{6}{8}$.



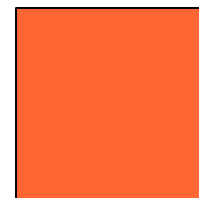
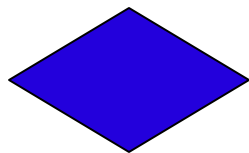
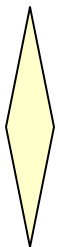
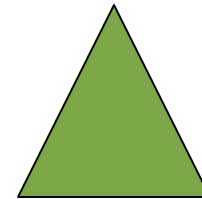
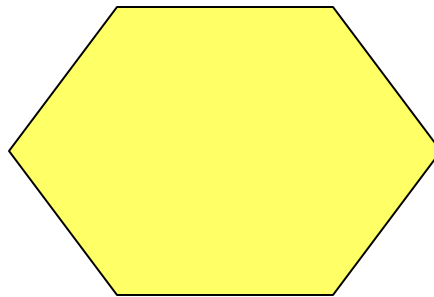
Modeling of Equivalence Fractions

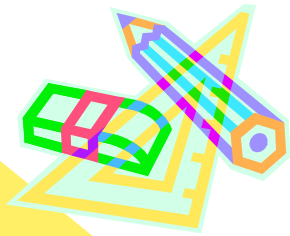
- Give each student an outline of four squares on a blank paper or a set of four cutout squares.
- We are planting gardens on each square. Fold your squares to illustrate the following gardens on the squares.
 - Jan will plant $\frac{1}{4}$ of her garden with tulips and $\frac{1}{4}$ of her garden with roses.
 - Ben will plant $\frac{1}{2}$ of his garden with sunflowers.
 - Dan will plant $\frac{3}{8}$ of his garden with roses and $\frac{1}{8}$ of his garden with sunflowers.
 - Sara will plant $\frac{2}{8}$ of her garden with tulips and the same amount with sunflowers.



Pattern Blocks Online

- http://www.arcytech.org/java/patterns/patterns_j.shtml





Teaching for Mathematical Understanding

Number and Operations
"Comparing Fractions"



Concepts or Rules?

Children have a strong mindset about numbers that causes them difficulties with the relative size of fractions. In their experience, larger numbers mean "more." The tendency is to transfer this whole number concept to fractions: Seven is more than four, so sevenths should be more than fourths.

The inverse relationship between number of parts and size of parts cannot be told, but must be a creation of each student's own thought process.

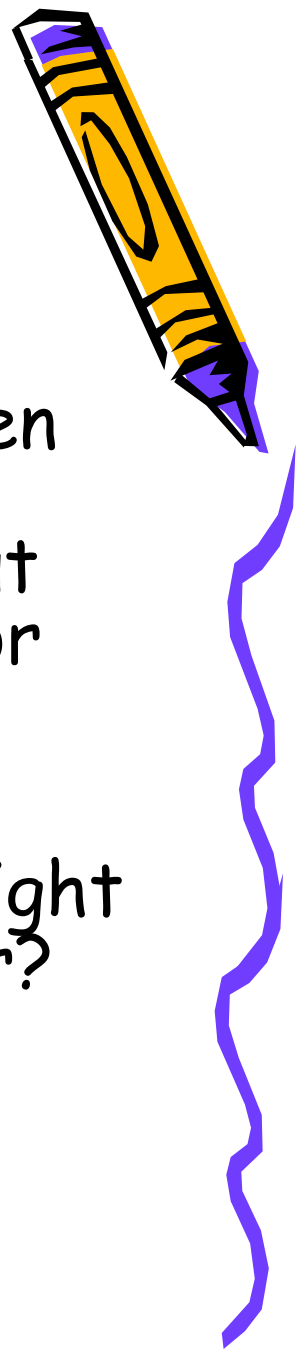
John A. Van De Walle (2007)



