TASK 8: Complex Operations Name: Key

With vectors, each form (component form and magnitude-direction form) had advantages and isadvantages for operations such as addition, subtraction, and scalar multiplication. The same is true for complex numbers. Let's see what the advantages and disadvantages of each form are.

1. First, find the modulus and argument (magnitude and direction) of each of the following complex numbers, then write each in polar form. We will be using these complex numbers throughout the task, so be careful! ***Round angles to the nearest whole number.***

	$3 + 4i$	$2i$	$-5 - 12i$	$4 - 4i$
modulus	5	2	13	$4\sqrt{2}$
argument	$53°$	$90°$	$247°$	$315°$
polar form	$5 \text{ cis } 53°$	$2 \text{ cis } 90°$	$13 \text{ cis } 247°$	$4\sqrt{2} \text{ cis } 315°$

2. Use what you know about complex numbers to simplify each of these **addition** and **subtraction** expressions in standard form. Complete the table. The first one has been done for you.

expression in standard form	$(3 + 4i) + (2i)$	$(3 + 4i) + (-5 - 12i)$	$(-5 - 12i) - (4 - 4i)$
answer in standard form $a + bi$	$3 + 6i$	$-2 - 8i$	$-9 - 8i$
modulus of answer	$3\sqrt{5}$	$2\sqrt{17}$	$\sqrt{145}$
argument of answer	$63°$	$256°$	$222°$
answer in polar form $r \text{ cis } \theta$	$3\sqrt{5} \text{ cis } 63°$	$2\sqrt{17} \text{ cis } 256°$	$\sqrt{145} \text{ cis } 222°$
original in polar form	$5 \text{ cis } 53° + 2 \text{ cis } 90°$	$5 \text{ cis } 53° + 13 \text{ cis } 247°$	$13 \text{ cis } 247° + 4\sqrt{2} \text{ cis } 315°$

3. Use what you know about complex numbers to simplify each of these **multiplication** expressions in standard form. Complete the table. The first one has been done for you.

expression in standard form	$(3 + 4i)(2i)$	$(3 + 4i)(-5 - 12i)$	$(-5 - 12i)(4 - 4i)$
answer in standard form $a + bi$	$-8 + 6i$	$-15 - 36i - 20i - 48i^2$ $33 - 56i$	$-20 + 20i - 48i + 48i^2$ $-68 - 28i$
modulus of answer	10	65	$52\sqrt{2}$
argument of answer	143°	301°	202°
answer in polar form r cis θ	10 cis 143°	65 cis 301°	$52\sqrt{2}$ cis 202°
original in polar form	(5 cis 53°)(2 cis 90°)	(5 cis 53°)(13 cis 247°)	(13 cis 247°)($4\sqrt{2}$ cis 315°)

4. Use what you know about complex numbers to simplify each of these **division** expressions in standard form. Complete the table. The first one has been done for you.

expression in standard form	$\dfrac{2i}{3 + 4i}$	$\dfrac{-5 - 12i}{3 + 4i}\left(\dfrac{3 - 4i}{3 - 4i}\right) = \dfrac{-15 + 20i - 36i + 48i^2}{9 + 16}$
answer in standard form $a + bi$	$\frac{8}{25} + \frac{6}{25}i$	$\frac{-63}{25} \; \frac{-16i}{25}$
modulus of answer	$\frac{2}{5}$	$\frac{13}{5}$
argument of answer	37°	194°
answer in polar form r cis θ	$\frac{2}{5}$ cis 37°	$\frac{13}{5}$ cis 194°
original in polar form	$\dfrac{2 \text{ cis } 90°}{5 \text{ cis } 53°}$	$\dfrac{13 \text{ cis } 247°}{5 \text{ cis } 53°}$

As we saw with vectors, there are advantages and disadvantages to each form—component and magnitude-direction form. The same is true with complex numbers: some operations are easy to perform in standard (rectangular) form, whereas others are easy in polar form.