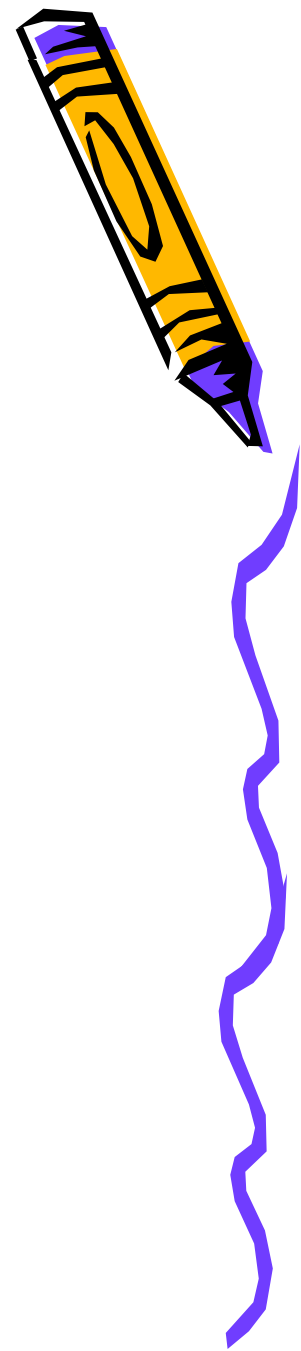
Comparing Fractions

Assume for a moment that you have been taught no rules about comparing two fractions. Suspend what you know about common denominators, cross products or decimal conversions.

Examine each pair of fractions. How might you determine which fraction is greater? Be prepared to tell how you decided.



Comparing Fractions Using Concepts, Not Rules



A.	$\frac{4}{5}$	or	$\frac{4}{9}$
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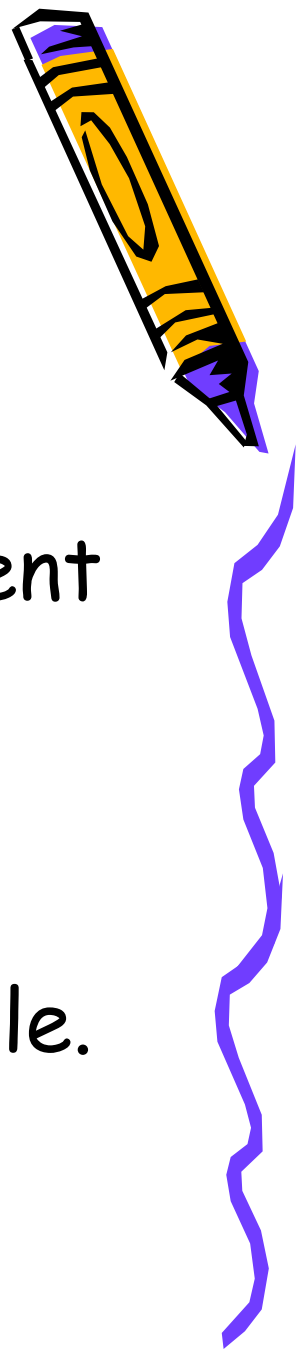


Comparing Fractions Using Concepts, Not Rules

Which fraction in each pair is greater?

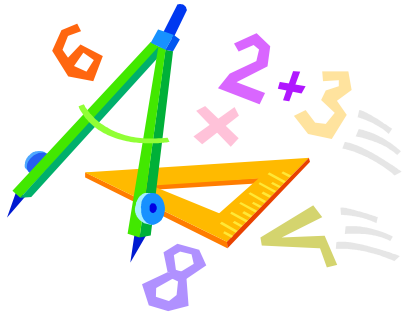
A.	$\frac{4}{5}$	or	$\frac{4}{9}$	G.	$\frac{7}{12}$	or	$\frac{5}{12}$
B.	$\frac{4}{7}$	or	$\frac{5}{7}$	H.	$\frac{3}{5}$	or	$\frac{3}{7}$
C.	$\frac{3}{8}$	or	$\frac{4}{10}$	I.	$\frac{5}{8}$	or	$\frac{6}{10}$
D.	$\frac{5}{3}$	or	$\frac{5}{8}$	J.	$\frac{9}{8}$	or	$\frac{4}{3}$
E.	$\frac{3}{4}$	or	$\frac{9}{10}$	K.	$\frac{4}{6}$	or	$\frac{7}{12}$
F.	$\frac{3}{8}$	or	$\frac{4}{7}$	L.	$\frac{8}{9}$	or	$\frac{7}{8}$

Conceptual Patterns for Comparing Fractions



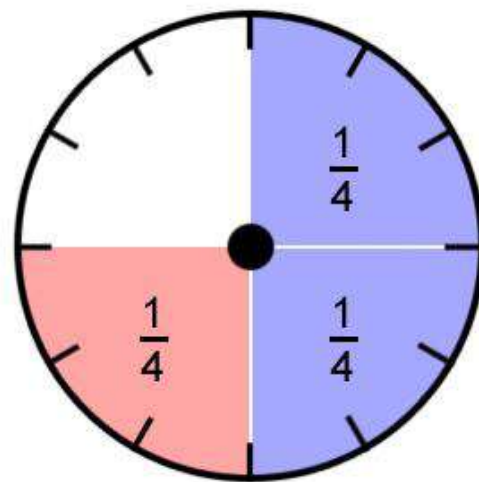
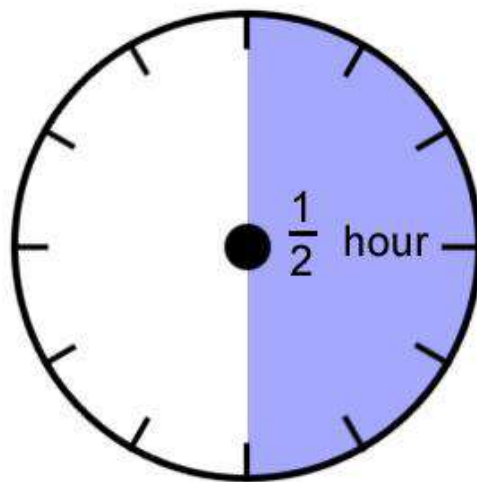
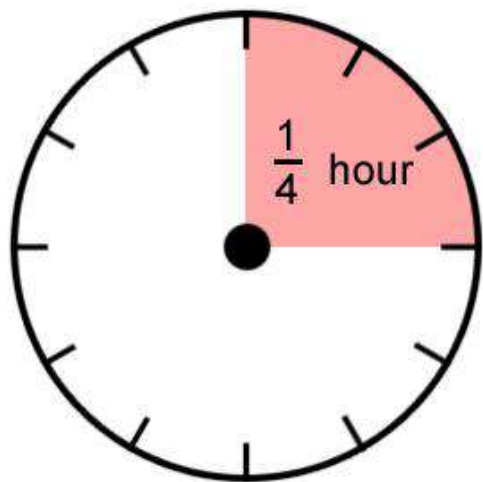
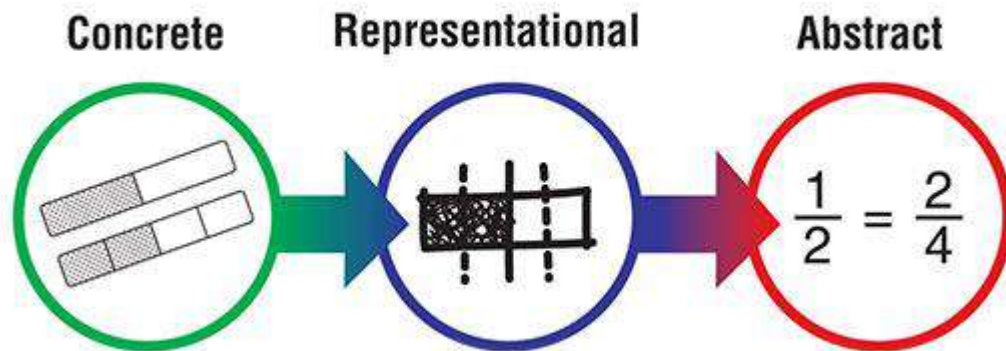
- More of the same size parts.
- Same number of parts, but different sizes.
- More or less than one-half or one whole.
- Distance from one-half or one whole.