5. Investigate your tables from #2, #3, and #4, and see if you can find quick methods in either (or both) forms to perform each operation. Write instructions in each box explaining how to perform each operation. The first column has been completed for you.

	addition (#2)	subtraction (#2)
standard form $a + bi$	Easy! :) Add real parts. Add imaginary parts.	Easy! :) Subtract real parts Subtract imaginary parts
polar form $r \text{ cis } \theta$	Convert to standard form first, then follow standard form instructions.	Convert to standard form first, then follow standard form instructions.

	multiplication (#3)	division (#4)
standard form $a + bi$	Convert to polar form first then follow polar form instructions ... OR dist. property	Convert to polar form first then follow polar form instructions OR multiply both num. & den. by denominator's conjugate
polar form $r \text{ cis } \theta$	Multiply moduli Add arguments * Subtract multiples of 360° if ∠ is greater than 360°	Divide moduli Subtract arguments * Add multiples of 360° if ∠ is less than 0°

	exponentiation (#6-10)
standard form $a + bi$	Convert to polar form first. Then raise r to power and degree to power.
polar form $r \text{ cis } \theta$	Raise r to power Raise degree to power

← We will complete this part of the table after the next part of our task.

Now let's take a look at how we can find powers of complex numbers.

6. Use your summary table from #5 to help you multiply (5 cis 20°)(5 cis 20°)(5 cis 20°).

125 cis 60°

7. Just as we can write (5)(5)(5) as 5^3, we can write (5 cis 20°)(5 cis 20°)(5 cis 20°) as $(5 \text{ cis } 20°)^3$. Using patterns you notice, explain how to find $(2 \text{ cis } 10°)^{17}$ without actually multiplying everything out. raise 2^{17} & multiply 10 × 17 to get the correct modulus & argument.

8. Explain why this process for finding powers of complex numbers in polar form makes sense based on the processes you wrote in #5 for adding and multiplying complex numbers. Because repeated multiplication is easiest w/ exponentials.

✓ 9. Go back to your table in #5 and complete the last column: exponentiation.

We know how to write radicals using exponent notation. For example, $\sqrt[4]{5} = 5^{1/4}$. We can therefore use what we summarized in #8 to find n^{th} roots of complex numbers:

10. Write $\sqrt[3]{3+4i}$ using exponent notation. Use the processes you summarized in #9 to find $\sqrt[3]{3+4i}$.

divided 53/3

$\sqrt[3]{3+4i}$ is really $(3+4i)^{1/3} = (5 \text{ cis } 53°)^{1/3} = \sqrt[3]{5} \text{ cis } 17.7°$

11. Use your summary table from #5 to help you perform the following operations:

a. $(3+6i)+(4-5i)$ $7+i$

b. (6 cis 30°) – (4 cis 120°) $(5.19+3i) - (-2+3.46i)$

$(7.19 - 0.46i)$

c. (8 cis 135°)(10 cis 210°) (80 cis 345°)

d. $(4-3i)(-7-24i)$ $-28-96i+21i+72i^2$

$-28-96i+21i-72$

$-100-75i$

e. $\frac{10\text{ cis }330°}{2\text{ cis }135°}$ 5 cis 195°

f. $\frac{6+8i}{3-4i}\left(\frac{3+4i}{3+4i}\right) = \frac{18+24i+24i+32i^2}{9+16} = \frac{-14+48i}{25} = \frac{-14}{25}+\frac{48i}{25}$

Multiplication, division, and exponentiation can require an extra step to make the argument between 0° and 360°.

12. Use your summary table from #5 to help you multiply (6 cis 150°)(3 cis 240°) quickly. Why is your answer not in typical polar form? 18 cis 390°

B/c the angle is > 360°

13. Adjust your answer to make the argument an angle between 0° and 360°. Explain your process. 18 cis 30° b/c 390° has a coterminal ∠ w/ the same location by 390° - 360° = 30°

14. Use your summary table from #5 to divide $\frac{6\text{ cis }150°}{3\text{ cis }240°}$.

Why is your answer not in typical polar form? 2 cis -90°

B/c the angle is < 0°

15. Adjust your answer to make the argument an angle between 0° and 360°.

Explain your process. 2 cis 270° b/c -90° has a coterminal ∠ in the same location as -90° by 360° + -90° = 270°

16. Use your summary table from #5 to find $(2 \text{ cis } 120°)^{14}$.

Why is your answer not in typical polar form? 16384 cis 1680°
The angle is greater than 360°