Putting Words into Action - Investigating Word Problems





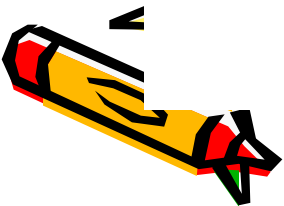
$$\frac{1}{4}$$

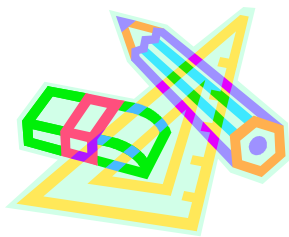
+

$$\frac{1}{2}$$

=

$$\frac{3}{4}$$





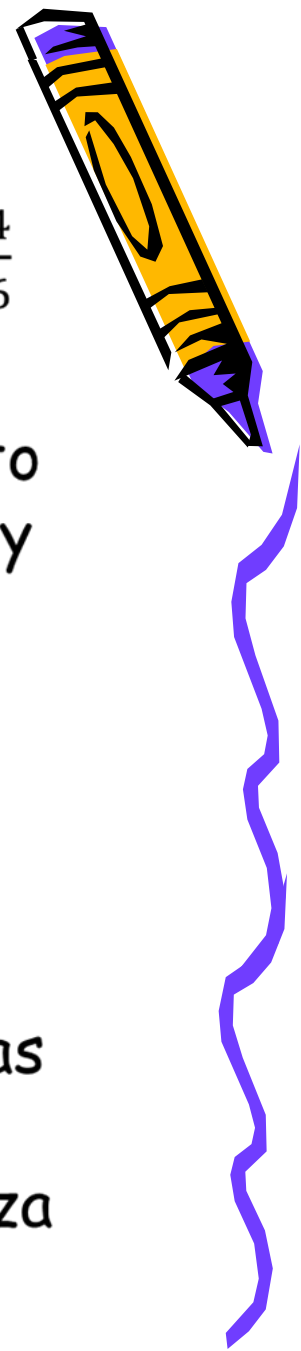
Teaching for Mathematical Understanding using the CPA Model

Concrete - Pictorial -
Abstract

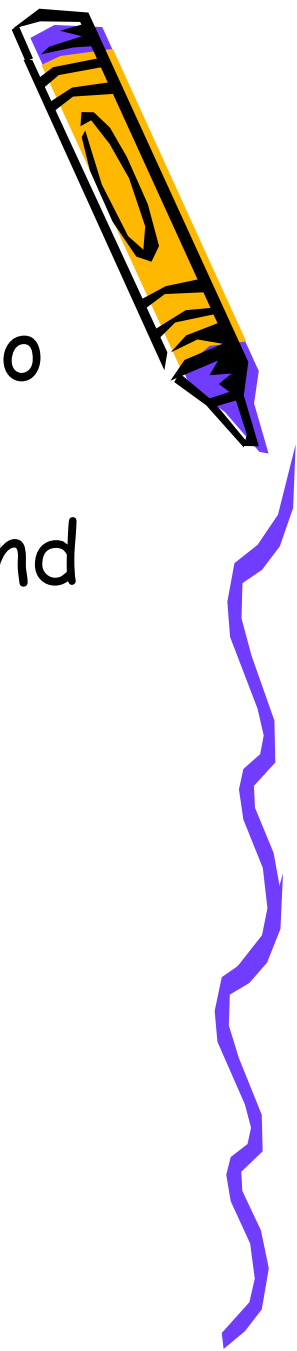


• Word Problems

- (1) Are the fractions $\frac{4}{6}$ and $\frac{2}{3}$ greater than, less than, or equal to each other? Justify your answer.
- (2) The class ordered two large pizzas. Half of first pizza was eaten and $\frac{5}{6}$ of the second pizza was eaten. What is the total amount of pizza eaten?



Group Activity

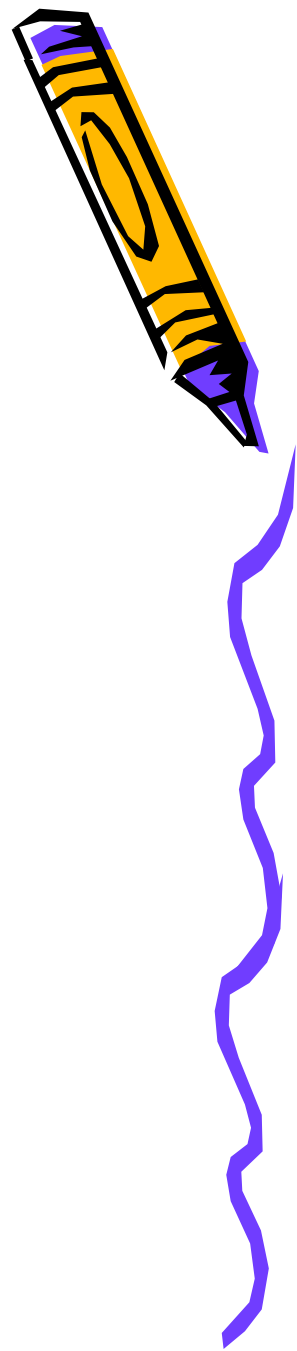


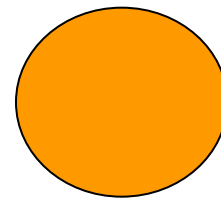
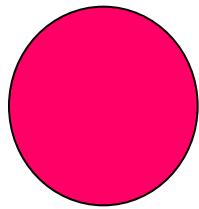
- Divide your participants groups into three different groups - circle fractions, rectangular fractions, and number lines.
- As the presenter displays the problem, use your manipulative to model the problem.
- Be prepared to share your illustration.



FRACTION ACTIVITIES

- Addition
- Subtraction
- Multiplication
- Division



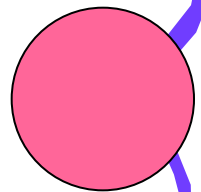
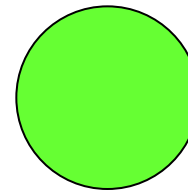
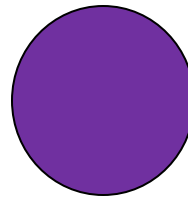
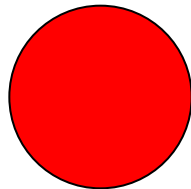
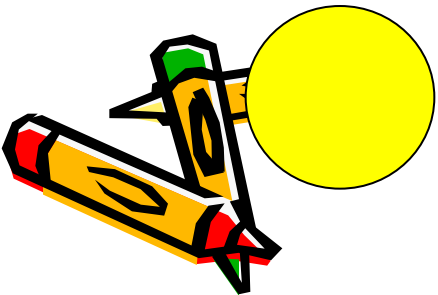


Addition and Subtraction

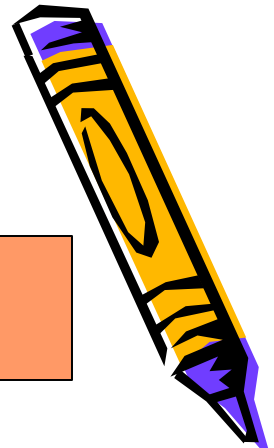
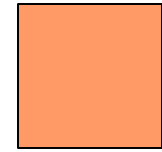
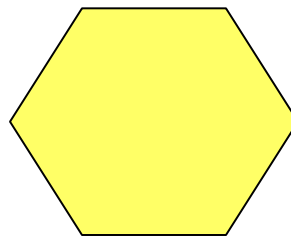
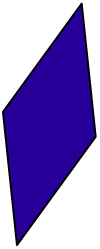
• $\frac{1}{2} + \frac{1}{2}$