17. Adjust your answer to make the argument an angle between 0° and 360°.

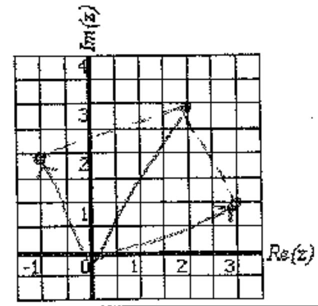
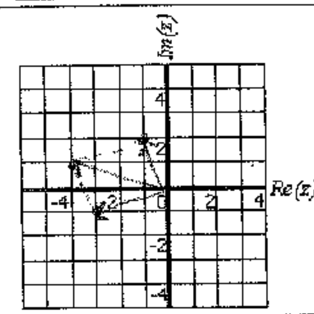
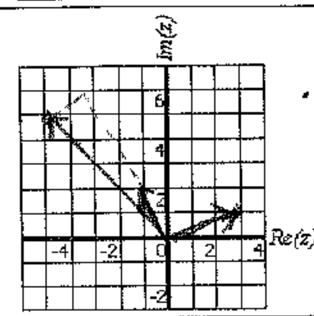
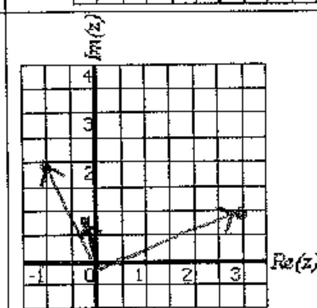
Explain your process. 16384 cis 240° b/c 1680° has the same location as 240° b/c 1680 - multiple 360° rotations is 240°

18. In your table from #5, write the additional step you had to perform and when it would be necessary.

See addition w/ asterik added in chart.

- This will make your summary tables complete.

19. Use your summary tables to represent each of the following problems on the next page and their answers on the complex plane:

question	show your work here	answer	on the complex plane
parts in standard form: $(-1 + 2i) + (3 + i)$ parts in polar form: $(\sqrt{5} \operatorname{cis} 117^\circ) + (\sqrt{10} \operatorname{cis} 18^\circ)$	$(-1 + 3) \quad (2i + i)$ $2 + 3i$ $\sqrt{13} \operatorname{cis} 56^\circ$	in standard form: $2 + 3i$ in polar form: $\sqrt{13} \operatorname{cis} 56^\circ$	
parts in standard form: $(-1 + 2i) - (3 + i)$ parts in polar form: $(\sqrt{5} \operatorname{cis} 117^\circ) - (\sqrt{10} \operatorname{cis} 18^\circ)$	$(-1 - 3) \quad 2i + -i$ $-4 + i$ $\sqrt{17} \operatorname{cis} 166^\circ$	in standard form: $-4 + i$ in polar form: $\sqrt{17} \operatorname{cis} 166^\circ$	
parts in standard form: $(-1 + 2i)(3 + i)$ parts in polar form: $(\sqrt{5} \operatorname{cis} 117^\circ)(\sqrt{10} \operatorname{cis} 18^\circ)$	$-3 - i + 6i + 2i^2$ $-3 + 5i - 2$ $(\sqrt{5})(\sqrt{10}) = \sqrt{50} = 5\sqrt{2}$ $117^\circ + 18^\circ = 135^\circ$	in standard form: $-5 + 5i$ in polar form: $5\sqrt{2} \operatorname{cis} 135^\circ$	
parts in standard form: $(-1 + 2i) \div (3 + i)$ parts in polar form: $(\sqrt{5} \operatorname{cis} 117^\circ) \div (\sqrt{10} \operatorname{cis} 18^\circ)$	$\frac{-1+2i}{3+i}\left(\frac{3-i}{3-i}\right)$ $\frac{-3 + i + 6i - 2i^2}{9 + 1}$ $\frac{\sqrt{5}}{\sqrt{10}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \quad 117 - 18 = 99$	in standard form: $-\frac{1}{10} + \frac{7}{10}i$ in polar form: $\frac{1}{\sqrt{2}} \operatorname{cis} 99^\circ$	
parts in standard form: $\sqrt{-1 + 2i}$ parts in polar form: $(\sqrt{5} \operatorname{cis} 117^\circ)^{1/2}$	$\sqrt{5}^{1/2} \operatorname{cis} 117/2$ $5^{1/4} \operatorname{cis} 58.5^\circ$ $5^{1/4} \cos 58.5 = 0.79$ $5^{1/4} \sin 58.5 = 1.27$	in standard form: $0.79 + 1.27i$ in polar form: $1.5 \operatorname{cis} 58.5^\circ$