• $\frac{7}{8} - \frac{3}{8}$

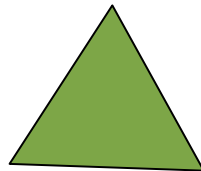
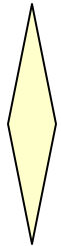
• $\frac{2}{3} + \frac{1}{6}$



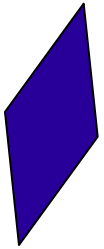
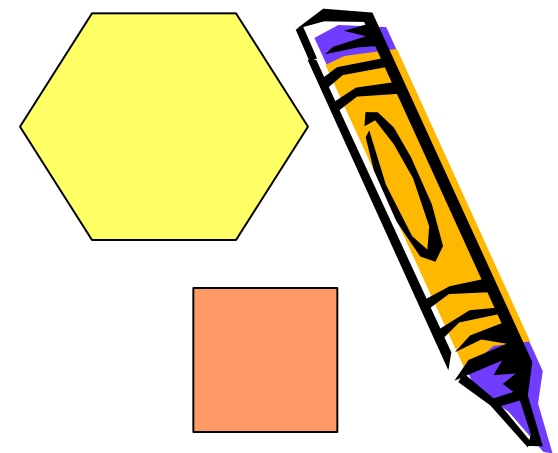
Pattern Blocks



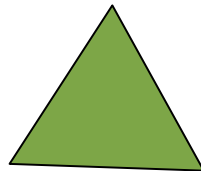
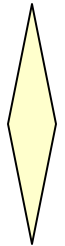
- Explore with the blocks. Be ready to identify relationships between and among the blocks.
- Build a triangle using four green blocks.
- What part of the large triangle does one green block represent?
- Now cover two green blocks with one blue block. What part of the large triangle is blue?



Pattern Blocks Fraction Puzzles

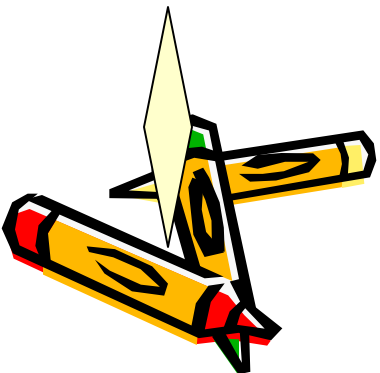
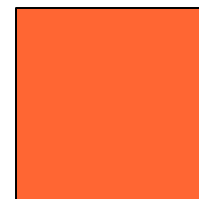
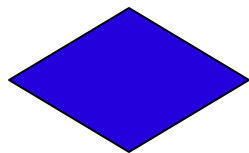
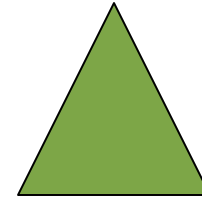
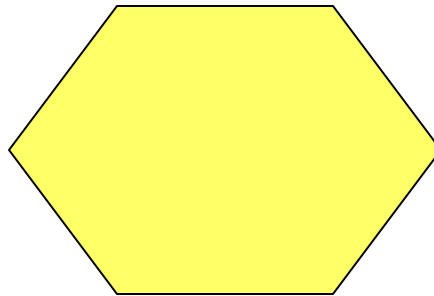
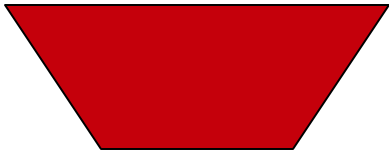


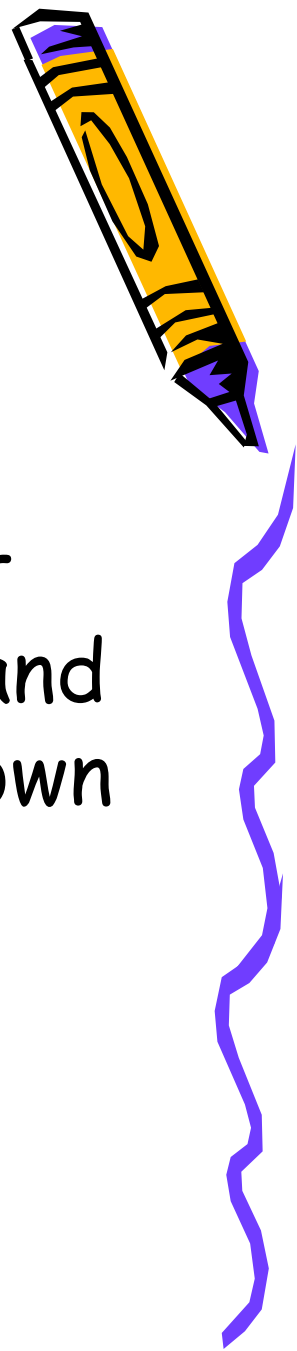
- Puzzle 1: Build a triangle that is one-third green and two-thirds red.
- Puzzle 2: Build a triangle that is two-thirds red, one-ninth green, and two-ninths blue.
- Puzzle 3: Build a parallelogram that is three-fourths blue and one-fourth green.
- Puzzle 4: Build a parallelogram that is two-thirds blue and one-third green.



Pattern Blocks Online

- http://www.arcytech.org/java/patterns/patterns_j.shtml





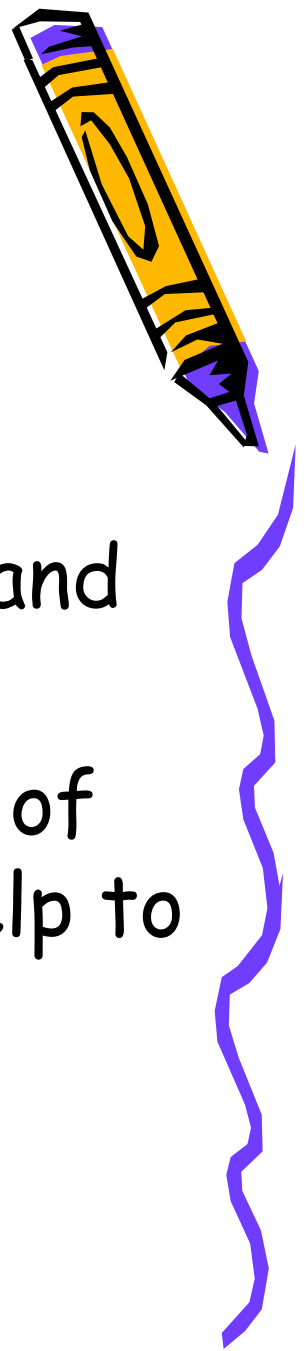
Enduring Understandings

The **quality** and **quantity** of student **experiences** increase the number and strength of connections among known information, leading to greater **understanding**.

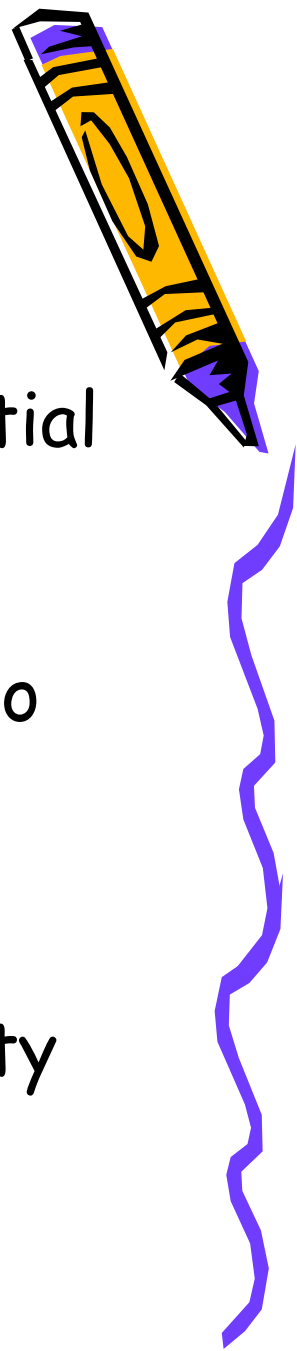


Essential Questions

- Why are the differences between **knowing/doing** and **understanding** important in mathematics teaching and learning?
- How does effective implementation of **Georgia Standards of Excellence** help to facilitate student understanding?



Revisiting the Essential Questions

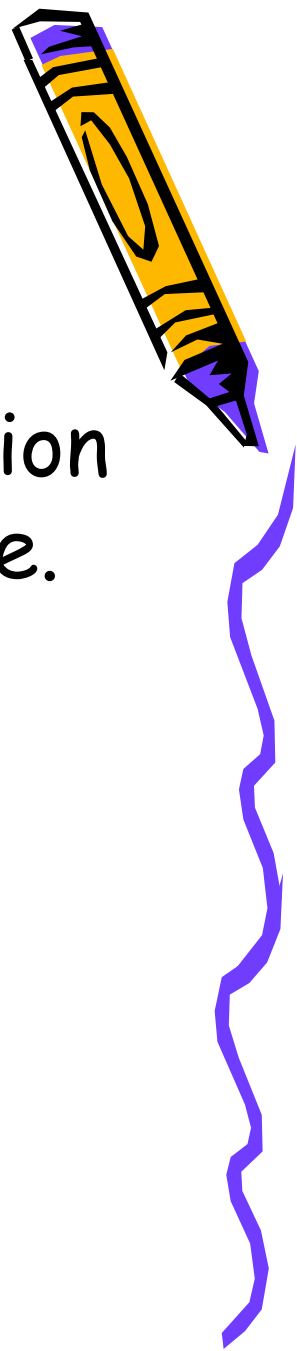


- In your group, you will revisit each Essential Question.
- Jot down the first thoughts that come to mind about the Essential Question.
- When all groups have responded, we will examine the charts and have an opportunity for sharing.

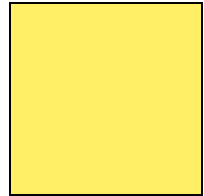
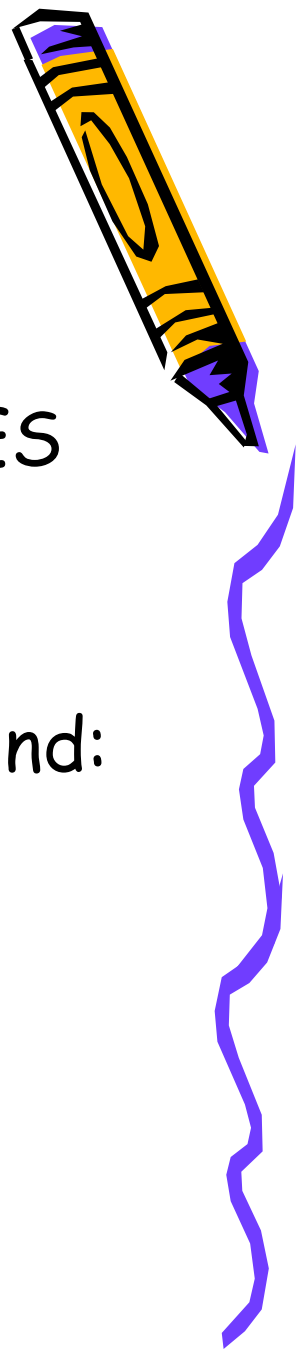


Teaching for Mathematical Understanding

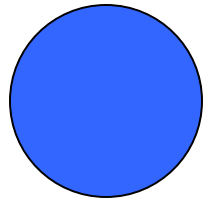
Please complete the course evaluation and clean up around your workspace.



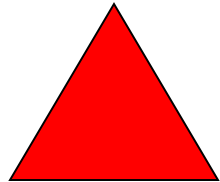
Ticket Out the Door



Something I learned that **SQUARES** with my beliefs:



A question going **AROUND** in my mind:



Three important **POINTS** to remember:



Ticket Out the Door

As a result of this training, I will:

- Stop
- Continue
- Start

